So, there are 15 possible 4-combinations of a set containing 3 items if repetition is allowed.

TUTORIAL 1

Pigeonhole Principle :

Q. Find the minimum number 'n' of integers to be selected from $S = \{1, 2, \ldots, 9\}$ so that :

a) The sum of two of the n integers is even.

→ We know,

sum of two even integers is even, also
sum of two odd integers is even

On the other hand, if we select an even & an odd integer their sum is odd, so two integers is not sufficient.

If we take three integers from set S, then atleast two of them must have same parity.

Consider the subsets, $\{1, 3, 5, 7, 9\}$ & $\{2, 4, 6, 8\}$ as pigeonholes.

Hence $n = 3$.

b) The difference of two of the 'n' integers is 5.

consider the five subsets
$\{1, 6\}, \{2, 7\}, \{3, 8\}, \{4, 9\}, \{5\}$
of S as pigeonholes.

We need $x - y = 5$ — (1)

So if we select one integer from each subset there is a possibility that we might not get the solution in (1).

Hence $n = 6$ will guarantee that two integers will belong to one of the subsets and their difference will be 5.

Q. Find the minimum number of students needed to guarantee that 5 of them belong to the same class (Freshman, Sophomore, Junior, Senior)

→ Here, the $n = 4$ classes (F, S, J, Senior) are the pigeonholes &

$$k + 1 = 5$$

so $k = 4$

Thus any $kn + 1$ students, i.e.

$$kn + 1 = 4 \times 4 + 1$$
$$= 16 + 1$$
$$= 17 \text{ students}$$

Among minimum 17 students (pigeons) 5 of them belong to the same class.

Q. Let L be a list of the 26 letters in the English alphabet (which consist of A, E, I, O, U and 21 consonants).

a) ~~Sol~~ Show that L has a sublist consisting of 4 or more consecutive consonants.

→ Sol: The five letters partition 'L' into $n = 6$ sublists (pigeonholes) of consecutive consonants.

Here, $k + 1 = 4$

$\therefore$ $k = 3$

Hence $kn + 1 = 3 \times 6 + 1$
$$= 18 + 1$$
$$= 19 \qquad (19 < 21)$$

Hence some sublist has ~~four~~ at least 4 consecutive consonants.

b) Assuming L begins with a vowel, say A, show that L has sublist consisting of 5 or more consonants.

Sol^n.

Since L begins with a vowel, the remainder of the vowel partition L into $n = 5$ sublists.

Here $k + 1 = 5$

$\therefore \quad k = 4$

Now, $kn + 1$ will be

$= 4 \times 5 + 1$

$= 20 + 1$

$= 21$

Thus some sublist has at least five consecutive consonants.

Q. Let p be "It is cold" & let q be "it is raining". Give a sample verbal sentence which describes each of the following statements.

(a) $\sim p$: It is not cold

(b) $p \wedge q$: It is cold & it is raining

(c) $p \vee q$: It is cold or it is raining

(d) $q \vee \sim p$: It is raining or it is not cold.

Q. Determine the validity of the following argument.

→ If 7 is less than 4, then 7 is not a prime number.

7 is not less than 4

7 is a prime number

Sol^n. Let p : 7 is less than 4

q : 7 is not a prime number

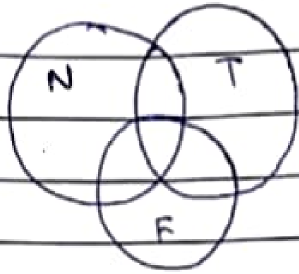
$\therefore \; p \to q$

$\underline{\sim p}$

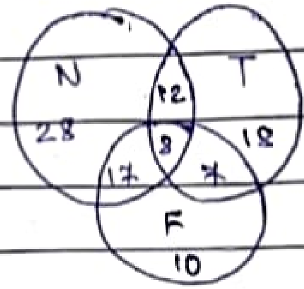
$\therefore \sim q$ by Modus Tollen.

$\therefore$ The given argument is valid argument.

Q. In a survey of 120 people, it was found that 65 read Newsweek magazine, 20 read both Newsweek & Time, 45 read Time, 25 read both Newsweek & Fortune, 42 read Fortune, 15 read both Time & Fortune, 8 read all three magazines.



(a)



(b)

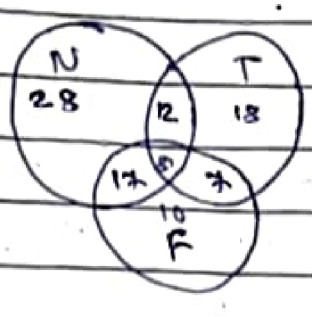
a) Find the number of people who read at least one of these magazines.

Let N be the set of people who read newspaper
Let T be the set of people who read time
Let F be the set of people who read fortune

Given $n(N) = 65$, $n(N \cap T) = 20$, $n(T) = 45$
$n(N \cap F) = 25$, $n(F) = 42$, $n(T \cap F) = 15$
$n(T \cap F \cap N) = 8$

$$n(A \cup T \cup F) = \{n(N) + n(T) + n(F)\} - \{n(N \cap T) + n(T \cap F) + n(N \cap F)\} + n(N \cap T \cap F)$$
$$= \{65 + 45 + 42\} - \{20 + 15 + 25\} + 8$$
$$= 152 - 60 + 8$$
$$= 100$$

b) Fill in the correct number of people in each of the eight regions of the Venn diagram where N, T & F denote the set of people who read Newsweek, Time & Fortune, resp



c) Find the no. of people who read exactly one magazine.
No. of people who exactly read one magazine
$= n(N) + n(T) + n(F) - \{2(N \cap F) + 2(N \cap T) + 2(T \cap F)\} + 3n(N \cap T \cap F)$
$= 65 + 42 + 45 - 2\{25 + 15 + 20\} + 3(8)$
$= 152 - 60 + 24$
$= 92 + 24$
$= 116$
∴ the no. of people who read exactly one magazine are 116.

Q. List the elements of the following sets; here $N = \{1, 2, 3, \ldots\}$

a) $A = \{x : x \in N, 3 < x < 12\}$
⇒ $A\{4, 5, 6, 7, 8, 9, 10, 11\}$

b) $B = \{x : x \in N, x \text{ is even}, x < 15\}$
⇒ $B\{2, 4, 6, 8, 10, 12, 14\}$

c) $C = \{x : x \in N, 4 + x = 3\}$
$C\{\phi\}$

Q. Find the no. of math students at a college taking at least one of the languages French, German & Russian given the following data:
65 study F, 20 study F & G, 45 study G, 25 study F & R, 8 study all 3 languages, 42 study R, 15 study G & R

Find $n(F \cup G \cup R)$.

Sol:

$$n(F \cup G \cup R) = \{n(F) + n(G) + n(R)\} - \{n(F \cap G) + n(G \cap R) + n(F \cap R)\} + n(F \cap G \cap R)$$

$$= (65 + 42 + 45) - (20 + 25 + 15) + 8$$

$$= 152 - 60 + 8$$

$$\therefore n(F \cup G \cup R) = 100$$

Q. Let A, B, C, D denote resp, art, biology, chemistry & drama courses, Find the number N of students in dormitory given the data,

→ $n(A) = 12$ $n(B) = 20$ $n(C) = 20$ $n(D) = 8$

$n(A \cap B) = 5$ $n(A \cap C) = 7$ $n(A \cap D) = 4$ $n(B \cap C) = 16$

$n(B \cap D) = 4$ $n(C \cap D) = 3$ $n(A \cap B \cap C) = 3$ $n(A \cap B \cap D) = 2$

$n(B \cap C \cap D) = 2$ $n(A \cap C \cap D) = 3$ $n(A \cap B \cap C \cap D) = 2$

$n(\overline{A \cup B \cup C \cup D}) = 71$

~~By P~~ Sol:

By Principle of Inclusion - Exclusion,

$$n(A \cup B \cup C \cup D) = \{n(A) + n(B) + n(C) + n(D)\} - \{n(A \cap B) + n(A \cap C) + n(A \cap D) + n(B \cap C) + n(B \cap D) + n(C \cap D)\} + \{n(A \cap B \cap C) + n(A \cap B \cap D) + n(B \cap C \cap D) + n(A \cap C \cap D)\} - n(A \cap B \cap C \cap D)$$

$$= \{12 + 20 + 20 + 8\} - \{5 + 7 + 4 + 16 + 4 + 3\} + \{3 + 2 + 2 + 3\} - 2$$

$$= 60 - 39 + 10 - 2$$

$$= 29$$

$$N = n(A \cup B \cup C \cup D) + n(\overline{A \cup B \cup C \cup D})$$

$$N = 29 + 71$$

$$N = 100$$

Date

Q. Suppose among 32 people who save paper or bottle (or both) for recycling, there are 30 who save paper and 14 who save bottles. Find the number m of people who a) save both b) save only paper c) save only bottles.

a) → Sol^n $n(P) = 30$, $n(B) = 14$, $n(P \cup B) = 32$

$$n(P \cap B) = n(P) + n(B) - n(P \cup B)$$
$$= 30 + 14 - 32$$
$$= 44 - 32$$
$$= 12$$

b)

$$n(\text{save only paper}) = n(P) - n(P \cap B)$$
$$= 30 - 12$$
$$= 18$$

c)

$$n(\text{save only bottles}) = n(B) - n(P \cap B)$$
$$= 14 - 12$$
$$= 2$$

Q. There are 22 female students & 18 male students in a classroom. Find the total number of + students.

→ $n(F) = 22$ $n(M) = 18$, $n(F \cap M) = 0$

$$\therefore n(F \cup M) = n(F) + n(F) - n(F \cap M)$$
$$= 22 + 18 - 0$$
$$n(F \cup M) = 40$$

Total no. of '+' students = 40.