Numericals: Unit 1

P 1

Calculate the group and phase velocities for the wave packet corresponding to a relativistic particle.

P 2

(a) Estimate the energy of the electrons that we need to use in an electron microscope to resolve a separation of 0.27 nm.

(b) In a scattering of 2 eV protons from a crystal, the fifth maximum of the intensity is observed at an angle of 30^0. Estimate the crystal's planar separation.

P3

Show that for a free particle, the uncertainty relation can also be written as

$$\Delta\lambda\, \Delta x \geq \lambda^2 / 4\pi \quad (\text{use}: \Delta x\, \Delta p_x \geq \hbar / 2)$$

P4

(a) Calculate the final size of the wave packet representing a free particle after traveling a distance of 100 m for the following four cases where the particle is

(i) a 25 eV electron whose wave packet has an initial width of 10^{-6} m

(ii) a 25 eV electron whose wave packet has an initial width of 10^{-8} m,

(iii) a 100 MeV electron whose wave packet has an initial width of 1 mm, and

(iv) a 100 g object of size 1 cm moving at a speed of 50 m s^{-1}.

(b) Estimate the times required for the wave packets of the electron in (i) and the object in (iv) to spread to 10 mm and 10 cm, respectively. Discuss the results obtained.

P5

An "excited" atom gives up its excess energy by emitting a photon of characteristic frequency. The average period that elapses between the excitation of an atom and the time it radiates is 1.0×10^{-8} s. Find the inherent uncertainty in the frequency of the photon.

P6

Estimate the lifetime of the excited state of an atom whose natural width is 3×10^{-4} eV; you may need the value $h = 6.626 \times 10^{-34}$ J s = 4.14×10^{-15} eV s.

P7

A measurement establishes the position of a proton with an accuracy of 1.00×10^{-11} m. Find the uncertainty in the proton's position 1.00 s later.

P8

A typical atomic nucleus is about 5.0×10^{-15} m in radius. Use the uncertainty principle to place a lower limit on the energy an electron must have if it is to be part of a nucleus. Or why Electrons cannot be present in the nucleus?

P9

Show that for those waves whose angular frequency ω and wave number k obey the dispersion relation $k^2c^2 = \omega^2 = constant$, the product of the phase and group velocities is equal to c^2,

$v_g v_{ph} = c^2$, where c is the speed of light.

P10

An electron is confined to a potential well of width 10 nm. Calculate the minimum uncertainty in its velocity

P11

If the kinetic energy of an electron known to be about 1 eV, must be measured to within 0.0001 eV, what accuracy can its position be measured simultaneously?

P12

An electron and a 150 gm base ball are traveling at a velocity of 220 m/s, measured to an accuracy of 0.005 %. Calculate and compare uncertainty in position of each

P13

Find the phase and group velocities of the de Broglie waves of

an electron whose speed is 0.900c.

P14

Verify that the uncertainty principle can be expressed in the form (1) $\Delta L\ \Delta\theta \geq \hbar/2$, where L is the uncertainty in the angular momentum of a particle and θ is the uncertainty in its angular position. (Hint: Consider a particle of mass m moving in a circle of radius r at the speed v, for which L =mvr) and (2) $\Delta E\ \Delta t \geq \hbar/2$

P14

Show that for a free particle, the uncertainty relation can also be written as

$\Delta\lambda\, \Delta x \geq \lambda^2 / 4\pi$ (use : $\Delta x\, \Delta p_x \geq \hbar / 2$)