Spring Representation and Calculations

Representation

A spring is represented by a line segment with endpoints $p_1(x_1, y_1)$ and $p_2(x_2, y_2)$. The initial length of the spring is L.

Vector Representation

The displacement vector $\mathbf{V}$ is defined as:

$$\begin{aligned} \Delta x &= x_2 - x_1 \\ \Delta y &= y_2 - y_1 \\ \mathbf{V} &= \Delta x\,\mathbf{i} + \Delta y\,\mathbf{j} \end{aligned}$$

The magnitude of $\mathbf{V}$ (current length M) is:

$$M = \|\mathbf{V}\| = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

The unit vector of $\mathbf{V}$ is:

$$\hat{\mathbf{V}} = \frac{\mathbf{V}}{M} = \frac{\Delta x}{M}\,\mathbf{i} + \frac{\Delta y}{M}\,\mathbf{j}$$

Force Calculation (Hooke's Law)

The restoring force F is given by Hooke's Law:

$$F = -k\,x$$

where k is the spring constant (assuming $k = 1$):

$$F = -x$$

The displacement x is the difference between the current length M and the initial length L:

$$x = M - L$$

Therefore, the force is:

$$F = -(M - L) = L - M$$

Force Vector

The force vector $\mathbf{F}$ is:

$$\begin{aligned}\mathbf{F} &= F\,\hat{\mathbf{V}} \\ &= (L - M)\left(\frac{\Delta x}{M}\,\mathbf{i} + \frac{\Delta y}{M}\,\mathbf{j}\right)\end{aligned}$$

The force components are:

$$\begin{aligned}F_x &= (L - M)\frac{\Delta x}{M} \\ F_y &= (L - M)\frac{\Delta y}{M}\end{aligned}$$

Damping

Considering damping, the damped force components are:

$$\begin{aligned}F'_x &= F_x - \text{damping\_constant} \cdot v_x \\ F'_y &= F_y - \text{damping\_constant} \cdot v_y\end{aligned}$$

Newton's Second Law and Updates

Newton's second law states:

$$\mathbf{F} = m\mathbf{a}$$

Assuming $m = 1$ and a time step $t = 1$, we have $\mathbf{a} = \mathbf{v}$, thus $\mathbf{F} = \mathbf{v}$.
Velocity updates (at point 2):

$$\begin{aligned}v_x &\leftarrow v_x + F'_x \\ v_y &\leftarrow v_y + F'_y\end{aligned}$$

Velocity updates (at point 1):

$$\begin{aligned}v_x &\leftarrow v_x - F'_x \\ v_y &\leftarrow v_y - F'_y\end{aligned}$$

Position updates (assuming $t = 1$, and therefore $v = d$):

$$\begin{aligned}\Delta x &\leftarrow \Delta x + v_x \\ \Delta y &\leftarrow \Delta y + v_y\end{aligned}$$