

COEP Technological University, Pune
Department of Mathematics
MA : Linear Algebra (LA)
F.Y. B.Tech. Semester I (Computer Branch)
Academic Year: 2023-24
Tutorial 6 : Basis and change of basis
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1. Find bases for the following subspaces of \mathbb{R}^5 .

$$W_1 = \{(a_1, a_2, a_3, a_4, a_5) \in \mathbb{R}^5 \mid a_1 - a_3 - a_4 = 0\} \text{ and}$$

$$W_2 = \{(a_1, a_2, a_3, a_4, a_5) \in \mathbb{R}^5 \mid a_2 = a_3 = a_4 \text{ and } a_1 + a_5 = 0\}.$$

What are the dimensions of W_1 and W_2 .

2. Let u, v , and w be distinct vectors of a vector space V . Show that if $\{u, v, w\}$ is a basis for V , then $\{u + v + w, v + w, w\}$ is also a basis for V .
3. (a) Find the coordinate matrix of $x = (12, 6)$ in \mathbb{R}^2 relative to the basis $B' = \{(4, 0), (0, 3)\}$.
(b) Find the coordinate matrix of $x = (-17, 22)$ in \mathbb{R}^2 relative to the basis $B' = \{(-5, 6), (3, -2)\}$.
4. For $B = \{(1, 3), (-2, -2)\}$, $B' = \{(-12, 0), (-4, 4)\}$ and $[x]_{B'} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$,
 - a) find the transition matrix from B to B' ,
 - (b) find the transition matrix from B' to B ,
 - (c) verify that the two transition matrices are inverses of each other, and
 - (d) find the coordinate matrix $[x]_B$, given the coordinate matrix $[x]_{B'}$.

Note: If you find any mistake please upload corrected question and your solution on moodle for others to follow/check.