C

(Printed Pages 4)

Roll No. _____

24/1156

B.C.A. (First Semester) Examination, 2024

Fifth Paper

Mathematics-I

Time : Two Hours]   *[Maximum Marks : 75*

Note : Attempt **all** sections as per instructions.

Section-A

(Very Short Answer Type Questions)

Note : Attempt all the **05 (five)** questions. Each question carries **02 (two)** marks and answer of each question should not exceed 50 words. $5 \times 2 = 10$

1. (a) Find the inverse of matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 9 \end{pmatrix}$.

   (b) Find $\lim\limits_{x \to 0} \dfrac{\sqrt{1+x}-1}{x}$.

   (c) Differentiate $x^{\sin x}$ w.r.t. x.

   (d) Define beta and gamma function.

   (e) If $\vec{r} = 2\hat{i} - 3\hat{j} + 6\hat{k}$, then find the unit vector in the direction of $\vec{r}$.

Section-B

(Short Answer Type Questions)

Note : Attempt **05 (five)** questions out of total 08 (Eight) questions. Each question carry **05 (five)** marks and answer of each question should not exceed 100 words. $5 \times 5 = 25$

2. (a) Find the rank of matrix.
   $$A = \begin{pmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{pmatrix}$$

   (b) Prove that the set $S = \{(1, 0, 0), (1, 0, 1), (0, 1, 0)\}$ is linearly independent set in $\mathbb{R}^3$.

   (c) Evaluate $\lim\limits_{x \to 0} \left(\dfrac{\tan x}{x} \right)^{1/x}$.

   (d) Obtain the Maclurin's series expansion of $e^x \cos x$.

2

(e) Prove that $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$.

(f) Find the value of the Constant 'a' such that the vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} + a\hat{j} + 5\hat{k}$ one coplanar.

(g) Prove that $|\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}| = 2[\vec{a},\vec{b},\vec{c}]$.

(h) Trace the curve $y^2 x = a^2$.

Section-C

(Long Answer Type Questions)

Note : Attempt any **02 (two)** questions out of total **04 (four)** questions. Each question carries **20 (twenty)** marks and answer of each question should not exceed 400 words. $2 \times 20 = 40$

3. (a) For what values of a, the system of equations :

$x+y+z=1$, $x+2y+4z=a$, $x+4y+10z=a^2$

has a solution and solve them completely in each case. $10+10=20$

(b) If $y = e^{m\cos^{-1}x}$, then prove that $(1-x^2)$ $y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$ Find $y_{n(0)}$. $10+10=20$

(c) If $I(m,n) = \int_0^{\pi/2} \cos^m x \cos nx\, dx$, then prove that $(m+n)\, I(m,n) = m\, I(m-1, n-1)$. Find the value of $I(4, 5.)$ $10+10=20$

(d) Define all types of discontinuity and examine the continuity of the function

$$f(x) = \begin{cases} x, & 0 \le x \le \frac{1}{2} \\ 1, & x = \frac{1}{2} \\ 1-x, & \frac{1}{2} < x \le 1 \end{cases} \quad \text{at } x = 0, \frac{1}{2}, 1.$$

Discuss the kind of discontinuity if any. $10+10=20$