16720A HW6 Report by Krishna Bairavi Soundararajan (Andrew ID: ksoundar)

1. Lucas-Kanade Tracking

1.1.a)

Translation warp function can be represented by

$$\mathbf{W}(\mathbf{x};\mathbf{p}) = \mathbf{x} + \mathbf{p} \tag{1}$$

The above equation can be represented in matrix format as; On expanding equation (2) and applying Eq (1) to it; we can represent it as;

$$\begin{aligned} \mathbf{W}(\mathbf{x};\mathbf{p}) &= \begin{bmatrix} 1 & 0 & p_1 \\ 0 & 1 & p_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} x + p_1 \\ y + p_2 \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{W} + p_2 \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{W}_x(\mathbf{x};\mathbf{p}) \\ \mathbf{W}_y(\mathbf{x};\mathbf{p}) \end{bmatrix} \end{aligned} \tag{2}$$

On taking partial differentiation we observe that the resultant matrix is an identity matrix

$$\begin{aligned} \frac{\partial \mathbf{W}(\mathbf{x};\mathbf{p})}{\partial \mathbf{p}^T} &= \begin{bmatrix} \frac{\partial \mathbf{W}_x(\mathbf{x};\mathbf{p})}{\partial p_i} & \frac{\partial \mathbf{W}_x(\mathbf{x};\mathbf{p})}{\partial p_2} \\ \frac{\partial \mathbf{W}_y(\mathbf{x};\mathbf{p})}{\partial p_j} & \frac{\partial \mathbf{W}_y^{(\mathbf{x};\mathbf{p})}}{\partial p_2} \end{bmatrix} \\ &= \begin{bmatrix} \frac{\partial(x+p_1)}{\partial p_1} & \frac{\partial(x+p_1)}{\partial p_2} \\ \frac{\partial(x+p_2)}{\partial p_1} & \frac{\partial(x+p_2)}{\partial p_2} \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned} \tag{3}$$

1.1.b) What we face is a minimization problem;

$$\arg\min_{\Delta p} \sum_{\mathbf{x} \in N} \| I_{t+1}(\mathbf{x}; \mathbf{p} + \Delta \mathbf{p}) - I_t(\mathbf{x}) \|_2^2 \tag{4}$$

On expansion using taylor series, we can write the above equation as

$$\arg\min_{\Delta p} \sum_{\mathbf{x} \in N} \left\| I_{t+1}\left(\mathbf{x}'\right) + \frac{\partial I_{t+1}\left(\mathbf{x}'\right)}{\partial \mathbf{x}'^T} \frac{\partial W(\mathbf{x}; \mathbf{p})}{\partial \mathbf{p}^T} \Delta \mathbf{p} - I_t(\mathbf{x}) \right\|_2^2 \tag{5}$$

On rearranging the above equation to find A, we get

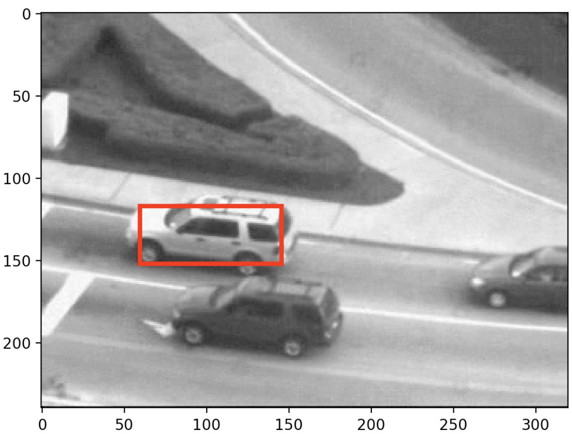
$$\arg\min_{\Delta p} \sum_{\mathbf{x} \in N} \left\| \frac{\partial I_{t+1}\left(\mathbf{x}'\right)}{\partial \mathbf{x}'^T} \frac{\partial W(\mathbf{x}; \mathbf{p})}{\partial \mathbf{p}^T} \Delta \mathbf{p} - \left(I_t(\mathbf{x}) - I_{t+1}\left(\mathbf{x}'\right)\right) \right\|_2^2 \tag{6}$$

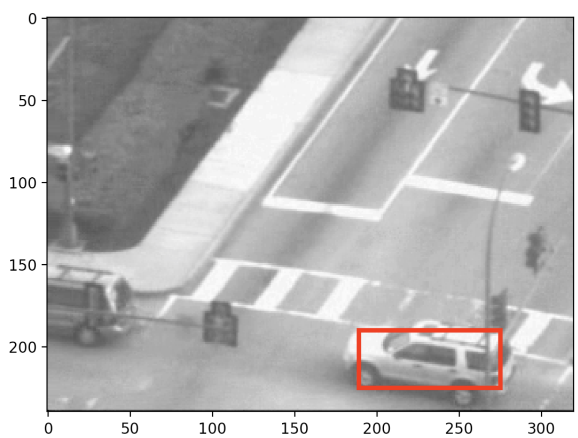
From the above equation we can derive at A and b

$$\begin{aligned} \mathbf{A} &= \frac{\partial I_{t+1}(\mathbf{x})}{\partial \mathbf{x}'^T} \frac{\partial W(\mathbf{x}; \mathbf{p})}{\partial \mathbf{p}^T} \\ \mathbf{b} &= I_t(\mathbf{x}) - I_{t+1}\left(\mathbf{x}'\right) \end{aligned} \tag{7}$$

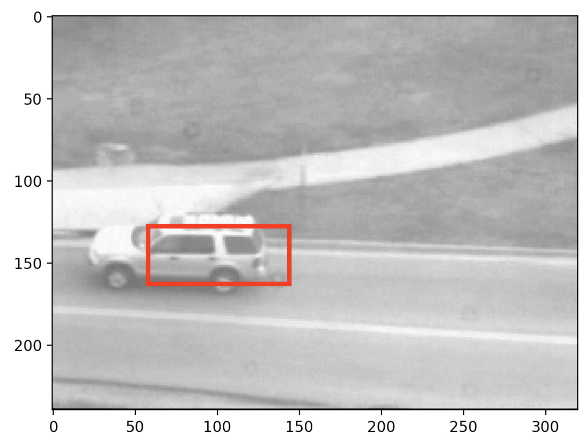
1.1.c) For a unique condition is that $\mathbf{A}^T\mathbf{A}$ must be full rank. $\left[\det\left(\mathbf{A}^T\mathbf{A}\right) \neq 0\right]$

1.3)

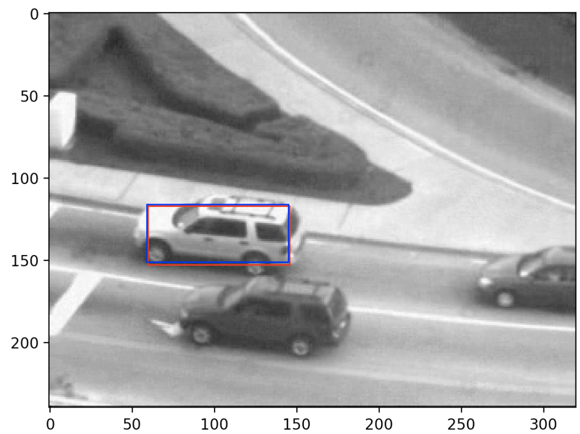


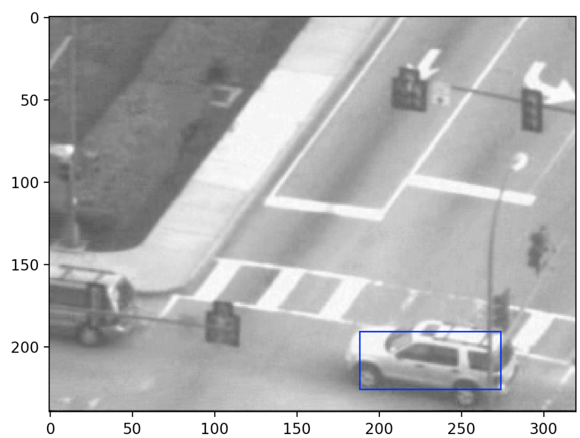



0
50
100
150
200
0
50
100
150
200
250
300


0
50
100
150
200
0
50
100
150
200
250
300

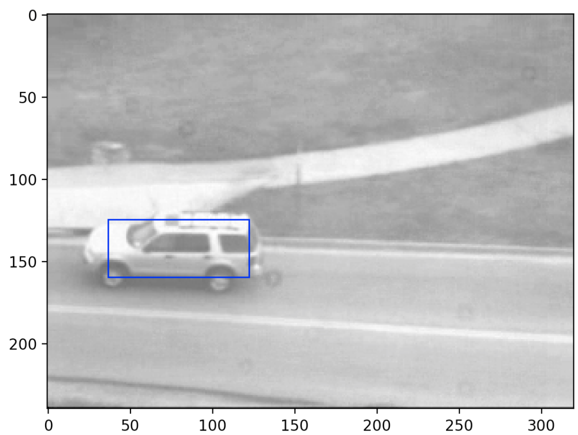
1.4)


0
50
100
150
200
0
50
100
150
200
250
300


0
50
100
150
200
0
50
100
150
200
250
300


0
50
100
150
200
0
50
100
150
200
250
300


0
50
100
150
200
0
50
100
150
200
250
300


0
50
100
150
200
0
50
100
150
200
250
300

2. Affine Motion Subtraction

2.3)

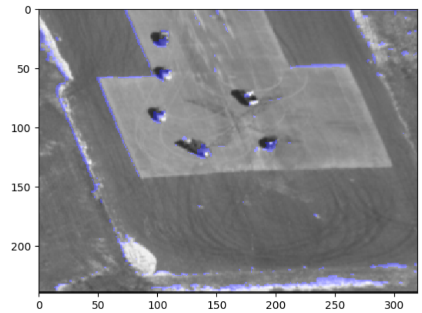


Figure 1: Frame 30

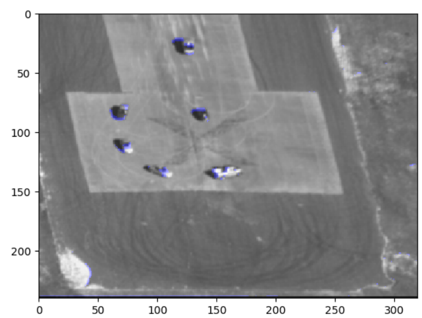


Figure 2: Frame 60

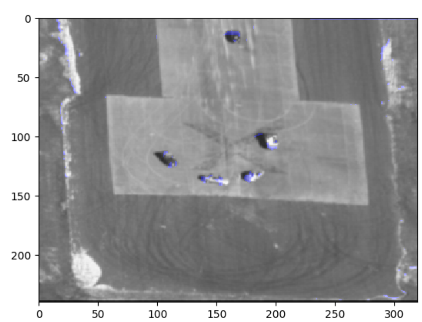


Figure 3: Frame 90

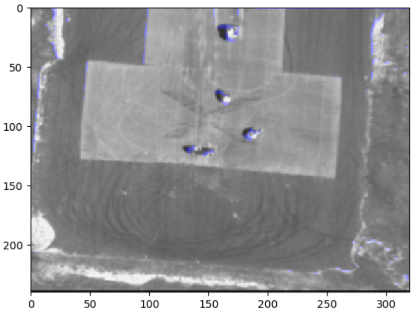


Figure 4: Frame 120

3. Efficient Tracking

3.1)

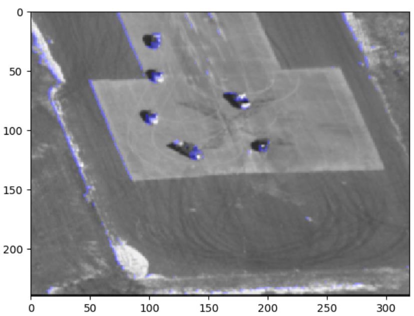


Figure 5: Frame 30

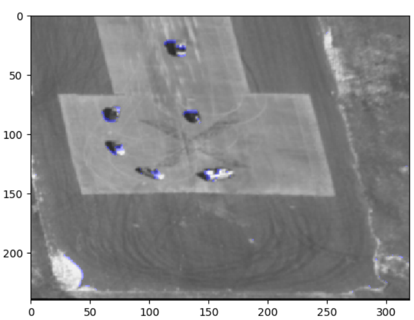


Figure 6: Frame 60

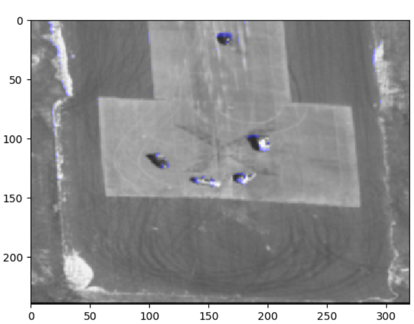


Figure 7: Frame 90

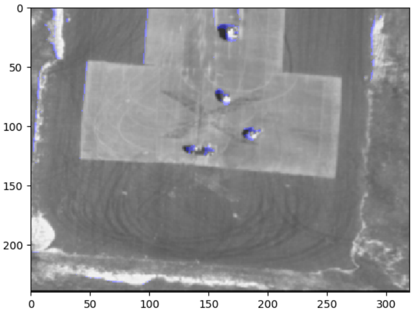


Figure 8: Frame 120

Inverse Composition is better than Lucas Kanade, this is because in Lucas Kanade traditional approach we find matrix A inside the loop which turns out to be computationally expensive, whilst, in case of Inverse Composition approach, this happens once before the loop.