A string 'x' is said to be accepted by a FA $M = \{Q, \Sigma, \delta, q_0, F\}$ if $\delta(q_0, x) = p$ for some $p \in F$. The language is accepted by M, $L(M) = \{x \mid \delta(q_0, x) \text{ is in } F\}$.

→ The language is a regular set (regular) if it is the set accepted by some M.

→ The languages accepted by FA are described by expressions called regular expressions.

→ **Kleene Closure:-**

A clean closure of 'L' denoted as L^* is a set,

$$L^* = \bigcup_{i=0}^{\infty} L^i$$

Ex:- If $\Sigma = \{a, b\}$, then

$$L^* = \{\epsilon, a, b, ab, aabb, abba, \ldots\}$$

→ **Positive closure:-**

The +ve closure of 'L' denoted by L^+ is a set,

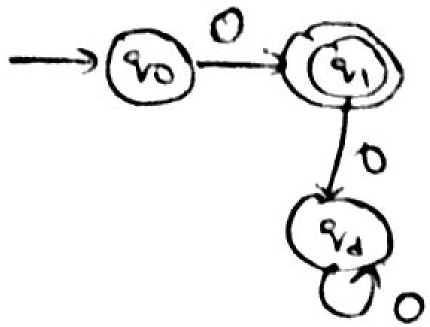
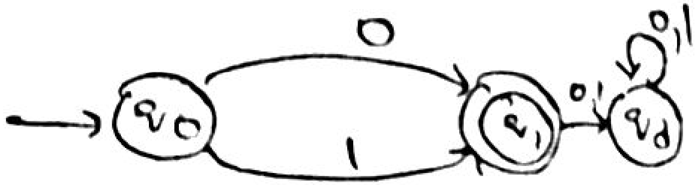
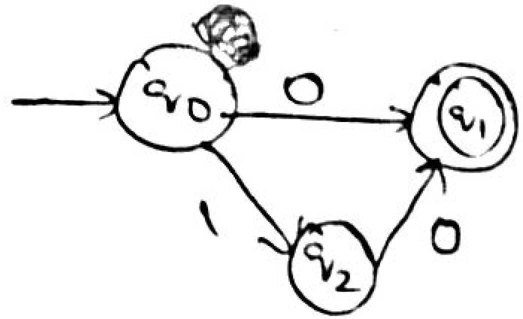
$$L^+ = \bigcup_{i=1}^{\infty} L^i$$

Ex:- let $L_1 = \{10, 1\}$, $L_2 = \{011, 11\}$, then write concatenation of L_1 & L_2

$$L_1 L_2 = \{10011, 1011, 1011, 111\}$$

$$L_1^* = \{\epsilon, 10, 1, 1010, 111, 101, 110, \ldots\}$$

Regular Expressions :-

RE	R Set	Automata
0	{0}	
1	{1}	
0110	{0110}	
0+1	{0,1}	
0^*	{ε, 0, 00, 000..}	
0+10	{0,10}	

Q: RE for $L = \{aa, aaaa, aaaaaa, \ldots\}$

$\Rightarrow aa(aa)^*$.

Q: RE for $L = \{0+1+2\}$

$\Rightarrow 0+1+2$.

Q: RE for language L over $\{0,1\}$ / every string begins with 00 and ends with 11.

$\Rightarrow 00(0+1)^*11$.

Q: RE over $\{a,b\}$ containing exactly 2 a's.

$\Rightarrow b^*ab^*ab^*$.

Q: RE for

(i) $\{0^i 1^j 2^k \mid i,j,k \geq 1\}$

$\Rightarrow 00^* 11^* 22^*$

(ii) $\{0^i 1^j 2^k \mid i,j,k \geq 0\}$

$\Rightarrow 0^* 1^* 2^*$

(iii) $\{0^i 1^j 2^k \mid i,j \geq 1; k \geq 0\}$

$\Rightarrow 00^* 11^* 2^*$

(iv) 10^{th} symbol from right is one.

$\Rightarrow (1+0)^* 1 (0+1)^9$.

Conversion of regular expressions to finite automata (NFA − ∈):-

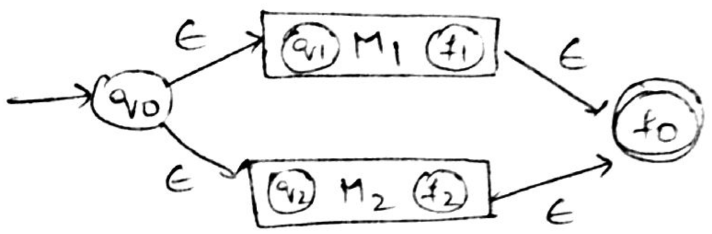
Theorem:-

Let 'r' be a RE, then there exists an NFA with ∈ transitions that accepts $L(r)$.

Proof:- $L(M) = L(r)$.

Case(i):- If r_1 and r_2 are two different RE's, we define M_1 for r_1 and M_2 for r_2.

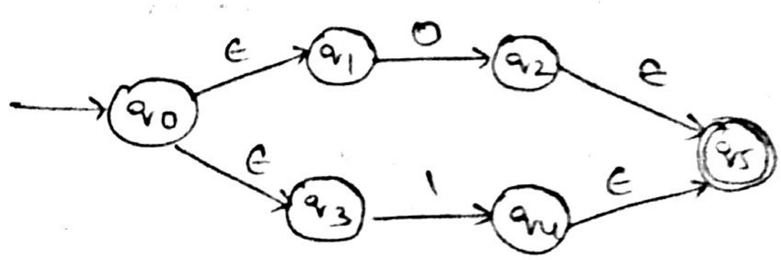
$r = r_1 + r_2$ is defined as follows:



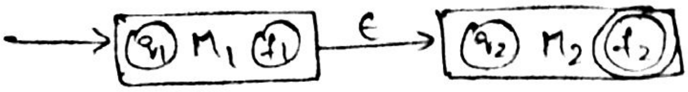
There is a path labelled x in M from 'q_0' to 'f_0' if and only if there is a path labelled in M_1 (or) in M_2.

$$L(M) = \{x + y \mid x \text{ is in } L(M_1) \text{ or } y \text{ is in } L(M_2)\}$$

Ex:- NFA − ∈ for 0 + 1



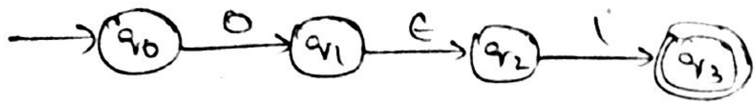
Case (ii):- $r = r_1 \cdot r_2$



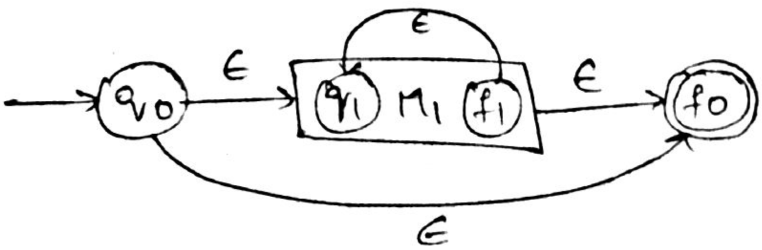
Every path in M from q_1 to f_2 is a path labelled by some string 'x' from q_1 to f_1 followed by the edge from f_1 to q_2 labelled ϵ followed by a path labelled some string 'y' from q_2 to f_2.

$$L = \{xy \mid x \text{ is in } L(M_1) \text{ and } y \text{ is in } L(M_2)\}$$

Ex:- NFA-ϵ for 01

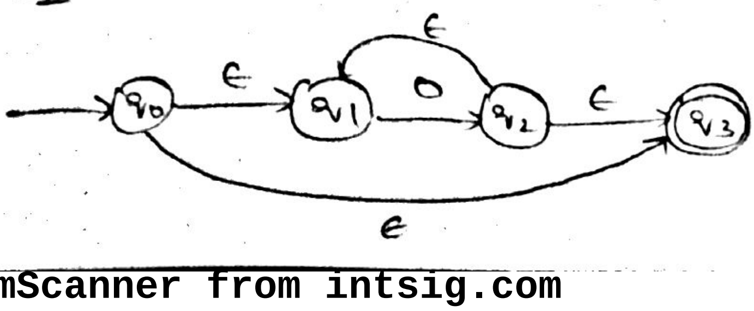


Case (iii):- $r = r_1^*$

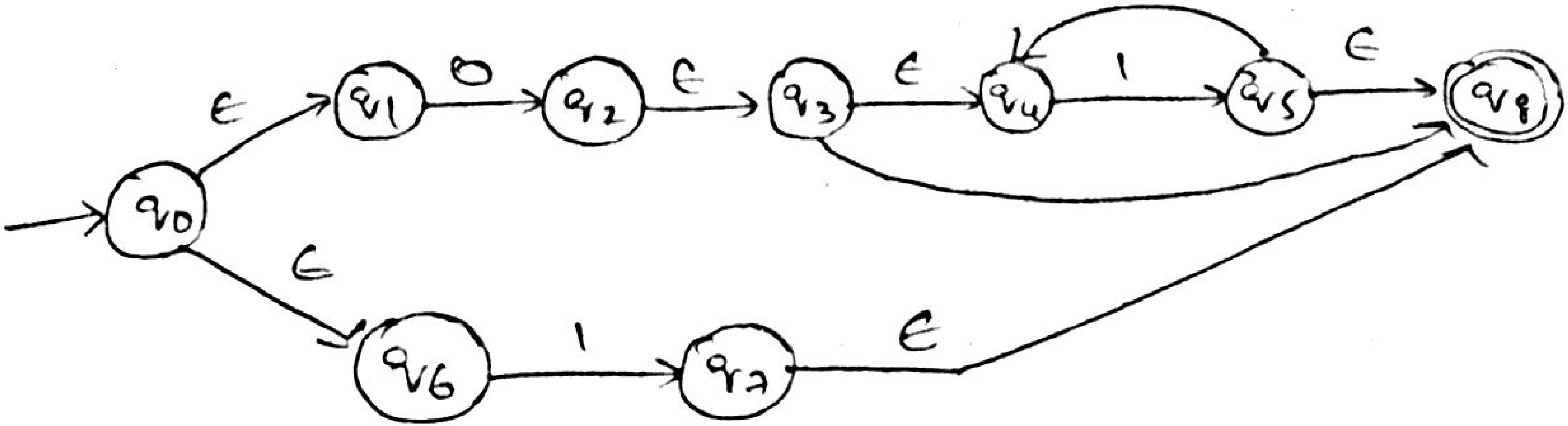


Any path from q_0 to f_0 consists either a path from q_0 to f_0 on ϵ (or) a path from q_0 to q_1 on ϵ followed by some number of paths from q_1 to f_1 then back to q_1 on ϵ, then to f_0 on ϵ.

Ex:- NFA-ϵ for 0^*



Ex:- NFA-ϵ for $01^* + 1$.



→ If we want to get NFA from regular expression, then

$RE \rightarrow NFA\text{-}\epsilon \rightarrow NFA.$

5|2|13

Identities for regular set:-

I_1: $\phi + R = R$

I_2: $\phi R = R\phi = \phi$

I_3: $\epsilon R = R\epsilon = R$

I_4: $\epsilon^* = \epsilon$ and $\phi^* = \epsilon$.

I_5: $R + R = R$

I_6: $R^* R^* = R^*$

I_7: $RR^* = R^* R = R^*$

I_8: $(R^*)^* = R^*$

I_9: $\epsilon + RR^* = R^* = \epsilon + R^* R$

I_{10}: $(PQ)^* P = P(QP)^*$

I_{11}: $(P+Q)^* = (P^* Q^*)^* = (P^* + Q^*)^*$

I_{12}: $(P+Q)R = PR + QR$

$R(P+Q) = RP + RQ$.

Note:- $(P+Q)^* \neq (P^* + Q^*)$

$(P+Q)^* \neq P^* Q^*$

Q: Give a RE representing a set L of strings in which every 0 is immediately followed by atleast two ones and P.T it is equivalent to $R = \epsilon + 1^* (011)^* (1^* (011)^*)^*$.

A: $L = \{\epsilon, 011, 1011, 011011, \ldots\}$

$\Rightarrow (1+011)^*$

Given $R = \epsilon + 1^*(011)^*(1^*(011)^*)^*$

$= (1^*(011)^*)^*$ $\quad (\because \epsilon + PP^* = P^*)$

$= (1+011)^*$ $\quad (\because (P^*Q^*)^* = (P+Q)^*)$

$\therefore$ HP

Q: Prove $(1+00^*1) + (1+00^*1)(0+10^*1)^*(0+10^*1) = 0^*1(0+10^*1)^*$

$\Rightarrow (1+00^*1)[\epsilon + (0+10^*1)(0+10^*1)^*]$

$\Rightarrow (1+00^*1)[(0+10^*1)^*]$ $\quad (\epsilon + PP^* = P^*)$

$\Rightarrow (\epsilon+00^*)1[(0+10^*1)^*]$ $\quad$ "

$\Rightarrow 0^*1[(0+10^*1)^*]$ $\quad$ " //

Ardens theorem:-

Let p and q be two RE's over Σ. If p does not contain Σ, then the following equation $r = q + rp$ has a unique solution given by $r = qp^*$.

Q: Conversion from DFA to regular expressions:

1. Derive RE for the following transition system.