```python
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        from copy import deepcopy
        import random
```

```python
In [2]: class DiscreteEnvironment():
            def __init__(self,start,stop):
                self.size=8
                self.grid=[[0 for i in range(self.size)] for j in range(self.size)]
                self.start=start
                self.state=self.start
                self.ter=stop
        #         self.grid[7][7]=10
                self.non_reward=-0.5
                self.ter_reward=10
                #change probability structure
        #         self.P=[[0.7,0.1,0.1,0.1],[0.1,0.7,0.1,0.1],[0.1,0.1,0.7,0.1],[0.1,0.1,0.1,0.7]]
                self.P=[[1,0,0,0],[0,1,0,0],[0,0,1,0],[0,0,0,1]]
                self.gamma=0.9
        #         self.actions=["up","down","right","left"]
                self.actions=[0,1,2,3]
            def step(self,action,update):
                temp_state=deepcopy(self.state)
                if(action==0):
                    if(self.state[1]>0):
                        temp_state[1]-=1
                if(action==1):
                    if(self.state[1]<self.size-1):
                        temp_state[1]+=1
                if(action==2):
                    if(self.state[0]<self.size-1):
                        temp_state[0]+=1
                if(action==3):
                    if(self.state[0]>0):
                        temp_state[0]-=1
        #         if(update==1):
        #             self.state=temp_state
                if(temp_state==self.ter):
                    return temp_state,self.ter_reward,1
                return temp_state,self.non_reward,0
            def reset(self):
                self.state=self.start
        #         self.grid=[[0 for i in range(self.size)] for j in range(self.size)]
```

```python
In [3]: class DiscreteAgent():
            def __init__(self,env):
                self.env=env
                self.sweep_no=0
                self.policy=[[1 for i in range(env.size)] for j in range(env.size)]
                self.another_policy=[0 for i in range(env.size)]
            def update(self):
                    pass
            def get_action(self,state):
                pass
```

```python
In [4]: class ValueIteration(DiscreteAgent):
            def update(self):
                previousreward = deepcopy(self.env.grid)
                is_converged=False
                error=0
                for i in range(0,self.env.size):
                    for j in range(0,self.env.size):
                        values=[]
                        for action in self.env.actions:
        #                     print("krishna")
                            self.env.state=[i,j]
                            ans=0
                            for action1 in self.env.actions:
                                self.env.state=[i,j]
                                state,r,x=self.env.step(action1,0)
        #                         print(str(action)+"->"+str(action1)+"->"+str(self.env.state)+"->"+str(state)+"->"+str(r)+"->"+str(self.env.P[action][action1]))
        #                         print(self.env.P[action][action1])
                                ans+=self.env.P[action][action1]*(r+previousreward[state[0]][state[1]]*self.env.gamma)
        #                         values.append(r+previousreward[state[0]][state[1]]*self.env.gamma)
                            values.append(ans)
        #                     print(values)
                        if([i,j]!=self.env.ter):
                            self.env.grid[i][j]=max(values)
                            self.policy[i][j]=values.index(max(values))
                        error=max(error,abs(previousreward[i][j]-self.env.grid[i][j]))
                if(error<0.01):
                    is_converged=True
                self.sweep_no+=1
                return self.sweep_no,is_converged
            def get_action(self,state):
                return self.policy[state[0]][state[1]]
```

```python
In [5]: env=DiscreteEnvironment([0,0],[7,7])
        # print(env.grid)
        agent=ValueIteration(env)
        # agent=PolicyIteration(env)
        # print(agent.sweep_no)
        sweep_no, is_converged = 0, False
        max_sweeps = 49
        returns=[0 for i in range(max_sweeps+1)]
        while sweep_no<max_sweeps:
            sweep_no,stop=agent.update()
            is_terminated=False
            state=env.start
            while not is_terminated:
                temp=state
                action = agent.policy[state[0]][state[1]]
                env.state=temp
                next_state, reward, is_terminated = env.step(action,0)
                if(next_state==temp):
        #             print(sweep_no)
        #             print("krishna")
                    reward=-5
                    returns[sweep_no]+=reward
                    break
                returns[sweep_no]+=reward
        #         print(reward)
                state=next_state
        #         print(sweep_no)
        print(returns)
```

```
[0, -5, -5, -5, -5, -5, -5, -5, -5, -5, -5, -5, -5, -5, 3.5, 3.5, 3.5, 3.5,
3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5,
3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5,
3.5, 3.5]
```

```python
In [6]: # agent.evaluate_policy()
        # while not is_converged:
        while sweep_no<max_sweeps:
            sweep_no, is_converged = agent.update()
        # print(env.grid)
        # print(sweep_no)
        print(agent.policy)
```

```
[[1, 1, 1, 1, 1, 1, 1, 2], [1, 1, 1, 1, 1, 1, 1, 2], [1, 1, 1, 1, 1, 1, 1,
2], [1, 1, 1, 1, 1, 1, 1, 2], [1, 1, 1, 1, 1, 1, 1, 2], [1, 1, 1, 1, 1, 1,
1, 2], [1, 1, 1, 1, 1, 1, 1, 2], [1, 1, 1, 1, 1, 1, 1, 1]]
```

```python
In [7]: class PolicyIteration(DiscreteAgent):
            def update(self):
                temp=deepcopy(self.policy)
                self.evaluate_policy()
                self.update_policy()
                self.sweep_no+=1
                if(self.policy==temp):
                    return self.sweep_no,True
                return self.sweep_no,False

            def evaluate_policy(self):
                is_converged=False
                while(is_converged==False):
        #         while(self.sweep_no<20):
                    error=0
                    previousreward = deepcopy(self.env.grid)
                    for i in range(0,self.env.size):
                        for j in range(0,self.env.size):
                            self.env.state=[i,j]
                            action=self.policy[i][j]
                            val=0
                            for action1 in self.env.actions:
                                self.env.state=[i,j]
                                state,r,x=self.env.step(action1,0)
                                val+=self.env.P[action][action1]*(r+previousreward[state[0]][state[1]]*self.env.gamma)
                            if([i,j]!=self.env.ter):
                                self.env.grid[i][j]=val
                            error=max(error,abs(previousreward[i][j]-self.env.grid[i][j]))
        #             print(error)
                    if(error<0.001):
                        is_converged=True

            def update_policy(self):
                for i in range(0,self.env.size):
                    for j in range(0,self.env.size):
                        values=[]
                        for action in self.env.actions:
                            val=0
                            for action1 in self.env.actions:
                                self.env.state=[i,j]
                                state,r,x=self.env.step(action1,0)
                                val+=self.env.P[action][action1]*(r+self.env.grid[state[0]][state[1]]*self.env.gamma)
                            values.append(val)
                        self.policy[i][j]=values.index(max(values))
            def get_action(self,state):
                return self.policy[state[0]][state[1]]
```

```python
In [8]: env1=DiscreteEnvironment([0,0],[7,7])
        agent1=PolicyIteration(env1)
        i=0
        stop=False
        sweep_no=0
        # max_sprint(hile(stop=False):
        returns1=[0 for i in range(max_sweeps+1)]
        while sweep_no<max_sweeps:
            sweep_no,stop=agent1.update()
            is_terminated=False
            state=env1.start
            while not is_terminated:
                temp=state
                action = agent1.policy[state[0]][state[1]]
                env1.state=temp
                next_state, reward, is_terminated = env1.step(action,0)
                if(next_state==temp):
        #             print(sweep_no)
        #             print("krishna")
                    reward=-5
                    returns1[sweep_no]+=reward
                    break
                returns1[sweep_no]+=reward
        #         print(reward)
                state=next_state
        #         print(sweep_no)
        print(returns1)
```

```
[0, -5, -5, -5, -5, -5, -5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.
5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.
5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.5, 3.
5, 3.5, 3.5, 3.5]
```

```python
In [9]: print(agent1.policy)
        # print(env1.grid)
        c=[[ 0 for i in range(agent.env.size)]for i in range(agent.env.size)]
        for i in range(8):
            for j in range(8):
                c[i][j]=agent.policy[i][j]-agent1.policy[i][j]
        print(c)
```

```
[[1, 1, 1, 1, 1, 1, 1, 2], [1, 1, 1, 1, 1, 1, 1, 2], [1, 1, 1, 1, 1, 1, 1,
2], [1, 1, 1, 1, 1, 1, 1, 2], [1, 1, 1, 1, 1, 1, 1, 2], [1, 1, 1, 1, 1, 1,
1, 2], [1, 1, 1, 1, 1, 1, 1, 2], [1, 1, 1, 1, 1, 1, 1, 1]]
[[0, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0,
0], [0, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0,
0, 0], [0, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 0]]
```

```python
In [10]: returns2=[0 for i in range(50)]
         for i in range(50):
             is_terminated=False
             state=env1.start
             while not is_terminated:
                 temp=state
                 action = random.randint(0,3)
                 env.state=temp
                 next_state, reward, is_terminated = env.step(action,0)
         #         print(str(state)+"->"+str(action)+"->"+str(next_state))
                 returns2[i]+=reward
                 state=next_state
         print(returns2)
```

```
[-192.5, -503.5, -24.0, -176.0, -84.0, -16.5, -506.5, -45.5, -18.0, -31.0,
-4.0, -27.0, -440.5, -5.0, -34.0, -29.5, -272.0, -224.5, -21.5, -32.5, -23.
0, -50.0, -75.5, -76.5, -260.0, -205.0, -218.5, -132.5, -243.5, -12.5, -58
8.0, -87.5, -67.5, -455.0, -93.5, -297.5, -26.0, -203.5, -15.0, -418.5, -4
3.5, -311.5, -459.5, -536.5, -431.5, -54.0, -9.0, -53.0, -16.5, -74.0]
```

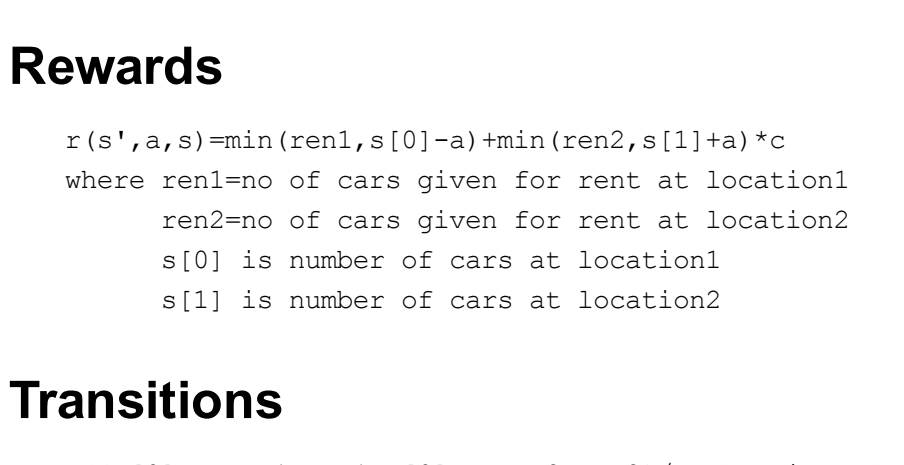
```python
In [11]: import matplotlib.pyplot as plt
         import matplotlib.patches as mpatches
         from matplotlib.colors import colorConverter as cc
         import numpy as np

         def plot_mean_and_CI(mean, lb, ub, color_mean=None, color_shading=None):
             # plot the shaded range of the confidence intervals
             plt.fill_between(range(mean.shape[0]), ub, lb,
                              color=color_shading, alpha=.5)
             # plot the mean on top
             plt.plot(mean, color_mean)
         returns=np.array(returns)
         returns1=np.array(returns)
         # generate 3 sets of random means and confidence intervals to plot
         ub0 = returns + np.random.random(50) + .5
         lb0 = returns - np.random.random(50) - .5

         ub1 = returns1 + np.random.random(50) + .5
         lb1 = returns1 - np.random.random(50) - .5

         returns3 =np.array(returns2)/100
         #Note:Random Agents returns scaled by 100 for better visualisation
         ub2 = returns3 + np.random.random(50) + .5
         lb2 = returns3 - np.random.random(50) - .5

         # plot the data
         fig = plt.figure(1, figsize=(7, 2.5))
         plot_mean_and_CI(returns, ub0, lb0, color_mean='k', color_shading='k')
         plot_mean_and_CI(returns1, ub1, lb1, color_mean='b', color_shading='b')
         plot_mean_and_CI(returns3, ub2, lb2, color_mean='g--', color_shading='g')
```



States

```
{i,j} where i=[0,20],j=[0,20]
0 is dummy state introduced for conveinience
where i is number of cars in location1 and j is number of cars in locat
ion2
```

Actions

```
actions in state A=[-5,5]
Here maximum number of cars that can be moved from one location to othe
r
Instead of having two numbers to know from which location we moved to w
hich location,
We maintain a single number,negative quantity represents moving from lo
cation2 to location1 and vice versa
```

Rewards

```
r(s',a,s)=min(ren1,s[0]-a)+min(ren2,s[1]+a)*c
where ren1=no of cars given for rent at location1
      ren2=no of cars given for rent at location2
      s[0] is number of cars at location1
      s[1] is number of cars at location2
```

Transitions

```
p({s[0]-a-ren1+ret1,s[0]+a-ren2+ret2}/s,a)=poisson(ren1,f_req)*poisson
(ren2,s_req)*poisson(ret1, f_ret)*poisson(ret2, s_ret)
where
ren1=no of cars given for rent at location1=min(asked for rent,availabl
e after taking action a)
ren2=no of cars given for rent at location2=min(asked for rent,availabl
e after taking action a)
ret1=no of cars returned at location1
ret2=no of cars returned at location2
poisson(ren1,f_req) is probability of poission distribution for value r
en1 where lambda=f_req
poisson(ren1,f_req) is probability of poission distribution for value r
en2 where lambda=s_req
poisson(ren1,f_req)  is probability of poission distribution for value
 ret1 where lambda=f_ret
poisson(ren1,f_req) is probability of poission distribution for value r
et2 where lambda=s_ret
and f_ren,s_ren,f_ret,s_ret are given
```

Bellman update equation

```
let c1=a+ren1-ret1
    c2=a-ren2+ret2
    P=p({s[0]-c1,s[1]+c2}/s,a}
   Policy evaluation update:
     where a=poilcy(s)
```

$$v(s) = \{P * (r(s', a, s) + v(s'))\}$$

Policy update equation:

$$policy(s) = (a) for \max_{\forall a \in A}\{P * (r(s', a, s) + v(s'))\}$$

```python
In [108]: import  math
          class CarEnvironment():
              def __init__(self):
                  self.maxcars=20
                  self.maxmove=5
                  self.f_req=3
                  self.s_req=4
                  self.f_ret=3
                  self.s_ret=2
                  self.rent=10
                  self.cost=-2
                  self.up=9
                  self.prob=dict()
                  self.state=[0,0]
                  self.grid=[[0 for i in range(self.maxcars+1)] for j in range(self.maxcars+1)]
                  self.size=self.maxcars+1
                  self.actions=[i for i in range(-self.maxmove,self.maxmove+1)]
                  self.gamma=0.9
              #precalculating and storing in dictionary as this is required so many number of times and will become time consuming
              def poisson(self,x, l):
                  key = x * 10 + l
                  if key not in self.prob.keys():
                      self.prob[key] = np.exp(-l) * pow(l, x) / math.factorial(x)
                  return self.prob[key]
```

```python
In [109]: class PolicyIteration2(DiscreteAgent):
              def update(self):
                  temp=deepcopy(self.policy)
          #         print("krishna1")
                  self.evaluate_policy()
          #         print("krishna2")
                  self.update_policy()
          #         print("krishna3")
                  self.sweep_no+=1
                  if(self.policy==temp):
                      return self.sweep_no,True
          #         print("krishna4")
                  return self.sweep_no,False

              def evaluate_policy(self):
                  is_converged = False
                  #no of iterations to run for evaluating policy
                  sweep=0
                  while(sweep<30):
          #             print("kmt1")
                      error=0
                      #storing current environment state
                      prevstate=deepcopy(self.env.grid)
                      for i in range(self.env.size):
                          for j in range(self.env.size):
          #                     print("kmt2")
                              self.env.state=[i,j]
                              action=self.policy[i][j]
                              returns=0
                              #max cars at that location cant exceed 20
                              cars1 = int(min(self.env.state[0] - action, self.env.maxcars))
                              cars2 = int(min(self.env.state[1] + action, self.env.maxcars))
                              #checking for all possibilities of number of cars that could be rented(as poisson has all probabilties to infinity neglecting probabilities after 9)
                              for ren1 in range(0, self.env.up):
                                  for ren2 in range(0, self.env.up):
          #                             print("kmt3")
                                      rentprob = self.env.poisson(ren1, self.env.f_req) * self.env.poisson(ren2, self.env.s_req)
                                      totalren1 = min(cars1, ren1)
                                      totalren2 = min(cars2, ren2)
                                      rew = ((totalren1 + totalren2) * self.env.rent+self.env.cost* abs(action))*rentprob
                              #checking for all possibilities of no of cars returned as to know the next state
                                      for ret1 in range(0, self.env.up):
                                          for ret2 in range(0, self.env.up):
          #                                     print("kmt4")
                                              retprob = self.env.poisson(ret1, self.env.f_ret) * self.env.poisson(ret2,self.env.s_req)
                                              prob=retprob*rentprob
                                              cars1_end = min(cars1 - totalren1 + ret1, self.env.maxcars)
                                              cars2_end = min(cars2 - totalren2 + ret2, self.env.maxcars)
                                      #rew is r(s,a) and returns is total value of state
                                              returns += retprob * rew + prob*(self.env.gamma* prevstate[cars1_end][cars2_end])
                                      self.env.grid[i][j]=returns
                                      error=max(error,abs(prevstate[i][j]-self.env.grid[i][j]))
                      sweep+=1
                      if(error<0.01):
                          is_converged=True
              def update_policy(self):
                  for i in range(0,self.env.size):
                      for j in range(0,self.env.size):
                          vals = []
                          #for all states checking which action gives maximum possible value
                          for action in self.env.actions:
                              #taking only for possible actions because you cannot move more than nof of cars at that state
                              if ((action >= 0 and i >= action) or (action < 0 and j >= abs(action))):
                                  self.env.state=[i,j]
                                  returns=0
                                  cars1 = int(min(self.env.state[0] - action, self.env.maxcars))
                                  cars2 = int(min(self.env.state[1] + action, self.env.maxcars))
                                  for ren1 in range(0, self.env.up):
                                      for ren2 in range(0, self.env.up):
                                          rentprob = self.env.poisson(ren1, self.env.f_req) * self.env.poisson(ren2, self.env.s_req)
                                          totalren1 = min(cars1, ren1)
                                          totalren2 = min(cars2, ren2)
                                          rew = ((totalren1 + totalren2) * self.env.rent+self.env.cost* abs(action))*rentprob
                                          for ret1 in range(0, self.env.up):
                                              for ret2 in range(0, self.env.up):
                                                  retprob = self.env.poisson(ret1, self.env.f_ret) * self.env.poisson(ret2,self.env.s_req)
                                                  prob=retprob*rentprob
                                                  cars1_end = min(cars1 - totalren1 + ret1, self.env.maxcars)
                                                  cars2_end = min(cars2 - totalren2 + ret2, self.env.maxcars)
                                                  returns += retprob * rew + prob*(self.env.gamma* self.env.grid[cars1_end][cars2_end])
                                  vals.append(returns)
                          bestAction = vals.index(max(vals))
                          self.policy[i] [j] = self.env.actions[bestAction]
              def get_action(self,state):
                  return self.policy[state[0]][state[1]]
```

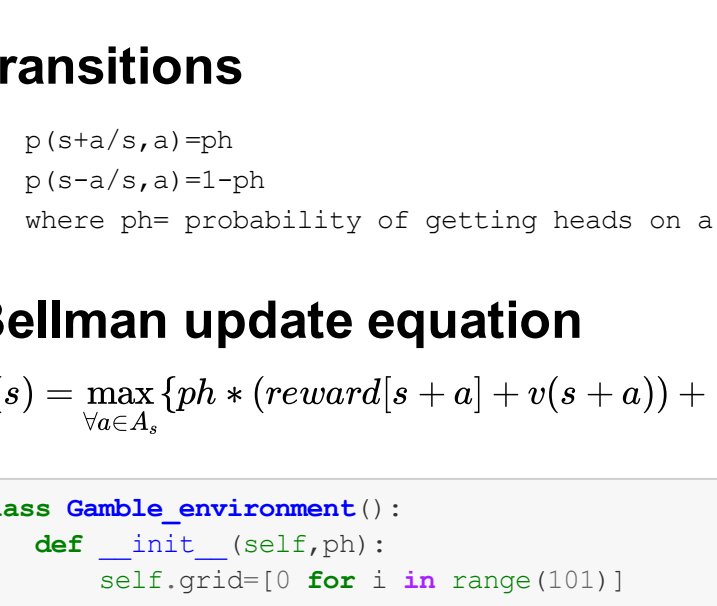
```python
In [110]: env4=CarEnvironment()
          agent4=PolicyIteration2(env4)
          is_converged=False
          sweep_no=0
          max_sweeps=1
          while not is_converged:
              sweep_no,is_converged=agent4.update()
```

```
krishna1
krishna2
krishna3
krishna4
krishna1
krishna2
krishna3
krishna4
krishna1
krishna2
krishna3
krishna4
krishna1
krishna2
krishna3
```

```python
In [112]: # print(env4.grid)
          # print(agent4.policy)
          # import numpy
          from mpl_toolkits import mplot3d

          import numpy as np
          import matplotlib.pyplot as plt


          fig = plt.figure()
          ax = plt.axes(projection="3d")
          z_points = []
          x_points = []
          y_points = []
          for i in range(21):
              for j in range(21):
                  x_points.append(i)
                  y_points.append(j)
                  z_points.append(env4.grid[i][j])
          ax.scatter3D(x_points, y_points, z_points, c=z_points);
          plt.show()
          #similarly plot for policy also
```



States

```
{1,2,..99}
but as you told i added two dummy states for convinience 0 and 100
```

Actions

```
actions in state A_s={0,1,..min(s,100-s)}
```

Rewards

```
r(s',a,s)=1 if s'=100
         =0 otherwise
```

Transitions

```
p(s+a/s,a)=ph
p(s-a/s,a)=1-ph
where ph= probability of getting heads on a toss
```

Bellman update equation

$$v(s) = \max_{\forall a \in A_s}\{ph * (reward[s+a] + v(s+a)) + (1-ph) * (reward[s-a] + v(s -$$

```python
In [118]: class Gamble_environment():
              def __init__(self,ph):
                  self.grid=[0 for i in range(101)]
                  self.state=1
                  self.p=ph
                  self.ter=100
                  self.size=101
                  self.actions=[i for  i in range(101)]
                  self.rewards=[0 for i in range(101)]
                  self.rewards[100]=1
                  self.gamma=1
```

```python
In [119]: class ValueIteration1(DiscreteAgent):
              def update(self):
                  previous = deepcopy(self.env.grid)
                  is_converged=False
                  error=0
                  for i in range(self.env.size):
                      values=[]
                      for action in self.env.actions:
                          ans=0
                          if(action in range(1,min(i,100-i)+1)):
                              self.env.state=i
                              ans+=self.env.p*(self.env.rewards[i+action]+previous[i+action]*self.env.gamma)+(1-self.env.p)*(self.env.rewards[i-action]+previous[i-action]*self.env.gamma)
                              values.append(ans)
                      if(i!=self.env.ter and i!=0):
                          self.env.grid[i]=max(values)
                      error=max(error,abs(previous[i]-self.env.grid[i]))
                  if(error<0.0001):
                      is_converged=True
                  self.sweep_no+=1
                  return self.sweep_no,is_converged
              def get_action(self,state):
          #         for state in range(1,101):
                  cur=self.env.grid
                  p=self.env.p
                  val=0
                  r=0
                  for action in self.env.actions:
                      if(action <=min(state,100-state) and action>=1):
                          ans=(p*(self.env.rewards[state+action]+cur[state+action])+(1-p)*(self.env.rewards[state-action]+cur[state-action]))
                          if(ans>val):
                              val=ans
                              r=action
                  return r
```

```python
In [120]: env2=Gamble_environment(0.25)#change value to 0.15 and 0.65 for their plots
          # print(env.grid)
          agent2=ValueIteration1(env2)
          sweep_no, is_converged = 0, False
          max_sweeps = 1000
          while not is_converged:
          # while sweep_no<max_sweeps:
              sweep_no, is_converged = agent2.update()
          # print(env2.grid)(
          policy=[]
          for i in range(1,101):
          #     agent2.another_policy=agent2.get_action(i)
              policy.append(agent2.get_action(i))
          print(policy)
```

```
[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2,
```