COEP Technological University

(COEP Tech)

A Unitary Public University of Government of Maharashtra

(MA-19002) Linear Algebra

Program : F.Y.B.Tech. Sem. I (All Branches) | Academic Year : 2022-23

Examination : End Semester Exam | Maximum Marks : 60

Date: 2/3/2023 | Time: 2.30pm to 5.30pm

Division: **Student MIS Number :** 612203012

Instructions :

1. Write your MIS Number on Question Paper.
2. Rough work/Calculations/marking anything on question paper is not allowed.
3. Mobile phones and programmable calculators are strictly prohibited.
4. Figures to the right indicate the course outcomes and maximum marks.
5. Unless mentioned, symbols and notations have their usual standard meanings.
6. Any essential result, formula or theorem assumed for answering questions must be clearly stated.
7. **Answers to all sub parts of each question should be written together. Also, write question number with sub part number while writing answers. For example, Q.III(B)(1)...**

Attempt all questions:

Question [I](15 marks)

A. State whether true or false:

1. For subspace W of a vector space V, zero element of W need not be the same as that of V. TRUE
2. Every linearly dependent set contains the zero vector. TRUE
3. The system $(\lambda-3)x+y=0,\ x+(\lambda-3)y=0$ has a non-trivial solution when $\lambda=-2$. TRUE

[CO1][$1\times 3=$ **3** marks]

B. Attempt **any one** of the following:

1. Determine values of a for which the system has (i) no solution, (ii) infinitely many solutions:

$$x+2y-3z=4,\ 3x-y+5z=2,\ 4x+y+(a^2-14)z=a+2.$$

2. Let $W=\{(x+2y,\ y,\ -x+3y)\mid x,y\in\mathbb{R}\}$. Prove that W is a subspace of $\mathbb{R}^3$. Also find its basis and dimension.

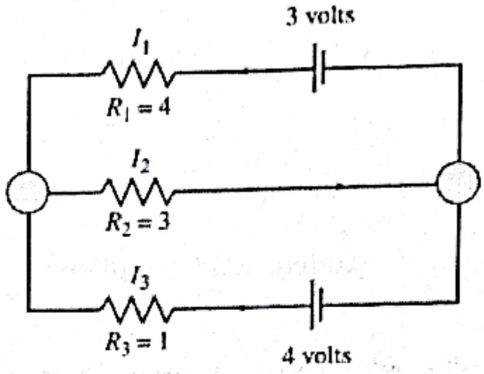
[CO3][4 marks]

C. Solve the following system using Gauss elimination method:
$x_1-x_2+x_3=5,\ x_1+x_2+x_3=-1,\ 9x_1+3x_2+x_3=1.$ [CO3][3 marks]

P.T.O

D. Check whether the set $S = \{(-1,2,3),\ (2,5,7),\ (3,7,10)\}$ is linearly dependent or independent in $\mathbb{R}^3$. [CO3][3 marks]

E. Generate a system of linear equations for determining the currents $I_1,\ I_2,\ I_3$ in the following network. (*Only write the system of equations. Do not solve it.*)



[CO5][2 marks]

Question [II](15 marks)

A. 1. Write the standard basis for the vector space of all polynomials of degree less than or equal to 2.

2. Is it true that "A set containing only one vector is linearly independent"? Give reason.

3. Define row space of an $m \times n$ matrix A.

[CO1][$1 \times 3 = 3$ marks]

B. 1. Let u and v be vectors in an inner product space V such that $\|u\| = 3$, $\|u+v\| = 4$, $\|u-v\| = 6$. What must be the value of $\|v\|$?

2. Verify Cauchy-Schwarz inequality for $< f, g >= \int_{-\pi}^{\pi} f(x)g(x)dx$, where $f(x) = \sin x$ and $g(x) = \cos x$.

[CO2][$2 \times 2 = 4$ marks]

C. Attempt **any one** of the following:

1. Find a basis for the subspace of $\mathbb{R}^4$ spanned by the set S, where S is as below.
$S = \{(-3,2,5,28), (-6,1,-8,-1), (14,-10,12,-10), (0,5,12,50)\}$.

2. Determine whether the given system of linear equations is consistent. If yes, write its solution in the form $X = X_h + X_p$, where X_p is a particular solution of the system $AX = B$, whereas X_h is a solution of the corresponding homogeneous system.

$$x + 2y - 3z = 1,\ 2x + 5y - 8z = 4,\ 3x + 8y - 13z = 7.$$

[CO3][3 marks]

D. Prove that if $S = \{v_1, v_2, \ldots, v_n\}$ is an orthogonal set of nonzero vectors in an inner product space V, then S is linearly independent. [CO4][2 marks]

E. Find the matrix A of the quadratic form associated with the equation $5x^2 - 2xy + 5y^2 + 10x - 17 = 0$. Then find an orthogonal matrix P such that $P^T AP$ is a diagonal matrix. Also, find equation of the conic in which xy-term has been eliminated. [CO5][3 marks]

Question [III](15 marks)

A. 1. Define linear transformation.

2. Consider the linear transformation $T(x, y) = (x+y,\ x-y)$. Find the image of $(3, -4)$ under T.

3. Let $T : M_{2,2} \to M_{2,2}$ be the zero transformation. Find range(T).

[CO1][$1 \times 3 = $ **3 marks**]

B. Attempt **any two** of the following:

1. Determine whether the linear transformation $T(x, y) = (x + 2y, 3y)$ is invertible. If yes, find its inverse. If not, justify your answer.

2. Find the standard matrix for the stated composition of linear transformations in $\mathbb{R}^3$. A rotation of 30^o about x-axis followed by a contraction with factor $k = 0.25$.

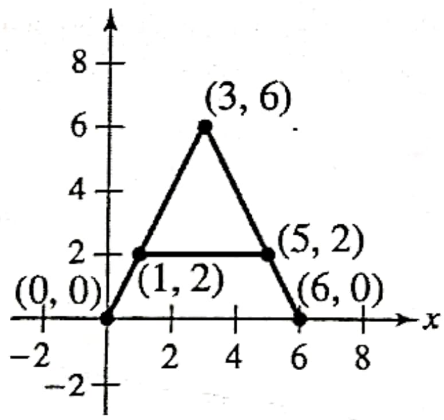
3. Find a basis for the kernel of linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ given by $T(v) = Av$, where matrix $A = \begin{bmatrix} 1 & -1 & -2 \\ -1 & 2 & 3 \end{bmatrix}$. Also find the nullity of T.

[CO2, 3][$3 \times 2 = $ **6 marks**]

C. For a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$, if rank(T) $= 2$, find its nullity and also give the geometric description of the kernel and range of T. [CO3][**2 marks**]

D. Let $T : V \to W$ be a linear transformation. Then prove that T is one-to-one if and only if $\ker(T) = \{0\}$. [CO4][**2 marks**]

E. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $T(x, y) = (x+y, y)$. Sketch the image of the following figure under T.



[CO5][**2 marks**]

Question [IV](15 marks)

A. 1. Let A be a 7×7 matrix with $\lambda = -1$ as its only eigenvalue, then eigenvalues of the matrix $B = 5A^3 - 2A + 3I$ are

a) 0 b) -1 c) 6 d) Can't find because we don't know what matrix B is.

2. State true or false: "Every 10×10 symmetric matrix with real entries is diagonalizable."

P.T.O

3. Complete the following theorem statement:
   Let A be an $n \times n$ matrix. Then A is orthogonally diagonalizable and has real eigenvalues if and only if A is $\cdots$.

[CO1][$1 \times 3 = $ **3 marks**]

B. Attempt **any two** of the following:

1. Let A be a 2×2 matrix with tr(A) $= -1$ and det(A) $= -6$. Is matrix A diagonalizable ? Why?
2. Show that A and A^t have the same eigenvalues.
3. Prove that if λ is an eigenvalue of an invertible matrix A, and x is a corresponding eigenvector, then $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}, and x is a corresponding eigenvector.

[CO2][$2 \times 2 = $ **4 marks**]

C. Consider the matrix $A = \begin{bmatrix} -2 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$.

1. Find the eigenvalues and eigenvectors of A.
2. Can A be diagonalizable? Justify your answer using part 1.

[CO3][**4 marks**]

D. Let A be an $n \times n$ symmetric matrix. Prove that eigenvectors of A corresponding to distinct eigenvalues are orthogonal. [CO4][**2 marks**]

E. A student of certain college is said to be in stage S_1, if he gets semester GPA (SGPA) < 3.0 and in stage S_2, if his SGPA ≥ 3.0. As a result of observations from the college, it is found that if a student is in stage S_1 in a semester, then he/she studies hard in the next semester to achieve stage S_2 with the probability 0.8. But if a student is in stage S_2, then the probability of him/her falling in stage S_1 is 0.3. We are assuming that student's achievement in a semester is determined only by the performance from the previous SGPA.

The probabilities here can be arranged in a matrix given by $M = \begin{array}{cc} S_1 & S_2 \end{array}$ $\begin{array}{c} \\ \begin{bmatrix} 0.2 & 0.3 \\ 0.8 & 0.7 \end{bmatrix} \begin{array}{c} S_1 \\ S_2 \end{array} \end{array}$, called the *transition matrix* for the Markov chain. This matrix can be used to find the probabilities of a student being in S_1 or S_2 at later times. If the stage vector $S = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$, where p_i gives probability of a student being in stage S_i, $i = 1,\ 2$ in the current semester, then the matrix multiplication MS represents the stage vector of the student in the next semester.

So, if the student is in stage S_1 in the first semester, what is his/her probability of being in stage S_2 at the end of the second semester?
Also, prove that 1 is an eigenvalue of the matrix M. [CO5][**2 marks**]