EE2703 : Applied Programming Lab
Assignment 6B
Laplace Transformation

Krishna Somasundaram
EE19B147

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Objective

Use Concepts learnt in Digital Signal Processing to find DFT's of various functions and analyse them.

- Use FFT library in python to analyse some basic functions like sin and cos(x)
- Use it further to Analyse functions like cubic sinusoids, nested sinusoids, and the Gaussian function.
- Plot magnitude and Phase plots of aforementioned functions.

Section 1

The DFT sin(t) turns out to be:

$$y = Sin(t)$$

$$Y(w) = \frac{\delta(w-1) - \delta(w+1)}{2j}$$

The DFT cos(kt) turns out to be:

$$y = Cos(k*t)$$

$$Y(w) = \frac{\delta(w-k) + \delta(w+k)}{2}$$

where k is a constant. We can determine Y(w) for these simple functions as they can be rewritten as complex exponentials, as shown below.

$$y = Sin(t) = \frac{e^{jt} - e^{-jt}}{2j}$$

and

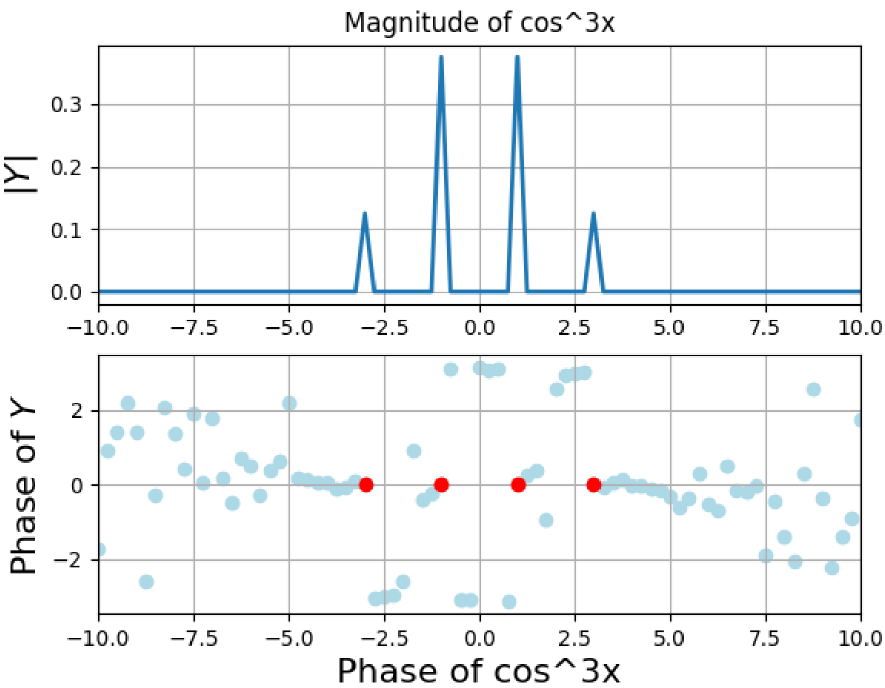
$$y = Cos(t) = \frac{e^{jt} + e^{-jt}}{2}$$

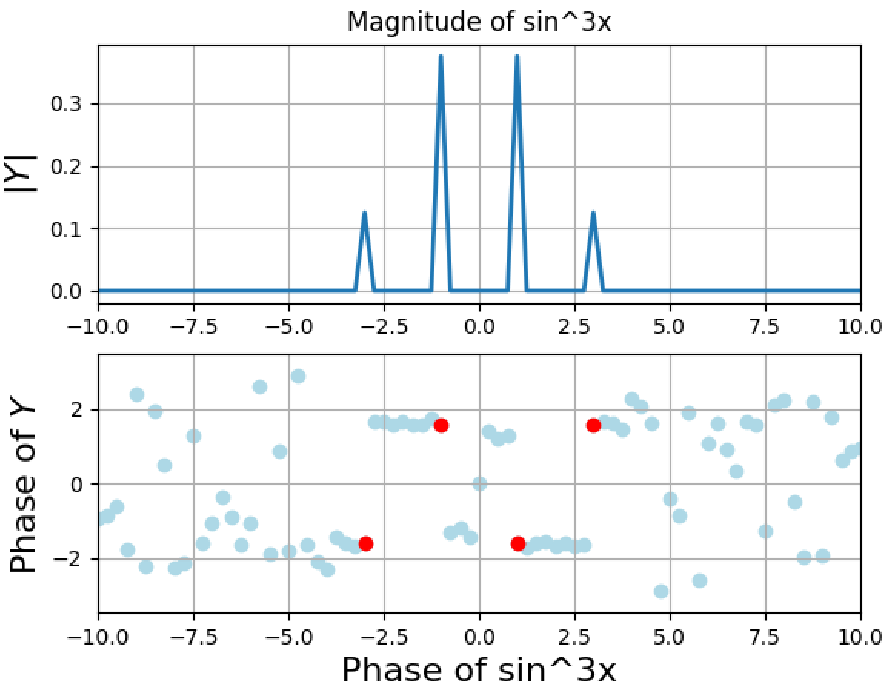
Let us perform a similar analysis for the upcoming questions as well.

Section 2

Let y = $Cos^3(x)$ and y = $Sin^3(x)$

We get the following Graphs.





This is Exactly as Expected! y can be written as:

$$y = Cos^3(x) = \frac{Cos(3x) - 3Cos(x)}{4}$$

and so

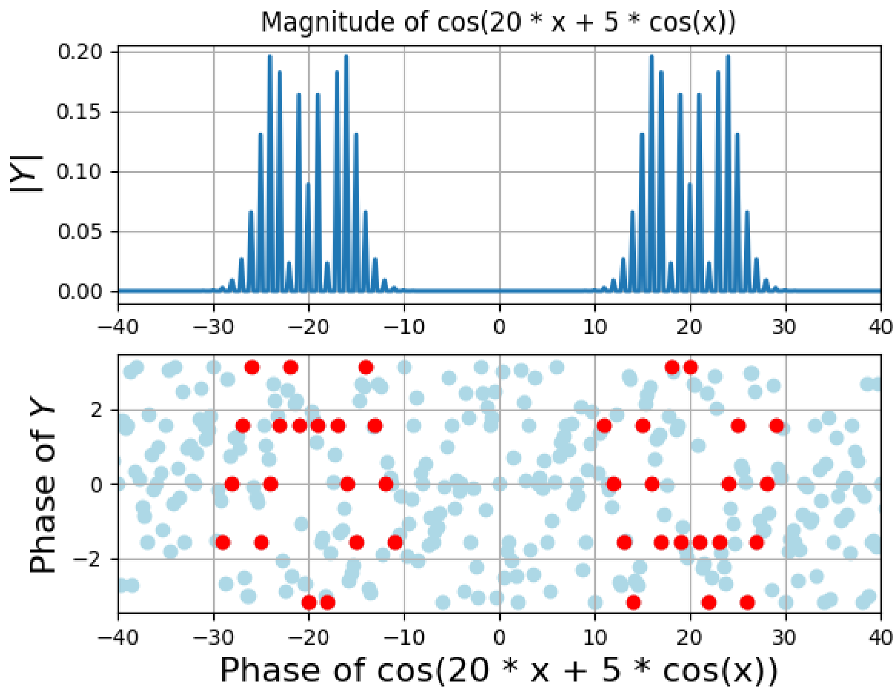
$$Y(w) = \frac{\delta(w-3) - \delta(w+3)}{8} - 3\frac{\delta(w-1) - \delta(w+1)}{8}$$

Similar work can be done and so explains the output for y = $Sin^3(x)$. Here, the only difference will be present in the Phase plot due to presence of j in the denominator.

Section 3

Let y = Cos(20x+5Cos(x))

We get the following plots.



The function is a bit difficult to analyse, but we can notice some interesting things about it. Firstly, all the points in the magnitude plot seem to be within a ±10 range of either 20 or -20. These are the points whose magnitude is not negligible. At these points, when we check the phase, they appear to be some integer multiple of $\pi/2$.

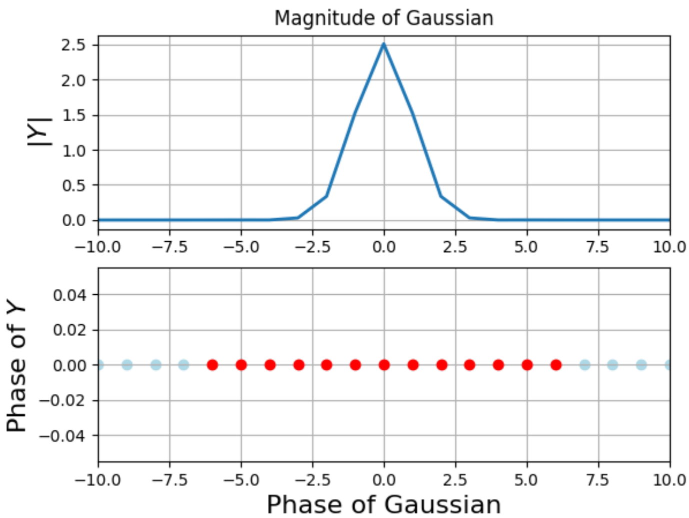
Section 4

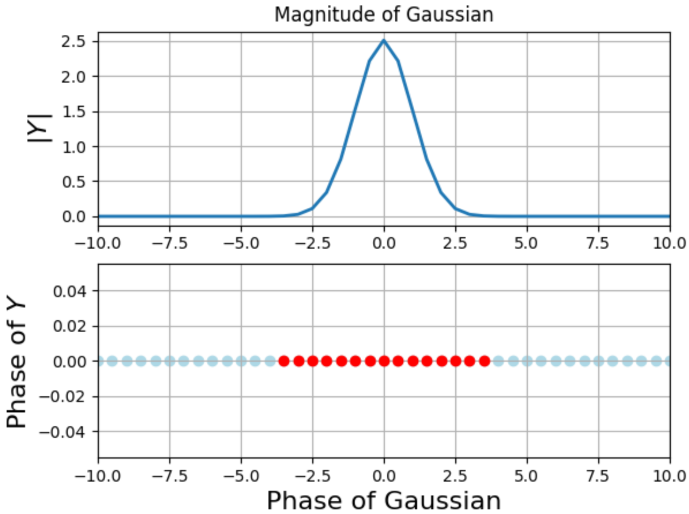
let y be the gaussian function. This case is interesting, as it is not frequency bandlimited. Thus, we will get some output whatever frequency we give it. Another thing to note is the CTFT of the gaussian function is another gaussian function, as shown below.

$$y = e^{\frac{-t^2}{2}}$$
$$Y(w) = \sqrt{2\pi} e^{\frac{-w^2}{2}}$$

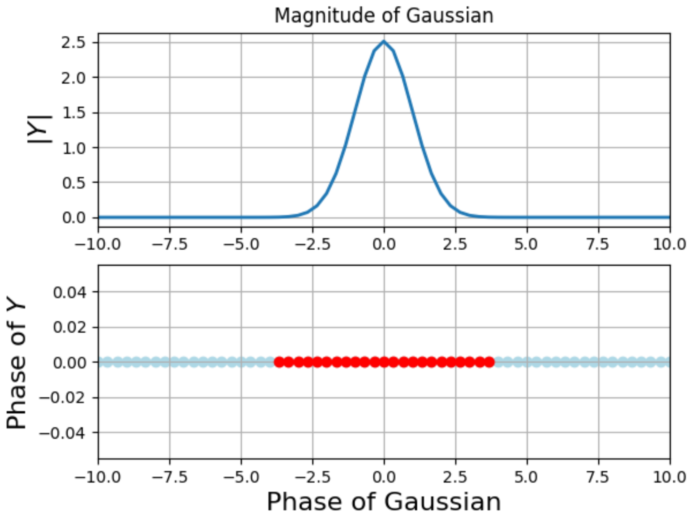
The question asks for what time range we will get the least error. Let us try a variety of time ranges to find this. The gaussian function itself has been normalized in order to achieve this. Also, I have taken the absolute value of the FFT and then FFT shifted it in order to find the minimum error. Here are the plots we get by varying the time range from ± 1 to ± 10 .

Now, let us plot the output of the system for a decay of 0.05 at various frequencies ranging from 1.4 to 1.6.

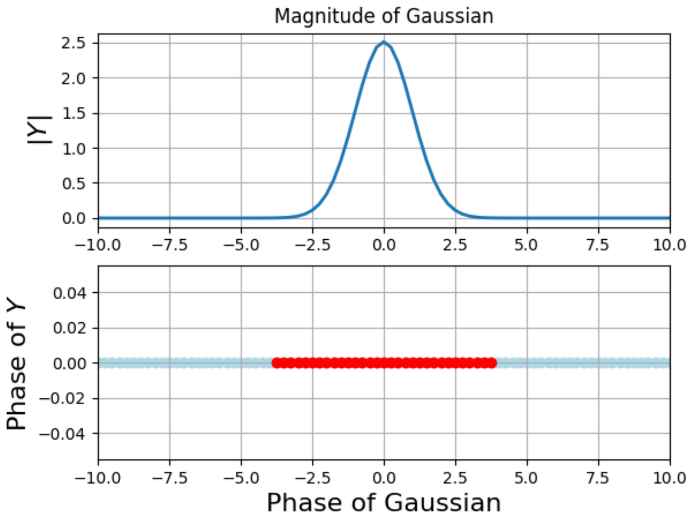




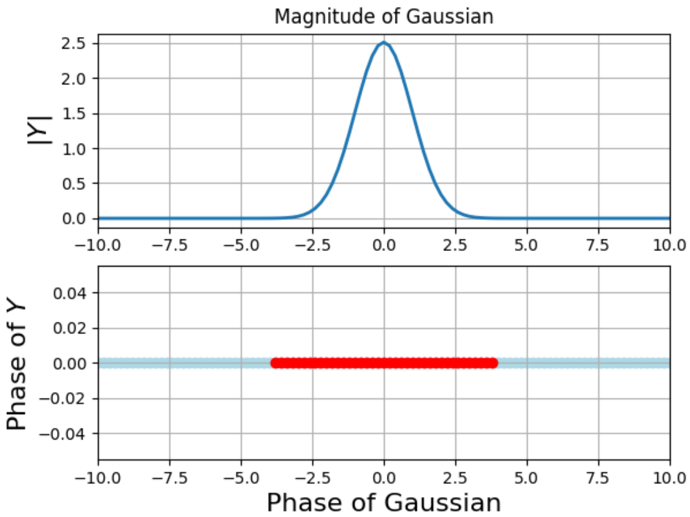
Magnitude of Gaussian
|Y|
Phase of Y
Phase of Gaussian

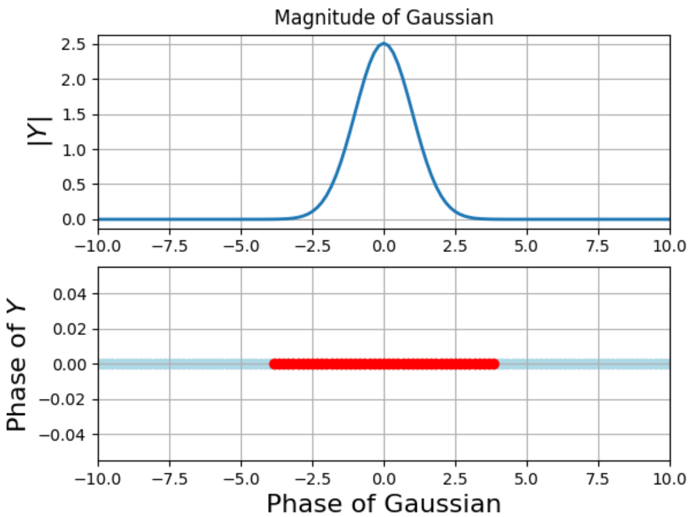


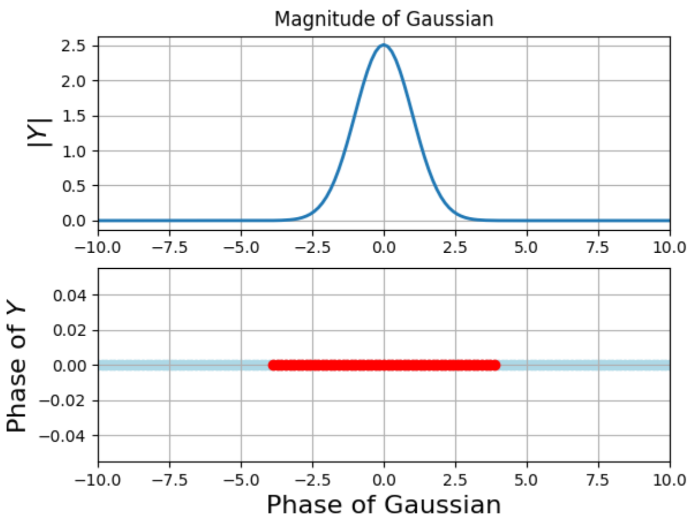
Magnitude of Gaussian
|Y|
Phase of Y
Phase of Gaussian

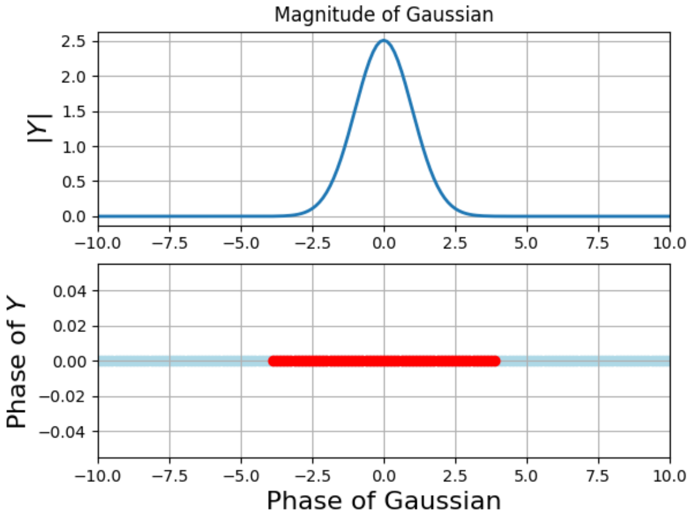


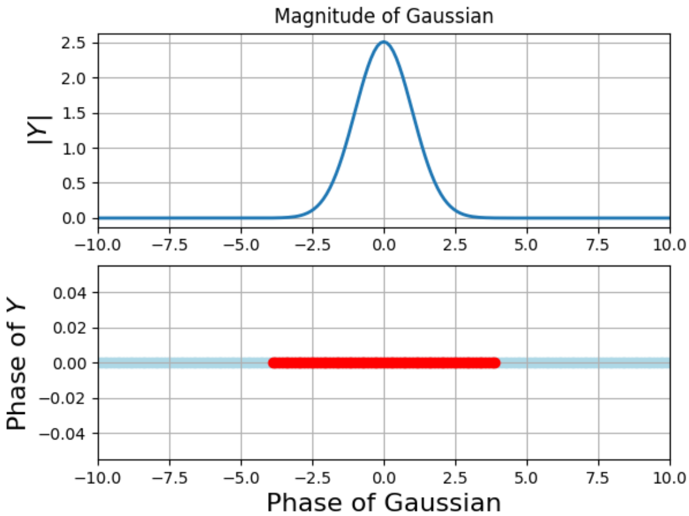
Magnitude of Gaussian
|Y|
Phase of Y
Phase of Gaussian

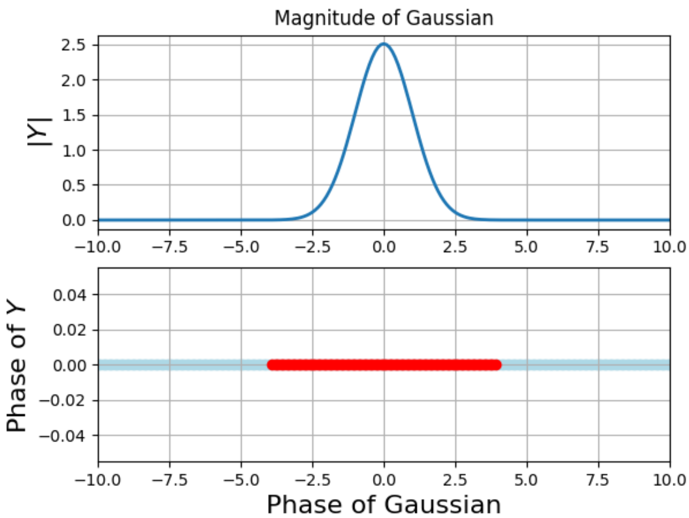

Magnitude of Gaussian
|Y|
Phase of Y
Phase of Gaussian


Magnitude of Gaussian
|Y|
Phase of Y
Phase of Gaussian


Magnitude of Gaussian
|Y|
Phase of Y
Phase of Gaussian


Magnitude of Gaussian
|Y|
2.5
2.0
1.5
1.0
0.5
0.0
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
Phase of Y
0.04
0.02
0.00
−0.02
−0.04
Phase of Gaussian


Magnitude of Gaussian
|Y|
2.5
2.0
1.5
1.0
0.5
0.0
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
Phase of Y
0.04
0.02
0.00
−0.02
−0.04
Phase of Gaussian


Magnitude of Gaussian
|Y|
2.5
2.0
1.5
1.0
0.5
0.0
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
Phase of Y
0.04
0.02
0.00
−0.02
−0.04
Phase of Gaussian

As I increased the time range, the graphs looked more and more like the real curve. Here are the generated Error values.

We get the max error for the time range 1 as 0.006496802750855624

We get the max error for the time range 2 as 1.5607475312151564e-09

We get the max error for the time range 3 as 2.5979218776228663e-14

We get the max error for the time range 4 as 8.881784197001252e-16

We get the max error for the time range 5 as 1.6653345369377348e-14

We get the max error for the time range 6 as 1.3988810110276972e-14

We get the max error for the time range 7 as 1.554312234475219e-14

We get the max error for the time range 8 as 9.921240527779468e-16

We get the max error for the time range 9 as 9.325873406851315e-15

We get the max error for the time range 10 as 8.881784197001252e-15

It looks like $\pm 4\pi$ gives the best results, however all of them after 2 are pretty much negligible.

Conclusion

In this Assignment, using FFT in python, we have analysed the Digital Fourier Transform of various signals, such as sinosuidal and gaussian functions. For the case of the nested sinosoids, we found that it only contained multiples of $\pi/2$ in the phase plot. For the gaussian function, we tried to find the error by comparing it with the CTFT of the same.