Procedure for finding the minimal function via K-maps (layman terms)

1. Convert truth table to K-map
2. Group adjacent ones: In doing so include the largest number of adjacent ones (Prime Implicants)

*Y*

*AB*

*CD* 00

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 |

01 11 10

1. Create new groups to cover all ones in the 00

map: create a new group only to include at 01

least once cell (of value 1 ) that is not

covered by any other group 11

1. Select the groups that result in the minimal

sum of products (we will formalize this 10

because its not straightforward)

# Reading the reduced K-map

*Y*

*AB*

*CD* 00

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 |

01 11 10

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*Y* = *AC* + *ABD* + *ABC* + *BD*

### Definitions: implicant, prime implicant, essential prime implicant

* Implicant: A product term that has non-empty intersection with on-set *F* and does not intersect with off-set *R* .

### Prime Implicant: An implicant that is not a proper subset of any other implicant i.e. it is not completely covered by any single implicant

* Essential Prime Implicant: A prime implicant with atleast one element that is not covered by one or more prime implicants.
  + Implicant: A product term that has non-empty intersection with on-set *F*

and does not intersect with off-set *R*

* + Prime Implicant: An implicant that is not a proper subset of any other implicant i.e. it is not completely covered by any single implicant

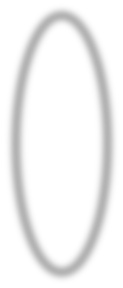
*Y*

*AB*

*CD* 00 01

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Q: Is this a prime implicant?

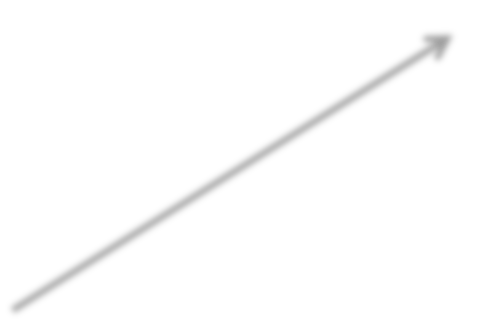


11

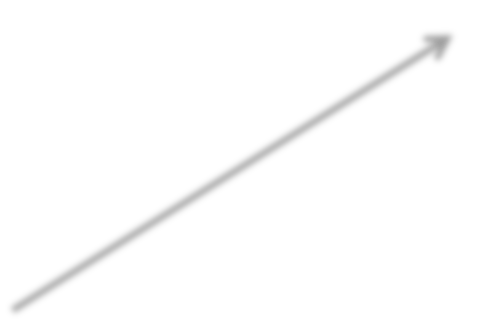
10

* 1. Yes

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 |



* 1. No
* Implicant: A product term that has non-empty intersection with on- set *F* and does not intersect with off-set *R*
* Prime Implicant: An implicant that is not a proper subset of any other implicant i.e. it is not completely covered by any single implicant



*Y*

*AB*

*CD* 00

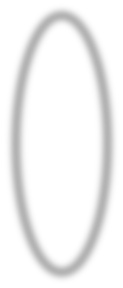
|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 |

01 11 10

Q: Is this a prime implicant?

00

A. Yes



01 B. No

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* Implicant: A product term that has non-empty intersection with on-set

*F* and does not intersect with off-set *R*

* Prime Implicant: An implicant that is not a proper subset of any other implicant i.e. it is not completely covered by any single implicant

*Y*

*AB*

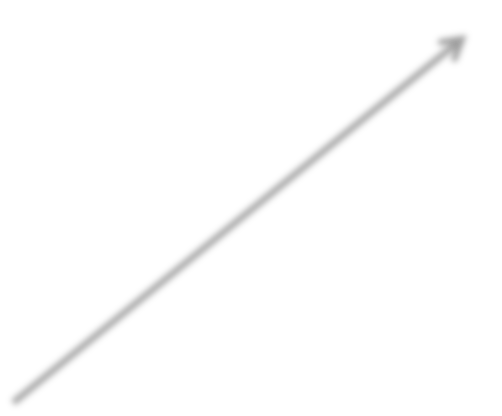
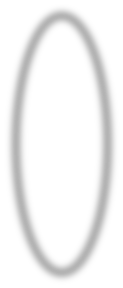
*CD* 00

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 00 | 1 | 0 | 0 | 1 |
| 01 | 0 | 1 | 0 | 1 |
| 11 | 1 | 1 | 0 | 0 |
| 10 | 1 | 1 | 0 | 1 |

01 11 10

Q: How about this one? Is it a prime implicant?

1. Yes



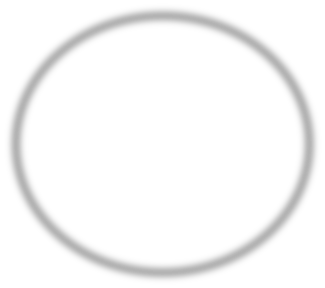
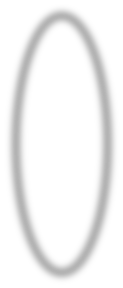
1. No

* Implicant: A product term that has non-empty intersection with on-set

*F* and does not intersect with off-set *R*

* Prime Implicant: An implicant that is not a proper subset of any other implicant i.e. it is not completely covered by any single implicant

*Y* Q: Is the red group a prime



*CD*

*AB*

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implicant?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 00 | 1 | 0 | 0 | 1 |
| 01 | 0 | 1 | 0 | 1 |
| 11 | 1 | 1 | 0 | 0 |
| 10 | 1 | 1 | 0 | 1 |

* 1. Yes
  2. No: Because it is covered by a larger group

*Y*

*AB*

*CD* 00

01 11 10

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 |

Q: Is the blue group an essential prime?

00

A. Yes

01 B. No

11

*Y*

*AB*

*CD* 00

01 11 10

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 |

Q: Is the blue group an essential prime?

00

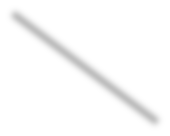
A. Yes

01 B. No

11

Q: Is the blue group a non-essential prime implicant?

00 01 11 10

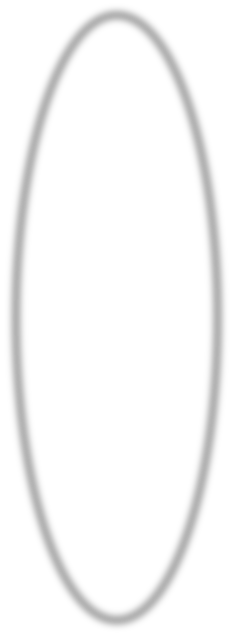


ab

cd

A. Yes

|  |  |  |  |
| --- | --- | --- | --- |
| 1 |  | 1 | 1 |
|  | 1 | 1 |  |
|  |  | 1 | 1 |
| 1 |  | 1 | 1 |



00 B. No

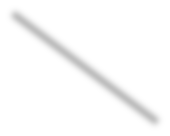
01

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Q: Is the blue group a non-essential prime implicant?

00 01 11 10

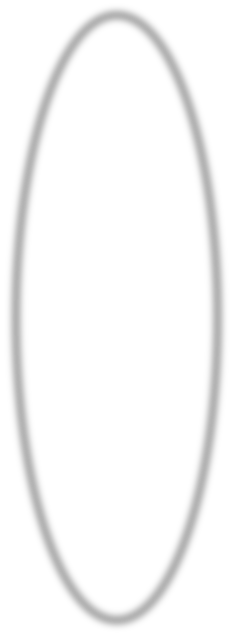


ab

cd

A. Yes

|  |  |  |  |
| --- | --- | --- | --- |
| 1 |  | 1 | 1 |
|  | 1 | 1 |  |
|  |  | 1 | 1 |
| 1 |  | 1 | 1 |



00 B. No

01

11

10

Procedure for finding the minimal function via K-maps (formal terms)

1. Convert truth table to K-map

*Y*

*AB*

*CD* 00

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 |

01 11 10

1. Include all essential primes

00

1. Include non essential primes as 01

needed to completely cover the onset 11

(all cells of value one)

10

*Y*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *Y* |
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | X |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | X |
| 1 | 0 | 1 | 1 | X |
| 1 | 1 | 0 | 0 | X |
| 1 | 1 | 0 | 1 | X |
| 1 | 1 | 1 | 0 | X |
| 1 | 1 | 1 | 1 | X |

*AB*

*CD* 00

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

01 11 10

00

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11

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*Y*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *Y* |
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | X |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | X |
| 1 | 0 | 1 | 1 | X |
| 1 | 1 | 0 | 0 | X |
| 1 | 1 | 0 | 1 | X |
| 1 | 1 | 1 | 0 | X |
| 1 | 1 | 1 | 1 | X |

*AB*

*CD* 00

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 0 | X | 1 |
| 0 | X | X | 1 |
| 1 | 1 | X | X |
| 1 | 1 | X | X |

01 11 10

00

01

11

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*Y*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *Y* |
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | X |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | X |
| 1 | 0 | 1 | 1 | X |
| 1 | 1 | 0 | 0 | X |
| 1 | 1 | 0 | 1 | X |
| 1 | 1 | 1 | 0 | X |
| 1 | 1 | 1 | 1 | X |

*AB*

*CD* 00

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 0 | X | 1 |
| 0 | X | X | 1 |
| 1 | 1 | X | X |
| 1 | 1 | X | X |

00

01

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01 11 10

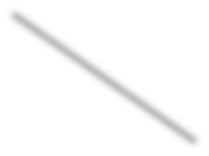
*Y* = *A* + *BD* + *C*

Given *F(a,b,c,d)* = Σm (0, 1, 2, 8, 14)

*D(a,b,c,d)* = Σm (9, 10)

* 1. Draw K-map

00 01 11 10



ab

cd

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

00

01

11

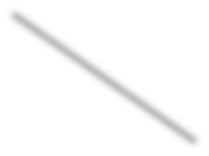
10

Given *F(a,b,c,d)* = Σm (0, 1, 2, 8, 14)

*D(a,b,c,d)* = Σm (9, 10)

1. Draw K-map

00 01 11 10



ab

cd

|  |  |  |  |
| --- | --- | --- | --- |
| 0 | 4 | 12 | 8 |
| 1 | 5 | 13 | 9 |
| 3 | 7 | 15 | 11 |
| 2 | 6 | 14 | 10 |

00

01

11

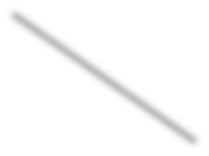
10

Given *F(a,b,c,d)* = Σm (0, 1, 2, 8, 14)

*D(a,b,c,d)* = Σm (9, 10)

1. Draw K-map

00 01 11 10



ab

cd

|  |  |  |  |
| --- | --- | --- | --- |
| 0  1 | 4  0 | 12  0 | 8  1 |
| 1  1 | 5  0 | 13  0 | 9  X |
| 3  0 | 7  0 | 15  0 | 11  0 |
| 2  1 | 6  0 | 14  1 | 10  X |

00

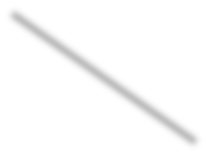
01

11

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1. Draw K-map
2. Identify Prime implicants
3. Identify Essential Primes

00 01 11 10



ab

cd

|  |  |  |  |
| --- | --- | --- | --- |
| 0  1 | 4  0 | 12  0 | 8  1 |
| 1  1 | 5  0 | 13  0 | 9  X |
| 3  0 | 7  0 | 15  0 | 11  0 |
| 2  1 | 6  0 | 14  1 | 10  X |

00

01

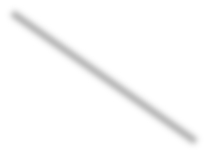
11

10

PI Q: How many primes (P) and essential primes (EP) are there?

1. Four (P) and three (EP)
2. Three (P) and two (EP)
3. Three (P) and three (EP)
4. Four (P) and Four (EP)

00 01 11 10



ab

cd

|  |  |  |  |
| --- | --- | --- | --- |
| 0  1 | 4  0 | 12  0 | 8  1 |
| 1  1 | 5  0 | 13  0 | 9  X |
| 3  0 | 7  0 | 15  0 | 11  0 |
| 2  1 | 6  0 | 14  1 | 10  X |

00

01

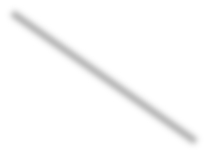
11

PI Q: Do the E-primes cover the entire on set?

1. Yes
2. No

f(a,b,c,d) = ?

00 01 11 10



ab

cd

|  |  |  |  |
| --- | --- | --- | --- |
| 0  1 | 4  0 | 12  0 | 8  1 |
| 1  1 | 5  0 | 13  0 | 9  X |
| 3  0 | 7  0 | 15  0 | 11  0 |
| 2  1 | 6  0 | 14  1 | 10  X |

00

01

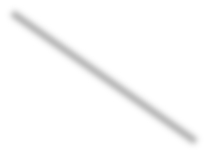
11

PI Q: Do the E-primes cover the entire on set?

1. Yes
2. No

f(a,b,c,d) = b’c’ + b’d’+ acd‘

00 01 11 10



ab

cd

|  |  |  |  |
| --- | --- | --- | --- |
| 0  1 | 4  0 | 12  0 | 8  1 |
| 1  1 | 5  0 | 13  0 | 9  X |
| 3  0 | 7  0 | 15  0 | 11  0 |
| 2  1 | 6  0 | 14  1 | 10  X |

00

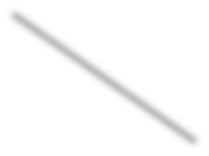
01

11

PI Q: Do the E-primes cover the entire on set?

1. Yes
2. No

00 01 11 10



ab

cd

|  |  |  |  |
| --- | --- | --- | --- |
| 0 | 4 | 12 | 8 |
| 1 | 5 | 13 | 9 |
| 3 | 7 | 15 | 11 |
| 2 | 6 | 14 | 10 |

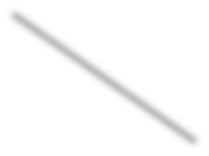
00

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00 01 11 10



ab

cd

|  |  |  |  |
| --- | --- | --- | --- |
| 0  1 | 4  1 | 12  0 | 8  0 |
| 1  X | 5  0 | 13  X | 9  0 |
| 3  1 | 7  0 | 15  1 | 11  X |
| 2  0 | 6  0 | 14  1 | 10  0 |

00

01

11

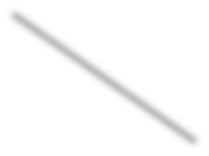
10

1. Prime implicants: Σm (0, 4), Σm (0, 1), Σm (1, 3), Σm (3, 11), Σm (14, 15),

Σm (11, 15), Σm (13, 15)

2. Essential Primes: Σm (0, 4), Σm (14, 15)

00 01 11 10



ab

cd

00

|  |  |  |  |
| --- | --- | --- | --- |
| 0  1 | 4  1 | 12  0 | 8  0 |
| 1  X | 5  0 | 13  X | 9  0 |
| 3  1 | 7  0 | 15  1 | 11  X |
| 2  0 | 6  0 | 14  1 | 10  0 |

01

11

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1. Prime implicants: Σm (0, 4), Σm (0, 1), Σm (1, 3), Σm (3, 11), Σm (14, 15), Σm

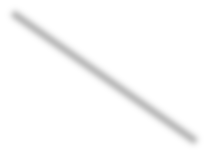
(11, 15), Σm (13, 15)

2.Essential Primes: Σm (0, 4), Σm (14, 15)

3.Min exp: Σm (0, 4), Σm (14, 15), (Σm (3, 11) or Σm (1,3) )

f(a,b,c,d) = a’c’d’+ abc+ b’cd (or a’b’d)

00 01 11 10



ab

cd

00

|  |  |  |  |
| --- | --- | --- | --- |
| 0  1 | 4  1 | 12  0 | 8  0 |
| 1  X | 5  0 | 13  X | 9  0 |
| 3  1 | 7  0 | 15  1 | 11  X |
| 2  0 | 6  0 | 14  1 | 10  0 |

01

11

10 32

# Reading

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