Enrolment No.: | | | | | | | | | | | | | | |

MIT-ADT UNIVERSITY
PUNE, INDIA
A leap towards World Class Education

End Semester Examinations – Nov-Dec 2019

MIT School of Engineering

I Semester B. Tech. / Integrated M. Tech. (All Programs)

18BTMT101: Linear Algebra and Calculus

Date : 14-12-2019 Max. Marks: 60

Time : 10:00 am to 12:30 pm

Instructions for the Students:

1. Assume suitable data if necessary.
2. Attempt Q.1 or Q.2, Q.3 or Q.4, Q.5 or Q.6, Q.7 or Q.8 and Q.9 or Q.10.
3. Use of nonprogrammable type of scientific calculator is allowed.
4. Do not write anything other than the Enrolment Number on the Question Paper.
5. Figures to right indicate the marks allotted to the questions.
6. Leave enough margin on all the sides and start each new question on a new page.

Q.1. **(A)** Show that the transformation $y_1 = 2x_1 - 2x_2 - x_3$; $y_2 = -4x_1 + 5x_2 + 3x_3$; $y_3 = x_1 - x_2 - x_3$ is regular. **[02]**

(B) Reduce the matrix $A = \begin{bmatrix} 3 & 2 & 5 & 2 & 7 \\ 1 & 1 & 2 & 3 & 5 \\ 3 & 3 & 6 & 9 & 15 \end{bmatrix}$ to its normal form and hence find its rank. **[05]**

(C) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$. **[05]**

OR

Q.2. **(A)** Apply the orthogonal property of the orthogonal matrix $\frac{1}{13}\begin{bmatrix} -12 & -5 \\ a & -12 \end{bmatrix}$ to find the value of a. **[02]**

(B) Investigate for what values of λ and μ the system of simultaneous equations, $x + y + z = 6$; $x + 2y + 3z = 10$; $x + 2y + \lambda z = \mu$ have (i) no solution (ii) an infinite Solution (iii) a unique solution. **[05]**

(C) Verify Cayley Hamilton Theorem for the matrix $A=\begin{bmatrix}0&1&2\\1&2&3\\3&1&1\end{bmatrix}$. Apply it to find A^{-1} of the given matrix. **[05]**

Q.3. **(A)** Check whether the set of all pairs of real numbers $(x,\ y)$ with the operation $(x_1,\ y_1)+(x_2,\ y_2)=(x_1+x_2,\ y_1-y_2)$ and $k(x,\ y)=(2kx,\ 2ky)$ is a vector space. **[04]**

(B) Determine whether the vectors (1, 2, 4), (2, –1, 3), (0, 1, 2), (–3, 7, 2) are linearly dependent or linearly independent, if dependent find the relation among them. **[04]**

(C) Apply Dimension Theorem to find the Nullity of the matrix $A=\begin{bmatrix}-1&3&2&0\\1&4&5&2\\2&1&3&0\end{bmatrix}$. **[04]**

OR

Q.4. **(A)** Check whether $W=\{(x,\ y,\ z)|\ x^2+y^2+z^2>0\},\ V=R^3$ is a subspace of R^3. **[04]**

(B) Find the basis and dimension for the solution space of the following system of equations: $x_1-4x_2+3x_3-x_4=0,\ 2x_1-9x_2+5x_3-3x_4=0$. **[04]**

(C) Determine whether the following vectors forms a basis for R^3 or not. **[04]**

$(-2,\ 2,\ 1),\ (1,\ -1,\ 4),\ (1,\ 1,\ 0)$

Q.5. **(A)** Evaluate: $\lim\limits_{x\to 0}\left[\frac{a}{x}-\cot\frac{x}{a}\right]$. **[04]**

(B) Apply Leibnitz's Theorem to prove $x^2y_{n+2}+(2n+1)xy_{n+1}+(n^2+1)y_n=0$, where $y=a\cos(\log x)+b\sin(\log x)$. **[04]**

(C) Verify the Lagrange's Mean Value Theorem for $f(x)=(x-1)(x-2)(x-3)$ in $(0,\ 4)$. **[04]**

OR

Q.6. **(A)** Evaluate: $\lim_{x\to 0}\dfrac{1-x^x}{x\log x}$. **[04]**

(B) Find the n[th] derivative of $y=\dfrac{x^4}{(x-1)(x-2)}$. **[04]**

(C) Verify the Cauchy's Mean Value Theorem for $f(x)=\sin x,\ \phi(x)=\cos x$ in $[\pi/6,\ \pi/3]$. **[04]**

Q.7. **(A)** Test the convergence of the series $\dfrac{2}{1}+\dfrac{3}{8}+\dfrac{4}{27}+\dfrac{5}{64}+\cdots+\cdots$. **[04]**

(B) Discuss the convergence of the series $\sum_{n=1}^{\infty}\dfrac{4\cdot 7\cdot 10\cdots\cdots(3n+1)}{1\cdot 2\cdot 3\cdots\cdots n}$. **[04]**

(C) Apply Taylor's theorem to express $f(x)=49+69x+42x^2+11x^3+x^4$ in ascending power of $(x+2)$. **[04]**

OR

Q.8. **(A)** Define Conditional and Absolute Convergence of the alternating series and test the convergence of the series $1-\dfrac{1}{2^2}+\dfrac{1}{3^2}-\dfrac{1}{4^2}+\dfrac{1}{5^2}\cdots$. **[04]**

(B) Applying standard expansion, prove that $\log(1+\sin x)=x-\dfrac{x^2}{2}+\dfrac{x^3}{6}-\dfrac{x^4}{12}+\dfrac{x^5}{24}+\cdots+\cdots$. **[04]**

(C) Find the expansion of $\tan^{-1}x$ and use it to express $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$ into the form of series up to x^7. **[04]**

Q.9. Solve **any two** of the following:

(A) Evaluate: **[06]**

(i) $\int_0^\infty \sqrt{y}e^{-y^3}\,dy$ **(ii)** $\int_0^\infty \frac{x^8\left(1+x^6\right)}{\left(1+x\right)^{24}}\,dx$

(B) If $I_n=\int_{\pi/4}^{\pi/2}\cot^n\theta\ d\theta$, prove that $I_n=\frac{1}{n-1}-I_{n-2}$. Hence evaluate $\int_{\pi/4}^{\pi/2}\cot^6\theta\ d\theta$. **[06]**

(C) Find the Fourier series of $f(x)=\pi^2-x^2,\ \ -\pi\le x\le\pi,\ f(x+2\pi)=f(x)$ and hence deduce the that $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots=\frac{\pi^2}{12}$. **[06]**

OR

Q.10. Solve **any two** of the following**:**

(A) Evaluate $\int_0^{\pi/2}\sqrt{\sin\theta}\,d\theta\int_0^{\pi/2}\sqrt{\cos\theta}\ d\theta$. **[06]**

(B) If $I_n=\int_0^{\pi/2}x^n\left(\sin x+\cos x\right)dx$, prove that $I_n=\left(\frac{\pi}{2}\right)^n+n\left(\frac{\pi}{2}\right)^{n-1}-n\left(n-1\right)I_{n-2}$. **[06]**

(C) For the periodic function $f(x)=\begin{cases}\pi x & 0\le x\le 1\\ \pi\left(2-x\right) & 1\le x\le 2\end{cases}$ of period 2, show that in the interval $0\le x\le 2$, $f\left(x\right)=\frac{\pi}{2}-\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{1}{\left(2n+1\right)^2}\cos\left(2n+1\right)\pi x$ **[06]**

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