CSE166 Assignment #:
Due Date:

Please list all contributors to the assignment below.
Staple this sheet to the front of your submission.

1 Student Name(s):

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CSE166 - Image Processing - Homework 7

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November 30, 2006

1. GW, Problem 10.13.
   (a) The coordinates of point 1 are $x = 0$, $y = 0$, which means the Hough transform equation becomes $\rho = 0$. θ can be anything, so this is a straight line on the ρ axis.
   (b) Yes, there is only one point where both x and y are zero.
   (c) When $\theta = 90^o$, the Hough transform equation becomes $y = \rho$, and when $\theta = -90^o$, the Hough transform equation becomes $-y = \rho$.

2. GW, Problem 10.14.
   (a)

$$\cos(\theta) = \frac{\rho}{x} \quad \rightarrow \quad x = \frac{\rho}{\cos(\theta)}$$
$$\cos(90 - \theta) = \sin(\theta) = \frac{\rho}{y} \quad \rightarrow \quad y = \frac{\rho}{\sin(\theta)}$$
$$a = \frac{y}{x} = \frac{\rho/\sin(\theta)}{\rho/\cos(\theta)} = \frac{\cos(\theta)}{\sin(\theta)} = \cot(\theta) \quad \rightarrow \quad \theta = \cot^{-1}(a)$$
$$b = y = \frac{\rho}{\sin(\theta)} \quad \rightarrow \quad \rho = \frac{b}{\sin(\theta)} = \frac{b}{\sin(\cot^{-1}(a))}$$

   (b)

$$y = -2x + 1 \quad \rightarrow \quad a = -2 \qquad b = 1$$
$$\theta = \cot^{-1}(a) = \cot^{-1}(-2) = -26.5651$$
$$\rho = \frac{b}{\sin(\cot^{-1}(a))} = \frac{1}{\sin(\cot^{-1}(-2))} = -2.2361$$

4. GW, Problem 11.18.

$$e_{ms} = \sum_{j=k+1}^{n} \lambda_j \rightarrow k = 2 \rightarrow e_{ms} = \sum_{j=3}^{6} \lambda_j = 280$$
$$e_{ms_{MAX}} = \sum_{j=k+1}^{n} \lambda_j \rightarrow k = 0 \rightarrow e_{ms_{MAX}} = \sum_{j=1}^{6} \lambda_j = 4421$$
$$\%\,\text{error} = \frac{e_{ms}}{e_{ms_{MAX}}} = 6.3\%$$

5. Consider the 2×2 matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

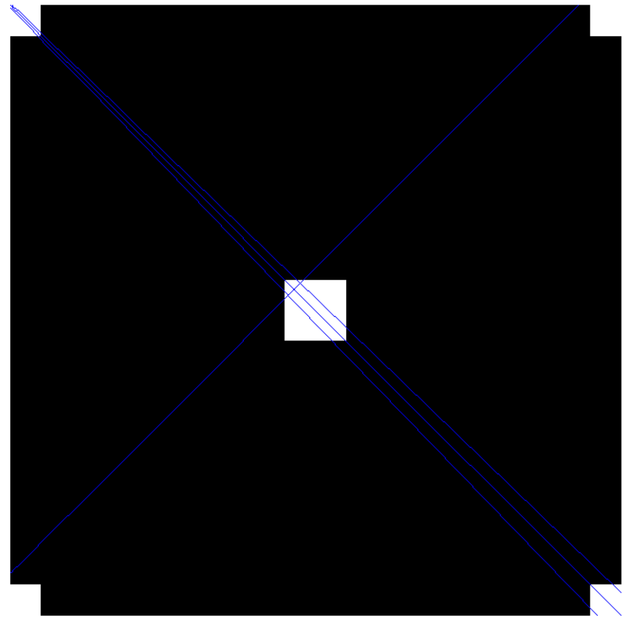
   Show that the inverse is given by

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

$$\det(A) = ad - bc$$
$$A^{-1}A = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{1}{ad-bc} \begin{bmatrix} da - bc & db - bd \\ -ca + ac & -cb + ad \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$
$$AA^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad-bc} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad-bc} \begin{bmatrix} ad - bc & -ab + ba \\ cd - dc & -cb + da \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

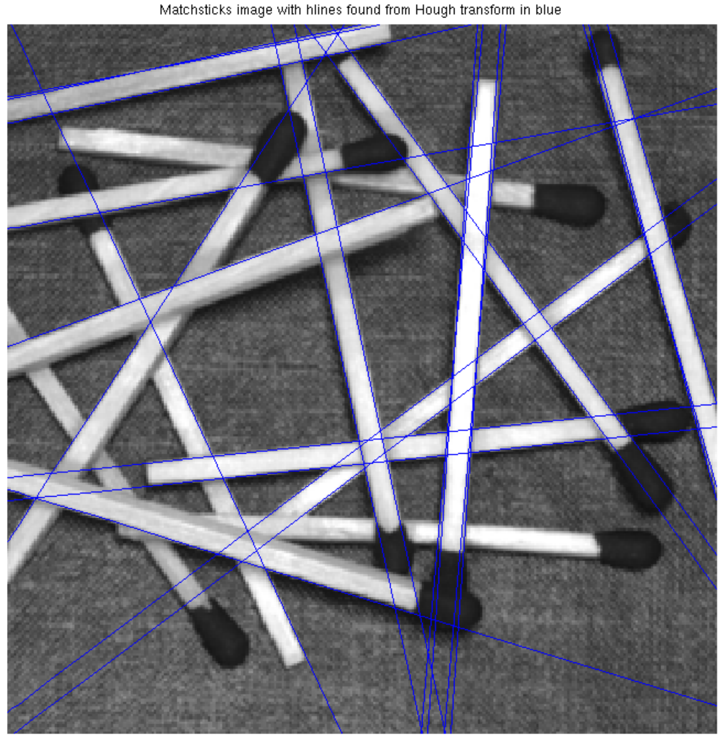
Hough transform of test image with 5 white dots

Test image with lines found from Hough Transform in blue



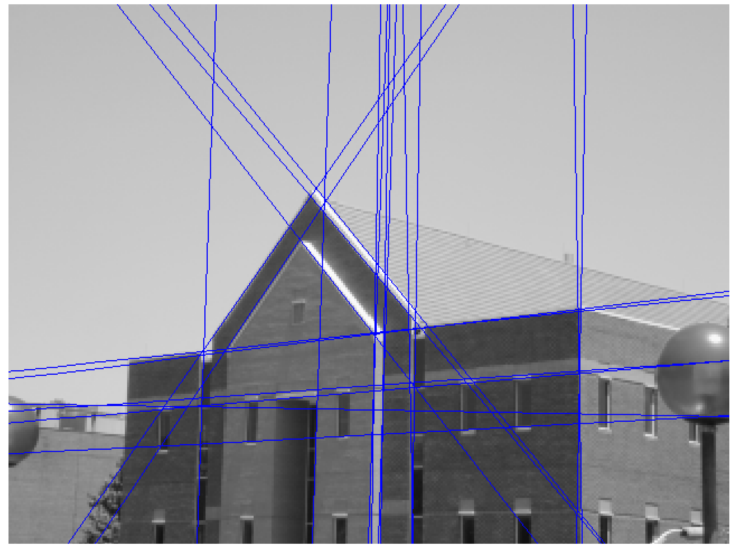
Hough transform of matchsticks image

Matchsticks image with hlines found from Hough transform in blue

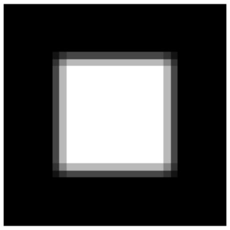


Hough transform of house image

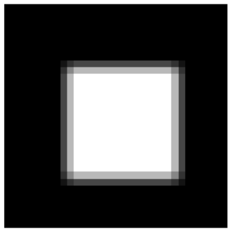
House image with lines found from Hough transform in blue



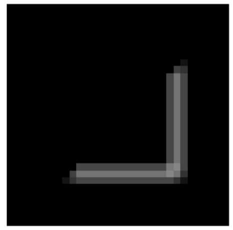
blurred white square centered on black background



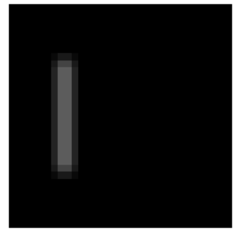
square shifted one pixel down and right



derivative of brightness with respect to t (time)

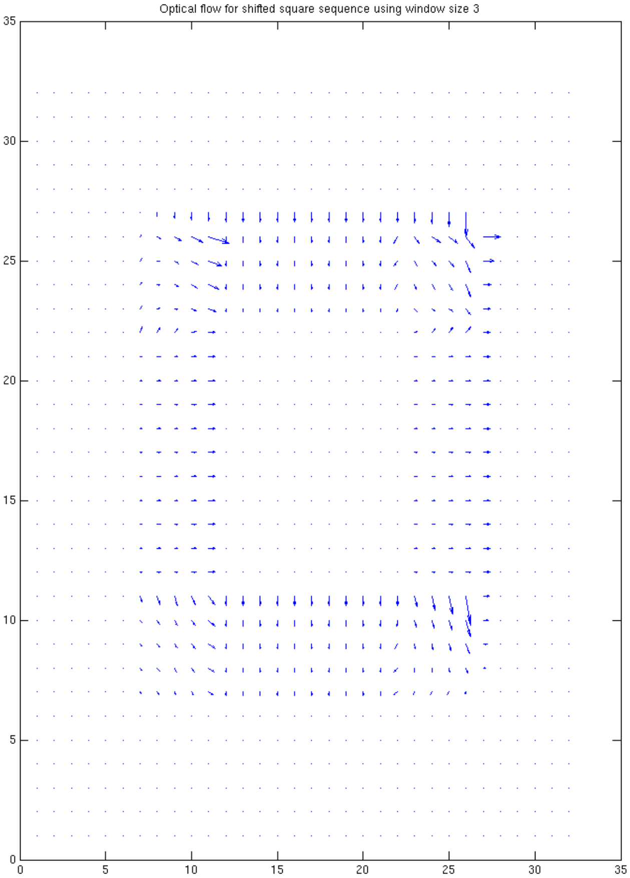


derivative of brightness with respect to x axis

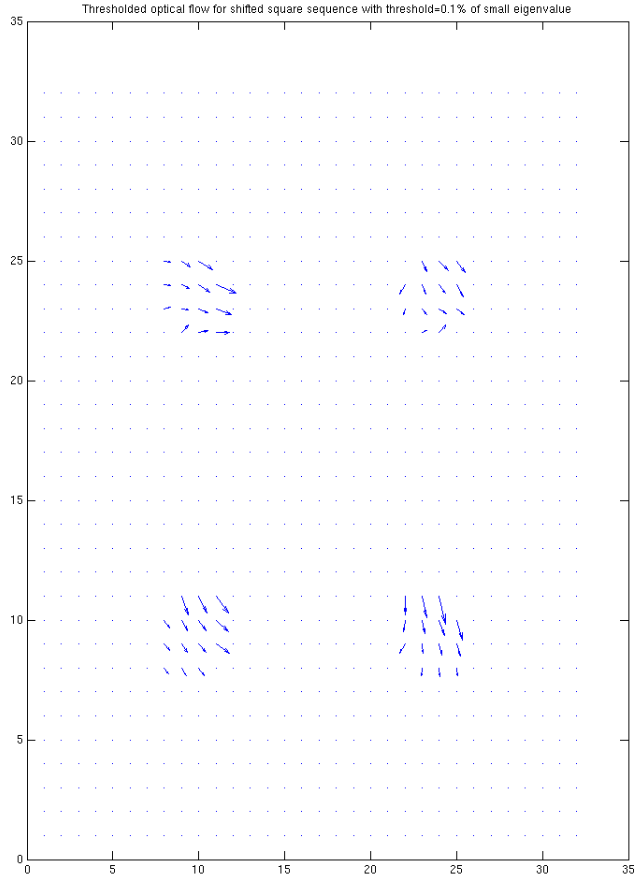


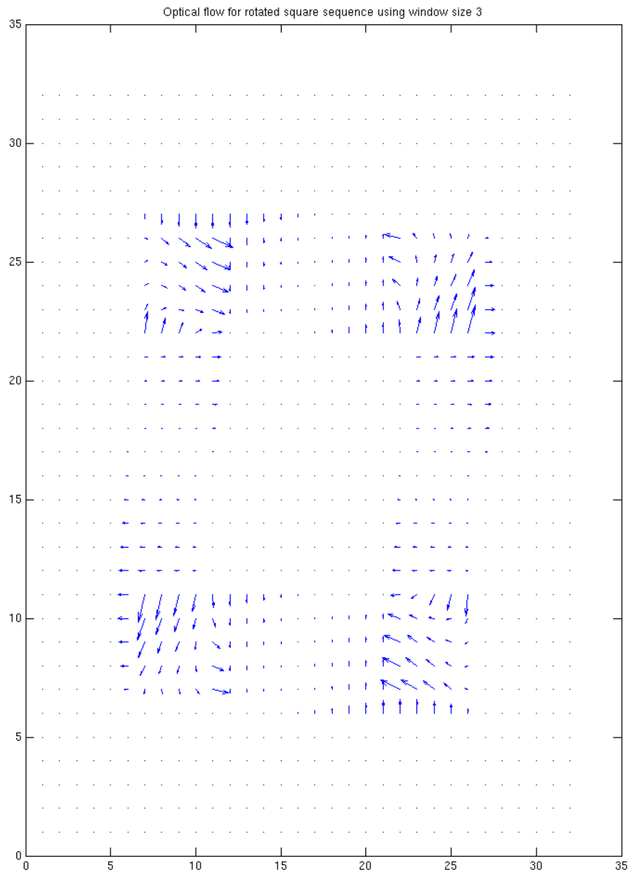
derivative of brightness with respect to y axis

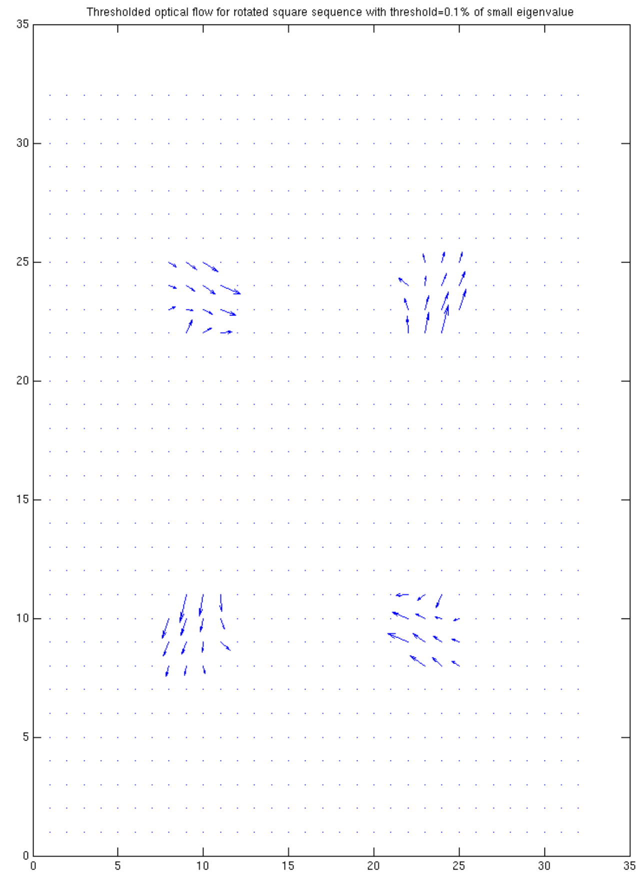
Optical flow for shifted square sequence using window size 3



Thresholded optical flow for shifted square sequence with threshold=0.1% of small eigenvalue




Optical flow for rotated square sequence using window size 3
0
5
10
15
20
25
30
35


Thresholded optical flow for rotated square sequence with threshold=0.1% of small eigenvalue
0
5
10
15
20
25
30
35

```matlab
% Author: <njoffe@ucsd.edu> Nitay Joffe
% Date: 11/30/2006
% Class: CSE 166 - Image Processing
% Homework: 7
% Problem: 1 - Hough Transform
% Question: a

% (a) Implement the Hough Transform (HT) using the (\rho, \theta)
%     parameterization as described in GW Section 10.2.2. Use accumulator cells
%     with a resolution of 1 degree in \theta and 1 pixel in \rho.
function [hough_space,rho_max,theta_max] = hough_transform(binary_image)
  theta_min=-90;
  theta_max=90;
  theta=theta_min:theta_max;
  cosine_theta=cosd(theta);
  sine_theta=sind(theta);

  [ones_y,ones_x]=find(binary_image);
  ones_size=numel(ones_x);
  for i=1:numel(ones_x)
    rho(i,:)=round(ones_x(i)*cosine_theta+ones_y(i)*sine_theta);
  end
  [image_rows,image_columns]=size(binary_image);
  diagonal_distance=sqrt(image_rows^2+image_columns^2);
  rho_max=round(sqrt(2)*diagonal_distance);

  rho=rho+rho_max+1;
  theta=theta+theta_max+1;

  rho_size=rho_max*2+1;
  theta_size=numel(theta);
  hough_space=zeros(rho_size,theta_size);
  for i=1:ones_size
    for j=1:theta_size
      hough_space(rho(i,j),theta(j))=hough_space(rho(i,j),theta(j))+1;
    end
  end
end
```

```
% Author: <njoffe@ucsd.edu> Nitay Joffe
% Date: 11/30/2006
% Class: CSE 166 - Image Processing
% Homework: 7
% Problem: 1 - Hough Transform
% Questions: b,c,d
clear;

% (b) Produce a simple 11 x 11 test image made up of zeros with 5 ones in it,
%     arranged like the 5 points in GW Figure 10.20(a).
test_image=zeros(11,11);
test_image(1,1)=1;
test_image(6,6)=1;
test_image(11,1)=1;
test_image(1,11)=1;
test_image(11,11)=1;

% Compute and display its HT; the result should look like GW Figure 10.20(b).
[test_image_ht,rho_max,theta_max]=hough_transform(test_image);

% Now threshold the HT to find the (\rho, \theta)-coordinates of cells with more
% than 2 votes and plot the corresponding lines in (x, y)-space on top of the
% original image.
[rho,theta]=find(test_image_ht>2);
rho=rho-rho_max-1;
theta=theta-theta_max-1;

[test_image_y_limit test_image_x_limit]=size(test_image);
test_image_line_functions=cell(numel(rho),1);
for i=1:numel(rho)
  test_image_line_functions{i}=@(x)(rho(i)-x.*cosd(theta(i)))/sind(theta(i));
end

% (c) Load in the matchstick image in GW Figure 8.02(a) and shrink it to half
%     its size using I=imresize(I,0.5,'bil','crop');.
matchsticks_image=imread('Fig8.02(a).jpg');
matchsticks_image=imresize(matchsticks_image,0.5,'bil');

% Compute and display its edges using the Sobel operator with default threshold
% settings, i.e. BW=edge(I,'sobel');.
BW=edge(matchsticks_image,'sobel');

% Now compute and display the HT of BW. As before, threshold the HT and plot the
% corresponding lines atop the original image; this time, use a threshold of 50%
% of the maximum accumulator count over the entire HT.
[matchsticks_ht,rho_max,theta_max]=hough_transform(BW);

threshold=max(max(matchsticks_ht))/2;
[rho,theta]=find(matchsticks_ht>threshold);
rho=rho-rho_max-1;
theta=theta-theta_max-1;

[matchsticks_y_limit matchsticks_x_limit]=size(BW);
matchsticks_line_functions=cell(numel(rho),1);
for i=1:numel(rho)
  matchsticks_line_functions{i}=@(x)(rho(i)-x.*cosd(theta(i)))/sind(theta(i));
end

% (d) Repeat the previous step for another image of your choice. The image can
%     be from the textbook or elsewhere, but its size must be at least 128x128
%     and it should contain several extended straight lines.
house_image=imread('Fig10.10(a).jpg');
house_image=imresize(house_image,0.2,'bil');
```

```
BW=edge(house_image,'sobel');
[house_image_ht,rho_max,theta_max]=hough_transform(BW);

threshold=max(max(house_image_ht))/2;
[rho,theta]=find(house_image_ht>threshold);
rho=rho-rho_max-1;
theta=theta-theta_max-1;

[house_y_limit house_x_limit]=size(BW);
house_line_functions=cell(numel(rho),1);
for i=1:numel(rho)
  house_line_functions{i}=@(x)(rho(i)-x.*cosd(theta(i)))/sind(theta(i));
end


figure;
subplot(1,2,1);
imshow(test_image_ht);
title('Hough transform of test image with 5 white dots');
subplot(1,2,2);
imshow(test_image);
hold on;
for i=1:numel(test_image_line_functions)
  fplot(test_image_line_functions{i},[1 test_image_x_limit 1 test_image_y_limit]);
end
title('Test image with lines found from Hough Transform in blue');

figure;
subplot(1,2,1);
imshow(matchsticks_ht);
title('Hough transform of matchsticks image');
subplot(1,2,2);
imshow(matchsticks_image);
hold on;
for i=1:numel(matchsticks_line_functions)
  fplot(matchsticks_line_functions{i},[1 matchsticks_x_limit 1 matchsticks_y_limit]);
end
title('Matchsticks image with hlines found from Hough transform in blue');

figure;
subplot(1,2,1);
imshow(house_image_ht);
title('Hough transform of house image');
subplot(1,2,2);
imshow(house_image);
hold on;
for i=1:numel(house_line_functions)
  fplot(house_line_functions{i},[1 house_x_limit 1 house_y_limit]);
end
title('House image with lines found from Hough transform in blue');
```

```
% Author: <njoffe@ucsd.edu> Nitay Joffe
% Date: 11/30/2006
% Class: CSE 166 - Image Processing
% Homework: 7
% Problem: 2 - Lucas-Kanader optical flow
% Question: a

% Implement the Lucas-Kanade algorithm for measuring optical flow, as
% described in class. Allow the user to specify the size of the window used
% in enforcing the smoothness constraint. Use the quiver function to
% display the optical flow vectors. In addition, have your program return the
% two eigenvalues of the windowed image second moment matrix at each pixel.
function [u,v,eigenvalues_min,eigenvalues_max]=optical_flow(first_image,second_image,window_size)
  window_delta=(window_size-1)/2;
  [dx,dy]=gradient(first_image);
  dt=second_image-first_image;

  h=ones(window_size,1);
  sum_dx_squared=conv2(h,h,dx.*dx,'same');
  sum_dy_squared=conv2(h,h,dy.*dy,'same');
  sum_dx_times_dy=conv2(h,h,dx.*dy,'same');

  [rows,columns]=size(first_image);
  for i=1:rows
    for j=1:columns
      window_rows=max(1,i-window_delta):min(rows,i+window_delta);
      window_columns=max(1,j-window_delta):min(columns,j+window_delta);

      dx_window=dx(window_rows,window_columns);
      dy_window=dy(window_rows,window_columns);
      dt_window=dt(window_rows,window_columns);

      A=[dx_window(:),dy_window(:)];
      b=dt_window(:);

      scatter_matrix=[sum_dx_squared(i,j),sum_dx_times_dy(i,j);
                      sum_dx_times_dy(i,j),sum_dy_squared(i,j)];
      eigenvalues=eig(scatter_matrix);
      eigenvalues_min(i,j)=min(eigenvalues);
      eigenvalues_max(i,j)=max(eigenvalues);

      gradient_vector=-A\b;

      u(i,j)=gradient_vector(1);
      v(i,j)=-gradient_vector(2);
    end
  end
end
```

```
% Author: <njoffe@ucsd.edu> Nitay Joffe
% Date: 11/30/2006
% Class: CSE 166 - Image Processing
% Homework: 7
% Problem: 2 - Lucas-Kanader optical flow
% Questions: b,c,d
clear;

% (b) Construct two frames of a simple motion sequence as follows. Make a 16x16
%     white square centered on a black background of size 32 x 32.
square_image=zeros(32);
square_image(9:24,9:24)=1;

% Blur it with a Gaussian filter with \sigma = 1.
gaussian_filter=fspecial('gaussian',3,1);
blurred_square=conv2(square_image,gaussian_filter,'same');

% This image represents I(x, y, t).
I=blurred_square;

% Produce the second image, representing I(x, y, t + 1), by displacing the first
% image down one pixel and to the right one pixel.
I_shifted=circshift(I,[1 1]);

% Display each frame, as well as I_t and the two components of \gradient(I).
[shifted_dx shifted_dy]=gradient(I);
shifted_dt=I_shifted-I;

% (c) Compute and display the optical flow for the above sequence using a window
%     size of 5 x 5.
window_size=3;
[I_shifted_u,I_shifted_v,I_shifted_eigenvalues_min]=optical_flow(I,I_shifted,window_size);

% Since you know the "ground truth" displacement (i.e. u = 1, v = 1), comment
% on the accuracy of your measured optical flow at various points throughout the
% image.

% Demonstrate how, by applying a threshold on the eigenvalues, you can suppress
% the flow vectors at pixels that suffer from the aperture problem.
threshold_percentage=0.1;
threshold=threshold_percentage*max(max(I_shifted_eigenvalues_min));

I_shifted_u_thresholded=I_shifted_u;
I_shifted_v_thresholded=I_shifted_v;

[rows,columns]=size(I_shifted_u);
for i=1:rows
  for j=1:columns
    if I_shifted_eigenvalues_min(i,j)<=threshold
      I_shifted_u_thresholded(i,j)=0;
      I_shifted_v_thresholded(i,j)=0;
    end
  end
end

% (d) Construct a new sequence consisting of the original first frame and a
%     second frame produced by rotating the first one by 5degrees (use imrotate
%     with the 'bil' and 'crop' options).
I_rotated=imrotate(I,5,'bil','crop');

% Now repeat step 2c using this sequence.
[I_rotated_u,I_rotated_v,I_rotated_eigenvalues_min]=optical_flow(I,I_rotated,window_size);
```

```
threshold=threshold_percentage*max(max(I_rotated_eigenvalues_min));

I_rotated_u_thresholded=I_rotated_u;
I_rotated_v_thresholded=I_rotated_v;

[rows,columns]=size(I_rotated_u);
for i=1:rows
  for j=1:columns
    if I_rotated_eigenvalues_min(i,j)<=threshold
      I_rotated_u_thresholded(i,j)=0;
      I_rotated_v_thresholded(i,j)=0;
    end
  end
end


figure;
subplot(1,5,1);
imshow(I);
title('blurred white square centered on black background');
subplot(1,5,2);
imshow(I_shifted);
title('square shifted one pixel down and right');
subplot(1,5,3);
imshow(shifted_dt);
title('derivative of brightness with respect to t (time)');
subplot(1,5,4);
imshow(shifted_dx);
title('derivative of brightness with respect to x axis');
subplot(1,5,5);
imshow(shifted_dy);
title('derivative of brightness with respect to y axis');

figure;
subplot(1,2,1);
quiver(I_shifted_u,I_shifted_v);
title(['Optical flow for shifted square sequence using window size ' num2str(window_size)]);
subplot(1,2,2);
quiver(I_shifted_u_thresholded,I_shifted_v_thresholded);
title(['Thresholded optical flow for shifted square sequence with threshold=' num2str(threshold_percentage) '% o

figure;
subplot(1,2,1);
quiver(I_rotated_u,I_rotated_v);
title(['Optical flow for rotated square sequence using window size ' num2str(window_size)]);
subplot(1,2,2);
quiver(I_rotated_u_thresholded,I_rotated_v_thresholded);
title(['Thresholded optical flow for rotated square sequence with threshold=' num2str(threshold_percentage) '% o
```