Introduction to ML (CS771), 2024-2025-Sem-I		Total Marks	25
Quiz 4. November 11, 2024		Duration	45 minutes
Name		Roll No.	

Instructions:

1.	Clearly write your name (in block letters) and roll number in the provided boxes above.
2.	Write your final answers concisely in the provided space. You may use blue/black pen.
3.	We won't be able to provide clarifications during the quiz. If any aspect of some question appears ambiguous/unclear to you, please state your assumption(s) and answer accordingly.

Question 1: Write **T** or **F** for True/False in the box next to each question given below, with a brief (1-2 sentences at most) explanation in the provided space in the box below the question. Marks will be awarded only when the answer (T/F) and explanation both are correct. (**3 x 2 = 6 marks**)

1.1	EM or ALT-OPT will be required for doing MLE for the parameters of a supervised generative classification model with Gaussian class-conditionals.	

1.2	Projecting D dimensional inputs to a different co-ordinate system with D dimensions using linear PCA will incur zero loss of information.	

1.3	Kernel PCA can also be used for doing linear dimensionality reduction.	

Question 2: Answer the following questions concisely in the space provided below the question.

2.1	In 1-2 sentences, briefly state what distortion error is in the context of dimensionality reduction. Given N inputs $\{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N\}$ an encoder function f and a decoder function g, write down the expression of the total distortion error assuming squared Euclidean distance as the distortion error. **(3 marks)**

2.2	Given the top K eigenvectors $\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_K$ computed using a set of inputs $\{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N\}$, write down the expression of the K-dim projection $\boldsymbol{z}_n$ of any input $\boldsymbol{x}_n \in \mathbb{R}^D$. **(2 marks)**

2.3	Using appropriate notation, write down the general expression of the loss function (assuming squared Euclidean distance based loss) for a matrix factorization problem for an $N \times M$ matrix $\boldsymbol{X}$ that may have some missing entries and write the expression required for the task of matrix completion, i.e., for predicting the value of some missing entry X_{ij}. **(4 marks)**

2.4	For a K-component Gaussian mixture model with parameters $\Theta = \{\pi_k, \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k\}_{k=1}^K$, starting with the expression of the joint distribution $p(\boldsymbol{x}_n, \boldsymbol{z}_n \mid \Theta)$, show the steps that to obtain $p(\boldsymbol{x}_n \mid \Theta)$ and write down the final expression of $p(\boldsymbol{x}_n \mid \Theta)$. **(3 marks)**

2.5	For a latent variable model with data $\boldsymbol{X}$, latent variables $\boldsymbol{Z}$, and parameters Θ, EM computes the MLE of Θ by solving $\Theta_{MLE} = \text{argmax}_\Theta \, f(\boldsymbol{X} \mid \Theta)$. Write down the general expression of $f(\boldsymbol{X} \mid \Theta)$, clearly specifying and defining the various terms in the expression. **(3 marks)**

2.6	For an MLP with L hidden layers and nonlinearity g in each hidden layer, clearly and briefly write down the expressions that will be used to compute the real-valued output $\hat{y}_n$ for an input $\boldsymbol{x}_n \in \mathbb{R}^D$. You may use any terms/notation necessary for these expressions. **(4 marks)**