

Probability & Statistics

Model Answer MID SEMESTER

1st APRIL 2025

Q-1) ① ① what type of statistical data are used in educational research

Both of them

② ② which of the following is measure of Central Tendency

Median

③ ③ which measure of central tendency used for categorical data

Mode

④ ④ which descriptive statistics gives the highest weight to outliers

Mean

⑤ ⑤ Consider the poisson distribution for the tossing of a biased coin. The mean for this distⁿ is μ . The std. deviation for this distribution is given by —

$\sqrt{\mu}$

⑥ ⑥ True False

⑦ ⑦ let $X \sim N(\mu, \sigma^2)$ If $\mu^2 = \sigma^2$ then the value of $P(X < -\mu / X < \mu)$ in terms of cumulative function $N(0,1)$ is $[1 - P(Z \leq 2)]$

FALSE

Explanation: $X \sim N(\mu, \sigma^2)$
 $\mu^2 = \sigma^2$ ($\mu > 0$)



Indrayani Hospital & Cancer Institute

March 2002 : Shree Swami Narsimha Saraswati Charitable trust at Alandi offered a three and half acre plot in Alandi to start a charitable cancer hospital to Dr. Sanjay Deshmukh, a cancer surgeon who had operated a devotee of the Math.

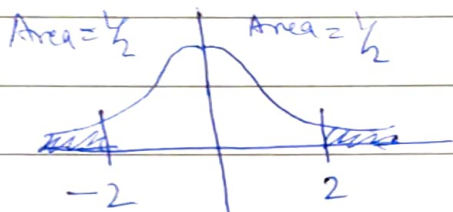
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$$P(X < -\mu / X < \mu) = ?$$

transforming to $Z \sim N(0,1)$

$$Z = \frac{X - \mu}{\sigma}$$

$$\sigma^2 = 1 \Rightarrow \sigma = 1, \mu > 0; \mu^2 = \sigma^2 (=) \mu = 1$$



when $x = -1$

$$Z = \frac{-1 - 1}{1} = -2$$

when $x = 1$

$$Z = \frac{1 - 1}{1} = 0$$

$$\therefore P(X < -\mu / X < \mu)$$

$$= P(Z < -2 / Z < 0)$$

$$= \frac{P(Z < -2 \cap Z < 0)}{P(Z < 0)} = \frac{P(Z < -2)}{P(Z < 0)}$$

$$= \frac{P(Z > 2)}{P(Z < 0)} = \frac{1 - P(Z < 2)}{1/2}$$

$$= 2[1 - P(Z < 2)]$$

③ For a Binomial variable x , If $x_1, x_2, x_3, \dots, x_n$ are independent & identically, distributed sample from the distribution of x with sum $y = \sum_{i=1}^n x_i$ then the distribution of y as $n \rightarrow \infty$ can be approximated as Bernoulli distribution.

FALSE

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© There exists a Binomial distribution with mean 5 and standard deviation 3.

FALSE

© For X be a poisson variate, then for Poisson distⁿ with λ mean and std. deviations are different.

TRUE

© The central Limit Theorem guarantees that the population is normal whenever n is sufficiently large.

FALSE

Q 2) (A) Let X be the amount of time a postal clerk spend on his/her customers. The time is known to have an Exponential distribution with the average amount of time equal to four minutes. Find the prob. that a clerk spend four to five minutes with a randomly selected customer. [3M]

Ans: let $X \sim$ Exponential random variable

Given $\mu = 4$ min

$$\Rightarrow \text{decay parameter } \lambda = \frac{1}{\mu} = \frac{1}{4} = 0.25$$

$$\sigma = \text{S.D} = \mu = 0.25 \Rightarrow \sigma = 0.25$$

$$\text{To find } P(4 < X < 5) = P(X < 5) - P(X < 4)$$

$$\text{Now } P(X < x) = 1 - e^{-\lambda x}$$

$$\Rightarrow P(X < 5) = 1 - e^{-(0.25)5} = 0.713$$

$$\& P(X < 4) = 1 - e^{-(0.25)4} = 0.6321$$

$$\Rightarrow P(4 < X < 5) = 0.7135 - 0.6321 = 0.0814$$



Indrayani Hospital & Cancer Institute

May 2003 : Shri Narsimha Saraswati Medical Foundation (SNSMF) was formed with the intent to provide state of art health care facility, particularly in cancer treatment, to poor class maintaining their dignity, in Alandi.

Q.2) (b) In order to identify which 3 cities receive the most sunshine. We simply compare the frequencies betⁿ each. First re-order the cities in order from the most to least hrs of sunshine then calculate their relative freq.

| Country | City | freq. | Relative. |
|------------|----------|-------|-----------------------------|
| Malta | Valletta | 3054 | $3054/24446 \approx 12.5\%$ |
| Greece | Athens | 2773 | $2773/24446 \approx 11.3\%$ |
| Italy | Cagliari | 2726 | 11.2% |
| Spain | | 2696 | 11% |
| Montenegro | | 2481 | 10.1% |
| Portugal | | 2468 | 10.1% |
| Bulgaria | | 2177 | 8.9% |
| Serbia | | 2112 | 8.6% |
| Georgia | | 2046 | 8.4% |
| Croatia | | 1913 | 7.8% |
| Total | | 24446 | 100% |

2M

To calculate cumulative freq. we add them of top three cities we get

$\frac{1}{2}M$ $12.5\% + 11.3\% + 11.2\% = 35\%$

$\frac{1}{2}M$ \therefore Top 3 cities in list receiving more than a quarter of the total sunshine hrs.



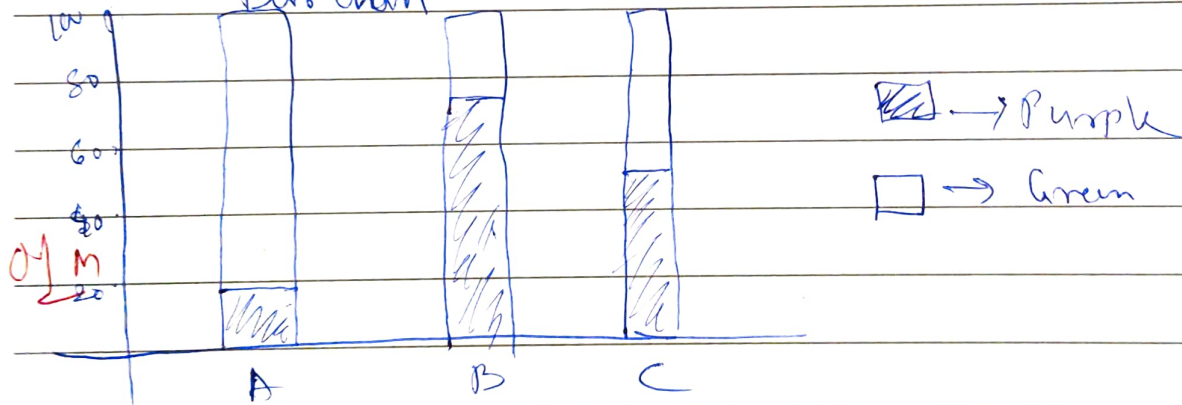
Q 2) (c) First in order to calculate row & column frequencies, will calculate totals of rows & columns.

| Proposition | Purple party | Green Party | Total |
|-------------|--------------|-------------|-------|
| A | 13 | 54 | 67 |
| B | 60 | 20 | 80 |
| C | 27 | 26 | 53 |
| Total | 100 | 100 | 200 |

Row frequency & chart

| Proposition | Purple Party | Green Party | Total |
|-------------|--------------|-------------|-------|
| A | 19% | 81% | 100% |
| B | 75% | 25% | 100% |
| C | 51% | 49% | 100% |
| | 50% | 5% | 100% |

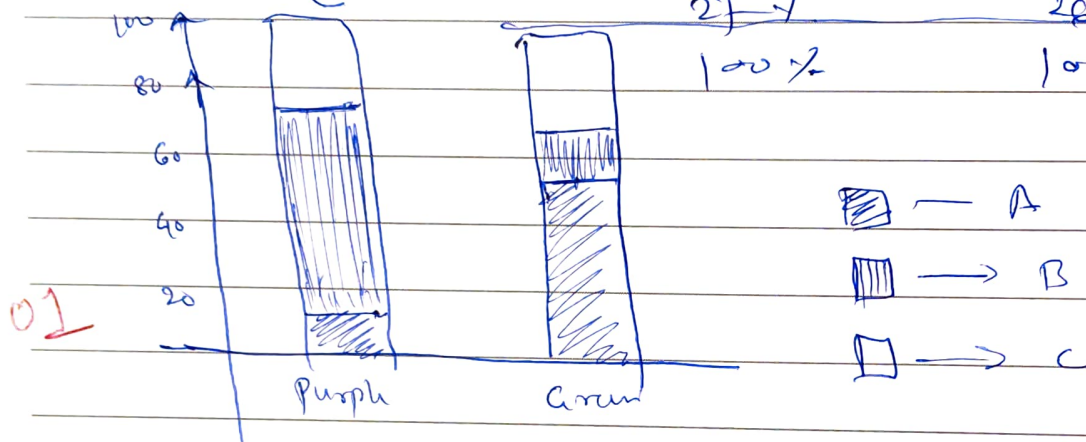
Bar chart



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The column freq. & chart is

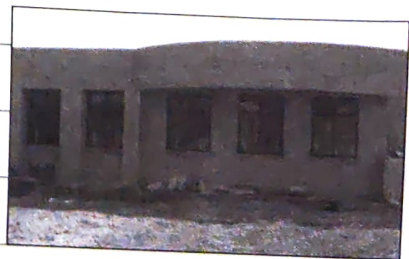
| Proposition | Purple Party | Green Party | Total |
|-------------|--------------|-------------|-------|
| A | 13 % | 54 % | 34 % |
| B | 60 % | 20 % | 40 % |
| C | 27 % | 26 % | 27 % |
| | 100 % | 100 % | 100 % |




Q.3) (A) $X \sim B(n, p)$, $n = 300$, $p = 0.53$
 $np \geq 5$ & $nq \geq 5$ \therefore approximately to normal
 $\mu = np = 300 \times 0.53 = 159$
 $\sigma = \sqrt{npq} = \sqrt{300 \times 0.53 \times 0.47}$
 $= 8.6447$

$Y \sim N(159, 8.6447)$
 (a) $P(X > 150) \approx P(X \geq 149.5)$
 $= 0.8641$

(b) $P(X \leq 160) \approx P(X \leq 160.5)$
 $= 0.5689$



 **Indrayani Hospital & Cancer Institute**

April 2008 : Their efforts bore fruit as the Indrayani Hospital and Cancer Institute opened its doors to the public. It was a day of celebration and hope, as the first patient received care within its walls.

OR.

$$\textcircled{1} P(X > 155) \approx P(X > 155.5) \\ = 0.6572$$

$$\textcircled{2} P(X \leq 147) \approx P(X < 146.5) \\ = 0.0741$$

$$Q.3) \textcircled{B} X \sim (70, 0.95) \quad \& \quad Y \sim (70, 0.05)$$

For X ; $n > 50$ But $np = 70 \times 0.95 = 66.5$
 $np > 5$

Thus X can not be approximated.

However Y , $n > 50$

$$np = 70 \times 0.05 = 3.5 < 5$$

1M 1) So Y can be approximated by a
Poisson distribution $Y \sim P_0(3.5)$

$$1M \textcircled{2} P(X = 67) P(Y = 3) = \frac{e^{-3.5} (3.5)^3}{3!}$$

$$= 0.2158$$

$$1M \textcircled{3} P(X \geq 67) = P(Y \leq 3) = 0.5366$$

Q.3) \textcircled{C} 1) Let X be poisson variate.

R-command

plot(dpois(x=1:50, lambda=3))



Indrayani Hospital & Cancer Institute

May 2008: A fully equipped pharmacy was established, ensuring that patients had access to vital medications.

2) $Y \leftarrow \text{dbinom}(0:25, \text{size}=20, \text{prob}=0.5)$
 $\text{plot}(0:25, Y, \text{type}="h")$