

HW3

1. def insertion_sort(A, n):

 for i in range(1, n):

 key = A[i]

 j = i - 1

 while j ≥ 0 and key < A[j]:

 A[j + 1] = A[j]

 j -= 1

 A[j + 1] = key

 return A

There are 2 important lines for the time complexity analysis.
The first is the outer for loop which will take $n-1$ iterations
bc it goes for the range 0 to n . The other line is the
while loop which gives us 3 possible cases that determines
the time complexity

Best case of while loop: if the array is already sorted, then
the while loop will not run, which is why the best case
of insertion sort is $O(n)$ time.

Average case: on average when sorting it makes sense for the
while loop to execute exactly $\frac{n}{2}$ times which is half. Since
this while loop is inside the for loop, we get $O((n-1)\left(\frac{n}{2}\right))$
which simplifies to $O(n^2)$ since that is the biggest degree.

Worst case: If array is sorted in reverse order, the while
loop will run n times, so we get $O((n-1)(n))$ which
is just $O(n^2)$ for the worst case as well.

1. W3

1. Insertion sort is known for being an in-place sorting algorithm meaning that there is no extra space needed to sort it. The only space used is to create some variables, so the space complexity will be $O(1)$.

Matrix-Multiplication (A, B)

1. if cols(A) ≠ rows(B)

1 · time

1 · time

raise ValueError

1 · time

rows_A

1 · time

cols_A

1 · time

cols_B

1 · time

result

1 · time

for i from 1 to rows_A do:

$(n+1)$ time

for j from 1 to cols_B do:

$n(n+1)$ time

sum

$(n)(n)$ time

for k from 1 to cols_A do: $n(n)(n+1)$ time

sum

$n(n)(n)$ time

result[i][j]

$n(n)$ time

return result

$n(n)$ time

$$7 + (n+1) + (n^2+n) + (n^2) + (n^3+n^2) + (n^3) + (n^2) + (n^2)$$

$$2n^3 + 5n^2 + n + 8 \text{ time}$$

$$ax^3 + bx^2 + cx + d$$

This means that the time complexity is cubic. In this case what n represents is rows_A, cols_B, cols_A runtime meaning that we get $O(\text{rows_A} \cdot \text{cols_B} \cdot \text{cols_A})$ runtime.

The space complexity should be constant b/c we do have variables but all are constant time.