* D.M. Unit 2 Tutorial *

Q1. A class contains 10 students with 6 men and 4 women. Find the number n of ways to:

(a) Select a 4 member committee from the students

(b) Select a 4 member committee with 2 men & 2 women.

(c) Select a president, vice president and treasurer.

Sol: (a) Since order does not count in committee

There are "10 choose 4" such committees. That is

$$n = C(10,4) = \binom{10}{4} = \frac{10.9.8.7}{4.3.2.1} = 210$$

(b) By the product rule.

$$n = \binom{6}{2}\binom{4}{2} = \frac{6.5}{2.1} \cdot \frac{4.3}{2.1} = 15(6) = 90$$

(c) $n = P(6,3) = 6.5.4 = 120$

Q2. A box contains 8 blue socks and 6 red socks. Find the number of ways two socks can be drawn from the box if:

(a) They can be any color.

(b) They must be the same color.

Sol: (a) There are "14 choose 2" ways to select 2 of the 14 socks

Thus

$$n = C(14,2) = \binom{14}{2} = \frac{14.13}{2.1} = 91$$

(b) There are $C(8,2)=28$ ways to choose 2 of 8 blue socks. $C(6,2)=15$ ways to choose 2 of the 4 red socks. thus

$$n = 28 + 15 = 43$$

Q6. State the essential difference between permutations and combination with example.

Sol: Order counts with permutations, such as words, sitting in a row, and electing a president, vice president and the treasurer. Order does not count with combinations such as committes and teams (without counting position). The product rule is usually used with permutations since the choice for each of the orderd position may be veiwed as a sequence of events.

Q7. Find a] $P(7,3)$, b] $P(14,2)$.

Sol: Recall $P(n,r)$ has r factors beginning with n

a] $P(7,3) = 7 \cdot 6 \cdot 5 = 210$

b] $P(14,2) = 14 \cdot 13 = 182$.

Find the number m of ways that 7 people can arrange themselves:

a] In a row of chairs;

b] Around a circular table;

a] Here $m = P(7,7) = 7!$ ways.

b] One person can sit at any place at the table. The other 6 people can arrange themselves in 6! ways around the table; that is $m = 6!$

This is an example of a circular permutation. In general, n objects can be arranged in a circle in $(n-1)!$ ways

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Q9. Find the number n of distinct permutations that can be formed from all the letters of each word.
(a) THOSE ; (b) UNUSUAL ; (c) SOCIOLOGICAL

Sol: (a) $n = 5! = 120$

(b) $n = \frac{7!}{3!} = 840$

(c) $n = \frac{12!}{3!2!2!2!} = 1,99,58,400$

Q10. A class contain 8 students. Find the number n of sample of size 3:
(a) With replacement (b) Without replacement.

Sol: (a) Each student in the ordered sample can be choosen in 8 ways; hence there are

$n = 8.8.8 = 8^3$.
$= 512$

(b) The first student in the sample can be chosen in 8 ways, the second in 7 ways, and the last in 6 ways. Thus there are $n = 8.7.6 = 336$

Q11 Find n if $P(n,2) = 72$

Sol: $P(n,2) = n(n-1) = n^2 - n$, Thus, we get
$n^2 - n = 72$
$(n-9)(n+8) = 0$
Since n must be positive, the only answer is $n = 9$

Q12. Find the number of committe of 5 with a given chairperson that can be selected from 12 people.

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Sol: No. of Persons = 5 + 1 (chairperson)

$\therefore$ no. of committees that can be selected is

$^{12}C_6 = 924$

Q4. Seven women and nine men are on the faculty the mathematics department at a school.

(a) How many ways are there to select a committee of five members of the department if at least one women must be on the committee.

(b) How many ways are there to select a committee of five members of the department if at least one women and at least one men must be on the committee.

Sol: (a) There are 16 faculty members in the department

$\therefore$ $C(16,5)$ combination of 5 people is possible

However, $C(9,5)$ of them are pure men.

$\therefore$ $C(16,5) - C(9,5)$

$= \frac{16!}{5!\,11!} - \frac{9!}{5!\,4!}$

$= 4{,}368 - 24$

$= 4{,}344$

(b) Now, there are $C(16,5)$ total $- C(9,5)$ no women $- C(7,5)$

$= \frac{16!}{5!\,11!} - \frac{9!}{5!\,4!} - \frac{7!}{5!\,2!}$

$= 4{,}368 - 24 - 21$

$= 4{,}368 - 45$

$= 4{,}323$

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Q5. How many ways are there to select 12 countries in the UN to serve on a council of 3 are selected from a block of 45, 4 are selected from a block of 57 and the others are selected from the remaining 69 countries.

Sol: Now, remaining no. of selected countries from block
So,

$= 12-(3+4) \Rightarrow 12-7 \Rightarrow 5$

Now, the no. of ways to select the no. of countries

$= {}^{45}C_3 \times {}^{57}C_4 \times {}^{57}C_5$

$= 14190 \times 395010 \times 4187106$

$= 2346953264$

Q12. How many ways are there for 10 women and six men to stand in a line so that no two men stand next to each other.

Sol: First consider the positions of the women, then the positions of then men.
The possible ways are there to arrange ten women in a row is

$10P10 = 10!$

$= 3628800$

As per given condition is no two men stand next to each other. Hence we need to find how many ways we can arrange 66 men in the 11 possible places

$11P6 = 11 \times 10 \times 9 \times 8 \times 7 \times 6 = 332640$

$11P6 = 11 \times 10 \times 9 \times 8 \times 7 \times 6 = 332640$

$\therefore 332640 \times 332640 = 1,207,084,032,000$

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Q13 How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other

Sol: There will be 9 places for 5 women to occupy, they can do it in $9C_5$ ways more over, they can arrange themselves in 5! ways.

Also, the man can be arranged in 8! ways
so, total no. of ways = $9C_5 \cdot 8! \cdot 5!$
= 609638400 ways

Q14 What is the coefficient of x^12 y^13 in the expansion of (2x-3y)^25?

Sol: To solve this problem, we first note that the expression equals $(2x-3y)^{25}$ using the binomial theorem we have

$$(2x+(-3y))^{25} = \sum_{j=0}^{25} \binom{25}{j} (2x)^{25-j} (-3y)^j$$

So, to find the coefficient of $x^{12}y^{13}$ we can write.

$$\binom{25}{13} 2^{13} (-3)^{13} = \frac{25!}{13!\,12!} 2^{12} 3^{13}$$

15. How many ways are there to select four pieces of fruit from a bowl containing apples, oranges and pears if the order in which the pieces are selected does not matter, only the type of fruit and not the individual piece matters, and there are at least four pieces of each type of fruit in the bowl.

Sol: There are 15 possible 4-combinations of a set containing 3 items