Semester-4

BCA 403

OT

(Optimization Techniques)

(According to Purvanchal University Syllabus)

"Full Line By Line Notes"

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Unit – 1

Linear Programming

Definition of LPP–

- A **linear programming** problem consists of a **linear** function to be maximized or minimized subject to certain constraints in the form of **linear** equations or inequalities.
- Linear Programming **Problems** (**LPP**) provide the method of finding such an optimized function along with/or the values which would optimize the required function accordingly. It is one of the most important Operations Research tools.

It consists for four basic components:

- Decision variables represent quantities to be determined
- Objective function represents how the decision variables affect the cost or value to be optimized (minimized or maximized)
- Constraints represent how the decision variables use resources, which are available in limited quantities
- Data quantifies the relationships represented in the objective function and the constraints

All linear programming problems must have following five characteristics:

(a) Objective function: There must be clearly defined objective which can be stated in quantitative way. In business problems the objective is generally profit maximization or cost minimization.

(b) Constraints: All constraints (limitations) regarding resources should be fully spelt out in mathematical form.

(c) Non-negativity: The value of variables must be zero or positive and not negative. For example, in the case of production, the manager can decide about any particular product number in positive or minimum zero, not the negative.

(d) Linearity: The relationships between variables must be linear. Linear means proportional relationship between two 'or more variable, i.e., the degree of variables should be maximum.

(e) Finiteness: The number of inputs and outputs need one to be finite. In the case of infinite factors, to compute feasible solution is not possible.

Graphical solution of two variable LPP–

2.3.1 Procedure for Solving LPP by Graphical Method

The steps involved in the graphical method are as follows.

Step 1 Consider each inequality constraint as an equation.

Step 2 Plot each equation on the graph as each will geometrically represent a straight line.

Step 3 Mark the region. If the inequality constraint corresponding to that line is O then the region below the line lying in the first quadrant (due to non-negativity of variables) is shaded. For the inequality constraint P sign, the region above the line in the first quadrant is shaded. The points lying in common region will satisfy all the constraints simultaneously. The common region, thus obtained, is called the feasible region.

Step 4 Assign an arbitrary value, say zero, for the objective function.

Step 5 Draw a straight line to represent the objective function with the arbitrary value (i.e., a straight line through the origin).

Step 6 Stretch the objective function line till the extreme points of the feasible region. In the maximization case, this line will stop farthest from the origin, passing through at least one corner of the feasible region. In the minimization case, this line will stop nearest to the origin and passes through at least one corner of the feasible region.

Step 7 Find the coordinates of the extreme points selected in step 6 and find th maximum or minimum value of Z.

Note: As the optimal values occur at the corner points of the feasible region, : is enough to calculate the value of the objective function of the corner points of th feasible region and select the one which gives the optimal solution, i.e., in the cas of maximization problem the optimal point corresponds to the corner point at whic the objective function has a maximum value and in the case of minimization, th corner point which gives the objective function the minimum value is the optima solution.

Example 2.9: Solve the following LPP by graphical method.

Minimize $Z = 20X_1 + 10X_2$

Subject ot $X_1 + 2X_2 \leq 40$

$3X_1 + X_2 \geq 30$

$4X_1 + 3X_2 \geq 60$

$X_1, X_2 \geq 0$

Solution: Replace all the inequalities of the constraints by equation

$X_1 + 2X_2 = 40$ If $X_1 = 0 \Rightarrow X_2 = 20$

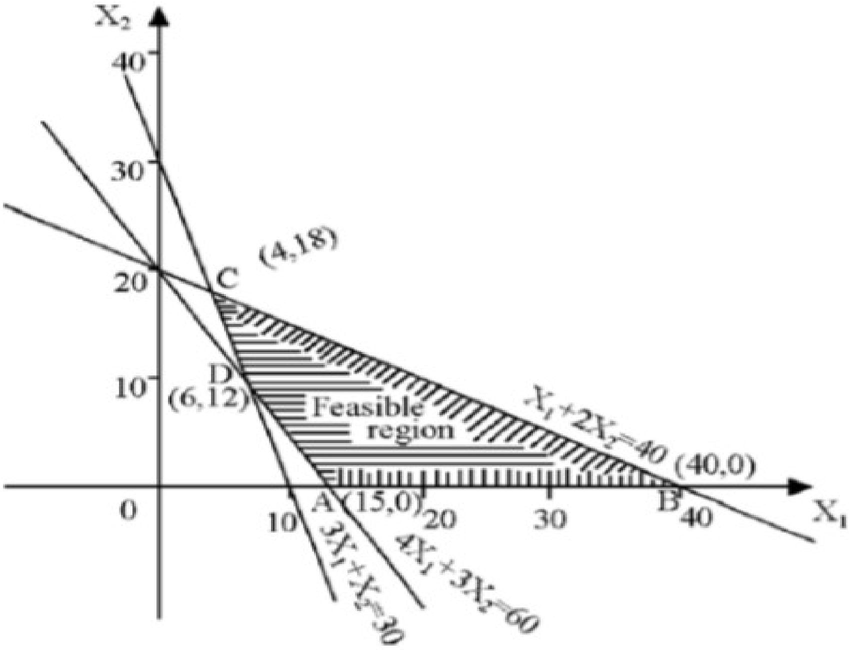
If $X_2 = 0 \Rightarrow X_1 = 40$

$\therefore X_1 + 2X_2 = 40$ passes through (0, 20) (40, 0)

$3X_1 + X_2 = 30$ passes through (0, 30) (10, 0)

$4X_1 + 3X_2 = 60$ passes through (0, 20) (15, 0)

Plot each equation on the graph.



The feasible region is $ABCD$.

C and D are the points of intersection of lines

$X_1 + 2X_2 = 40$, $3X_1 + X_2 = 30$ and

$4X_1 + 3X_2 = 60$, $X_1 + X_2 = 30$

On solving we get $C = (4, 18)$ $D = (6, 12)$

Corner points	Value of $Z = 20X_1 + 10X_2$
A (15, 0)	300
B (40, 0)	800
C(4, 18)	260
D (6, 12)	240 (Minimum value)

$\therefore$ The minimum value of Z occurs at D (6, 12). Hence, the optimal solution is $X_1 = 6$, $X_2 = 12$.

General LPP problem–

2.3.2 General Formulation of LPP

The general formulation of the LPP can be stated as follows: Maximize or Minimize

$$Z - C_1X_1 + C_2X_2 + \dots + C_nX_n \qquad \dots(1)$$

Subject to m constraints

$$\left\{\begin{array}{l} a_{11}X_1 + a_{12}X_2 + \cdots + a_{1j}X_j + \cdots + a_{m}X_n \; (\le = \ge) b_1 \\ a_{21}X_1 + a_{22}X_2 + \cdots + a_{2j}X_j + \cdots + a_{2n}X_n \; (\le = \ge) b_2 \\ \vdots \\ a_{i1}X_1 + a_{i2}X_2 + \cdots + a_{1j}X_j + \cdots + a_{1n}X_n (\le = \ge) b_i \\ \vdots \\ a_{m1}X_1 + a_{m2}X_2 + \cdots + a_{mj}X_j + \cdots + a_{mn}X_n (\le = \ge) b_m \end{array}\right\} \qquad \dots(2)$$

In order to find the values of n decision variables $X_1 X_2 \dots X_n$ to maximize or minimize the objective function and the non-negativity restrictions

$$X_1 \ge 0,\ X_2 \ge 0 \ge X_n \ge 0 \qquad \dots(3)$$

Canonical and standard form of LPP–

2.3.5 Canonical or Standard Forms of LPP

The general LPP can be put in the following forms, namely canonical and standard forms.

In the *standard form*, irrespective of the objective function, namely maximize or minimize, all the constraints are expressed as equations. Moreover, RHS of each constraint and all variables are non-negative.

Characteristics of the Standard Form

(i) The objective function is of maximization type.

(ii) All constraints are expressed as equations.

(iii) The right hand side of each constraint is non-negative.

(iv) All variables are non-negative.

In the *canonical form*, if the objective function is of maximization, all the constraints other than non-negativity conditions are '≤' type. If the objective function is of minimization, then all the constraints other than non-negative condition are '≥' type.

Characteristics of the Canonical Form

(i) The objective function is of maximization type.

(ii) All constraints are of (≤) type.

(iii) All variables X_i are non-negative.

Note:

(i) Minimization of a function Z is equivalent to maximization of the negative expression of this function, i.e., Min Z = –Max (–Z).

(ii) An inequality in one direction can be converted into an inequality in the opposite direction by multiplying both sides by (-1).

(iii) Suppose we have the constraint equation

$$a_{11}x_1+a_{12}x_2+\ldots\ldots+a_{1n}x_n=b_1$$

This equation can be replaced by two weak inequalities in opposite directions

$$a_{11}x_1+a_{12}x_2+\ldots\ldots+a_{m}x_n\le b_1$$
$$a_{11}x_1+a_{12}x_2+\ldots\ldots+a_{1n}x_n\ge b_1$$

(iv) If a variable is unrestricted in sign, then it can be expressed as a difference of two non-negative variables, i.e., if X_1 is unrestricted in sign, then $X_1 = X_1' - X_1''$, where X_1' , $X_1' - X_1''$ are ≥ 0.

(v) In the standard form, all the constraints are expressed in equation, which is possible by introducing some additional variables called slack variables and surplus variables so that a system of simultaneous linear equations is obtained. The necessary transformation will be made to ensure that $b_i \ge 0$.

Simplex methods and artificial variable–

- Simplex method is an iterative procedure for solving LPP in a finite number of steps.
- This method provides an algorithm which consists of moving from one vertex of the region of feasible solution to another in such a manner that the value of the objective function at the succeeding vertex is less or more as the case may be than at the previous vertex.
- This procedure is repeated and since the number of vertices is finite, the method leads to an optimal vertex in a finite number of steps or indicates the existence of unbounded solution.

Artificial Variable :

- In order to use the simplex method on problems with mixed constraints, we turn to a device called an artificial variable.
- This variable has no physical meaning in the original problem and is introduced solely for the purpose of obtaining a basic feasible solution so that we can apply the simplex method.
- An artificial variable is a variable introduced into each equation that has a surplus variable. To ensure that we 7 q p consider only basic feasible solutions, an artificial variable is required to satisfy the nonnegative constraint.

Sensitivity analysis–

- Sensitivity Analysis deals with finding out the amount by which we can change the input data for the output of our linear programming model to remain comparatively unchanged.
- This helps us in determining the sensitivity of the data we supply for the problem. If a small change in the input (for example in the change in the availability of some raw material) produces a large change in the optimal solution for some model, and a corresponding small change in the input for some other model doesn't affect its optimal solution as much, we can

conclude that the second problem is more robust then the first. The second model is less sensitive to the changes in the input data.

Problem of degeneracy & Concept of duality–

Every LPP (called the primal) is associated with another LPP (called its dual).

Either of the problem can be considered as primal with the other as dual. The importance of the duality concept is due to **two main reasons:**

(i) If the primal contains a large number of constraints and a smaller number of variables, the labour of computation can be considerably reduced by converting it in to the dual problem and then solving it.

(ii) (ii) The interpretation of the dual variables from the cost or economic point of view, proves extremely useful in making future decisions in the activities being programmed.

Formulation of Dual Problem

For formulating a dual problem, first we bring the problem in the canonical form. The following changes are used in formulating the dual problem.

(1) Change the objective function of maximization in the primal into that of minimization in the dual and vice versa.

(2) The number of variables in the primal will be the number of constraints in the dual and vice versa.

(3) The cost coefficients C1, C2 ... Cn in the objective function of the primal will be the RHS constant of the constraints in the dual and vice versa.

(4) In forming the constraints for the dual, we consider the transpose of the body matrix of the primal problem.

(5) The variables in both problems are non-negative.

(6) If the variable in the primal is unrestricted in sign, then the corresponding constraint in the dual will be an equation and vice versa.

Unit – 2

Transportation problems

Introduction to transportation model–

- Transportation problems are one of the subclasses of LPPs in which the objective is to transport various quantities of a single homogenous commodity that are initially stored at various origins, to different destinations in such a way that the transportation cost is minimum.

Matrix form of TP–

A typical transportation problem is shown in Fig. 9. It deals with sources where a supply of some commodity is available an destinations where the commodity is demanded. The classic statement of the transportation problem uses a matrix with the rows representing sources and columns representing destinations. The algorithms for solving the problem are based on this matrix representation. The costs of shipping from sources to destinations are indicated by the entries in the matrix. If shipment is impossible between a given source and destination, a large cost of M is entered. This discourages the solution from using such cells. Supplies and demands are shown along the margins of the matrix. As in the example, the classic transportation problem has total supply equal to total demand.

	D1	D2	D3	Supply
S1	3	1	M	5
S2	4	2	4	7
S3	M	3	3	3
Demand	7	3	5	

Figure 9. Matrix model of a transportation problem.

Application of TP model–

- Application of Transportation Problem:
  - Minimize shipping costs
  - Determine low cost location

- Find minimum cost production schedule
- Military distribution system

Assignment problems–

Suppose there are n jobs to be performed and n persons are available for doing these jobs. Assume that each person can do one job at a time, though with varying degrees of efficiency. Let c_{ij} be the cost if the i^{th} person is assigned to the j^{th} job. The problem is to find an assignment (which job should be assigned to which person on a one-one basis) so that the total cost of performing all the jobs is minimum. Problems of this type are known as assignment problems.

The assignment problem can be stated in the form of a non cost matrix $[c_{ij}]$ of real numbers as given in the following table.

Jobs

$$\textbf{Persons}\quad \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ \\ i \\ \\ n \end{array} \begin{array}{ccccc} 1 & 2 & 3\ldots & j\ldots & n \\ \hline c_{11} & c_{12} & c_{13} & c_{1j} & c_{1n} \\ c_{21} & c_{22} & c_{23} & c_{2j} & c_{2n} \\ c_{32} & c_{32} & c_{23} & c_{3j} & c_{3n} \\ & & & & \\ c_{i1} & c_{i2} & c_{i2} & c_{ij} & c_{in} \\ \vdots & & & & \\ c_{n1} & c_{n2} & c_{n3}\cdots & c_{nj}\cdots & c_{nn} \end{array}$$

Mathematical Formulation–

Mathematical Formulation of the Assignment Problem

Mathematically, the assignment problem can be stated as

$$\sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \quad i = 1, 2, \ldots n$$
$$j = 1, 2, \ldots n$$

Subject to the restrictions

$$x_{ij} = \begin{cases} 1 \text{ if the } i^{th} \text{ person is assigned } j^{th} \text{ job} \\ 0 \text{ if not} \end{cases}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ (one job is done by the i}^{th} \text{ person)}$$

and $\sum_{i=1}^{n} x_{ij} = 1$ (only one person should be assigned the j^{th} job)

Where x_{ij} denotes that the j^{th} job is to be assigned to the i^{th} person.

Difference between Transportation Problem and Assignment Problem

Transportation problem	*Assignment problem*
1. No. of sources and number of destinations need not be equal. Hence, the cost matrix is not necessarily a square matrix.	Since assignment is done on one-to-one basis, the number of sources and the number of destinations are equal. Hence, the cost matrix must be a square matrix.
2 x_{ij}, the quantity to be transported from ith origin to jth destination can take any possible positive values, and satisfies the rim requirements.	x_{ij}, the jth job is to be assigned to the ith person and can take either the value 1 or 0.
3. The capacity and the requirement value is equal to ai and bj for the ith source and jth destinations (i=1, 2m j=1,2... n)	The capacity and the requirement value is exactly 1 i.e., for each source of each destination the capacity and the requirement value is exactly 1.
4. The problem is unbalanced if the total supply and the total demand are not equal.	The problem is unbalanced if the cost matrix is not a square matrix.

Finding IBFS–

An initial basic feasible solution to
a transportation problem can be found by any one of the three

following methods:
(i) North west corner rule (NWC)
(ii) Least cost method (LCM)
(iii) Vogel's approximation method (VAM)

North West Corner Rule

Step 1 Starting with the cell at the upper left corner (North west) of the transportation matrix we allocate as much as possible so that either the capacity of the first row is exhausted or the destination requirement of the first column is satisfied i.e., $X_{11} = \min (a_1,b_1)$.

Step 2 If $b_1 > a_1$, we move down vertically to the second row and make the second allocation of magnitude $x_{22} = \min(a_2\ b_1 - x_{11})$ in the cell (2, 1).

If $b_1 < a_1$, move right horizontally to the second column and make the second allocation of magnitude $X_{12} = \min (a_1, X_{11}[b_1)$ in the cell (1, 2).

If $b_1 = a_1$ there is a tie for the second allocation. We make the second allocations of magnitude $x_{12} = \min (a_1 - a_1, b) = 0$ in cell (1, 2).

or $x_{21} = \min (a_2 - a_1 - b_1) = 0$ in cell (2, 1).

Step 3 Repeat steps 1 and 2 moving down towards the lower right corner of the transportation table until all the rim requirements are satisfied.

Example 3.1 Obtain the initial basic feasible solution of a transportation problem whose cost and rim requirement table is the following.

Origin\Destination	D_1	D_2	D_3	*Supply*
O_1	2	7	4	5
O_2	3	3	1	8
O_3	5	4	7	7
O_4	1	6	2	14
Demand	7	9	18	34

Solution Since $\Sigma ai = 34 = \Sigma bj$ there exists a feasible solution to the transportation problem. We obtain initial feasible solution as follows.

The first allocation is made in the cell (1, 1) the magnitude being $x_{11} = \min (5, 7) = 5$. The second allocation is made in the cell (2, 1) and the magnitude of the allocation is given by $x_{21} = \min (8, 7 - 5) = 2$.

	D_1	D_2	D_3	Supply
O_1	2 (5)	7	4	~~5~~ 0
O_2	3 (2)	3 (6)	1	~~8~~ ~~6~~ 0
O_3	5	4 (3)	7 (4)	~~7~~ ~~4~~ 0
O_4	1	6	2 (14)	~~14~~ 0
Demand	~~7~~ ~~2~~ 0	~~9~~ ~~3~~ 0	~~18~~ ~~14~~ 0	34

The third allocation is made in the cell (2, 2) the magnitude x_{22} = min (8 – 2, 9) = 6.

The magnitude of fourth allocation is made in the cell (3, 2) given by min (7, 9 – 6) = 3.

The fifth allocation is made in the cell (3, 3) with magnitude x_{33} = min (7 – 3, 14) = 4.

The final allocation is made in the cell (4, 3) with magnitude x_{43} = min (14, 18 – 4) = 14.

Hence, we get the initial basic feasible solution to the given T.P. and is given by

$X_{11} = 5; X_{21} = 2; X_{22} = 6; X_{32} = 3; X_{33} = 4; X_{43} = 14$

Total cost = 2 e 5 – 3 e 2 – 3 e 6 – 3 e 4 – 4 e 7 – 2 e 14

=10 – 6 – 18 – 12 – 28 –28 = Rs 102.

Least Cost or Matrix Minima Method

Step 1 Determine the smallest cost in the cost matrix of the transportation table. Let it be C_{ij}. Allocate x_{ij} = min (a_i, b_j) in the cell (i, j).

Step 2 If $x_{ij} = a_i$ cross off the i^{th} row of the transportation table and decrease b_j by a_i. Then go to step 3.

If $x_{ij} = b_j$ cross off the j^{th} column of the transportation table and decrease a_i by bj. Go to step 3.

If $x_{ij} = a_i = b_j$ cross off either the i^{th} row or the j^{th} column but not both.

Step 3 Repeat steps 1 and 2 for the resulting reduced transportation table until all the rim requirements are satisfied. Whenever the minimum cost is not unique, make an arbitrary choice among the minima.

Example 3.3 Obtain an initial feasible solution to the following TP using Matrix Minima Method.

	D_1	D_2	D_3	D_4	*Supply*
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
Demand	4	6	8	6	24

Solution Since $\Sigma a_i = \Sigma b_j = 24$, there exists a feasible solution to the TP using the steps in the least cost method, the first allocation is made in the cell (3, 1) the magnitude being $x_{31} = 4$. This satisfies the demand at the destination D_1 and we delete this column from the table as it is exhausted.

	D_1	D_2	D_3	D_4	Supply
O_1	1	2 6	3	4	~~6~~ 0
O_2	4	3	2 2	0 6	~~8~~ ~~2~~
O_3	0 4	2	2 6	1	~~10~~ ~~6~~
Demand	~~4~~ 0	~~6~~ 0	~~8~~ ~~2~~ 0	~~6~~ 0	24

The second allocation is made in the cell (2, 4) with magnitude x_{24} = min (6, 8) = 6. Since it satisfies the demand at the destination D_4, it is deleted from the table. From the reduced table, the third allocation is made in the cell (3, 3) with magnitude x_{33} = min (8, 6) = 6. The next allocation is made in the cell (2, 3) with magnitude x_{23} of min (2, 2) = 2. Finally, the allocation is made in the cell (1, 2) with magnitude x_{12} = min (6, 6)\6. Now all the rim requirements have been satisfied and hence, the initial feasible solution is obtained.

The solution is given by

$$x_{12} = 6, x_{23} = 2, X_{24} = 6, X_{31} = 4, X_{33} = 6$$

Since the total number of occupied cells = 5 < $m + n - 1$.

We get a degenerate solution.

Total cost = $6 \times 2 + 2 \times 2 + 6 \times 0 + 4 \times 0 + 6 \times 2$

$= 12 + 4 + 12 =$ Rs 28.

Vogel's Approximation Method (VAM)

The steps involved in this method for finding the initial solution are as follows.

Step 1 Find the penalty cost, namely the difference between the smallest and next smallest costs in each row and column.

Step 2 Among the penalties as found in Step(1) choose the maximum penalty. If this maximum penalty is more than one (i.e., if there is a tie) choose any one arbitrarily.

Step 3 In the selected row or column as by Step(2) find out the cell having the least cost. Allocate to this cell as much as possible depending on the capacity and requirements.

Step 4 Delete the row or column which is fully exhausted. Again, compute the column and row penalties for the reduced transportation table and then go to Step (2). Repeat the procedure until all the rim requirements are satisfied.

Note If the column is exhausted, then there is a change in row penalty and vice versa.

Example 3.5 Find the initial basic feasible solution for the following transportation problem by VAM.

Note If the column is exhausted, then there is a change in row penalty and vice versa.

Example 3.5 Find the initial basic feasible solution for the following transportation problem by VAM.

		Destination				
		D_1	D_2	D_3	D_4	***Supply***
Origin	O_1	11	13	17	14	250
	O_2	16	18	14	10	300
	O_3	21	24	13	10	400
	Demand	200	225	275	250	950

Solution Since $\Sigma a_i = \Sigma b_j = 950$ the problem is balanced and there exists a feasible solution to the problem.

First, we find the row Σ column penalty P_I as the difference between the least and the next least cost. The maximum penalty is 5. Choose the first column arbitrarily.

In this column, choose the cell having the least cost name (1, 1). Allocate to this cell with minimum magnitude (i.e.(250, 200) = 200.) This exhausts the first column. Delete this column. Since a column is deleted, then there is a change in row penalty P_{II} and column penalty P_{II} remains the same. Continuing in this manner, we get the remaining allocations as given in the following table below.

I allocation

	D_1	D_2	D_3	D_4	Supply	P_I
O_1	11 (200)	13	17	14	50 ~~250~~	2
O_2	16	18	14	10	300	4
O_3	21	24	13	10	400	3
Demand	~~200~~ 0	225	275	250		
P_I	5□	5	3	0		

II allocation

	D_2	D_3	D_4	Supply	P_{II}
O_1	13 (50)	17	14	~~500~~	3
O_2	18	14	10	300	4
O_3	24	13	10	400	3
Demand	~~225~~ 175	275	250		
P_{II}	5□	3	0		

III allocation

	D_2	D_3	D_4	Supply	P_{III}
O_2	18 (175)	14	10	~~300~~ 125	4
O_3	24	13	10	400	3
Demand	~~175~~ 0	275	250		
P_{III}	6□	1	0		

IV allocation

	D_3	D_4	Supply	P_{IV}
O_2	14	10 (125)	~~125~~ 0	4 ←
O_3	13	10	400	3
Demand	275	~~250~~ 125		
P_{IV}	1	0		

V allocation

	D_3	D_4	Supply	P_V
O_3	13 (275)	10	~~400~~ 125	3
Demand	~~275~~ 0	125		
P_V	13□	10		

VI allocation

	D_4	Supply	P_{VI}
O_3	10 (125)	~~125~~ 0	10 ←
Demand	~~125~~ 0		
P_{VI}	10		

Finally, we arrive at the initial basic feasible solution which is shown in the following table.

	D_1	D_2	D_3	D_4	Supply
O_1	11 (200)	13 (50)	17	14	250
O_2	16	18 (175)	14	10 (125)	300
O_3	21	24	13 (275)	10 (125)	400
Demand	200	225	275	250	

There are 6 positive independent allocations which equals to $m + n - 1 = 3 + 4 - 1$. This ensures that the solution is a non-degenerate basic feasible solution.

$\therefore$ The transportation cost

$= 11 \times 200 + 13 \times 50 + 18 \times 175 + 10 \times 125$

$+ 13 \times 275 + 10 \times 125 =$ Rs 12,075.

Degeneracy–

In a TP, if the number of non-negative independent allocations is less than m]n[1, where m is the number of origins (rows) and n is the number of destinations (columns) there exists a degeneracy. This may occur either at the initial stage or at the subsequent iteration.

To resolve this degeneracy, we adopt the following steps.

(1) Among the empty cell, we choose an empty cell having the least cost which is of an independent position. If this cell is more than one, choose any one arbitrarily.

(2) To the cell as chosen in step (1) we allocate a small positive quantity ∈> 0. The cells containing ∈ are treated like other occupied cells and degeneracy is removed by adding one (more) accordingly. For this modified solution, we adopt the steps involved in MODI method till an optimum solution is obtained.

Unbalanced transportation problem–

The given TP is said to be unbalanced if $\Sigma a_i \neq \Sigma b_j$, i.e., if the total supply is not equal to the total demand.

There are two possible cases.

Case i : $\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$

If the total supply is less than the total demand, a dummy source (row) is included in the cost matrix with zero cost; the excess demand is entered as a rim requirement for this dummy source (origin). Hence, the unbalanced transportation problem can be converted into a balanced TP.

Case ii : $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$

i.e., the total supply is greater than the total demand. In this case, the unbalanced TP can be converted into a balanced TP by adding a dummy destination (column) with zero cost. The excess supply is entered as a rim requirement for the dummy destination.

Unit – 3

Sequencing models & Related problems

Sequencing problem–

- When a number of jobs are given to be done and they require processing on two or more machines, the main concern of a manager is to find the order or sequence to perform these jobs. We shall consider the sequencing problems in respect of the jobs to be performed in a factory and study the method of their solution. Such sequencing problems can be broadly divided in two groups. In the first one, there are n jobs to be done, each of which requires processing on some or all of the k different machines.

Processing n jobs through two machine–

Example

There are nine jobs, each of which must go through two machines P and Q in the order PQ, the processing times (in hours) are given below:

Machine	Job(s)								
	A	B	C	D	E	F	G	H	I
P	2	5	4	9	6	8	7	5	4
Q	6	8	7	4	3	9	3	8	11

Find the sequence that minimizes the total elapsed time T. Also calculate the total idle time for the machines in this period.

Solution

The minimum processing time on two machines is 2 which correspond to task A on machine P. This shows that task A will be preceding first. After assigning task A, we are left with 8 tasks on two machines

Machine	B	C	D	E	F	G	H	I
P	5	4	9	6	8	7	5	4
Q	8	7	4	3	9	3	8	11

Minimum processing time in this reduced problem is 3 which correspond to jobs E and G (both on machine Q). Now since the corresponding processing time of task E on machine P is less than the corresponding processing time of task G on machine Q therefore task E will be processed in the last and task G next to last. The situation will be dealt as

A							G	E

The problem now reduces to following 6 tasks on two machines with processing time as follows:

Machine	B	C	D	F	H	I
P	5	4	9	8	5	4
Q	8	7	4	9	8	11

Here since the minimum processing time is 4 which occurs for tasks C and I on machine P and task D on machine Q. Therefore, the task C which has less processing time on P will be processed first and then task I and task D will be placed at the last i.e., 7th sequence cell.

The sequence will appear as follows:

A	C	I				D	E	G

The problem now reduces to the following 3 tasks on two machines

Machine	B	F	H
P	5	8	5
Q	8	9	8

In this reduced table the minimum processing time is 5 which occurs for tasks B and H both on machine P. Now since the corresponding time of tasks B and H on machine Q are same i.e. 8. Tasks B or H may be placed arbitrarily in the 4th and 5th sequence cells. The remaining task F can then be placed in the 6th sequence cell. Thus the optimal sequences are represented as

A	I	C	B	H	F	D	E	G

or

A	1	C	H	B	F	D	E	G

Further, it is also possible to calculate the minimum elapsed time corresponding to the optimal sequencing A → I → C → B → H → F → D → E → G.

Job Sequence	Machine A		Machine B	
	Time In	Time Out	Time In	Time Out
A	0	2	2	8
I	2	6	8	19
C	6	10	19	26
B	10	15	26	34
H	15	20	34	42
F	20	28	42	51
D	28	37	51	55
E	37	43	55	58
G	43	50	58	61

Hence the total elapsed time for this proposed sequence staring from job A to completion of job G is 61 hours .During this time machine P remains idle for 11 hours (from 50 hours to 61 hours)and the machine Q remains idle for 2 hours only (from 0 hour to 2 hour).

Processing n jobs through three machine–

Example

There are five jobs (namely 1,2,3,4 and 5), each of which must go through machines A, B and C in the order ABC. Processing Time (in hours) are given below:

Jobs	1	2	3	4	5
Machine A	5	7	6	9	5
Machine B	2	1	4	5	3
Machine C	3	7	5	6	7

Find the sequence that minimum the total elapsed time required to complete the jobs.

Solution

Here Min Ai = 5; Bi = 5 and Ci =3 since the condition of Min. Ai ≥ Max. Bi is satisfied the given problem can be converted into five jobs and two machines problem.

Jobs	$G_i = A_i + B_i$	$H_i = B_i + C_i$
1	7	5
2	8	8
3	10	9
4	14	11
5	8	10

The Optimal Sequence will be

2 5 4 3 1

Total elapsed Time will be

Jobs	Machine A		Machine B		Machine C	
	In	Out	In	Out	In	Out
2	0	7	7	8	8	15
5	7	12	12	15	15	22
4	12	21	21	26	26	32
3	21	27	27	31	32	37
1	27	32	32	34	37	40

Min. total elapsed time is 40 hours.
Idle time for Machine A is 8 hrs. (32-40)
Idle time for Machine B is 25 hours (0-7, 8-12, 15-21, 26-27, 31-32 and 34-40)
Idle time for Machine C is 12 hours (0-8, 22-26.)

Processing n jobs through m machine–

Find an optimal sequence for the following sequencing problem of four jobs and five machines when passing is not allowed, of which processing time (in hours) is given below:

Job	Machine				
	A	B	C	D	E

1	7	5	2	3	9
2	6	6	4	5	10
3	5	4	5	6	8
4	8	3	3	2	6

Also find the total elapsed time.
Solution
Here Min. Ai = 5, Min. Ei = 6
Max. (Bi, Ci, Di) = 6, 5, 6respectively
Since Min. Ei = Max. (Bi, Di) and Min. Ai = Max. Ci satisfied therefore the problem can be converted into 4 jobs and 2 fictitious machines G and H as follows:

Job	Fictitious Machine	
	$G_i = A_i + B_i + C_i + D_i$	$H_i = B_i + C_i + D_i + E_i$
1	17	19
2	21	25
3	20	23
4	16	14

The above sequence will be:

1 3 2 4

Total Elapsed Time Corresponding to Optimal Sequence can be obtained as follows:

	Machine A		Machine B		Machine C		Machine D		Machine E	
Job	In	Out	In	Out	In	Out	In	Out	In	Out
1	0	7	7	12	12	14	14	17	17	26
3	7	12	12	16	16	21	21	27	27	35
2	12	18	18	24	24	28	28	33	35	45
4	18	26	26	29	29	32	33	35	45	51

Thus the minimum elapsed time is 51 hours.

Idle time for machine A = 25 hours(26-51)

Idle time for machine B = 33 hours(0-7,16-18,24-26,29-51)

Idle time for machine C = 37 hours(0-12,14-16,21-24,28-29,32-51)

Idle time for machine D = 35 hours (0-14,17-21,27-28,35-51)
Idle time for machine E = 18 hours (0-17,26-27)

Traveling salesman problem–

- The Traveling Salesman Problem (TSP) is a problem in combinatorial optimization studied in operations research and theoretical computer science. Given a list of cities and their pair wise distances, the task is to find a shortest possible tour that visits each city exactly once.
- The TSP has several applications even in its purest formulation, such as planning, logistics, and the manufacture of microchips.

Unit – 4

PERT & CPM

PERT–

- PERT is an acronym for Program (Project) Evaluation and Review Technique, in which planning, scheduling, organizing, coordinating and controlling uncertain activities take place. The technique studies and represents the tasks undertaken to complete a project, to identify the least time for completing a task and the minimum time required to complete the whole project. It was developed in the late 1950s. It is aimed to reduce the time and cost of the project.

CPM–

- Developed in the late 1950s, Critical Path Method or CPM is an algorithm used for planning, scheduling, coordination and control of activities in a project. Here, it is assumed that the activity duration is fixed and certain. CPM is used to compute the earliest and latest possible start time for each activity.

Comparison Chart

BASIS FOR COMPARISON	PERT	CPM
Meaning	PERT is a project management technique, used to manage uncertain activities of a project.	CPM is a statistical technique of project management that manages well defined activities of a project.
What is it?	A technique of planning and control of time.	A method to control cost and time.
Orientation	Event-oriented	Activity-oriented
Evolution	Evolved as Research & Development project	Evolved as Construction project
Model	Probabilistic Model	Deterministic Model
Focuses on	Time	Time-cost trade-off
Estimates	Three time estimates	One time estimate
Appropriate for	High precision time estimate	Reasonable time estimate
Management of	Unpredictable Activities	Predictable activities
Nature of jobs	Non-repetitive nature	Repetitive nature
Critical and Non-critical activities	No differentiation	Differentiated

BASIS FOR COMPARISON	PERT	CPM
Suitable for	Research and Development Project	Non-research projects like civil construction, ship building etc.
Crashing concept	Not Applicable	Applicable

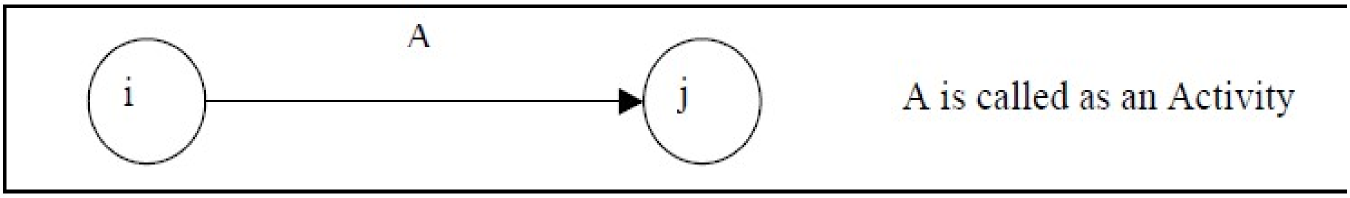
Network and basic components–

PERT / CPM networks contain two major components

i. Activities, and
ii. Events

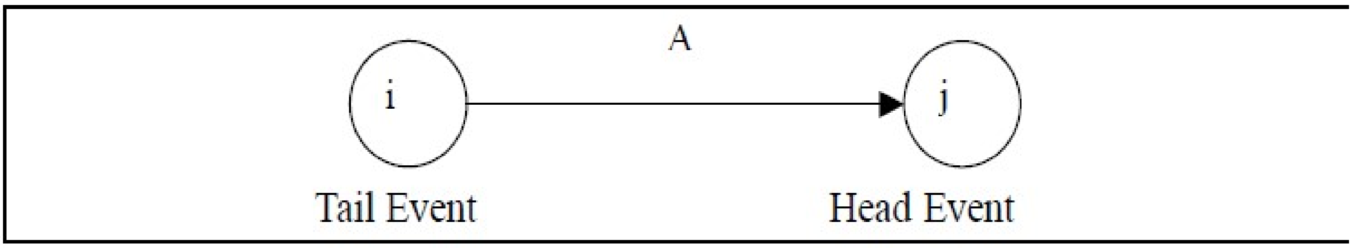
Activity*:* An activity represents an action and consumption of resources (time, money, energy) required to complete a portion of a project. Activity is represented by an arrow.

An Activity



Event*:* An event (or node) will always occur at the beginning and end of an activity. The event has no resources and is represented by a circle. The ith event and jth event are the tail event and head event respectively.

An Event



Unit – 5

Dynamic Programming

Bellman's principal of optimality of Dynamic Programming–

Bellman's principle of optimality: An optimal policy (set of decisions) has the property that whatever the initial state and decisions are, the remaining decisions must constitute and optimal policy with regard to the state resulting from the first decision.

Mathematically, this can be written as:

fN(x)=max.[r(dn)+fN−1T(x,dn)]dn∈xfN(x)=max.[r(dn)+fN−1T(x,dn)]dn∈x

where fN(x)=the optimal return from an N-stage process when initial state is x

r(dn)=immediate return due to decisiondn

T(x,dn)=the transfer function which gives the resulting state

{x}=set of admissible decisions

This equation is also known as a dynamic programming equation. It represents a necessary condition for optimality associated with the mathematical optimization method known as dynamic programming. It writes the value of a decision problem at a certain point in time in terms of the payoff from some initial choices and the value of the remaining decision problem that results from those initial choices. This breaks a dynamic optimization problem into simpler subproblems.

Multistage decision problem & Its solution by Dynamic Programming–

- The dynamic programming approach for solving multistage decision-making problems is presented.
- A general formulation is given followed by models of deterministic, stochastic, and adaptive versions of a particular multistage decision-making problem.
- The dynamic programming approach for solving multistage decision-making problems is presented. A general formulation is given followed by models of deterministic, stochastic, and adaptive versions of a particular multistage decision-making problem.
- Dynamic programming is a useful mathematical technique for making a sequence of interrelated decisions. It provides a systematic procedure for determining the optimal combination of decisions. In contrast to linear programming, there does not exist a standard mathematical formulation of "the" dynamic programming problem.

Simple Difference between DPP & LPP –

- Dynamic programming is a way of solving problems by breaking them down into simpler subproblems. It is essentially a 'smart' recursion. Often extra work doesn't have to be repeated if solutions to subproblems are cached after they are solved. Dynamic programming has a confusing name which traces back to its roots as a field studied by operations researchers.

  Linear programming is a mathematical method (and associated algorithms) for maximizing or minimizing a function subject to a series of linear constraints.

D.P. Algorithm–

The solution of a multistage problem by dynamic programming involves the following steps.

Step 1: Identify the decision variables and specify the objective function to be optimized under certain limitations, if any.

Step 2: Decompose the given problem into a number of smaller sub-problems. Identify the state variable at each stage.

Step 3: Write down the general recursive relationship for computing the optimal policy. Decide whether forward or backward method is to be followed to solve the problem.

Step 4: Construct appropriate stages to show the required values of the return function at each stage.

Step 5: Determine the overall optimal policy or decisions and its value at each stage. There may be more than such optimal policy.

Note: *Some Topics are missing here…*