EE2703 : Applied Programming Lab
Assignment 4
Fourier Approximations

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Aim

Introduction

In mathematics, a Fourier series is a periodic function composed of harmonically related sinusoids, combined by a weighted summation. With appropriate weights, one cycle (or period) of the summation can be made to approximate an arbitrary function in that interval (or the entire function if it too is periodic). As such, the summation is a synthesis of another function. The process of deriving weights that describe a given function is a form of Fourier analysis.

Assignment

Question 1

Let us plot $f(x) = e^x$ and $f(x) = cos(cos(x))$

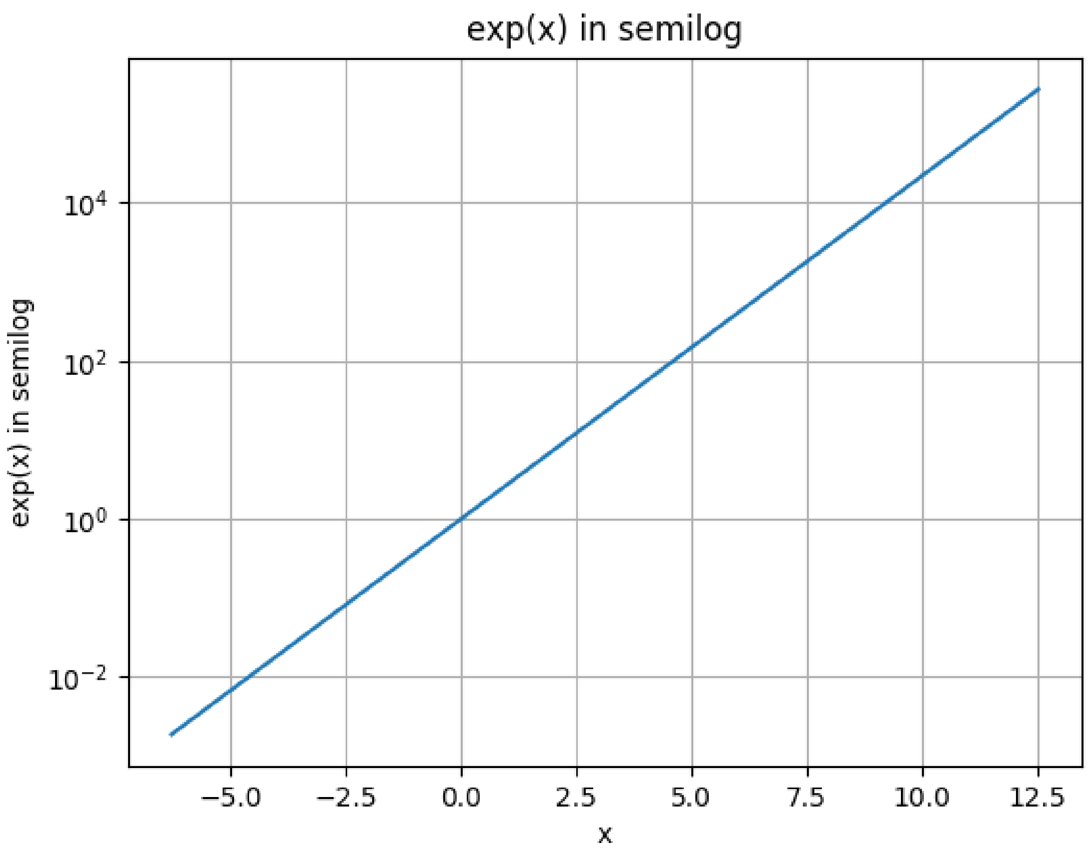

Figure 1: e^x in semilog plot

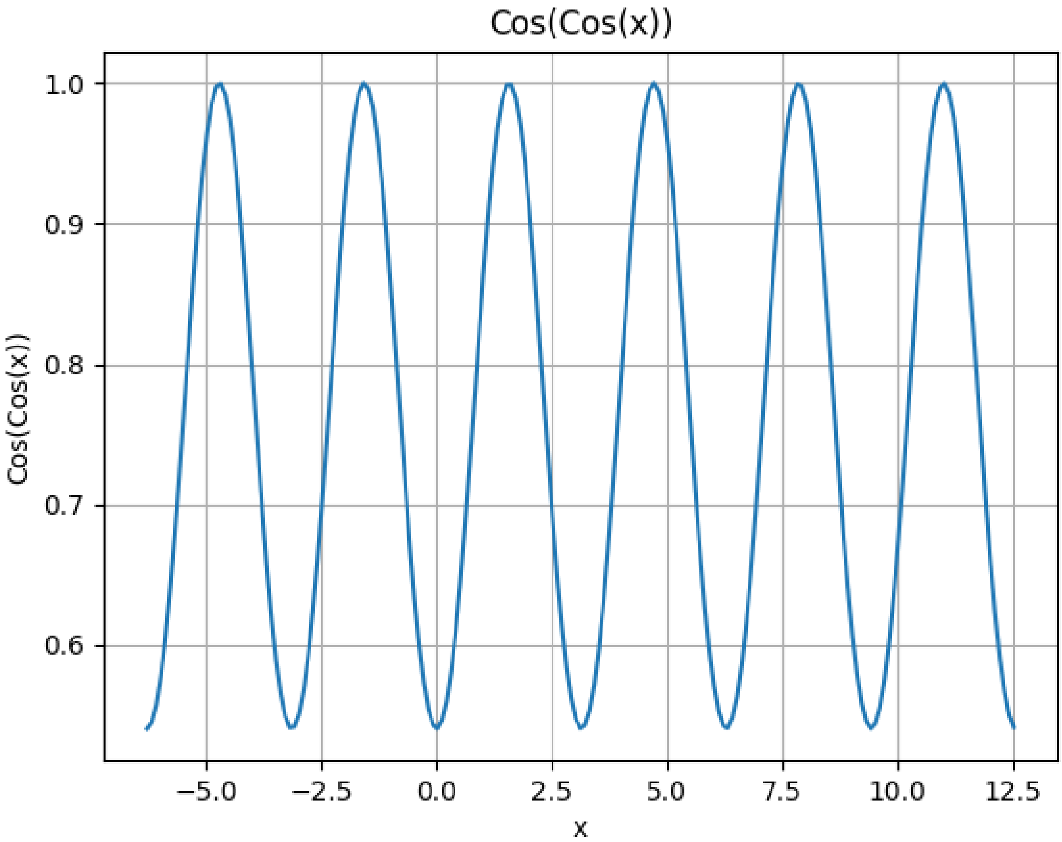

Figure 2: Cos(Cos(x))

Clearly, $cos(cos(x))$ is periodic while e^x is not.

Question 2

The fourier series coefficients are given by:

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)cos(nx)\,dx$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)sin(nx)\,dx$$

We use the above formulas to determine 51 fourier coefficients for both of the functions that we are analysing.

Question 3

Now, we will determine the c matrix, which is obtained from the a and b coefficients as:

$$\begin{pmatrix} a_0 \\ a_1 \\ b_1 \\ a_2 \\ b_2 \\ . \\ . \\ a_n \\ b_n \end{pmatrix}$$

Now, we plot the c matrix of e^x and $cos(cos(x))$ in semilog and loglog plots

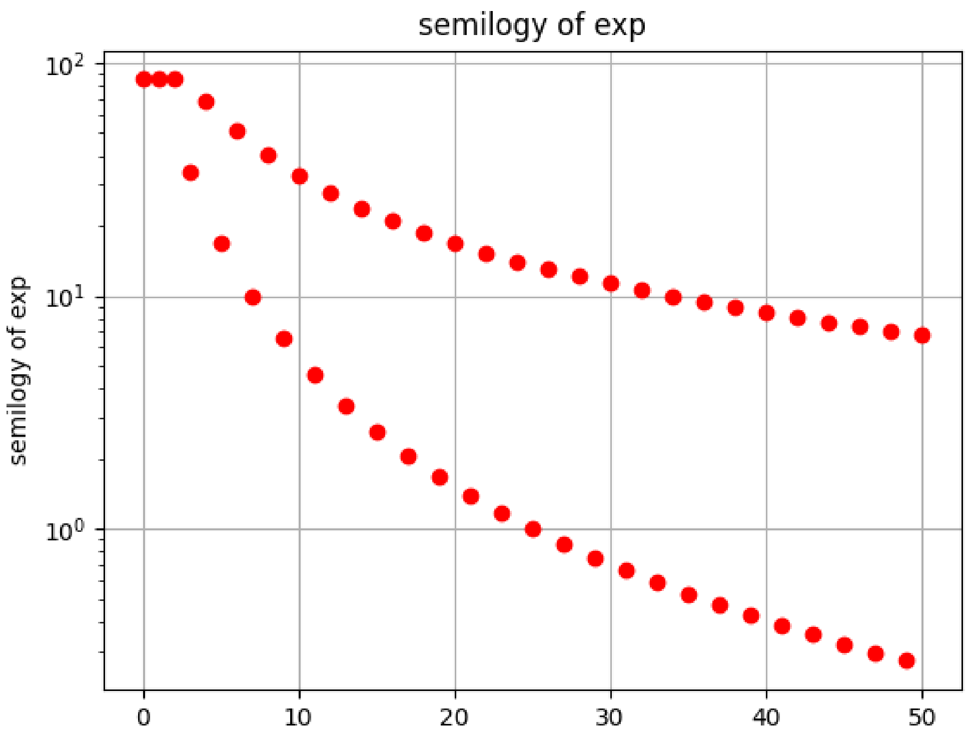

Figure 3: semilog plot of e^x

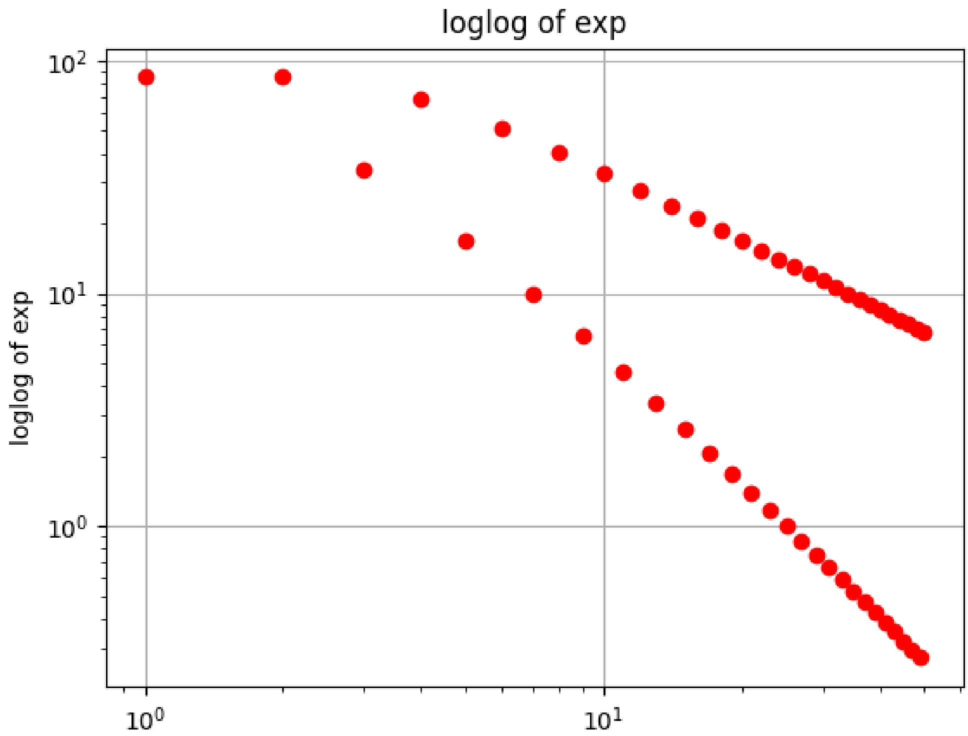

Figure 4: loglog plot of e^x

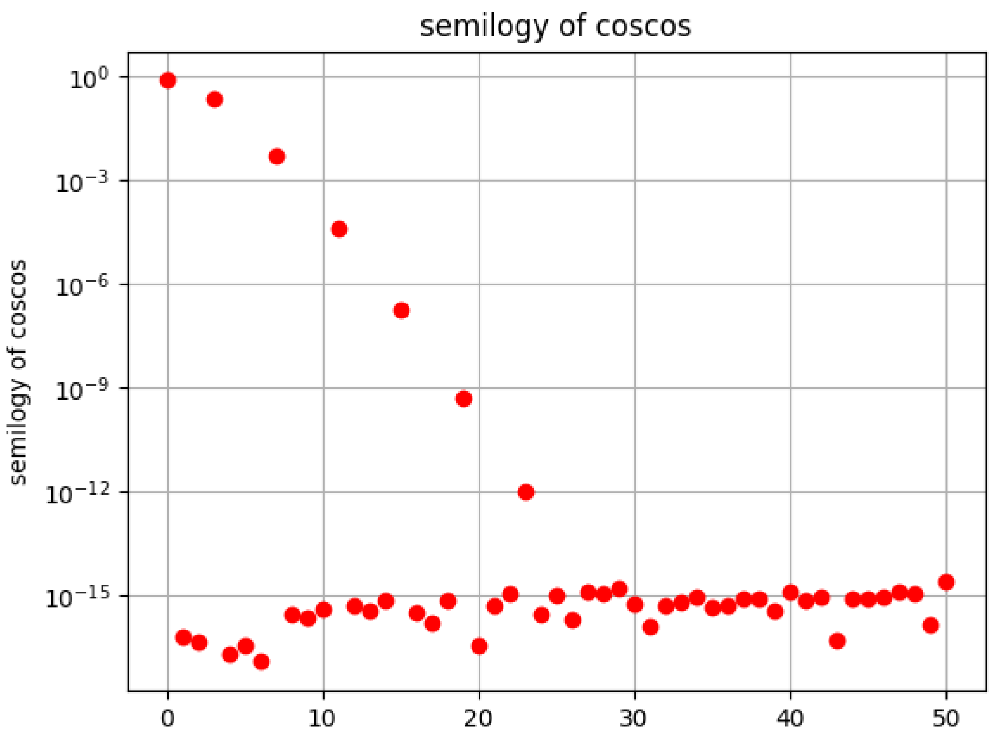

Figure 5: semilog plot of Cos(Cos(x))

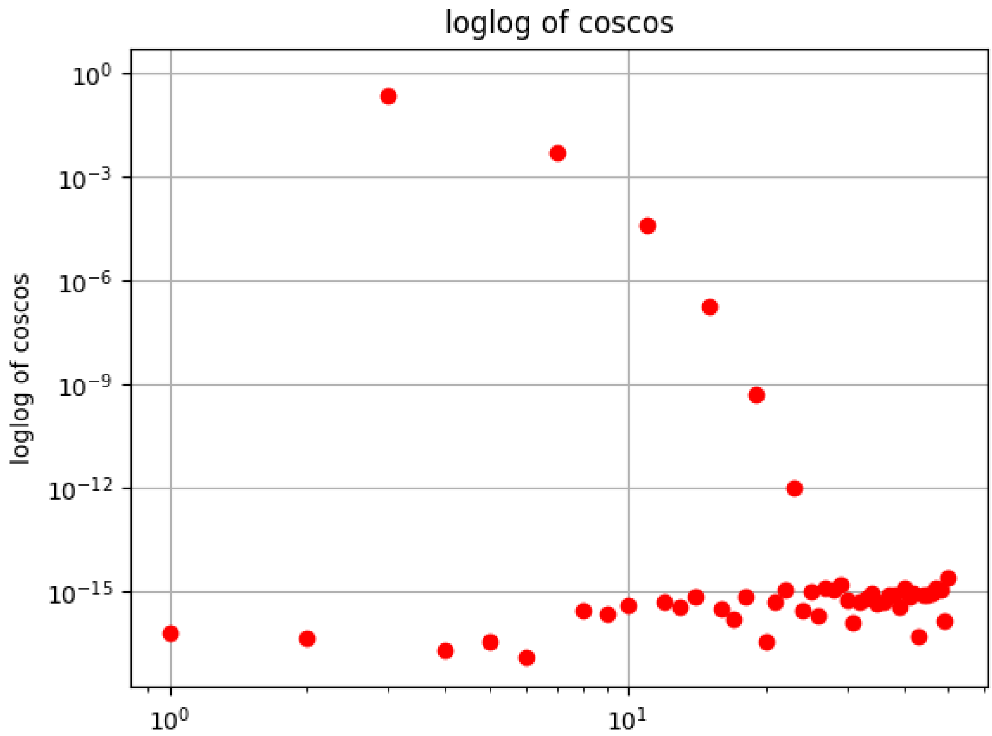

Figure 6: loglog plot of Cos(Cos(x))

- The b_n coefficients of cos(cos(x)) are expected to be zero as it is an even function and b_n are the coefficients of the sin in the Fourier Series. Hence the odd sinusoidal components are expected to be zero.
- e^x is an increasing function, and non-periodic. Thus, it has a higher dependency on coefficients obtained from higher coefficients. On the other hand Cos(Cos(x)) is periodic, and so the magnitude of its coefficients taper of very quickly as n increases.
- The coefficients of e^x are proportional to $\frac{1}{n^2+1}$, Thus for large n,

$$log(|a_n|), log(|a_n|) \propto -2log(n)$$

. Hence, the loglog plots are linear. For Cos(Cos(x)), log of the coefficients turn out to be $\propto -n$. Thus, the semilog plot of Cos(Cos(x)) is linear.

Question 4

Next we use the Least squared method to again determine the coefficients.

$$a_0 + \sum_{n=1}^{25} a_n cos(nx_i) + \sum_{n=1}^{25} b_n sin(nx_i) \approx f(x_i)$$

So we solve it as a matrix problem.

$$\begin{pmatrix} 1 & \cos x_1 & \sin x_1 & \dots & \cos 25x_1 & \sin 25x_1 \\ 1 & \cos x_2 & \sin x_2 & \dots & \cos 25x_2 & \sin 25x_2 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & \cos x_{400} & \sin x_{400} & \dots & \cos 25x_{400} & \sin 25x_{400} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ b_1 \\ \dots \\ a_{25} \\ b_{25} \end{pmatrix} = \begin{pmatrix} f(x_1) \\ f(x_2) \\ \dots \\ f(x_{400}) \end{pmatrix}$$

The left Matrix is denoted by A, and the rightmost one by b, thus we solve $Ac = b$, where c are the fourier coefficients.

Question 5

Now, let us plot and compare the approximate fourier coefficients with the ones obtained via the least squares method w.r.t the coefficients found via direct integration.

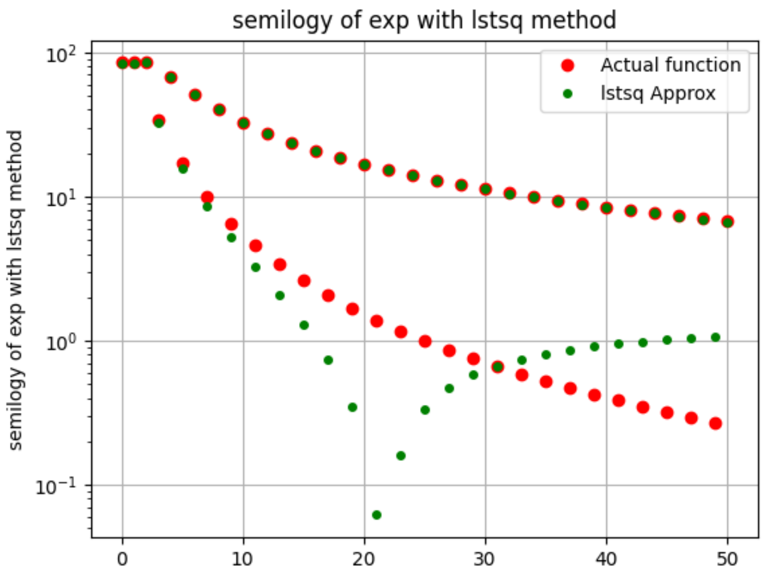

Figure 7: semilog plot of e^x

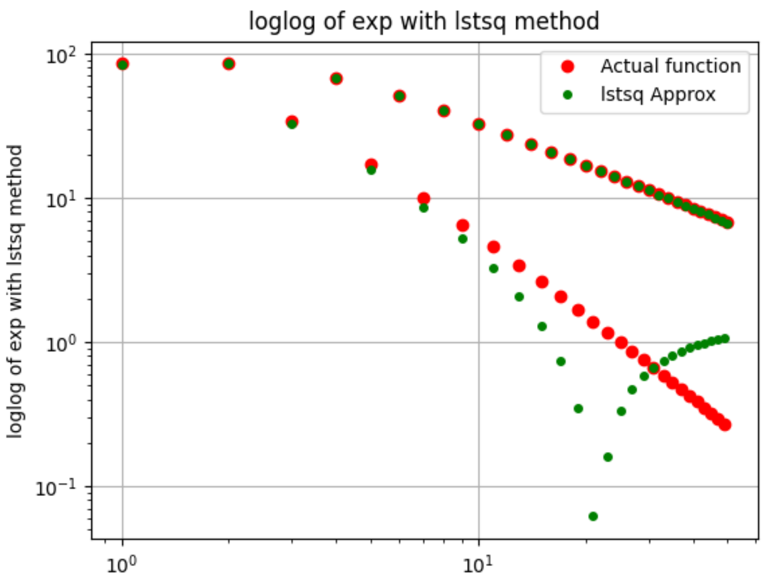

Figure 8: loglog plot of e^x

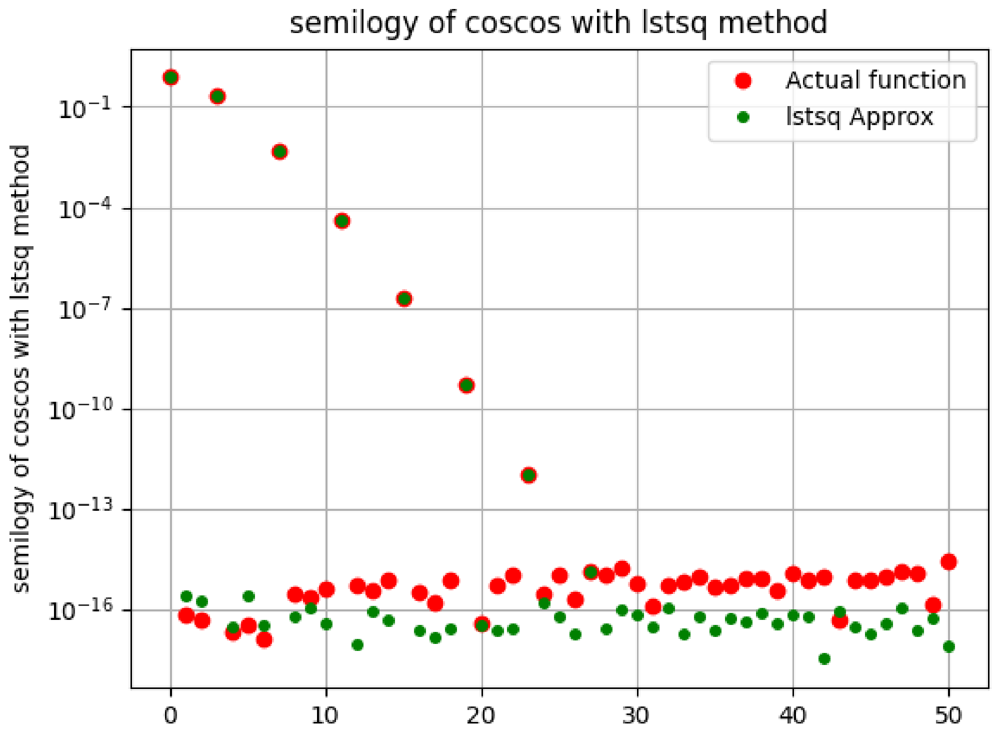

Figure 9: semilog plot of Cos(Cos(x))

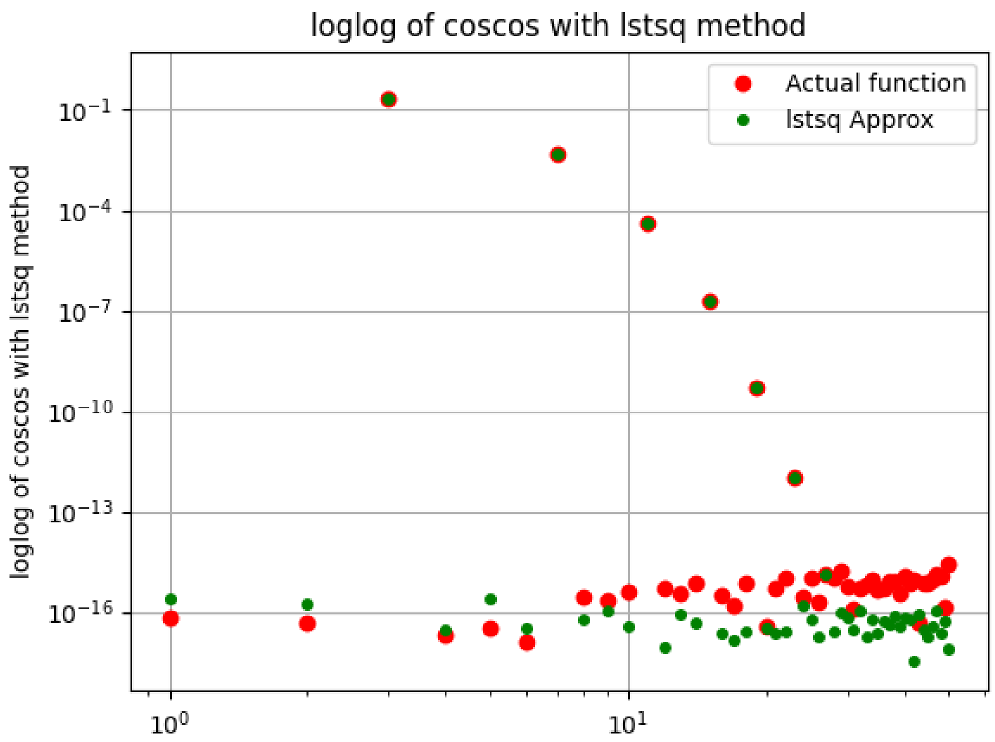

Figure 10: loglog plot of Cos(Cos(x))

Question 6

The maximum absolute error between the coefficients obtained from analysis equation and the coefficients obtained from least square method for the function e^x was found to be 1.33273087033551.

The maximum absolute error between the coefficients obtained from analysis equation and the coefficients obtained from least square method for the function cos(cos(x)) was found to be $2.597698681166561 \times 10^{-15}$

As we can see, the error in approximation is very negligible in the Cos(Cos(x)) case as compared with exp(x) case. The reason being Cos(Cos(x)) depends greatly on the coefficients which are obtained at lower frequencies, while on the other hand exp(x) has significant weight attributed to those obtained at higher frequencies as well. This is known as Gibbs phenomenon.

Question 7

Now, let us multiply A with the c matrix obtained by lstsq method to get a set of values, and compare these values with the actual functions themselves.

$$Ac=b$$

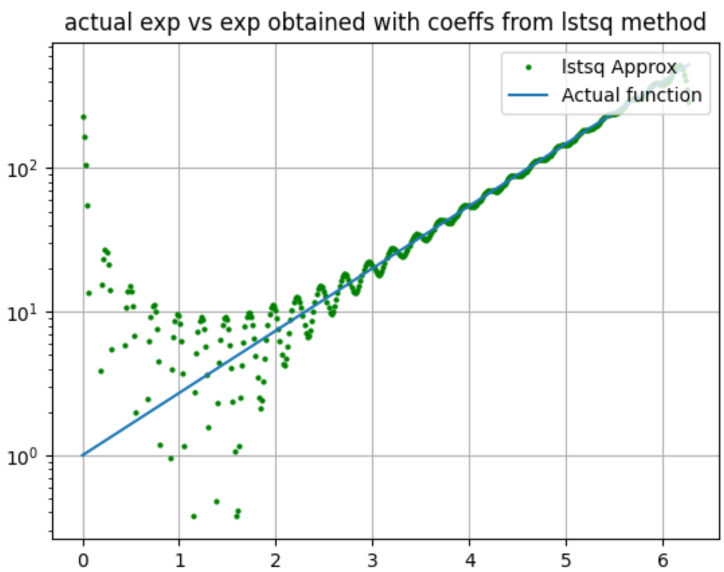

Figure 11: actual exp vs exp obtained with coeffs from lstsq method

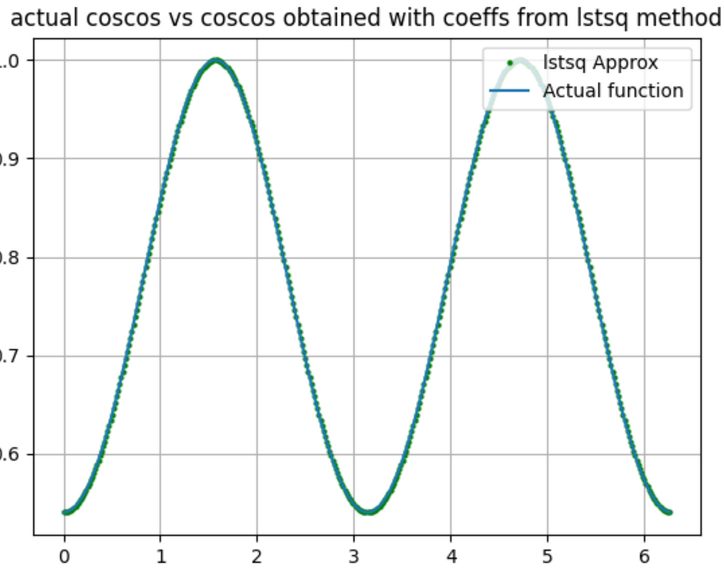

Figure 12: actual coscos vs coscos obtained with coeffs from lstsq method

Conclusion:

Fourier Coefficients of cos(cos(x)) and e^x is calculated by both Analysis and Least Square method. Their respective graphs have been plotted as well. We observe that the presence of discontinuities and higher frequencies made the fourier series approximations for e^x significantly diverge from its function compared to the function cos(cos(x)).