Practical -1 :

No.
(1)
(2)
(3)
(4)

(Q1) $\lim_{x\to a}\left[\frac{\sqrt{a+2x}}{\sqrt{3a+x}} - \frac{\sqrt{3x}}{2\sqrt{x}}\right]$

$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}\;2\sqrt{x}} \times \frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{3a+x}+\sqrt{3x}} \times \frac{\sqrt{a+2x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}\right.$

$\lim_{x\to a} \frac{(a+2x-3x)(\sqrt{3a+x}+2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x}+\sqrt{3x})}$

$\frac{1}{3}\lim_{x\to a} \frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(a-x)(\sqrt{a+2x}+\sqrt{3x})}$

$\frac{1}{3}\lim_{x\to a} \frac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}$

$= \frac{1}{3} \times \frac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$

$= \frac{1}{3} \times \frac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$

$= \frac{1}{3} \times \frac{4\sqrt{a}}{2\sqrt{3}\sqrt{a}} = \frac{2}{3\sqrt{3}}$

(2) $\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$

$\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right)$

$\lim_{y\to 0} \frac{a+y-a}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$

$\lim_{y\to 0} \frac{y}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$

$= \frac{1}{\sqrt{a+0}\,(\sqrt{a+0}+\sqrt{a})}$

$= \frac{1}{\sqrt{a}\,(\sqrt{a}+\sqrt{a})}$

$= \frac{1}{\sqrt{a}\,(2\sqrt{a})} = \frac{1}{2a}$

(3) $\lim_{x\to \pi/6} \frac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$

By substituting $x - \pi/6 = h$

$x = h + \pi/6$

where $h \to 0$

$$\lim_{h\to 0} \frac{\cos(h+\pi/6) - \sqrt{3}\sin(h+\pi/6)}{\pi - 6(h+\pi/6)}$$

$$\lim_{h\to 0} \frac{\cos h \cdot \cos \pi/6 \cdot \sin h \cdot \sin \pi/6 - \sqrt{3}\sin h \cdot \cos \pi/6 + \cos h \cdot \sin \pi/6}{\pi - 6\left(\frac{6h+\pi}{6}\right)}$$

$$\lim_{h\to 0} \frac{\cos h \cdot \sqrt{3}/2 \cdot \sin h \cdot 1/2 - 2/\sqrt{3}\,(\sin h \cdot \sqrt{3}/2 + \cos h \cdot 1/2)}{\pi - 6h + \pi}$$

$$\lim_{h\to 0} \frac{\cos \frac{\sqrt{3}}{2} h \cdot \sin \frac{h}{2} - \sin \frac{\sqrt{3}}{2} h - \cos \frac{\sqrt{3}}{2} h}{-6h}$$

$$\lim_{h\to 0} \frac{-\sin \frac{4h}{2}}{-6h}$$

$$\lim_{h\to 0} \frac{\sin 4h}{h}$$

$$\frac{1}{3} \lim_{h\to 0} \frac{\sin h}{h} = \frac{1}{3} \times 1 = \frac{1}{3}.$$

(4) $$\lim_{x\to\infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}\right]$$

By rationalizing numerator and denominator both

$$\lim_{x\to\infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}\right] \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+3} + \sqrt{x^2-3}} \times \left.\frac{\sqrt{x^2-3} + \sqrt{x^2+1}}{\sqrt{x^2-3} + \sqrt{x^2+1}}\right]$$

$$\lim_{x\to\infty} \left[\frac{(x^2+5-x^2+3)\,(\sqrt{x^2+3} + \sqrt{x^2+1})}{(x^2+3-x^2+1)\,(\sqrt{x^2+5} + \sqrt{x^2-3})}\right]$$

$$= \lim_{x\to\infty} \frac{\cancel{8}^{\,4}\,(\sqrt{x^2+3} + \sqrt{x^2+1})}{\cancel{4}_{\,1}\,(\sqrt{x^2+5} + \sqrt{x^2-3})}$$

$$4 \lim_{x\to\infty} \frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)} + \sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)} + \sqrt{x^2\left(1+\frac{3}{x^2}\right)}}$$

After applying limit we get;

$= 4.$

(5) $$f(x) = \begin{cases} \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}, & \text{for } 0 < x \le \frac{\pi}{2} \\[2ex] = \dfrac{\cos x}{\pi - 2x}, & \text{for } \frac{\pi}{2} < x < \pi \end{cases} \quad \text{at } x$$

$$f(\pi/2) = \frac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}} \qquad \therefore f(\pi/2) = 0$$

f at $x = \frac{\pi}{2}$ define.

(ii) $\lim_{x \to \pi/2^+} f(x) = \lim_{x \to \pi/2} \frac{\cos x}{\pi - 2x}$

By substituting method:

$x - \frac{\pi}{2} = h$

$x = h + \pi/2$ where $h \to 0$

$\lim_{h \to 0} \frac{\cos(h + \pi/2)}{\pi - 2(h + \pi/2)}$

$\lim_{h \to 0} \frac{\cos(h + \pi/2)}{\pi - 2\left(\frac{2h + \pi}{2}\right)}$

$\lim_{h \to 0} \frac{\cos h \cdot \cos \pi/2 - \sin h \cdot \sin \pi/2}{-2h}$

$\lim_{h \to 0} \frac{\cancel{\cos h \cdot 0} - \sin h}{-2h}$

$= \lim_{h \to 0} \frac{-\sin h}{-2h}$

$\frac{1}{2} \lim_{h \to 0} \frac{\sin h}{h} = \frac{1}{2}$

(b). $\lim_{x \to \pi/2^-} f(x) = \lim_{x \to \pi/2^-} \frac{\sin 2x}{\sqrt{1 - \cos 2x}}$ using $\sin 2x = 2 \sin x \cdot \cos x$

$\lim_{x \to \pi/2^-} \frac{2 \sin x \cdot \cos x}{\sqrt{2 \sin^2 x}}$

$\lim_{x \to \pi/2^-} \frac{2 \sin x \cdot \cos x}{\sqrt{2} \sin x}$

$\lim_{x \to \pi/2^-} \frac{2 \cos x}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}} \lim_{x \to \pi/2^-} \cos x$

$\therefore$ LHL $\neq$ RHL

$\therefore$ f is not continous at $x = \pi/2$

5.

(ii) $f(x) = \begin{cases} \frac{x^2 - 9}{x - 3} & 0 < x < 3 \\ = x + 3 & 3 \le x \le 6 \\ = \frac{x^2 \cancel{- 9}}{x + 3} & 6 \le x < 9 \end{cases}$ at

at $x = 3$

$\to$ (i) $f(3) = \frac{x^2 - 9}{x - 3} = 0$

f at $x = 3$ define

(iii) $\lim_{x\to 3^+}$

$f(3) = 3+3 = 3+3 = 6$

f is defined at $x=3$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3) = 6$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3}$

$\therefore LHL = RHL$

f is continous at $x=3$

for $x=6$

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9} = 3$

(2). $\lim_{x\to 6^+} \frac{x^2-9}{x+3}$

$\lim_{x\to 6^+} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$

$\lim_{x\to 6^+} (x-3) = 6-3 = 3$

$\lim_{x\to 6^-} x+3 = 3+6 = 9$.

$\therefore LHL \neq RHL$

f is not continous.

6)

(i) $f(x) = \frac{1-\cos 4x}{x^2}$ $\quad x<0$

$= k$ $\quad x=0$ $\Big\}$ at $x=0$

Soln:- f is continous at $x=0$

$\lim_{x\to 0} f(x) = f(0)$

$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k$.

$\lim_{x\to 0} 2\frac{\sin^2 2x}{x^2} = k$.

$2\lim_{x\to 0} \frac{\sin^2 x}{x^2} = k$

$2\lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$.

$2(2)^2 = k$

$k = 8$

(ii) $f(x) = (\sec 2x)^{\cot^2 x}$ $\quad x \neq 0$

$= k$ $\quad x = 0$ $\Big\}$ at $x=0$

Soln:-

$f(x) = (\sec 2x)^{\cot^2 x}$

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(using $\tan^2 x - \sec^2 x = 1$ & $\cot^2 x = 1/\tan^2 x$
$\sec^2 x = 1 + \tan^2 x$)

$$\therefore \lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$$

$$\lim_{x\to 0} (1+\tan^2 x)^{\frac{1}{\tan^2 x}}$$

We know that
$$\lim_{x\to 0} (1+px)^{1/px} = e$$

$= e$

$K = e$

$$f(x) = \frac{\sqrt{3}-\tan x}{\pi - 3x} \quad x \neq \pi/3$$
$$= K \quad x = \pi/3$$
at $x = \pi/3$

$$x - \cancel{\frac{\pi}{3}} = h$$

$x = h + \pi/3$ where $h \to 0$

$$f(\pi/3 + h) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\lim_{h\to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

{using 040 $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$}

$$\lim_{h\to 0} \frac{\sqrt{3} - \dfrac{\tan \pi/3 + \tan h}{1 - \tan \pi/3 \cdot \tan h}}{\pi - \pi - 3h}$$

$$\lim_{h\to 0} \frac{\dfrac{\sqrt{3}\left(1 - \tan\frac{\pi}{3} \cdot \tan h\right) - \left(\tan\frac{\pi}{3} + \tan h\right)}{1 - \tan \pi/3 \cdot \tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{(\sqrt{3} - \sqrt{3} \times \sqrt{3} \cdot \tan h) - (\sqrt{3} + \tan h)}{1 - \tan \pi/3 \cdot \tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{(\cancel{\sqrt{3}} - 3\tan h \cancel{-\sqrt{3}} - \tan h)}{1 - \sqrt{3}\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{-4\tan h}{-3h(1 - \sqrt{3}\tan h)}$$

$$\lim_{h\to 0} \frac{4\tan h}{3h(1-\sqrt{3}\tan h)}$$

$$\frac{4}{3}\lim_{h\to 0}\frac{\tan h}{h} \cdot \lim_{h\to 0}\frac{1}{(1-\sqrt{3}\tan h)}$$

$$= \frac{4}{3}\cdot\frac{1}{(1-\sqrt{3}(0))}$$

$$= \frac{4}{3}(1/1) = \underline{\underline{4/3}}.$$

(7)

(i) $f(x) = \left.\begin{cases} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ = 9 & x = 0 \end{cases}\right\}$ at $x = 0$

$$f(x) = \frac{1-\cos 3x}{x\tan x}$$

$$\lim_{x\to 0} = \frac{2\sin^2 3/2\,x}{x\tan x}$$

$$\lim_{x\to 0} = \frac{\dfrac{2\sin^2\frac{3x}{2}}{x^2}\times x^2}{\dfrac{x\cdot\tan x}{x^2}\times x^2}$$

$$\lim_{x\to 0}\frac{(3/2)^2}{1} = 2\times\frac{9}{4} = 9/2$$

$$\lim_{x\to 0} f(x) = \frac{9}{2} \qquad 9 = f(0)$$

$\therefore f$ is not continous at $x = 0$

Redefine function

$f(x) = \left\{\begin{array}{ll} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ \dfrac{9}{2} & x = 0 \end{array}\right\}$

Now $\lim_{x\to 0} f(x) = f(0)$

f has removable discontunity at $x = 0$

7 (ii)

$f(x) = \left.\begin{cases} \dfrac{(e^{3x}-1)\sin x^{\circ}}{x^2} & x \neq 0 \\ = \dfrac{\pi}{6} & x = 0 \end{cases}\right\}$ at $x = 0$

$$\lim_{x\to 0}\frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$\lim_{x\to 0} \frac{e^{3x}-1}{x} \cdot \lim_{x\to 0}\frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

eq $= \lim_{x \to 0} \dfrac{2\sin^2 x/2}{x^2}$

eq $= 2 \lim_{x \to 0} \left(\dfrac{\sin x/2}{x}\right)^2 =$

multiplying with 2 at numerator & denominator

$= 1 + 2 \times 1/4 = 3/2 = \underline{\underline{f(0)}}$

(7) $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $\quad x = \pi/2$

f(0) is continuous at $x = \pi/2$

$$\lim_{x \to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$\lim_{x \to \pi/2} \frac{2 - 1 + \sin x}{\cos^2 x\,(\sqrt{2} + \sqrt{1+\sin x})}$$

$$\lim_{x \to \pi/2} \frac{1 + \sin x}{1 - \sin^2 x\,(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x \to \pi/2} \frac{1}{(1 - \sin x)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \frac{1}{2(\sqrt{2}+\sqrt{2})} = \frac{1}{2(2\sqrt{2})} = \frac{1}{4\sqrt{2}}$$

09/12/19

$\therefore f(\pi/2) = 1/4\sqrt{2}$

Practical · 02 ·

TOPIC : Derivation . .

Q1) Show that the following function defined from IR to IR are differentiable.

(i) $\cot x$ (ii) $\operatorname{cosec} x$ (iii) $\sec x$

Q2) If $f(x) = 4x+1$, $x \le 2$
$= x^2+1$, $x > 0$, at $x = 2$ then

find f is differentiable or not?

Q3) If $f(x) = 4x+7$; $x < 3$
$= x^2+3x+1$; $x \ge 3$ at $x = 3$, then find f

is differentiable or not?

Q4) If $f(x) = 8x-5$, $x \le 2$
$= 3x^2-4x+7$, $x > 2$ at $x = 2$ then find

is differentiable or not.

Q1

(i) $\cot x$

$f(x) = \cot x$

$$f(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x-a}$$

$$= \lim_{x \to a} \frac{1/\tan x - 1/\tan x}{x-a}$$

$$= \lim_{x \to 0} \frac{\tan a - \tan x}{1 \cdot (-a) \tan x \cdot \tan a}$$

put $x - a = h$
$x = a + h$
as $x \to a$, $h \to a$

$$f(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a) \tan(a+h) \cdot \tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h) \cdot \tan a}$$

Formula: $\tan(A-B) = \dfrac{\tan A - \tan B}{1 - \tan A \cdot \tan B}$

$\tan A \cdot \tan B = \tan(A-B)\,(1 + \tan A \cdot \tan B)$

(iii) $\sec x$

$f(x) = \sec x$

$$D.f(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to a} \frac{\sec x - \sec a}{x - a}$$

$$= \lim_{x \to a} \frac{1/\cos x - 1/\cos a}{x-a}$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a) \cdot \cos a \cdot \cos x}$$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$D.f(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cdot \cos(a+h)}$$

Formula:- $-2\sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$

$$\lim_{h \to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right) \cdot \sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cdot \cos(a+h)}$$

$$\lim_{h \to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right) \cdot \sin -\frac{h}{2}}{\cos a \cdot \cos(a+h) \times -h/2} \times -\frac{1}{2}$$

$$= \frac{-1}{2} \times \frac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cdot \cos(a+0)}$$

$$= \frac{-1}{2} \times 2 \frac{\sin a}{\cos a \times \cos a}$$

$= \tan a \cdot \sec a.$

(Q2) $= 4x + 1$, $x \leq 2$
$= x^2 + 5$, $x > 0$ } $x = 2$

LHD:

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x + 1 - (4 \times 2 + 1)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4x + 1 - 9}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x - 8}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4(x-2)}{(x-2)} = 4$$

$Df(2^-) = 4$

RHD:

$Df(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$

$\lim_{x \to 2^+} \frac{x^2-4}{x-2}$

$\lim_{x \to 2^+} \frac{(x+2)(x-2)}{(x-2)}$

$= 2+2 = 4$

$Df(2^+) = 4$

$\therefore$ RHD = LHD

f is differentiable at $x = 2$

(Q.3) $f(x) = 4x+7, \quad x < 3$
$\quad x^2+3x+1, \quad x \geq 3 \Big\} \; x = 3$

Soln:

RHD:

$Df(3^+) = \lim_{x \to 3^+} \frac{f(x)-f(3)}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x+1-(3^2+3\times3+1)}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+6x-3x-18}{x-3}$

$= \lim_{x \to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$

$= \lim_{x \to 3^+} \frac{(x+6)(x-3)}{x-3} = 3+6 = 9$

$Df(3^+) = 9$

LHD = $Df(3^-)$

$= \lim_{x \to 3^-} \frac{f(x)-f(3)}{x-3}$

$= \lim_{x \to 3^-} \frac{4x+7-19}{x-3}$

$= \lim_{x \to 3^-} \frac{4x-12}{x-3}$

$= \lim_{x \to 3} \frac{4(x-3)}{(x-3)}$

$Df(3^+) = 4$

RHD $\neq$ LHD

F is not differentiable at $x=3$

(Q4) If $f(x) = 8x-5, \quad x \leq 2$
$= 3x^2-4x+7, \quad x > 2 \Big\} \; x = 2$

Soln:-

$f(2) = 8\times2-5 = 16-5 = 11$

RHD:

$Df(2^+) = \lim_{x \to 2^+} \frac{f(x)-f(2)}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x - 2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x - 2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x - 2}$

$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x - 2}$

$= \lim_{x \to 2^+} \frac{(3x+2)(x-2)}{(x-2)}$

$= 3 \times 2 + 2 = 8$

$D.f(2^+) = 8$

LHD:

$D.f(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$

$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$

$= \lim_{x \to 2^-} \frac{8x - 16}{x - 2}$

$= \lim_{x \to 2^-} \frac{8(x-2)}{(x-2)}$

$= 8$

LHS = RHS

f is differentiable at $x = 3$.

AK 27/01/2020

Practical 3 ::

TOPIC : Application of Derivatives.

(1) Find the intervals in which function is increasing or decreasing.

(i) $f(x) = x^3 - 5x - 11$
(ii) $f(x) = x^2 - 4x$
(iii) $f(x) = 2x^3 + x^2 - 20x + 4$
(iv) $f(x) = x^3 - 27x + 5$
(v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

(2) Find the intervals in which function is concave upwards.

1) $y = 3x^2 - 2x^3$
2) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$
3) $y = x^3 - 27x + 5$
4) $y = 69 - 24x - 9x^2 + 2x^3$
5) $y = 2x^3 + x^2 - 20x + 4$

Soln:-

(Q1)

(i) $f(x) = x^3 - 5x - 11$

$\therefore f'(x) = 3x^2 - 5$

$\therefore$ f is increasing iff $f'(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x-\sqrt{5/3})(x+\sqrt{5/3})>0$

$x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

and f is decreasing iff $f'(x)<0$

$3x^2-5<0$

$3(x^2-5/3)<0$

$(x-\sqrt{5/3})(x+\sqrt{5/3})<0$

$x \in (-\sqrt{5/3}, \sqrt{5/3})$

12) $f(x) = x^2-4x$

$f'(x) = 2x-4$

$\therefore f(x)$ is increasing iff $f'(x)>0$

$2x-4>0$

$2(x-2)>0$

$x-2>0$

$x \in (2, \infty)$

and f is decreasing iff $f'(x)<0$

$\therefore 2x-4<0$

$2(x-2)<0$

$x-2<0$

$x \in (-\infty, 2)$

13) $f(x) = 2x^3+x^2-20x+4$

$f'(x) = 6x^2+2x-20$

$\therefore f$ is increasing iff $f'(x)>0$

$\therefore 6x^2+2x-20>0$

$2(3x^2+x-10)>0$

$3x^2+x-10>0$

$3x(x+2)-5(x+2)$

$(3x-5)(x+2)>0$

$x \in (-\infty, -2) \cup (5/3, \infty)$

and f is decreasing iff $f'(x)<0$

$\therefore 6x^2+2x-20<0$

$2(3x^2+x-10)<0$

$3x^2+x-10<0$

$3x^2+6x-5x-10<0$

$3x(x+2)-5(x+2)<0$

$(x+2)(3x-5)<0$

$x \in (-2, 5/3)$

14) $f(x) = x^3-27x+5$

$f'(x) = 3x^2-27$

$\therefore f$ is increasing iff $f'(x)>0$

$3(x^2-9)>0$

$(x-3)(x+3)>0$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$3(x^2 - 9) < 0$

$(x-3)(x+3) < 0$

$+ \quad - \quad +$

$-3 \quad 3$

$x \in (-3, 3)$.

15) $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 - 18x - 24 > 0$

$6(x^2 - 3x - 4) > 0$

$x^2 - 4x + x - 4 > 0$

$x(x-4) + 1(x-4) > 0$

$(x-4)(x+1) > 0$

$+ \quad - \quad +$

$-1 \quad 4$

$x \in (-\infty, -1) \cup (4, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 - 18x - 24 < 0$

$6(x^2 - 3x - 4) < 0$

$x^2 - 4x + 1(x-4) < 0$

$(x-4)(x+1) < 0$

$+ \quad - \quad +$

$x \in (-1, 4)$

Q2

(1) $y = 3x^2 - 2x^3$

$\therefore f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

f is concave upward if $f''(x) > 0$

$(6 - 12x) > 0$

$12(6/12 - x) > 0$

$x - 1/2 > 0$

$x > 1/2$

$\therefore f''(x) > 0$

$x \in (1/2 - \infty)$

(2) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward if $f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$12(x^2 - 3x + 2) > 0$

$x^2 - 2x - x + 2 > 0$

$x(x-2) - 1(x-2) > 0$

$(x-2)(x-1) > 0$

$+ \quad - \quad +$

$1 \quad 2$

$x \in (-\infty, 1) \cup (2, \infty)$

$y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$

$\therefore x \in (0, \infty)$

(4) $y = 69 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 64$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x + 8$

f is concave upward iff $f''(x) > 0$

$\therefore 12x - 18 > 0$

$\therefore 12(x - 18/12) > 0$

$\therefore x - 3/2 > 0 \quad \therefore x > 3/2$

$\therefore x \in (3/2, \infty)$

5. $y \neq 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$12x + 2 > 0$

$12(x + 2/12) > 0$

(ii) $f(x) = 3 - 5x^3 + 3x^5$

$\therefore f'(x) = -15x^2 + 15x^4$

Consider,

$f'(x) = 0$

$-15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$f''(1) = -30 + 60$

$= 30 > 0$

$\therefore f$ has minimum value at $x = 1$

$\therefore f(1) = 3 - (5)(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$

$\therefore f$ has maximum value at $x = -1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3$

$= 5$

$\therefore f$ has the maximum value 5 at $x = -1$ and the minimum value 1 at $x = 1$

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Practical 4

Aim:- Application of Derivative of Newton's method

Q1

(i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - \frac{32}{x^3}$

Now consider,

$f'(x) = 0$

$2x - \frac{32}{x^3} = 0$

$2x = \frac{32}{x^3}$

$x^4 = \frac{32}{2}$

$x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + 96/2^4$

$= 2 + \frac{96}{16}$

$= 2 + 6 = 8 > 0$

f has a minimum value at $x = 2$

$\therefore f(2) = 2^2 + \frac{16}{2^2}$

$= 4 + 16/4$

$= 4 + 4$

$= 8$

$f''(-2) = 2 + \frac{96}{-2^4}$

$= 2 + \frac{96}{16}$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = -2$

$\therefore$ Function reaches minimum value at $x = 2$ and $x = -2$

Q2.

(i) $f(x) = x^3 - 3x^2 - 55x + 9.5$

$f'(x) = 3x^2 - 6x - 55$

By Newton's method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 0 + \frac{9.5}{55}$$

$x_1 = 0.1727$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727) - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$f'(x_1) = 3(0.1727)^3 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= -55.9467$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.1727 - \frac{0.0829}{-55.9467}$$

$= 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) + 55$

$= 0.0879 - 1.0272 - 55$

$= \underline{-55.9393}$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.1712 + \frac{0.0011}{55.9393}$$

$\boxed{= 0.1712}$

The root of the equation is $\underline{0.1712}$

(ii) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial approximation

$\therefore$ By Newton's method;

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 3 - \frac{6}{23}$$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$
$= 20.5528 - 10.9568 - 9$
$= 0.596$
$f'(x_1) = 3(2.7392)^2 - 4$
$= 22.5096 - 4$
$= 18.5096$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 2.7392 - \frac{0.596}{18.5096}$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$
$= 19.8386 - 10.8284 - 9$
$= 0.0102$
$f'(x_2) = 3(2.7071)^2 - 4$
$= 21.9851 - 4$
$= 17.9851$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 2.7071 - \frac{0.0102}{17.9851}$

$= 2.7071 - 0.0056$
$= 2.7015$

$f'(x_3) = 3(2.7015)^2 - 4$
$= 21.8943 - 4$
$= 17.8943$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$
$= 19.7158 - 10.806 - 9$
$= -0.0901$

$x_4 = 2.7015 + \frac{0.0901}{17.8943}$

$= 2.7015 + 0.0050$
$= \boxed{2.7065}$

(3) $f(x) = x^3 - 1.8x^2 - 10x + 17$
$f'(x) = 3x^2 - 3.6x - 10$
$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$
$= 1.8 - 10 + 17$
$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$
$= 8.7.2 - 20 + 17$
$= -2.2$

Let $x_0 = 2$ be initial approximation
By Newton's method

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = x_0 - \frac{f(x_2)}{f'(x_2)}$

$= 2 - \frac{2.2}{5.2}$

$= 2 - 0.4230 = 1.577$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$
$= 3.9219 - 4.4764 - 15.77 + 17$
$= 0.6755$
$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$
$= 7.4608 - 5.6772 - 10$
$= -8.2164$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 1.577 + \frac{0.6755}{8.2164}$

$= 1.577 + 0.0822$
$= 1.6592$
$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$
$= 4.5892 - 4.9708 - 16.618 + 17$
$= 0.0004$
$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$
$= 8.2347 - 5.9$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$
$= 8.2847 - 5.9824 - 10$
$= -7.6977$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

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$= 1.8618 + \frac{0.0004}{7.6977} = \boxed{1.6618}$

equation is 1.6618.

Practical 5

Aim:- Integration.

(i) $\int \frac{dx}{\sqrt{x^2+2x-3}}$

$= \int \frac{dx}{\sqrt{x^2+2x-3}}$

$= \int \frac{1}{\sqrt{x^2+2x(1)-3}} \, dx$

$= \int \frac{1}{\sqrt{(x^2+2x(1))-3}} \, dx$

$= \int \frac{1}{\sqrt{x^2+2x(1)+(1)^2-(1)^2-3}} \, dx$ (Adding and subtracting)

$= \int \frac{1}{\sqrt{(x+1)^2-(1)^2-3}} \, dx$

$= \int \frac{1}{\sqrt{(x+1)^2-4}} \, dx$

$= \int \frac{1}{\sqrt{(x+1)^2-(2)^2}} \, dx$

$= \log|x + \sqrt{x^2+a^2}| + c.$

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substitute $t4$ with $u/2$.

$= \int \frac{u/2 \times \sin(u)}{8}\, du$

$= \int \frac{u \times \sin(u)}{2} / 8 \, du$

$= \int \frac{u \times \sin(u)}{16} \, du$

$= \frac{1}{16} \int u \times \sin(u)\, du$

$= \frac{1}{16} \left(u \times (-\cos u)\right) - \int -\cos(u)\, du$

$= \frac{1}{16} \times \left(u \times (-\cos u)\right) + \int \cos(u)\, du$

$= \frac{1}{16} \times \left(u \times (-\cos(u)) + \sin(u)\right)$

Return the substitution $u = 2t4$

$= 1/16 \times (2t4 \times (-\cos(2t4)) + \sin(2t4))$

$= -\frac{t4 \times \cos(2t4)}{8} + \frac{\sin(2t4)}{16} + C.$

i) $\int \sqrt{x}\,(x^2 - 1)\, dx$

$\int \sqrt{x}\, x^2 - \sqrt{x}\, dx$

$\int x^{1/2} \times x^2 - x^{1/2}\, dx$

$\int x^{5/2} - x^{1/2}\, dx$

$\int x^{5/2} - \int x^{1/2}\, dx$

$I1 = \frac{x^{5/2+1}}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7} = \frac{2x^3\sqrt{x}}{7}$

$I2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x^3}}{3}$

$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + C$

ii) $\int \frac{\cos x}{\sqrt[3]{\sin(x)^2}}\, dx$

$= \int \frac{\cos x}{\sin x^{2/3}}\, dx$

put $t = \sin(x)$

$t = \cos x$

$= \int \frac{\cos(x)}{\sin(x)^{3/2}} \times \frac{1}{\cos(x)}\, dt.$

$= \frac{1}{\sin x^{3/2}}\, dt$

$= \frac{1}{t^{2/3}}\, dt$

$x = \int \frac{1}{t^{2/3}}\, dt = \frac{-1}{(2/3 - 1)\, t^{2/3-1}}$

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$= \frac{-1}{-1/3\, t^{2/3-1}} = \frac{1}{1/3\, t^{-1/3}} = \frac{t^{1/3}}{1/3} = 3t^{1/3}$

$= 3\sqrt[3]{t}$

return substitution $t = \sin(x)$

$= 3\sqrt[3]{\sin(x)} + C$.

X $\int \frac{x^2-2x}{x^3-3x^2+1}\,dx$

put $x^3-3x^2+1 = dt$

$I = \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-3\times 2x}\,dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-6x}\,dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3(x^2-2x)}\,dt$

$= \int \frac{1}{x^3-3x^2+1} \times \frac{1}{3}\,dt$

$= \int \frac{1}{3(x^3-3x^2+1)}\,dt = \int \frac{1}{3t}\,dt$

$= \frac{1}{3}\int \frac{1}{t}\,dt \qquad \int \frac{1}{x}\,dx = \ln|x| + C$

$= \frac{1}{3}\times$

$=$

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$\int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

$\Rightarrow I = \int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

Let $\frac{1}{x^2} = t$

$x^{-2} = t$

$-\frac{2}{x^3}\,dx = dt$

$I = \frac{-1}{2}\int -\frac{2}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

$= -\frac{1}{2}\int \sin t$.

$= \frac{-1}{2}(-\cos t) + C$

$= \frac{1}{2}\cos t + C$

Resubstitution $t = 1/x^2$

$I = \frac{1}{2}\cos(1/x^2) + C$

$\Rightarrow \cancel{\int \frac{x^2-2x}{}}\ \int \cos^2 x \cdot \sin 2x\,dx$

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$= \int \cos^2 x \cdot \sin 2x\,dx$

$\cos^2 x = t$

$-2\cos x \cdot \sin x\,dx = dt$

$= -2\sin 2x\,dx = dt$

Practical 6

Aim :- Application of integration and numerical integration.

Find the length of the given curve.

$x = t - \sin t$, $y = 1 - \cos t$ $t \in [0, 2\pi]$

$$= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$x = t - \sin t$

$\frac{dx}{dt} = 1 - \cos t$

$y = 1 - \cos t$

$\frac{dy}{dt} = 0 - (-\sin t)$

$\frac{dy}{dt} = \sin t$

$$L = \int_0^{2\pi} \sqrt{(1 - \cos t + \sin t)^2 + (\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t}$$

$$= \int_0^{2\pi} \sqrt{2(1 - \cos t)}$$

$$= \int_0^4 \sqrt{\frac{4+9x}{4}}\, dx$$

$$= \frac{1}{2} \int_0^4 \sqrt{4+9x}\ dx$$

$$= \frac{1}{2} \left[\frac{(4+9x)^{1/2+1}}{1/2+1} \right]_0^4 \cdot dx$$

$$= \frac{1}{27} \left[(4 \times 9x)^{3/2} \right]_0^4$$

$$= \frac{1}{27} \left[(4+0)^{3/2} - (4+36)^{3/2} \right]$$

$$= \frac{1}{27} (4)^{3/2} - (40)^{3/2}$$

14) $\frac{dx}{dy} = 3\cos t$, $\frac{dy}{dx} = -3\sin t$

$$= \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\ dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\ dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\ \ dt$$

$$= \int_0^{2\pi} \sqrt{9(\sin^2 t + \cos^2 t)}\ dt$$

$$= \int_0^{2\pi} \sqrt{9(1)}\ dt$$

$$= \int_0^{2\pi} 3\ dt$$

$$= 3 \int_0^{2\pi} dt$$

$$= 3[x]_0^{2\pi}$$

$$= 3(2\pi - 0)$$

$$= 6\pi$$

15) $\frac{dx}{dy} = \frac{1}{6} \frac{d}{dy}(y^3) + 1/2 \frac{dx}{dy}(1/y)$

$$= \frac{1}{6} 3y^2 + \frac{1}{x}\left[-1/y^2\right)$$

$$= \frac{y^2}{2} - \frac{1}{2y^2}$$

$$= \frac{y^4-1}{2y^2}$$

$$= \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}$$

$$= \int_1^2 \sqrt{1 + \left(\frac{y^4-1}{2y^2}\right)^2}\ dy$$

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$$\int_1^2 \sqrt{\frac{(y^4+1)^2}{4y^4}}\, dy$$

$$= \int_1^2 \frac{(y^4+1)}{(2y^2)^2}\, dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$$

$$= \int_1^2 \frac{y^4}{2y^2}\, dy + \int_1^2 1/2y^2\, dy$$

$$= \frac{1}{2}\int_1^2 y\, dy + 1/2 \int \frac{1}{y^2}\, dy$$

$$= \frac{1}{2}\left[\frac{y^3}{3} - y^{-2}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{7}{3} + \frac{1}{2}\right]$$

$$= \frac{1}{2}\left[\frac{17}{6}\right]$$

$$= \frac{17}{12} \text{ units}$$

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12) ii) $\int_0^2 e^{x^2}\, dx$ with $n = 4$.

Soln:- $\int_0^4 x^2\, dx$

$h = \frac{4-0}{4} = 1$

x	0	1	2	3	4
y	0	1	4	9	16

$$\int_0^4 x^2\, dx = \frac{1}{3}[(y_0+y_3) + 4(y_1+y_2)\ 2(y_2)]$$

$$= \frac{1}{3}[16 + 4(10) + 8]$$

$$= \frac{64}{3}$$

$$\int_0^4 x^2\, dx = 21.533$$

Q III $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ with $n = 6$

$$h = \frac{\pi/3 - 0}{6} = \pi/18$$

$$\int_0^{\pi/2} \sqrt{\sin x} = \frac{h}{3}\left[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$= $ [illegible]

$$= \int_0^{\pi/2} \sqrt{\sin x}\, dx = \text{[illegible]}$$

Linear Differential Equation

Solve the following

(1) $\dfrac{dy}{dx} + \dfrac{1}{x} y = \dfrac{e^x}{x}$

$P(x) = \dfrac{1}{x}$ $\quad Q(x) = \dfrac{e^x}{x}$

$I.F. = e^{\int \frac{1}{x} dx}$

$y(IF) = \int Q(x)(IF)\, dx + c$

$= \int \dfrac{e^x}{x} \cdot x \, dx + c$

$= \int e^x \, dx + c$

$xy = e^x + c$

(2) $e^x \dfrac{dy}{dx} + 2e^x y = 1$

$\dfrac{dy}{dx} + 2\dfrac{e^x}{e^x} y = \dfrac{1}{e^x}$

$\dfrac{dy}{dx} + 2y = \dfrac{1}{e^x}$

$\dfrac{dy}{dx} + 2y = e^{-x}$

$y(IF) = \int Q(x)\,(IF)\,dx + C$.

$= \int 2x \cdot e^{-2x}\ e^{2x} + C$

$y e^{2x} = \int 2x + C = \underline{\underline{x^2 + C}}$.

6) $\sec^2 x \cdot \tan y\, dx + \sec^2 y \tan^2 y \tan x\, dy = 0$

$\sec^2 x \cdot \tan y\, dx = -\sec^2 y \tan x\, dy$

$= \dfrac{\sec^2 x}{\tan x}\, dx = -\dfrac{\sec^2 x}{\tan y}\, dy$

$= \int \dfrac{\sec^2 x}{\tan x}\, dx = -\int \dfrac{\sec^2 x}{\tan y}\, dy$

$= \log|\tan x| = -\log|\tan y| + C$

$= \log|\tan x + \tan y| = C$

$= \tan x + \tan y = e^C$

7) $\dfrac{dy}{dx} = \sin^2(x - y + 1)$

put $x - y + 1 = v$

$x - y + 1 = v$

$1 - \dfrac{dy}{dx} = \dfrac{dv}{dx}$

$\dfrac{1 - dv}{dx} = \dfrac{dy}{dx}$

$1 - \dfrac{dv}{dx} = \sin^2 v$

$\dfrac{dv}{dx} = 1 - \sin^2 v$

$\dfrac{dv}{dx} = \cos^2 v$

$\dfrac{dv}{\cos^2 x} = dx$

$\int \sec^2 x\, dv = \int dx$

$\tan v = x + C$

18) $\dfrac{dy}{dx} = \dfrac{2x + 3y - 1}{6x + 4y + 6}$

Soln:-

Put $2x + 3y = u$

$2 + 3\dfrac{dy}{dx} = \dfrac{du}{dx}$

$\dfrac{dy}{dx} = \dfrac{1}{3}\left(\dfrac{du}{dx} - 2\right)$

$= \dfrac{1}{3}\left(\dfrac{dv}{x} - 2\right) = \dfrac{1}{3}\left(\dfrac{v-1}{v+2}\right)$

$\dfrac{du}{dx} = \dfrac{v-1}{v+2} + 2$

$\dfrac{du}{dx} = \dfrac{v - 1 + 2v + 4}{v + 2}$

$= \frac{3u+3}{u+2}$

$= \frac{3(u+1)}{u+2}$

$= \int \frac{u+2}{v+1} \, dv = 3\,dx$

$= \int \frac{v+1}{u} \, dx + \int \frac{1}{v+1} \, dv = \int 3\,dx$

$v + \log|v| = 3x + C$

$2x + 3y + \log|2x + 3y + 1| = 3x + C$

$3y = x - \log|2x + 3y + 1| + C$

Practical - 8

Aim:- Euler's Method

(Q.1) $\frac{dy}{dx} = y + e^{x} - 2$ $\quad y(0) = 2 \quad h = 0.5$ find $y(2)$

Soln:-

$f(x) = y + e^{x} - 2$ $\quad x_0 = 0, \ y(0) = 2 \quad h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$\therefore y(2) = 9.8215$

(162) $\frac{dy}{dx} = 1+y^2$, $y(0)=0$, $h=0.2$ find $y(1)$?

$y_0 = 0$, $x_0 = 0$, $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.4111	0.9234
4	0.8	0.9234	1.8526	1.2939
5	1	1.2939		

$y(1) = \underline{1.2939}$

(163) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0)=1$, $h=0.2$ find $y(1)$?

$x_0 = 0$, $y(0) = 1$, $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2105
3	0.6	1.2105	0.7040	1.3513
4	0.8	1.3513	0.7694	1.5051
5	1	1.5051		

$\therefore y(1) = \underline{1.5051}$

) $\frac{dy}{dx} = 3x^2+1$ $y(1) = 2$ find $y(2)$ $h = 0.5$ & $h = 0.25$

Soln: $y_0 = 2$ $x_0 = 1$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.75	7.875
2	2	7.875		

$\therefore y(2) = 7.875$

) $y_0 = 2$ $x_0 = 1$ $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4219
2	1.5	4.4219	59.6569	19.3360
3	1.75	19.3360	1122.4426	299.9966
4	2	299.9960		

$y(2) = \underline{299.9960}$

067

(Q5) $\frac{dy}{dx} = \sqrt{xy} + 2 \quad y(1) = 1, \; h = 0.2$

$x_0 = 1, \; y_0 = 1 \quad h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	~~3.6~~ 1.6
1	1.2	~~3.6~~ 1.6		

$y(1.2) = \cancel{3.6}\; 1.6$

AK 20/01/2020

Aim:- Limits and Partial order derivatives

068

(Q1) Evaluate the following limits.

1) $\lim_{(x,y)\to(-4,-1)} \frac{x^3 - 3x + y^2 - 1}{xy + 5}$

$= \lim_{(x,y)\to(-4,-1)} \frac{x^3 - 3x + y^2 - 1}{xy + 5}$

Apply limit

$= \frac{(-4)^3 - 3(-4) + (-1)^2 - 1}{(-4)(-1) + 5}$

$= \frac{-64 + 12 + 1 - 1}{4 + 5} = \frac{-52}{9}$

(2) $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$

$= \lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$

Apply limit

$= \frac{(0+1)(2)^2 + (0)^2 - 4(2)}{2 + 3(0)}$

$= \frac{1(4 + 0 - 8)}{2}$

$= \frac{4-8}{2} = \frac{-4}{2} = -2$

(iii) $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2 - y^2 z^2}{x^3 - x^2 yz}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x^2 - y^2 z^2}{x^3 - x^2 yz}$

Apply Limit

$= \frac{(1)^2 - (1)^2 (1)^2}{(1)^3 - (1)^2 (1)(1)}$

$= \frac{1-1}{1-1} = \frac{0}{0}$

$\therefore$ Limit does not exist.

2) Find f_x, f_y for each of the following

i) $f(x,y) = xy e^{x^2+y^2}$

$f_x = y(1 \cdot e^{x^2+y^2}) + xy(e^{x^2+y^2} \cdot 2x)$

$= y e^{x^2+y^2} + 2x^2 y e^{x^2+y^2}$

$f_y = x(1 \cdot e^{x^2+y^2}) + xy(e^{x^2+y^2} \cdot 2y)$

$= x e^{x^2+y^2} + 2xy^2 e^{x^2+y^2}$

$f_x = y e^{x^2+y^2} + 2x^2 y e^{x^2+y^2}$

$f_y = x e^{x^2+y^2} + 2xy^2 e^{x^2+y^2}$

(ii) $f(x,y) = e^x \cos y$

$= f_x' = \cos y \, e^x$

$= f_y = e^x \cdot -\sin y$

$\therefore f_y = -\sin y \, e^x$

(iii) $f(x,y) = x^3 y^2 - 3x^2 y + y^3 + 1$

$f_x = y^2 \, 3x^2 - 3y \, 2x + 0 + 0$

$= 3x^2 y^2 - 6xy$

$f_y = x^3 \, 2y - 3x^2 + 3y^2$

$= 2x^3 y - 3x^2 + 3y^2$

3) Using definition find values of f_x, f_y at $(0,0)$ for $f(x,y) = \frac{2x}{1+y^2}$

$f_x(a,b) = \lim_{h\to 0} \frac{f(a+h, b) - f(a,b)}{h}$

$f_y(a,b) = \lim_{h\to 0} \frac{f(a, b+h) - f(a,b)}{h}$

According to given $(a,b) = (0,0)$

ean

$$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{2h - 0}{h} = \underline{\underline{2}}$$

$$f_y(0,0) = \lim_{h\to 0} \frac{f(0,h) - f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{0-0}{h} = 0$$

$\therefore f(x) = 2, \; f(y) = 0$

(Q4) Find all second order partial derivatives of f. Also verify whether $f_{xy} = f_{yx}$

$\therefore f_{xx} = \frac{d^2 f}{dx^2} \quad f_{yy} = \frac{d^2 f}{dy^2}$

Applying u/v rule

$$f_x = \frac{x^2(0-y) - (y^2 - xy)2x}{x^4}$$

$$= \frac{-x^2 y - 2xy^2 + 2x^2 y}{x^4}$$

$$\therefore f_x = \frac{x^2 y - 2xy^2}{x^4}$$

$$\therefore f_x = \frac{x^2 y - 2xy^2}{x^4}$$

$$f_{xx} = \frac{x^4(2xy - 2y^2) - (x^2 y - 2xy^2)(4x^3)}{x^8}$$

$$= \frac{2x^5 y - 2x^4 y^2 - (4x^5 y - 8x^4 y^2)}{x^8}$$

$$= \frac{2x^5 y - 2x^4 y^2 - (4x^5 y - 8x^4 y^2)}{x^8}$$

$$= \frac{2x^5 y - 2x^4 y^2 - 4x^5 y + 8x^4 y^2}{x^8}$$

$$= \frac{-2x^5 y + 6x^4 y^2}{x^8}$$

$$= \frac{6x^4 y^2 - 2x^5 y}{x^8}$$

$$f_{xx} = \frac{6y^2 - 2xy}{x^4}$$

$$f_y = \frac{1}{x^2}(2y - x)$$

$$f_{yy} = \frac{1}{x^2} \cdot 2 = \frac{2}{x^2}$$

$$f_{xy} = \frac{2y - x}{x^2}$$

$$= \frac{x^2(-1) - (2y - x)(2x)}{x^4}$$

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55 Find the linearization of $f(x,y)$ at given point.

(i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f(1,1) = \sqrt{1+1} = \sqrt{2}$

$f_x(1,1) = (x^2+y^2)^{1/2}$

$= \frac{1}{2\sqrt{x^2+y^2}} \, 2x + 0$

$= \frac{1}{\cancel{2}\sqrt{2}} \cancel{2} = \frac{1}{\sqrt{2}}$

$f_y(1,1) = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2y$

$= \frac{1}{\cancel{2}\sqrt{y}\cancel{2}\sqrt{2}} \cancel{2}y$

$= \frac{1}{\sqrt{2}}$

$L(x,y) = \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{x-1}{\sqrt{2}} + \frac{y-1}{\sqrt{2}}$

$= \sqrt{2} + \frac{x-1+y-1}{\sqrt{2}}$

$= \frac{2+x+y-1}{\sqrt{2}} = \frac{x+y}{\sqrt{2}}$

ii) $f(x,y) = 1-x+y \sin x$ at $(\pi/2, 0)$

$f(\pi/2, 0) = 1 - \frac{\pi}{2} + 0 \sin \pi/2$

$= 1 - \pi/2$

$f_x = 0 - 1 + \cos x$

$= \cos x - 1$

$= 0 - 1 = -1$

$f_y = 0 - 0 + \sin x$

$= \sin \pi/2 = 1$

$L(x,y) = 1 - \pi/2 + (-1)(x - \pi/2) + 1(y-0)$

$= 1 - \cancel{\frac{\pi}{2}} - x + \cancel{\frac{\pi}{2}} + y$

$= 1 - x + y$

iii) $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = 0$

$f_x = \frac{1}{x} + 0$

$= \frac{1}{1} = 1$

$f_y = \frac{1}{1} = 1$

$L(x,y) = 0 + (1)(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$ //

Practical - 10

Aim: Directional derivatives, gradient vector making, tangent & normal vector.

i) $f(x,y) = x + 2y - 3$ $a = (1,-1)$ $u = 3i - j$

Here, $u = 3i - j$ is not a unit vector

$|u| = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) : f(1,-1) = (1) + 2(-1) - 3 = 1 - 2 - 3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f\left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{h}{\sqrt{10}}\right)$

$f(a+hu) = \left(1 + \frac{3}{\sqrt{10}}\right) + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$

$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$

$= \frac{18h}{5} + 8$

$\therefore D_u f(a) = \lim_{h \to 0} \frac{\frac{18h}{5} + 8 - 8}{h}$

$= \frac{18h}{5}$

Q2

(i) $f(x,y) = x^y + y^x$

$f_x = y \cdot x^{y-1} + y^x \log y$

$f_y = x^y \log x + x y^{x-1}$

$\nabla f(x,y) = (f_x, f_y)$

$= (y x^{y-1} + y^x \log y, \ x^y \log x + x y^{x-1})$

$f(1,1) = (1+0, 1+0)$

$= (1,1)$

(ii) $f(x,y) = (\tan^{-1} x) \cdot y^2 \quad a = (1,-1)$

$f_x = \frac{1}{1+x^2} \cdot y^2$

$f_y = 2y \tan^{-1} x$

$\nabla f(x,y) = (f_x, f_y)$

$= (y^2/1+x^2, \ 2y \tan^{-1} x)$

$f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2)\right)$

$= \left(\frac{1}{2}, \frac{\pi}{4} \cdot \frac{(-2)}{1}\right)$

$= \left(\frac{1}{2}, -\frac{\pi}{2}\right)$

$f(x,y,z) = xyz - e^{x+y+z}, \quad a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, \ xz - e^{x+y+z}, \ xy - e^{x+y+z}$

$f(1,-1,0) = \left((-1)(0) - e^{1+(-1)+0}, \ (1)(0) - e^{1+(-1)+0}, \ (1)(-1) - e^{1+(-1)+0}\right)$

$= (0 - e^0, 0 - e^0, -1 - e^0)$

$= (-1, -1, -2)$

$x^2 \cos y + e^{xy} = 2 \quad at \ (1,0)$

$f_x = \cos y \, 2x + e^{xy} \, y$

$f_y = x^2(-\sin y) + e^{xy} \cdot x$

$(x_0, y_0) = (1, 0) \quad \therefore x_0 = 1,\ y_0 = 0$

eq^n of tanget
$f(x)(x - x_0) + f_y(y - y_0) = 0$

$f_x(x_0, y_0) = \cos 0 \cdot 2(1) + e^0 \cdot 0$
$= 1(2) + 0$
$= 2$

$f_y(x_0, y_0) = (1)(-\sin 0) + e^0 \cdot 1$
$= 0 + 1 \cdot 1$
$= 1$

$2(x-1) + 1(y-0) = 0$
$2x - 2 + y = 0$
$2x + y - 2 = 0$

eqn of normal
$= ax + by + c = 0$
$= bx + ay + d = 0$

$= 1(1) + 2(y) + d = 0$
$= 1 + 2y + d = 0$ at $(1, 0)$
$= 1 + 2(0) + d = 0$
$= d + 1 = 0$
$\therefore d = -1$

(iii) $x^2 + y \ldots + 2 = 0$ at $(2, -2)$
$f_x = 2x + 0 - 2 \cdot 1 + 0 + 0$
$= 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$
$= 2y + 3$

$(x_0, y_0) = (2, -2) \quad \therefore x_0 = 2,\ y_0 = -2$
$f_x(x_0, y_0) = 2(2) - 2 = 2$
$f_y(x_0, y_0) = 2(-2) + 3 = -1$

eq^n of tangent
$f_x(x - x_0) + f_y(y - y_0) = 0$
$2(x-2) + (-1)(y+2) = 0$
$2x - 2 - y - 2 = 0$
$2x - y - 4 = 0$ → It is required eg^n of tangent

Normal
$= ax + by + c = 0$
$bx + ay + b = 0$

$= -1(x) + 2(y) + d = 0$
$-x + 2y + d = 0$ at $(2, -2)$
$-2 + 2(-2) + d = 0$
$-2 - 4 + d = 0$
$-6 + d = 0$
$\therefore d = 6$

Find the local maxima & minima for the following

i $f(x,y) = 3x^2+y^2-3xy+6x-4y$

$f_x = 6x+0-3y+6-0$
$= 6x-3y+6$

$f_y = 0+2y-3x+0-4$
$= 2y-3x-4$

$f_x = 0$
$6x-3y+6=0$
$3(2x-y+2)=0$
$2x-y+2=0$
$2x-y=-2 \rightarrow ①$

$f_y = 0$
$2y-3x-4=0$
$2y-3x=4 \rightarrow ②$

multipy eq^n 1 with 2
$4x-\cancel{2y}=-4$
$\cancel{2y}-3x=4$
$x=0$

Substitute value of x in eq^n ①
$2(0)-y=-2$
$\cancel{-}y=\cancel{-}2 \quad \therefore y=2$
$\therefore$ Critical points are $(0,2)$

$r = f_{xx} = 6$
$t = f_{yy} = 2$
$s = f_{xy} = -3$
Here $r>0$
$= rt-s^2$
$= 6(2)-(-3)2$
$= 12-9$
$= 3>0$
$\therefore f$ has maximum at $(0,2)$
$3x^2+y^2-3xy+6x-4y$ at $(0,2)$
$3(0)^2+(2)^2-3(0)(2)+6(0)-4(2)$
$0+4-0+0-8$
$= -4$

ii $f(x,y) = 2x^4+3x^2y-y^2$
$f_x = 8x^3+6xy$
$f_y = 3x^2-2y$
$f_x = 0$
$\therefore 8x^3+6xy=0$
$2x(4x^2+3y)=0$
$4x^2+3y=0 \rightarrow ①$

$f_y = 0$
$3x^2-2y=0 \rightarrow ②$

multipy eq^n ① with 3
② with 4
$12x^2+9y=0$
$-12x^2-8y=0$
$1y=0$
$\therefore y=0$

Sub-stitute value of y in eq^n ①

$4x^2 + 3(0) = 0$

$4x^2 = 0$

$x = 0$

Critical point is $(0,0)$

$r = f_{xx} = 24x^2 + 6x$

$t = f_{yy} = 0 - 2 = -2$

$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

$\therefore r$ at $(0,0)$

$= 24(0) + 6(0) = 0$

$\therefore r = 0$

$rt - s^2 = 0(-2) - (s)^2$

$= 0 - 0 = 0$

$r = 0$ & $rt - s^2 = 0$

(nothing to say)

$f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0 \quad \therefore \quad 2x + 2 = 0$

$x = \frac{-2}{2} \quad \therefore x = -1$

$f_y = 0 \qquad -2y + 8 = 0$

$y = \frac{-8}{-2} 4 \qquad \therefore y = 4$

$\therefore$ Critical point is $(-1, 4)$

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$f(x,y)$ at $(-1,4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= 37 - 70 = -33$

AV
27/01/2020