1. Define an Inner Product Space.

2. State and Prove Pythagoras Theorem for inner product spaces.

3. State and prove Triangle inequality for inner product spaces.

4. State and Prove Parallelogram Law for inner product spaces.

5. Check whether the product $< u, v >$ of two vectors in a vector space V is an inner product on V.
   (a) $V = \mathbb{R}^3$, $< u, v >= u_1v_1 + 3u_2v_2 + 2u_3v_3$ where $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$.
   (b) $V = \mathbb{R}^2$, $< u, v >= u_1v_1 - 3u_2v_2$ where $u = (u_1, u_2)$ and $v = (v_1, v_2)$.
   (c) $V = M_{2,2}$, $< u, v >= u_1v_1 + u_2v_2 + u_3v_3$ where $u = \begin{bmatrix} u_1 & u_2 \\ u_3 & u_4 \end{bmatrix}$ and $v = \begin{bmatrix} v_1 & v_2 \\ v_3 & v_4 \end{bmatrix}$.

   (d) $V = C[-\pi, \pi]$, the space of continuous real-valued functions on the interval $[-\pi, \pi]$, $< u, v >= \int_{-\pi}^{\pi} u(t)v(t)\, dt$.

6. Use the inner product $< p, q >= p_1q_1 + 2p_2q_2 + p_3q_3$ on P_2, where
   $p(x) = p_1 + p_2x + p_3x^2$ and $q(x) = q_1 + q_2x + q_3x^2$, to find $< p, q >$, $\|p\|$, $\|q\|$, $d(p, q)$ and angle between p and q for the polynomials $p(x) = 1 - 2x - x^2$ and $q(x) = x - x^2$.

7. Let V be the vector space of $n \times n$ real symmetric matrices. Show that $< A, B >= trace(A^t B)$ is a positive definite scalar product on V.

8. Let u and v be vectors in an inner product space V such that $\|u\| = 3, \|u + v\| = 4, \|u - v\| = 6$. What must be the value of $\|v\|$?

Note: If you find any mistake please upload corrected question and your solution on moodle for others to follow/check.