COEP Technological University, Pune
Department of Mathematics
MA : Linear Algebra (LA)
F.Y. B.Tech. Semester I (Computer Branch)
Academic Year: 2023-24
Tutorial 3 : System of linear Equations
Faculty Name: Dr. C. M. Deshpande

1. Explain the equivalence of a system of m linear equations in n unknowns with the matrix equation $AX = B$. Classify the systems using A and B.

2. Write the three elementary row operations for any system of equations and hence state the elementary row operations for matrices.

3. Find the solution set (if exists) for the following systems of linear equations by using Gauss Elimination method.
   (a) $8y + 6z = -4;\ -2x + 4y - 6z = 18;\ x + y - z = 2.$
   (b) $10x + 4y - 2z = 14;\ -15x + y + 2z - 3w = 0;\ x + y + w = 6;\ -5x + 5y - 10z + 8w = 26.$
   (c) $-x + y = 1.5;\ 2x - 4y = 3.$

4. Solve the following systems using Gauss Jordan elimination method.
   (a) $x - y = 9;\ -x + y = 1.$
   (b) $x + y = 2;\ 3x - y = 0.$
   (c) $2x - 8y + 4z = 0;\ 3x - 10y + 7z = 0;\ 10y + 5z = 0.$

5. Consider the system of equations $x+y+2z = a;\ x+z = b;\ 2x+y+3z = c$. Show that for this system to be consistent, the constants a, b, and c must satisfy $c = a + b$.

6. Write the system of equations represented by the following matrices considering as (a) coefficient matrix A and (b) augmented matrix $A|B$:
   (i) $\begin{bmatrix} 4 & 0 & 6 \\ -1 & 1 & -1 \\ 2 & -4 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & -2 & 4 & 0 & 0 \\ -3 & 3 & -6 & 5 & 15 \\ 1 & -1 & 2 & 0 & 0 \end{bmatrix}$

7. Indicate whether the statement is always true or sometimes false. Justify your answer by giving a logical argument or a counterexample.

   (a) If a matrix is reduced to reduced row-echelon form by two different sequences of elementary row operations, the resulting matrices will be different.

   (b) If a matrix is reduced to row-echelon form by two different sequences of elementary row operations, the resulting matrices might be different.

   (c) If the reduced row-echelon form of the augmented matrix for a linear system has a row of zeros, then the system must have infinitely many solutions.

   (d) If three lines in the plane are sides of a triangle, then the system of equations formed from their equations has three solutions, one corresponding to each vertex.

   (e) A linear system of three equations in five unknowns must be consistent.

   (f) A linear system of five equations in three unknowns cannot be consistent.

   (g) If a linear system of n equations in n unknowns has n leading 1's in the reduced row-echelon form of its augmented matrix, then the system has exactly one solution.

   (h) If a linear system of n equations in n unknowns has two equations that are multiples of one another, then the system is inconsistent.

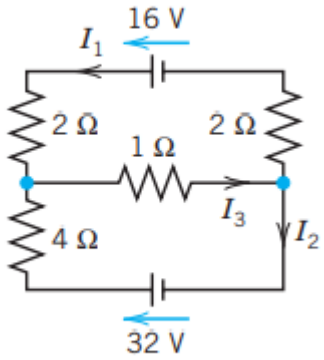
8. Find the polynomial passing through the points:

   (i) $(-1, 3), (0, 0), (1, 1), (4, 58)$

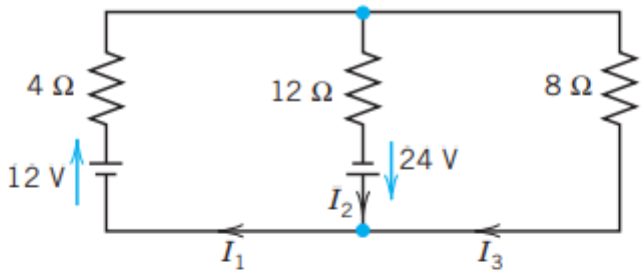
   (ii) $(1920, 106), (1930, 123), (1940, 132), (1950, 151)$. If this represents the census data, can you predict the population in the year 2000?

In Probs. 17–19, using Kirchhoff's laws (see Example 2) and showing the details, find the currents:

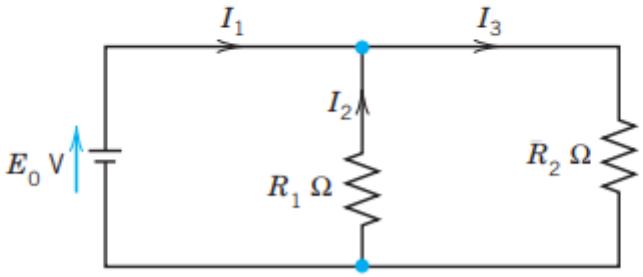
17.



18.

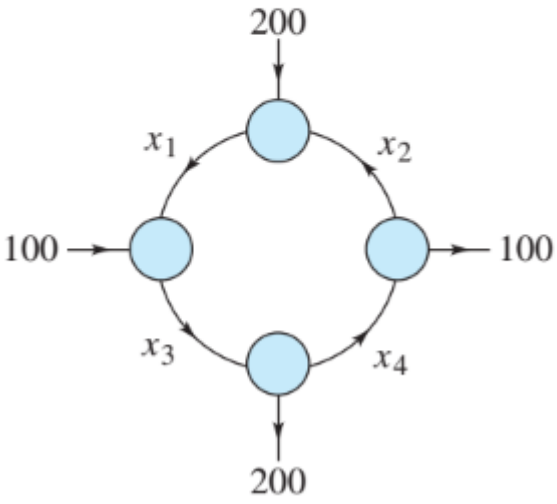


19.



The flow of traffic (in vehicles per hour) through a network of streets is shown in Figure 1.17.

(a) Solve this system for x_i, $i = 1, 2, 3, 4$.
(b) Find the traffic flow when $x_4 = 0$.
(c) Find the traffic flow when $x_4 = 100$.



The flow through a network is shown in Figure 1.24.

(a) Solve the system for x_i, $i = 1, 2, \ldots, 6$.

(b) Find the flow when $x_3 = 100$, $x_5 = 50$, and $x_6 = 50$.

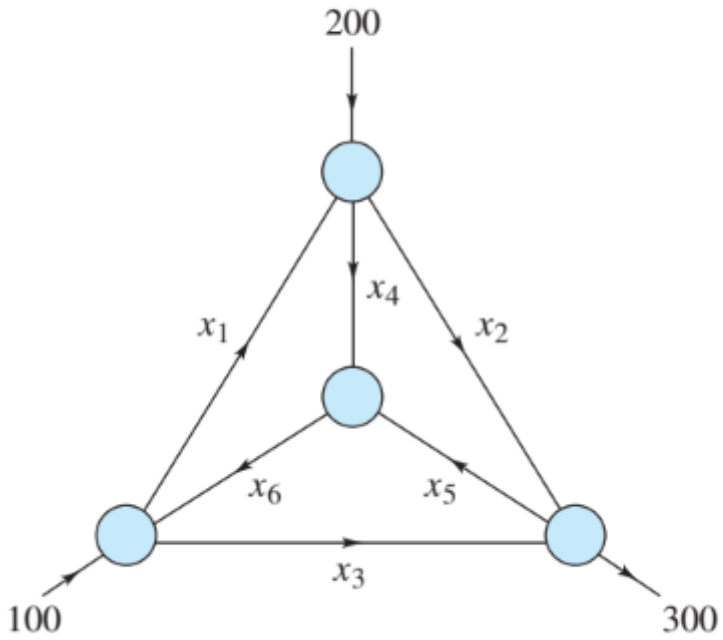


Figure 1.24

Note: If you find any mistake please upload corrected question and your solution on moodle for others to follow/check.