COEP Technological University

(COEP Tech)

A Unitary Public University of Government of Maharashtra

Linear Algebra for School of Computational Sciences

Program : F.Y.B.Tech.

Examination : End Semester Examination

Date: 24/11/2023

Academic Year : 2023-24

Maximum Marks : 60

Time: 10.00am-1.00pm

Instructions :

1. Write your MIS Number on question paper.
2. Do not write anything on the question paper.
3. Mobile phones and programmable calculators are strictly prohibited.
4. Exchange/sharing of stationary, calculator etc. is not allowed.
5. Figures to the right indicate the course outcomes and maximum marks.
6. Unless otherwise mentioned symbols and notations have their usual standard meanings.
7. Any essential result, formula or theorem assumed for answering questions must be clearly stated.
8. **Answer to all sub parts of each question should be written together. Also, write question number with sub part number while writing answers. For example, Q.I(a)(iv)...**

Attempt All the questions.

Question [I] Do as directed:

a) Fill in the blanks by choosing the most correct answer from the list below: [CO1,CO2][10]

has no solution, is a non-homogeneous system, has eigen value 1, minimum 5 linearly independent rows, is singular, has no solution or infinitely many solutions, subspaces of the same vector space, orthogonal, maximum 5 linearly independent columns, has distinct real eigen values, has unique solution, is a linear map from $\mathbb{R}^5$ to $\mathbb{R}^3$, has eigen value 2, has infinitely many solutions, has unique solution or infinitely many solutions, is a linear map from $\mathbb{R}^3$ to $\mathbb{R}^5$, is symmetric, is invertible, equal, the zero vector.

i) The system $BX = A$ __________

ii) The system $B_{3\times5}X = 0$ __________

iii) If a matrix has rank 5 then it has __________

iv) The system $B_{3\times5}X = C$ __________

v) If the matrix A satisfies $A^2 - 3A + 5I = 0$ then it __________

vi) Null space of A and range space of A^T are __________

vii) A linear transformation maps the zero vector to __________

viii) A matrix $A_{3\times5}$ __________

ix) A is diagonalizable if it __________

x) If 2 is an eigen value of A then $2A^2 - 3A - I$ __________

b) Say True or False. Justify your answers. [CO2,CO3][10]

i) Determinant of a matrix, as a function from the space of square matrices to the set of real numbers is a linear map.

ii) If all eigen values of A are real then A is symmetric.

iii) A set of non zero orthogonal vectors is linearly independent.

iv) $W = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 | x_1 + x_4 = 0\}$ is a three dimensional subspace of $\mathbb{R}^4$.

v) Vectors $A = \begin{pmatrix} 1 & -4 \\ -2 & 8 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 4 \\ -2 & 1 \end{pmatrix}$ are orthogonal in the standard inner product space of 2×2 real matrices.

Question [II] Solve ANY FIVE: **[CO2][10 marks]**

a) Give an example of a non trivial subspace of the vector space of polynomials of degree less equal 12.

b) The vectors $(-4/3, 8/3, -4, -32/3)$ and $(-16/3, -2, 4/3, -2/3)$ are orthogonal / parallel / of same length. Strike off the wrong answers and rewrite the complete statement. Justify.

c) Is $B = [3\ 4]^T$ in the column space of $A = \begin{pmatrix} -1 & 2 \\ 4 & 0 \end{pmatrix}$? Justify.

d) Find the coordinate matrix of $u = (-3, 4)$ relative to the orthonormal basis $B = \{(\sqrt{5}/5, 2\sqrt{5}/5), (-2\sqrt{5}/5, \sqrt{5}/5)\}$.

e) Find the matrix of $T(x, y, z, w) = (2x - w + 3z, y - 2z, 2w - x + z)$ relative to the standard bases.

f) Find a basis for the space of real skew-symmetric matrices of order 2×2.

Question [III] Solve ANY FOUR: **[CO3, CO4][20 marks]**

a) Find an orthonormal basis for the solution space of the system of homogeneous equations below:

$$x_1 - x_2 + x_3 + x_4 = 0 \quad ; \quad x_1 - 2x_2 + x_3 + x_4 = 0$$

b) State and prove the Cauchy-Schwarz inequality for an inner product space and hence prove that

$$\left(\sum_{i=1}^{n} x_i\right)^2 \le n \sum_{i=1}^{n} x_i^2$$

c) Let S be a subspace of $\mathbb{R}^n$. Define the orthogonal complement of S as $S^{\perp} = \{u \in \mathbb{R}^n \mid v \circ u = 0,\ \forall v \in S\}$. Prove that $S^{\perp}$ is a subspace of $\mathbb{R}^n$ and that $\mathbb{R}^n = S \bigoplus S^{\perp}$, i.e. every vector w in $\mathbb{R}^n$ can be uniquely expressed as a sum $v + u$ where $v \in S$ and $u \in S^{\perp}$. (This is called a direct sum decomposition of an inner product space i.e. every inner product space can be expressed as a direct sum of a subspace and it's orthogonal complement. Imagine any line passing through the origin as a subspace of $\mathbb{R}^2$ and the line passing through origin and perpendicular to this line as it's orthogonal complement!!)

d) Find eigen values and eigen vectors of $A = \begin{pmatrix} 1 & -1.5 & 2.5 \\ -2 & 6.5 & -10 \\ 1.5 & -4.5 & 8 \end{pmatrix}$. Is A diaginalizable?

e) Define kernel and range of a linear transformation and find the same for $T(x, y, z) = (x + 2y + z, x + 5z, z)$. Hence or otherwise conclude if T is invertible or not.

Question [IV] Solve ANY TWO: **[CO4, CO5][10 marks]**

a) Define an inner product space. Does there exist an inner product on the vector space $\mathbb{R}^3$ with respect to which the vectors $(4, 2, 0)$ and $(2, -2, 1)$ are orthogonal? Justify. Is this unique? Justify.

b) The number of calories burned by different body weights performing different types of aerobic exercises for a 20 minute time period are as below:
A 55kg male burns 109, 127 and 64 calories in bicycling, jogging and walking respectively while the same data for a 65 kg male reads 136, 159 and 79.
Express this information as a matrix and hence find the calories burnt by Aditya and Amit weighing 55kg and 65kg respectively if they walked for an hour followed by jogging for 10 minutes and cycling for 30 minutes.

c) Prove that every square matrix can be expressed uniquely as a sum of symmetric and skew symmetric matrix. Hence or otherwise prove that dimension of the space of square matrices of order n is n^2.