COEP Technological University, Pune Department of Mathematics
MA : Linear Algebra (LA)
F.Y. B.Tech. Semester I (Computer Branch) Academic Year: 2024-25
Tutorial 1 : Review of Matrix Algebra
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1. Define a matrix and classify matrices based on (i) it's order (size) and (ii) it's entries.

2. Define: Equality, Addition and Scalar multiple of a real matrix.

3. If A denotes a 15×15 matrix representing the distance in kms between 15 cities then obtain a matrix B which will represent the distance between the same 15 cities in miles.

4. State the necessary condition for addition of 2 real matrices. Is this sufficient?

5. Define a row vector and a column vector. Give an example of four forces in equilibrium.

6. Let $M = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ If $a_{ij} = 1$ then draw an arc(edge) joining (i, j) for each $i, j = 1, 2, 3, 4$. This figure is said to be a graph with vertices $1, 2, 3, 4$ and edges (i, j) such that $a_{ij} = 1$. Also write the set of edges.

7. State any one application of matrix application to justify the standard definition of matrix multiplication which is not so natural.

8. Find (a) $A + 3B$, (b) $2A - B$, (c) A^T (d) B^{-1}.
$A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix}$

9. Let A, B, C, D and E be matrices with the provided orders given as $A : 3 \times 4$, $B : 3 \times 4$, $C : 4 \times 2$, $D : 4 \times 2$, $E : 4 \times 3$. For the following matrices, if defined, determine the size of the matrix; if not defined, provide an explanation.

(a) $A+B$ (b) $C+E$ (c) $\frac{1}{2}D$ (d) $-4A$ (e) $A-2E$
(f) AC (g) BE (h) $E-2A$ (i) $2D+C$ (j) BE^T

10. Verify $AB = BA$ for the following matrices.

(a) $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, $B = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$

(b) $A = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$, $B = \begin{bmatrix} -5 & 0 \\ 0 & 4 \end{bmatrix}$

Is matrix multiplication commutative in general? Justify.

11. Find AB and BA, if they are defined.

(a) $A = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$ (b) $A = \begin{bmatrix} -1 & 3 \\ 4 & -5 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 0 & 7 \end{bmatrix}$

12. Show that a square matrix can be written as sum of a symmetric matrix and a skew-symmetric matrix.

13. Find inverse of the matrix (if exists).

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{bmatrix}$

14. State whether the following statements are True or False. Justify your answers with proper reason/ counter example.

(a) Matrix addition is commutative and associative.

(b) Matrix multiplication is neither commutative nor associative.

(c) If the matrices A, B, and C satisfy $AB = AC$, then $B = C$.

(d) $(AB)^T = A^T B^T$.

(e) $(A+B)^T = A^T + B^T$.

(f) Inverse of non-singular matrix is unique.

(g) Product of two invertible matrices is invertible.

(h) Sum of two singular matrices is singular.

(i) $(A^{-1})^T = (A^T)^{-1}$.

15. Let $A = \begin{matrix} a & b \\ c & d \end{matrix}$. Find products AS and SA for the matrix $S = \begin{matrix} 1 & x \\ 0 & 1 \end{matrix}$.
Describe in words the effect on A of this product.

16. Let A be a square matrix. (a) If $A^2 = O$ show that $I - A$ is invertible. (b) If $A^3 = O$, show that $I - A$ is invertible. (c) In general, if $A^n = O$ for some positive integer n, show that $I - A$ is invertible. [Hint: Think of the geometric series.] (d) Suppose that $A^2 + 2A + I = O$. Show that A is invertible.

17. Let $A = (a_{ij})_{2\times3}$, $B = (b_{ij})_{2\times3}$, $C = (c_{ij})_{3\times3}$, $D = (d_{ij})_{4\times3}$, $E = (e_{ij})_{2\times2}$. State at least one sum, one product and one associative law that is valid.

18. Justify why the matrix $\begin{bmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{bmatrix}$ is called the rotation matrix. Using this (i) prove the sum and difference formulae for $\sin(a \pm b)$ and $\cos(a \pm b)$ and (ii) write the matrix for rotation through $n\vartheta$.

Note: ~~If you find any mistake please upload corrected question and your solution on moodle for others to follow/check.~~

(h) Sum of two singular matrices is singular.

(i) $(A^{-1})^T = (A^T)^{-1}$.

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