COEP Technological University

CT 23004 Discrete Structures

Time: 60 min

Max marks: 20

Instructions

1. Answer all questions.
2. Show all necessary working.

Choose the appropriate option **[1 mark each]**

(1) How many of the following sets are countably infinite?
$\mathbb{N}-\mathbb{Q}$, $\mathbb{Z}-\mathbb{N}$, $\mathbb{R}-\mathbb{Z}$, $\mathbb{R}-\mathbb{Q}$
(a) 0 (b) 1 (c) 2 (d) 3

(2) Which of the following functions from $\mathbb{R}$ to $\mathbb{R}$ is not one-one?
(a) $\frac{2x-1}{x-2}$ (b) x^5-1 (c) x^3-3x^2+3x-1 (d) x^3-4x^2-3x-2

(3) If A=$\mathbb{Q}$ and $A \cup B$ is uncountable, which of these cannot be B?
(a) $\mathbb{Q}^c$ (b) $\mathbb{Z}$ (c) $[0,1]$ (d) $\mathbb{R}$

(4) Which of the following statements is true for the given relation?
a and b are people, aRb if a speaks a language that b speaks.
(a) It is not symmetric (b) It is not reflexive (c) It is not transitive (d) It is an equivalence relation

(5) Which of the following is not a function from the set $\{1,2,3\}$ to $\mathbb{R}$?
(a) $\{(1,2),(2,4),(3,6)\}$ (b) $\{(1,1),(3,4),(2,4)\}$ (c) $\{(1,2),(1,4),(2,5)\}$ (d) $\{(1,1),(2,2),(3,3)\}$

Section B: Solve any 5 **[3 marks each]**

1. State the biggest domain and codomain which are subsets of $\mathbb{R}$ for which the following relations are functions and the functions are onto:
(a) $y=\log(2x+1)$ (b) $y=\sqrt{3-x^2}$ (c) $y=\frac{1}{x+4}$

2. Show that any 2 distinct equivalence classes are disjoint i.e. if R is relation from A to B and X and Y are equivalence classes of R, then either X=Y or $X \cap Y = \phi$.

3. Give examples such that
   (a) $A \subset B$. A is countable but B is uncountable. [0.5 mark]
   (b) $A \subset B$. A is countable and B is also countable. [0.5 mark]
   (c) Function from $\mathbb{R}$ to $\mathbb{R}$ such that f is one-one but not onto. (Show that function is not onto) [1 mark]
   (d) Function from $\mathbb{R}$ to $\mathbb{R}$ such that f is onto but not one-one. (Show that function is not one-one) [1 mark]

4. Show that there exists a bijection between set of multiples of 2 and set of multiples of 3. Prove that the function is a bijection.

5. Prove by mathematical induction: for all natural numbers n≥ 4, $2^n \leq n!$

6. Show that the relation defined on $\mathbb{R}$ as aRb if b-a is an integer is an equivalence relation. Also find equivalence class 5 and $\sqrt{2}$.