Tutorial 2

Dirac notation and Bra-Ket algebra

1.

Which of the following sets of vectors in $\mathbb{R}^3$ are linearly independent?

(a) $\left\{ \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 8 \\ 4 \\ 8 \end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -7 \\ -8 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 6 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 7 \\ 0 \\ -2 \end{bmatrix} \right\}$

2.

Consider two different wave-functions $\Psi_m(x)$ and $\Psi_n(x)$. The condition for the wave-functions to be orthonormal is

a) $\int_{-\infty}^{+\infty} \Psi_m^*(x)\Psi_n(x)\,dx = \delta_{mn}$ b) $\int_{-\infty}^{+\infty} \Psi_m^*(x)\Psi_n(x)\,dx = 0$

c) $\int_{-\infty}^{+\infty} \Psi_m^*(x)\Psi_n(x)\,dx = 1$ d) $\Psi_m^*(x)\Psi_n(x) = \delta_{mn}$

3.

The non-zero commutator brackets are

a) $[z^2, p_x]$ b) $[x, p_x^3]$

c) $[y^2, p_y]$ d) $[p_x^2, p_z]$

4.

The pairs that must obey Heisenberg's uncertainty principle are

a) position and energy b) position and momentum

c) energy and time d) mass and energy

5. Evaluate [Lx,Ly] = ______ [Ly,Lz] = ______ [Lz,Lx] =____

6.

Consider the following eigenvalue equation: $\hat{O}g(x) = \lambda\, g(x)$, where the operator $\hat{O} = \left(-\frac{\partial^2}{\partial x^2} + x^2\right)$ and its eigenfunction is $g(x) = A\,x\,e^{-x^2/2}$. The eigenvalue λ is _____.

7.

Consider the two kets, $|\psi\rangle = \begin{pmatrix} 2i \\ 3+i \\ 3 \end{pmatrix}$, $|\phi\rangle = \begin{pmatrix} 4 \\ -3i \\ 2-i \end{pmatrix}$. Then $\langle\phi|\psi\rangle$ will be $ai + b$.

i) $a =$ _____ (Answer should be an integer)
ii) $b =$ _____ (Answer should be an integer)

8.

Consider the states $|\psi\rangle = 3i|\phi_1\rangle - 7i|\phi_2\rangle$ and $|\chi\rangle = -|\phi_1\rangle + 2i|\phi_2\rangle$, where $|\phi_1\rangle$ and $|\phi_2\rangle$ are orthonormal. Then $\langle\psi+\chi|\psi+\chi\rangle$ is _____. (Answer should be an integer)

9. Verify Schwarz inequality and triangular inequality in the above numerical.

10.

Consider a three-dimensional vector space spanned by an orthonormal basis $|1\rangle, |2\rangle, |3\rangle$. Kets $|\alpha\rangle$ and $|\beta\rangle$ are given by

$$|\alpha\rangle = i|1\rangle - 2|2\rangle - i|3\rangle, \quad |\beta\rangle = i|1\rangle + 2|3\rangle$$

Then the inner product $\langle\alpha|\beta\rangle = a + bi$, where

i) $a =$ _____ (Answer should be an integer)
ii) $b =$ _____ (Answer should be an integer)