1
Units and Measurement

The comparison of any physical quantity with its standard unit is called **measurement**.

Physical Quantities

All the quantities which can be measured directly or indirectly in terms of which the laws of Physics are described are called physical quantities.

Units

A standard amount of a physical quantity chosen to measure the physical quantity of the same kind is called a physical unit. It should be easily reproducible, internationally accepted.

Fundamental Quantities and Their Units

Those physical quantities which are independent to each other are called **fundamental quantities** and their units are called **fundamental units.**

Fundamental quantities	Fundamental units		
	Name	**Symbol**	**Definition**
Length	Metre	m	The metre is the length of the path travelled by light in vacuum during a time interval of 1/299,792,458 of a second. (*1983*)
Mass	Kilogram	kg	The kilogram is equal to the mass of the International prototype of the kilogram (a platinum-iridium alloy cylinder) kept at International Bureau of Weights and Measures at Sevres, near Paris, France. (*1889*)

Fundamental quantities	Fundamental units		
	Name	Symbol	Definition
Time	Second	s	The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom. (*1967*)
Electric current	Ampere	A	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section and placed 1 m apart in vacuum, would produce a force between these conductors equal to 2×10^{-7} N/m of length. (*1948*)
Thermodynamic temperature	Kelvin	K	The kelvin is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (*1967*)
Amount of substance	Mole	mol	The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kg of carbon-12. (*1971*)
Luminous intensity	Candela	cd	The candela is the luminous intensity in a given direction of a source that emits monochromatic radiation of frequency 540×10^{12} Hz and that has a radiant intensity in that direction of 1/683 watt per steradian. (*1979*)

Supplementary Fundamental Quantities and Their Units

Radian and steradian are two supplementary fundamental units. It measures plane angle and solid angle respectively.

Supplementary fundamental quantities	Supplementary units		
	Name	Symbol	Definition
Plane angle	Radian	rad	One radian is the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle. *i.e.* $d\theta = \frac{ds}{r}$
Solid angle	Steradian	sr	One steradian is the solid angle subtended at the centre of a sphere, by that surface of the sphere, which is equal in area, to the square of radius of the sphere. *i.e.* $d\Omega = \frac{dA}{r^2}$

Derived Quantities and Their Units

Those physical quantities which are derived from fundamental quantities are called **derived quantities** and their units are called **derived units** *e.g.* Velocity, acceleration, force, work etc.

Systems of Units

A system of units is the complete set of units, both fundamental and derived, for all kinds of physical quantities. The common system of units which is used in mechanics are given below:

(i) **CGS System** In this system, the unit of length is centimetre, the unit of mass is gram and the unit of time is second.

(ii) **FPS System** In this system, the unit of length is foot, the unit of mass is pound and the unit of time is second.

(iii) **MKS System** In this system, the unit of length is metre, the unit of mass is kilogram and the unit of time is second.

(iv) **SI System** The system of units, which is accepted Internationally for measurement is the System Internationaled units abbreviated as SI. This system contains seven fundamental units and two supplementary fundamental units.

Relationship between Some Mechanical SI Unit and Commonly Used Units

S.No.	Physical Quantities		Units
1.	Length	(a)	1 micrometre $= 10^{-6}$ m
		(b)	1 nanometre $= 10^{-9}$ m
		(c)	1 angstrom $= 10^{-10}$ m
2.	Mass	(a)	1 metric ton $= 10^{3}$ kg
		(b)	1 pound = 0.4537 kg
		(c)	1 amu $= 1.66 \times 10^{-23}$ kg
3.	Volume		1 litre $= 10^{-3}$ m^3
4.	Force	(a)	1 dyne $= 10^{-5}$ N
		(b)	1 kgf = 9.81 N
5.	Pressure	(a)	1 kgf-m^2 = 9.81 Nm^{-2}
		(b)	1 mm of Hg = 133 Nm^{-2}
		(c)	1 pascal = 1 Nm^{-2}
		(d)	1 atmosphere pressure = 76 cm of Hg $= 1.01 \times 10^{5}$ pascal

S.No.	Physical Quantities		Units
6.	Work and energy	(a)	$1 \text{ erg} = 10^{-7}$ J
		(b)	1 kgf-m = 9.81 J
		(c)	$1 \text{ kWh} = 3.6 \times 10^{6}$ J
		(d)	$1 \text{ eV} = 1.6 \times 10^{-19}$ J
7.	Power	(a)	$1 \text{ kgf- ms}^{-1} = 9.81$ W
		(b)	1 horse power = 746 W

Some Practical Units of Length

(i) $1 \text{ fermi} = 10^{-15}$ m

(ii) $1 \text{ X-ray unit} = 10^{-13}$ m

(iii) $1 \text{ astronomical unit} = 1.49 \times 10^{11}$ m (average distance between sun and earth)

(iv) $1 \text{ light year} = 9.46 \times 10^{15}$ m

(v) $1 \text{ parsec} = 3.08 \times 10^{16}$ m = 3.26 light year

Dimensions

Dimensions of any physical quantity are those powers to which the fundamental quantities are raised to express that quantity. The expression of a physical quantity in terms of its dimensions, is called its **dimensional formula**.

Dimensional Formula of Some Physical Quantities

S.No.	Physical Quantities	Dimensional Formula	MKS Units
1.	Area	$[L^2]$	m^2
2.	Volume	$[L^3]$	m^3
3.	Velocity	$[LT^{-1}]$	ms^{-1}
4.	Acceleration	$[LT^{-2}]$	ms^{-2}
5.	Force	$[MLT^{-2}]$	newton (N)
6.	Work or energy	$[ML^2T^{-2}]$	joule (J)
7.	Power	$[ML^2T^{-3}]$	$J\ s^{-1}$ or W (watt)
8.	Pressure or stress	$[ML^{-1}T^{-2}]$	Nm^{-2}
9.	Linear momentum or impulse	$[MLT^{-1}]$	$kg\ ms^{-1}$
10.	Density	$[ML^{-3}]$	$kg\ m^{-3}$
11.	Strain	dimensionless	unitless

S.No.	Physical Quantities	Dimensional Formula	MKS Units
12.	Modulus of elasticity	$[ML^{-1}T^{-2}]$	Nm^{-2}
13.	Surface tension	$[MT^{-2}]$	Nm^{-1}
14.	Velocity gradient	$[T^{-1}]$	s^{-1}
15.	Coefficient of viscosity	$[ML^{-1}T^{-1}]$	kg $m^{-1}s^{-1}$
16.	Gravitational constant	$[M^{-1}L^{3}T^{-2}]$	Nm^2/kg^2
17.	Moment of inertia	$[ML^2]$	kg -m^2
18.	Angular velocity	$[T^{-1}]$	rad/s
19.	Angular acceleration	$[T^{-2}]$	rad/s^2
20.	Angular momentum	$[ML^2T^{-1}]$	kg m^2s^{-1}
21.	Specific heat	$[L^2T^{-2}\theta^{-1}]$	kcal $kg^{-1}K^{-1}$
22.	Latent heat	$[L^2T^{-2}]$	kcal/kg
23.	Planck's constant	$[ML^2T^{-1}]$	J-s
24.	Universal gas constant	$[ML^2T^{-2}\theta^{-1}]$	J/mol-K

Homogeneity Principle

If the dimensions of left hand side of an equation are equal to the dimensions of right hand side of the equation, then the equation is dimensionally correct. This is known as **homogeneity principle**.

Mathematically, [LHS] = [RHS].

Applications of Dimensions

(i) To check the accuracy of physical equations.

(ii) To change a physical quantity from one system of units to another system of units.

(iii) To obtain a relation between different physical quantities.

Significant Figures

In the measured value of a physical quantity, the number of digits about the correctness of which we are sure plus the next doubtful digit, are called the significant figures.

Rules for Finding Significant Figures

(i) All non-zeros digits are significant figures, *e.g.* 4362 m has 4 significant figures.

(ii) All zeros occuring between non-zero digits are significant figures, *e.g.* 1005 has 4 significant figures.

(iii) All zeros to the right of the last non-zero digit are not significant, *e.g.* 6250 has only 3 significant figures.

(iv) In a digit less than one, all zeros to the right of the decimal point and to the left of a non-zero digit are not significant, *e.g.* 0.00325 has only 3 significant figures.

(v) All zeros to the right of a non-zero digit in the decimal part are significant, *e.g.* 1.4750 has 5 significant figures.

Significant Figures in Algebraic Operations

(i) **In Addition or Subtraction** In addition or subtraction of the numerical values, the final result should retain as many decimal places as there are in the number with the least places. *e.g.*

If $l_1 = 4.326$ m and $l_2 = 1.50$ m

Then, $l_1 + l_2 = (4.326 + 1.50)\ \text{m} = 5.826\ \text{m}$

As l_2 has measured upto two decimal places, therefore

$$l_1 + l_2 = 5.83\ \text{m}$$

(ii) **In Multiplication or Division** In multiplication or division of the numerical values, the final result should retain as many significant figures as there are in the original number with the least significant figures. *e.g.* If length $l = 12.5$ m and breadth $b = 4.125$ m.

Then, area $A = l \times b = 12.5 \times 4.125 = 51.5625\ \text{m}^2$

As l has only 3 significant figures, therefore

$$A = 51.6\ \text{m}^2$$

Rounding Off

The process of omitting the non significant digits and retaining only the desired number of significant-digits, incorporating the required modifications to the last significant digit is called rounding off the number.

Rules for Rounding Off a Measurement

(i) If the digit to be dropped is less than 5, then the preceding digit is left unchanged. *e.g.* 1.54 is rounded off to 1.5.

(ii) If the digit to be dropped is greater than 5, then the preceding digit is raised by one. *e.g.* 2.49 is rounded off to 2.5.

(iii) If the digit to be dropped is 5 followed by digit other than zero, then the preceding digit is raised by one. *e.g.* 3.55 is rounded off to 3.6.

(iv) If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by one, if it is odd and left unchanged if it is even. *e.g.* 3.750 is rounded off to 3.8 and 4.650 is rounded off to 4.6.

Error

The lack in accuracy in the measurement due to the limit of accuracy of the measuring instrument or due to any other cause is called an error. The difference between the measured value and the true value of a quantity is known as the error in the measurement.

Errors are usually classified as

1. Absolute Error

The difference between the true value and the measured value of a quantity is called absolute error. If $a_1, a_2, a_3, \ldots, a_n$ are the measured values of any quantity a in an experiment performed n times, then the arithmetic mean of these values is called the true value (a_m) of the quantity.

$$a_m = \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}$$

The absolute error in measured values is given by

$$\Delta a_1 = a_m - a_1$$
$$\Delta a_2 = a_m - a_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\Delta a_n = a_m - a_n$$

2. Mean Absolute Error

The arithmetic mean of the magnitude of absolute errors in all the measurement is called mean absolute error.

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + \ldots + |\Delta a_n|}{n}$$

3. Relative Error or Fractional Error

The ratio of mean absolute error to the true value is called relative error.

$$\text{Relative error} = \frac{\text{Mean absolute error}}{\text{True value}} = \frac{\overline{\Delta a}}{a_m}$$

4. **Percentage Error**

The relative error expressed in percentage is called percentage error.

$$\text{Percentage error} = \frac{\overline{\Delta a}}{a_m} \times 100\%$$

Combinations of Errors

(i) **Error in Addition or Subtraction** Let $x = a + b$ or $x = a - b$

If the measured values of two quantities a and b are $(a \pm \Delta a)$ and $(b \pm \Delta b)$, then maximum absolute error in their addition or subtraction is

$$\Delta x = \pm (\Delta a + \Delta b)$$

(ii) **Error in Multiplication or Division** Let $x = a \times b$ or $x = \frac{a}{b}$

If the measured values of a and b are $(a \pm \Delta a)$ and $(b \pm \Delta b)$, then maximum relative error

$$\frac{\Delta x}{x} = \pm \left(\frac{\Delta a}{a} + \frac{\Delta b}{b} \right)$$

(iii) **Error in Raised to a Power** Let $z = a^p b^q / c^r$

If the measured values of a, b and c are $(a \pm \Delta a)$, $(b \pm \Delta b)$ and $(c \pm \Delta c)$, then maximum error $\frac{\Delta z}{z} = p\left(\frac{\Delta a}{a}\right) + q\left(\frac{\Delta b}{b}\right) + r\left(\frac{\Delta c}{c}\right)$

Note The smallest value that can be measured by a measuring instrument is called least count of that instrument. *e.g.* A metre scale having graduation at 1 mm division scale spacing, has a least count of 1 mm or 0.1 cm.