1. Given two odd integers a, b, prove that $a + b$ is even.

   (*Hint:* Try to mathematically define the notions of even and odd, and use these definitions.)

2. Given a *connected* graph $G(V, E)$, with $|V| = n$ and $|E| = m$, show that $\sum_{v \in V} \deg(v) = 2m$.

   (*Hint:* Does the inductive proof we saw in class still work here?)

3. Given a *directed* graph $G(V, E)$, with $|V| = n$ and $|E| = m$, use induction to show that

   $$\sum_{v \in V} \deg_{in}(v) = \sum_{v \in V} \deg_{out}(v) = m$$

   , where $\deg_{in}(v)$ and $\deg_{out}(v)$, respectively, are the number of directed edges entering and leaving some vertex $v \in V$.

4. $G(V, E)$ is a graph in which every vertex has degree at least 2, that is, $\forall v \in V : \deg(v) \geq 2$. Show that G is cyclic.

   (*Hint:* Try writing an algorithm for constructing a cycle in such a graph G. These types of proofs are called *constructive proofs*.)