COEP Technological University

(COEP Tech)

A Unitary Public University of Government of Maharashtra

[MA-23005]- Probability and Statistics

Program : F.Y.B.Tech

Examination : End Semester

Branch:

Academic Year : 2023-24

Maximum Marks : 60

Student MIS Number : | | | | | | | | | | |

Instructions :

1. Write your MIS Number on Question Paper.
2. Writing anything on question paper and on statistical tables is not allowed.
3. Mobile phones and programmable calculators are strictly prohibited.
4. Exchange/Sharing of stationery, calculator etc. is not allowed.
5. Figures to the right indicate the course outcomes and full marks.
6. Any essential result, formula or theorem assumed for answering questions must be clearly stated.
7. Whenever necessary, use statistical tables provided by invigilator to do the statistical calculations.
8. Whenever necessary, write probability answers correct to four decimal places.

Attempt all the questions.

Q.1) Choose the correct alternative.

1) In one year, three awards (research, teaching, and service) will be given to a class of 25 graduate students in a Statistics department. If each student can receive at most one award, how many possible selections are there? [CO1,1M]

a)13800 b)12600 c)9600 d)12580

2) Given a Normal distribution with $\mu = 40$ and $\sigma = 6$, find the value of x that has 14% of area to the right [CO2,1M]

a)39.22 b)46.48 c)42.58 d)36.18

3) What will be the value of $f_{0.95}$ when degree of freedom are 6 and 10 ? [CO1,1M]

(a)2.9012 (b)0.2463 (c)0.3448 (d)0.2563

4) Which of the following distributions is used to compare two variances? [CO1,1M]

a) T–Distribution b) F–Distribution c) Normal Distribution d) Poisson Distribution

5) A company producing cereals offers a toy in every sixth cereal package in celebration of their 50th anniversary. A father immediately buys 20 packages, then what is the probability of finding 4 toys in the 20 packages? [CO2,1M]

a)0.2022 b)0.2423 c)0.2224 d)0.2126

6) Let X be a random variable with PDF given by [CO2,1M]

$$f(x) = \begin{cases} cx^2 & \text{for } |x| \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

then find the value of c

a) $\frac{5}{2}$ b) $\frac{7}{2}$ c) $\frac{3}{2}$ d) $\frac{9}{2}$

7) How many different letter arrangements can be made from the letters in the word STATISTICS. [CO3,1M]

a)12600 b)50400 c)40600 d)45400

8) A bag-I contains 4 white and 6 black balls while another bag-II contains 4 white and 3 black balls. One ball is drawn at random from one of the bags, and it is found to be black. find the probability that it was drawn from Bag-I. [CO3,1M]

a)0.5833 b)0.6242 c)0.3162 d)0.5262

9) Consider two independent chi-squared random variables which are χ^2_m and χ^2_n . The sum of these two random variables has degrees of freedom [CO4,1M]

a) $m+n$ b) n c)m d) $2(m^2+n^2)$

10) In Wilcoxon Signed Ranked test, the null hypothesis $\tilde{\mu} = \tilde{\mu}_0$ can be rejected in favor of the alternative $\tilde{\mu} < \tilde{\mu}_0$ only if [CO5,1M]

a)w_+ is large and w_- is small. b)w_+ is large and w_- is large.

c)w_+ is small and w_- is large. d)w_+ is small and w_- is small

Q.2) Attempt the following questions.

a) Prove that Binomial distribution tends to Poisson distribution under certain conditions. Suppose that, on average, 1 person in 1000 makes numerical error in preparing his or her income tax return. find the probability that 6 ,7, or 8 of them contains an error. [CO3,4M].

b) Show that S^2 is an unbiased estimator of the parameter σ^2. [CO4,4M]

c) The marks obtained by a number of students in a certain subject are approximately normally distributed with mean 65 and standard deviation 5. If 3 students are selected at random from this group, what is the probability that at least one of them would have scored above 75? [CO4,4M]

d) The following data represents the number of hours of flight training received by 18 students pilots from a certain instructor prior to their first solo flight: [CO5,4M]

9 12 18 14 12 14 12 10 16

11 9 11 13 11 13 15 13 14

Perform a sign test at 0.02 level of significance to test the instructor's claim that the median time required before his student's solo is 12 hours of flight training.

Q.3) Attempt the following questions.

a) Past experience indicates that the time required for high school seniors to complete a standardized test is a normal random variable with a standard deviation of 6 minutes. Test the hypothesis that $\sigma = 6$ against the alternative that $\sigma < 6$ if a random sample of the test times of 20 high school seniors has a standard deviation $s = 4.51$. Use a 0.05 level of significance. [CO3,4M]

b) Given a standard Normal distribution, find the value of k such that

1) P(Z>k)=0.3015. [CO2,2M]

2) P(k<Z<-0.18)=0.4197 [CO2,2M]

c) A random sample of 64 bags of white cheddar popcorn weighed, on average, 5.23 ounces with a standard deviation of 0.24 ounce. Test the hypothesis that $\mu = 5.5$ ounces against the alternative hypothesis, $\mu < 5.5$ ounces, at the 0.05 level of significance.Write an appropriate R command for the following test. [CO3,4M]

d) Test the hypothesis that the average content of containers of a particular lubricant is 10 liters if the contents of a random sample of 10 containers are 10.2 , 9.7 , 10.1 , 10.3 , 10.1 , 9.8 , 9.9 , 10.4 , 10.3 and 9.8 liters. Use a 0.01 level of significance and assume that the distribution of contents is normal. [CO3,4M]

Q.4) The proportion of adults living small town who are college graduates estimated to be p=0.6. To test the hypothesis, a random sample of 15 adults is selected . If the number of college graduates in the sample is anywhere from 6 to 12, we shall not reject the null hypothesis that p=0.6; otherwise we shall conclude that $p \neq 0.6$.

a) Evaluate α assuming $p = 0.6$. [CO3,2M]

b) Evaluate β for the alternatives p=0.5 and for p=0.7 [CO3,4M]

OR

Two samples are drawn from normal population. From the following data , test whether the two samples have the same variance at 5% level: [CO3,6M]

sample 1 : 60 65 71 74 76 82 85 87

sample 2 : 61 66 67 85 78 63 85 86 88 91

Q.5) A study is conducted to compare the lengths of time required by men and women to assemble a certain product.Past experience indicates that the distribution of times for both men and women is approximately normal but the variance of the times for women is less than that for the men. A random sample of times for 11 men and 14 women gives respective standard deviations 6.1 and 5.3. Test the hypothesis $\sigma_1^2 = \sigma_2^2$ that against the alternative that $\sigma_1^2 > \sigma_2^2$ [CO3,4M]

OR

The average life of a bread-making machine is 7 years, with a standard deviation of 1 year. Assuming that the lives of these machines follow approximately a normal distribution, find

a) the probability that the mean life of a random sample of 9 such machines falls between 6.4 and 7.2 years;

b) the value of $\bar{X}$ to the right of which 15% of the means computed from random samples of size 9 would fall. [CO4,4M]

Q.6) The length of time person speaks over phone follows exponential distribution with mean 6. What is the probability that a person will talk for [CO4,4M]

1) More than 8 minutes. Also write an appropriate R command to find probability.

2) Between 4 and 8 minutes. Also write an appropriate R command to find probability.

Q.7) Using statistical tables find the following

1) $P(-t_{0.005} < T < t_{0.01})$ when degree of freedom is 20.

2) Find k such that $P(k < T < 2.807) = 0.095$ for a random sample of size 24 from a normal population. [CO2,4M]

END