COEP Technological University

A Unitary Public University of Government of Maharashtra
(Formerly College of Engineering Pune (COEP))

END Semester Examination

(CT-23004) Discrete Structures

Course: B.Tech., Semester II

Branch: Computer Engineering

Academic Year: 2023-2024

Max Marks :60

Duration: 3 Hours

Date:

Instructions:

Student MIS No. 6 1 2 3 0 3 0 4 3

1. Figures to the right indicate the full marks.
2. Mobile phones and programmable calculators are strictly prohibited.
3. Writing anything on question paper is not allowed.
4. Exchange/Sharing of stationery, calculator etc. not allowed.
5. Write your MIS Number on Question Paper

			Marks	CO	PO
Q 1		Solve any 5			
	a	Find range and domain of the following functions: (i) $\frac{2}{x^2+4}$ (ii) $\sqrt{2x+4}$	2	2	2
	b	What is the smallest sample space that can be chosen from the office with 20 sectors so that at least 11 employees will be from the same sector?	2	2	2
	c	Give counter examples for each of the following statements: (i) $\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, x^2 + y^2 = 0$ (ii) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, xy = 1$ (iii) $\forall x \in \mathbb{N}$ x has factors other that x and 1 (iv) $\forall x \in \mathbb{Z}$ x is a square of an integer	2	1	2
	d	Consider the following operation on set of natural number: a*b = a+b+3, Determine whether the operation is associative.	2	2	2

	e	If $\|A \times B\| = 12$, what are possible cardinality of A and B?	2	2	2
	f	Draw Venn diagrams for the following: (i) $A - (B \cup C)$ (ii) $A \cap (B - C)$ (iii) $(A - B) \cap (B - C)$ (iv) $(C \cap B) - (C \cup A)$	2	2	2
Q 2		Solve any 5			
	a	Draw all possible conclusions based on the given statements. "Every traveler has a destination", "Welma is travelling on Wall Street", "Oscar has a destination in Rome", "Bary is not travelling", "Ria does not have any destination".	3	1	2
	b	There are 38 different time periods during which classes at the college can be scheduled. If there are 677 different classes, how many different rooms will be needed?	3	2	3
	c	Write the negation in English by writing each statement using quantifiers and negating. i. Some people like ice cream but everyone loves chocolate ii. Someone likes in ice cream and does not love chocolate.	3	1	2
	d	Using rules of logic, prove that $(p \rightarrow r) \vee (q \rightarrow r)$ is equivalent to $(p \wedge q) \rightarrow r$	3	1	3
	e	What is the minimum number of elements that should be selected from {1, 2, 3, 4, 5, 6} to guarantee that at least two of them add up to 7?	3	2	2
	f	Which of the following propositions are satisfiable? Justify. i. $(p \vee q \vee r) \wedge (\sim p \vee \sim q \vee \sim r)$ ii. $[p \rightarrow (q \wedge \sim p)] \vee [q \rightarrow (r \wedge \sim q)]$ iii. $\sim (p \vee (\sim p \wedge q))$	3	1	3
Q 3		Solve any 5			
	a	Suppose that A is a nonempty set, and f is a function that has A as its domain. Let R be the relation on A consisting of all ordered pairs (x, y) such that f(x) = f(y). i. Show that R is an equivalence relation on A. ii. What are the equivalence classes of R?	4	3	2
	b	How many bit strings of length 10 have i. exactly three 0s?	4	1	4

		ii. more 0s than 1s? iii. at least seven 1s? iv. at least three 1s?			
	c	If A∩B=A U B, then A=B WITHOUT using Venn diagrams.	4	2	2
	d	Give a function from Integers(Z) to natural numbers which is one-one but not onto. Show the function is one-one but not onto.	4	2	3
	e	There are 350 farmers in a large region. 260 farm beetroot, 100 farm yams, 70 farm radish, 40 farm beetroot and radish, 40 farm yams and radish, and 30 farm beetroot and yams. Let B, Y, and R denote the set of farms that farm beetroot, yams and radish respectively. Determine the number of farmers that farm beetroot, yams, and radish.	4	2	4
	f	Let G and H be groups and f: G→ H be a homomorphism. Prove that f(G) is a subgroup of H.	4	2,3	2
Q4		Solve any 3			
	a	Show that (Q,⊕, *) is a ring, Where a⊕b=a+b+1 and a*b=ab+a+b	5	3	4
	b	Suppose that A is a nonempty set, and f is a function that has A as its domain. Let R be the relation on A consisting of all ordered pairs (x, y) such that f(x) = f(y). i. Show that R is an equivalence relation on A. ii. What are the equivalence classes of R?	5	3	3
	c	Show that these statements about the integer x are equivalent: i. 3x + 2 is even ii. x + 5 is odd iii. x^2 is even	5	2	4
	d	A password for an must have 8 to 12 characters, where a character in the password can be a lowercase English letter, an uppercase English letter, a digit, or one of the six special characters:!, @, #, $, %, &. i. How many different passwords are available for this computer system? ii. How many of these passwords contain at least one occurrence of at least one of the six special characters? iii. Determine how long it takes a hacker to try every possible password, assuming that it takes one nanosecond for a hacker to check each possible password.	5	2	4