HW 5

1. $T(n) = 2T(n-1)+1$

$T(n-1) = 2T(n-2)+1$

$T(n) = 2\cdot(2T(n-2)+1)+1$

$T(n-2) = 2\cdot(n-3)+1$

$T(n) = 2\cdot(2\,(2T(n-3)+1)+1)+1$

$T(n) = 8T(n-3)+3$

$T(n) = 2^3 T(n-3)+3$

$T(n) = 2^k T(n-k)+k$

$n-k=0$

$n=k$

$T(n) = 2^n T(n-n)+n$

$T(n) = 2^n+n$

$O(2^n)$

Masters Theorem

$a=2>1$

Case 3

$\Theta(n^0 \cdot 2^{\frac{n}{1}}) = \Theta(2^n)$

2. $T(n) = 3T(n-1)+n$

Masters theorem:

$a=3$, $b=1$

Since $a=3>1$

Case 3

$\Theta(b^1 3^{\frac{n}{1}}) = \Theta(n3^n)$

3. $T(n) = 9T(n/2) + n^2$ Masters theorem

$a = 9, b = 2, K = 2, p = 0$

$\log_2 9 > 2$

case 1

$O(n^{\log_2 9})$

4. Masters theorem:

$T(n) = 100\,T(n/2) + n^{\log_2 cn+1}$

$a = 100, b = 2, K = \log_2 cn+1, p = 0$

$\log_2 100 = \log_2 cn+1$

if $(cn+1) < 100$ case 1

$O(n^{\log_2 100})$

if $(cn+1) = 100$ case 2

$O(n^{\log_2 cn+1} \log n)$

if $(cn+1) > 100$ case 3

$O(n^{\log_2 cn+1})$

5. $T(n) = 4T(n/2) + n^2 \log n$ $a = 4, b = 2, K = 2, p = +1$

$\log_2 4 = 2$

case 2

$O(n^2 \log^2 n)$

6. $T(n) = 5T(n/2) + \frac{n^2}{\log n}$

$a = 5, b = 2, K = 2, p = -1$

$\log_2 5 > 2$

case 1

$O(n^{\log_2 5})$

problem 2

yetAnotherfunc(n):	T(n)
if n > 1:	1
for (i=0; i<10n; i++)	n+1
do something	n
yetAnotherFunc(n/2);	T(n/2)
yetAnotherFunc(n/2);	T(n/2)

$T(n) = 2T(n/2) + 2n + 2$

Masters theorem:

$a=2, b=2, k=1, p=0$

$\log_2 2 = 1$

case 2

$\Theta(n \log n)$