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EENG 5307 H.W.-2

1. Problem 1: Solving a Linear System in Time & Laplace domains.
Study the following System Dynamics with input $u(t) = \begin{bmatrix} \delta(t) \\ \delta(t) \end{bmatrix}$ and initial condition $x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} u(t).$$

$$y(t) = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} x(t).$$

<u>Solution</u>

Considering the Eigen Values for the time domain approach

$$\text{determinant}(\lambda I - A) = 0, \quad A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$\det \begin{bmatrix} \lambda & -1 \\ +2 & \lambda+3 \end{bmatrix} = 0$$

$$\lambda(\lambda+3) + 2 = 0$$

$$\lambda^2 + 3\lambda + 2 = 0$$

$$\boxed{\lambda = -1, -2.} \qquad \lambda_1 = -1, \lambda_2 = -2.$$

Now finding the Eigen Vectors for Each Eigen Value.

$$AV_1 = \lambda_1 V_1$$

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$$\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix} = -1 \begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix}$$

Solving the. above matrices

$V_{12} = -V_{11}$ — ①

$-2V_{11} - 3V_{12} = -V_{12}$

using ① Equation.

$-2V_{11} - 3V_{12} = V_{11}$

$-\cancel{3}V_{11} = \cancel{3}V_{12}$ — ②.

Since ① & ② Equations are similar we choose.

a particular value

$$\boxed{V_{11} = 1 \;;\; V_{12} = -1}$$

$$V_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Now. consider the other Eigen Value.

$AV_2 = \lambda_2 V_2$

$$\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} V_{21} \\ V_{22} \end{bmatrix} = -2 \begin{bmatrix} V_{21} \\ V_{22} \end{bmatrix}$$

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$$V_{22} = -2V_{21} \quad -(3)$$

$$-2V_{21} - 3V_{22} = -2V_{22}$$

$$-2V_{21} = V_{22} \quad -(4).$$

Similarly we use. $V_{21} = 1$ For ~~which~~. ~~X~~ Equations (3) & (4) ~~have~~ are Similar.

$$\boxed{V_{21} = 1 \;;\; V_{22} = -2.}$$

$$V_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}.$$

$$e^{At} = \cancel{\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} e^{-t} & \\ & e^{-2t} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}^{-1}} \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}^{-1}$$

$$e^{At} = \begin{bmatrix} e^{-t} & e^{-2t} \\ -e^{-t} & -2e^{-2t} \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ 2e^{-2t} - 2e^{-t} & 2e^{-2t} - e^{-t} \end{bmatrix}$$

$$y(t) = Ce^{At}x(0) + C\int_0^t e^{A(t-\tau)} BU(\tau)d\tau + DU(t)$$

Here $x(0) = 0$; $y(t) = C\int_0^t e^{A(t-\tau)} BU(\tau)d\tau$

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$$y(t) = C\int_0^t e^{At}\cdot e^{-A\tau} B\,U(\tau)\,d\tau$$

$$= C e^{At} B \int_0^t e^{-A\tau} U(\tau)\,d\tau$$

Here

$$U(\tau) = \delta(\tau)$$

$$= C e^{At} B \int_0^t e^{-A\tau}\delta(t)\,d\tau$$

impulse function will be '1' at only one point

$$= Ce^{At}B\left[e^{(0)}\right] = Ce^{At}B\begin{bmatrix}1\\1\end{bmatrix}$$

considering the Vector here

$$= \begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}2e^{-t}-e^{-2t} & e^{-t}-e^{-2t}\\ 2e^{-2t}-2e^{-t} & 2e^{-2t}-e^{-t}\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}$$

$$= \begin{bmatrix}4e^{-t}-2e^{-2t} & 2e^{-t}-2e^{-2t}\\ e^{-2t} & e^{-2t}\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}$$

$$= \begin{bmatrix}6e^{-t}-4e^{-2t} & 2e^{-t}\\ 2e^{-2t} & 0\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}$$

$$y(t) = \begin{bmatrix}8e^{-t}-4e^{-2t}\\ 2e^{-2t}\end{bmatrix}$$

y(t) for time domain & Laplace domain are Same.

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Laplace Domain.

$$y(t) = \mathcal{L}^{-1}\{[C(SI-A)^{-1}B + D]U(S)\} + \mathcal{L}^{-1}\{C(SI-A)^{-1}\}x(0).$$

given $x(0) := \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

hence $y(t) = \mathcal{L}^{-1}\{[C(SI-A)^{-1}B + D]\,U(S)\}$

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}; \quad B = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}; \quad C = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}; \quad D = 0.$$

$$SI - A = \begin{bmatrix} S & -1 \\ 2 & S+3 \end{bmatrix}$$

$$(SI-A)^{-1} = \frac{1}{(S^2+3S+2)}\begin{bmatrix} S+3 & 1 \\ -2 & S \end{bmatrix} = \frac{1}{(S+1)(S+2)}\begin{bmatrix} S+3 & 1 \\ -2 & S \end{bmatrix}$$

$$C(SI-A)^{-1} = \frac{1}{(S+1)(S+2)}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} S+3 & 1 \\ -2 & S \end{bmatrix}$$

$$= \frac{1}{(S+1)(S+2)}\begin{bmatrix} 2S+6 & 2 \\ S+3-2 & 1+S \end{bmatrix} = \frac{1}{(S+1)(S+2)}\begin{bmatrix} 2S+6 & 2 \\ S+1 & 1+S \end{bmatrix}$$

$$C(SI-A)^{-1}B = \frac{1}{(S+1)(S+2)}\begin{bmatrix} 2S+6 & 2 \\ S+1 & 1+S \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \frac{1}{(S+1)(S+2)}\begin{bmatrix} 2S+8 & 2S+4 \\ 2S+2 & 0 \end{bmatrix}$$

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$$\left[C(SI-A)^{-1}B+D\right] = \frac{1}{(S+1)(S+2)}\begin{bmatrix} 2S+8 & 2S+4 \\ 2S+2 & 0 \end{bmatrix}$$

$$\mathcal{L}^{-1}\left\{C(SI-A)^{-1}B+D\right\}U(S)$$

$$U(S) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$y(t) = \mathcal{L}^{-1}\left\{\begin{bmatrix} \frac{2S+8}{(S+1)(S+2)} & \frac{2\cancel{(S+2)}}{(S+1)\cancel{(S+2)}} \\ \frac{2\cancel{(S+1)}}{\cancel{(S+1)}(S+2)} & 0 \end{bmatrix}\right\}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\mathcal{L}^{-1}\left(\frac{2S+8}{(S+1)(S+2)}\right) = \mathcal{L}^{-1}\left\{\frac{A}{S+1} + \frac{B}{S+2}\right\}$$

Solving for A & B

$$2S+8 = A(S+2) + B(S+1)$$

at $S = -2 \Rightarrow B = -2$

at $S = -1 \Rightarrow A = 3$

$$\mathcal{L}^{-1}\left\{\begin{bmatrix} \frac{3}{S+1} - \frac{2}{S+2} & \frac{2}{S+1} \\ \frac{2}{S+2} & 0 \end{bmatrix}\right\}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6e^{-t} - 4e^{-2t} & 2e^{-t} \\ 2e^{-2t} & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 8e^{-t} - 4e^{-2t} \\ 2e^{-2t} \end{bmatrix}$$

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(2). Reconsider the above system dynamics (1) with initial condition. $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ please find the zero i/p response. you can choose either the time domain (or) the Laplace domain approach.

Considering the Laplace domain approach.

$$Y(s) = C[sI - A]^{-1} x[0] + [C(sI-A)^{-1} B + D] U(s)$$

Here, we are considering the "zero i/p response".

$$Y(s) = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2+3s+2} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2+3s+2} \begin{bmatrix} 2s+6 & 2 \\ s+1 & s+1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2+3s+2} \begin{bmatrix} 2s+8 \\ 2s+2 \end{bmatrix}$$

$$Y(s) = \begin{bmatrix} \dfrac{2(s+4)}{s^2+3s+2} \\ \dfrac{2(s+1)}{(s+1)(s+2)} \end{bmatrix}$$

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Applying inverse Laplace domain for the above form tp obtain $y(t)$

$$\therefore L^{-1}(y(s)) = L^{-1}\begin{bmatrix} \dfrac{2(s+4)}{(s+1)(s+2)} \\[2ex] \dfrac{2(s+1)}{(s+1)(s+2)} \end{bmatrix}$$

$$L^{-1}\left(\frac{2(s+4)}{(s+1)(s+2)}\right) = \frac{A}{s+1} + \frac{B}{s+2}$$

$$= \frac{6}{s+1} + \frac{(-4)}{s+2}$$

$$= 6e^{-t} - 4e^{-2t}$$

$$L^{-1}\left(\frac{2(s+1)}{(s+1)(s+2)}\right) = L^{-1}\left(\frac{2}{s+2}\right)$$

$$= 2e^{-2t}$$

$$\therefore y(t) = \begin{bmatrix} 6e^{-t} - 4e^{-2t} \\ 2e^{-2t} \end{bmatrix}$$

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Problem 2: Linear Algebra & matrix operation.

1) Find the inverse of the matrix $A = \begin{bmatrix} s+1 & 2 \\ 2 & s+3 \end{bmatrix}$

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} s+3 & -2 \\ -2 & s+1 \end{bmatrix}$$

$$\Rightarrow |A| = \{(s+1)(s+3) - 4\}$$

$$= \{s^2 + 3s + s + 3 - 4\}$$

$$= \{s^2 + 4s - 1\}$$

$$A^{-1} = \frac{1}{s^2+4s-1} \begin{bmatrix} s+3 & -2 \\ -2 & s+1 \end{bmatrix}$$

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$$A^{-1} = \begin{bmatrix} \dfrac{s+3}{s^2+4s-1} & \dfrac{-2}{s^2+4s-1} \\ \dfrac{-2}{s^2+4s-1} & \dfrac{s+1}{s^2+4s-1} \end{bmatrix}$$

2- Find the determinant and inverse of the matrix

$$A = \begin{bmatrix} s-1 & 2 & -3 \\ 1 & s-2 & 1 \\ 0 & 1 & s \end{bmatrix}$$

$$\underline{\det\{A\}} = (s-1)\begin{vmatrix} s-2 & 1 \\ 1 & s \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ 0 & s \end{vmatrix} - 3\begin{vmatrix} 1 & s-2 \\ 0 & 1 \end{vmatrix}$$

$$= (s-1)(s^2-2s-1) - 2(s) - 3(1)$$

$$= (s-1)(s^2-2s-1) - 2s - 3$$

$$= s^3 - \cancel{2s^2} - s - s^2 + \cancel{2s^2} + 1 - \cancel{2s} - 3 + \cancel{2s} + 2s^2$$

$$\det\{A\} = s^3 - 3s^2 - 3s - 2$$

$$A^{-1} = \frac{1}{\det\{A\}}\left(\text{matrix of minors}\right)$$

$$\cancel{\text{matrix of minors}} = \begin{bmatrix} (s-1)(s(s-2)-1) & 2((1)(s)-1(0)) & -3((1)(1)-(s-2)(0)) \\ 1((s)\cancel{(s-2)}(2)-(-3)(1)) & (s-2)((s)(s-1)-(-3)(0)) & 1((1)(s-1)-(2)(0)) \\ 0((2)(1)-(-3)(s-2)) & 1((1)(s-1)-(-3)(1)) & s((s-1)(s-2)-(2)(1)) \end{bmatrix}$$

$$= \begin{bmatrix} (s-1)(s^2-2s-1) & 2s & -3 \\ 2s+3 & (s-2)(s)(s-1) & s-1 \\ 0 & s+2 & s(s-1)(s-2)-2s \end{bmatrix}$$

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$\cancel{A^{-1} =}$

matrix of minors $= \begin{bmatrix} (s-2)s-1 & 1(s)-1(0) & 1(1)-(s-2)0 \\ 2(s)-(-3)(1) & s(s-1)-(-3)(0) & 1(s-1)-2(0) \\ 2(1)-(-3)(s-2) & (1)(s-1)-(1)(-3) & (s-1)(s-2)-(1)(2) \end{bmatrix}$

$$= \begin{bmatrix} s^2-2s-1 & s & 1 \\ 2s+3 & s^2-s & s-1 \\ 2+3s-6 & s-1+3 & s^2-2s-s+\cancel{2}-\cancel{2} \end{bmatrix}$$

$$= \begin{bmatrix} s^2-2s-1 & s & 1 \\ 2s+3 & s^2-s & s-1 \\ 3s-4 & s+2 & s^2-3s \end{bmatrix}$$

$$A^{-1} = \frac{1}{s^3-3s^2-s-2} \begin{bmatrix} s^2-2s-1 & -s & 1 \\ -(2s+3) & s^2-s & -(s-1) \\ 3s-4 & -(s+2) & s^2-3s \end{bmatrix}^T$$

$$= \frac{1}{s^3-3s^2-s-2} \begin{bmatrix} s^2-2s-1 & -2s-3 & 3s-4 \\ -s & s^2-s & -s-2 \\ 1 & -s+1 & s^2-3s \end{bmatrix}$$

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3. Find the Eigen Values & corresponding Eigen Vectors of the matrix

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$$A = \begin{bmatrix} 5 & -3 \\ -6 & 2 \end{bmatrix}$$

we use the following Equation for finding 'λ' [Eigen Values]

$$\Rightarrow |\lambda I - A| = 0$$

$$\Rightarrow (\lambda - 2)(\lambda - 5) - (3)(6) = 0$$

$$\Rightarrow \lambda^2 - 2\lambda - 5\lambda + 10 - 18 = 0$$

$$\Rightarrow \lambda^2 - 7\lambda + 10 - 18 = 0$$

$$\Rightarrow \lambda^2 - 7\lambda - 8 = 0$$

$$\Rightarrow \lambda^2 - 8\lambda + \lambda - 8 = 0$$

$$\Rightarrow \lambda(\lambda - 8) + 1(\lambda - 8) = 0$$

$$\Rightarrow (\lambda - 8)(\lambda + 1) = 0$$

$$\Rightarrow \underline{\lambda = 8, -1}$$

$$\lambda_1 = 8, \ \lambda_2 = -1$$

Finding the Eigen Vectors for Each Eigen Value.

$$AV_1 = \lambda_1 V_1$$

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$$\begin{bmatrix} 5 & -3 \\ -6 & 2 \end{bmatrix} \cdot \begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix} = 8 \begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix}$$

Solving the above matrices.

$$5V_{11} - 3V_{12} = 8V_{11}$$

$$\cancel{3}V_{11} = -\cancel{3}V_{12}$$

$$V_{11} = -V_{12} \quad - (1)$$

$$-6V_{11} + 2V_{12} = 8V_{12}$$

Since

$$-\cancel{6}V_{11} = \cancel{6}V_{12} \quad - (2)$$

~~Solving~~ Equations (1) & (2) are similar we choose a particular Value.

$$\boxed{V_{11} = 1; \; V_{12} = -1} \qquad V_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Now consider the other Eigen Value.

$$AV_2 = \lambda_2 V_2$$

$$\begin{bmatrix} 5 & -3 \\ -6 & 2 \end{bmatrix} \begin{bmatrix} V_{21} \\ V_{22} \end{bmatrix} = -1 \begin{bmatrix} V_{21} \\ V_{22} \end{bmatrix}$$

$$5V_{21} - 3V_{22} = -V_{21} \Rightarrow \overset{2}{\cancel{6}}V_{21} = \cancel{3}V_{22}$$

$$V_{22} = 2V_{21} \quad - (3)$$

$$-6V_{21} + 2V_{22} = -V_{22} \Rightarrow -\overset{2}{\cancel{6}}V_{21} = -\cancel{3}V_{22}$$

$$2V_{21} = V_{22} \quad - (4)$$

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Similarly we choose $V_{21} = 1$

hence $V_{21} = 1$, $V_{22} = 2$.

$$V_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\cancel{V = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}} \qquad V = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$$

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4. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 1 & 1 \\ 3 & 3 & 4 & 5 \end{bmatrix}$

Performing Row operations to find the rank.

(i) $R_3 \rightarrow R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

(ii) $R_3 \rightarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The rank of the matrix is '2'

5. Find the rank of the matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 1 \\ 0 & 3 & 4 \\ 0 & 1 & 1 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 1 \\ 0 & 3 & 4 \\ 0 & 1 & 1 \end{bmatrix}$$

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operations

$R_2 \to 2R_1 - R_2$ $\Rightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & -1 & 5 \\ 0 & 3 & 4 \\ 0 & 1 & 1 \end{bmatrix}$

$R_2 \to R_2 + R_4$ $\Rightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & 0 & 6 \\ 0 & 3 & 4 \\ 0 & 1 & 1 \end{bmatrix}$

$R_2/2$ $\Rightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & 0 & 3 \\ 0 & 3 & 4 \\ 0 & 1 & 1 \end{bmatrix}$

$R_3 \to R_3 - 3R_4$ $\Rightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & 0 & 3 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$R_2 \to R_2 - 3R_3$ $\Rightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

The no. of identical ~~rows~~ colounns are three hence the rank of the matrix is given by '3'

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Problem 3: Stability and System poles

Check if the following Systems are internally asymptotically stable, internally marginally stable, or BIBO stable based on the given information.

1. Transfer function $T = \dfrac{\begin{bmatrix} 1 & s-2 \\ 0 & 1 \end{bmatrix}}{(s+2)(s+3)}$

$\Delta(s)$ $|sI - A|$ $(s+2)(s+3)$

Equating & finding the roots

$s = -2, -3$

The roots are in the open left half plane. hence not BIBO

The Asymptotically Stability is unknown. since we don't know the complete System.

marginal Stable is also known.

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2. $T = \dfrac{1}{(s+2)(s-3)}$

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Given transfer function, the poles of the transfer function are $s = -2$, $s = 3$

Here, one pole is on the left half plane and one is on the right half plane.

Hence we can infer that the system is not asymptotically stable, neither marginal stable, nor BIBO stable

Asymptotical stability - NO.

Marginal stability - NO

BIBO - NO.

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3. System matrix $A = \begin{bmatrix} 5 & -3 \\ -6 & 2 \end{bmatrix}$

$$|sI - A| = \left| \begin{bmatrix} s-5 & 3 \\ 6 & s-2 \end{bmatrix} \right|$$

$$(s-5)(s-2) - (3)(6)$$

$$s^2 - 7s + 10 - 18 = 0$$

$$s^2 - 7s - 8 = 0$$

$$s^2 - 8s + s - 8 = 0$$

$$s(s-8) + 1(s-8) = 0$$

$$(s+1)(s-8) = 0$$

$$s = -1, 8$$

No poles on $j\omega$ axis Not Marginally stable.

System is Not Asymptotically stable.

Hence not B.I.B.O.

4. System dynamics in the State Space form.

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & \alpha \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} u(t).$$

$$y(t) = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$$

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To find out the poles consider

$$|SI - A| = 0$$

$$\left|\begin{bmatrix} s & 0 & 0 \\ 0 & s+2 & 1 \\ 0 & 0 & s-\alpha \end{bmatrix}\right| = 0 \quad \Rightarrow \quad s(s+2)(s-\alpha) = 0$$

poles are $s=0$, $s=-2$, $s=\alpha$

As one of pole is on the $j\omega$ axis an origin, hence we can infer that the system is not asymptotical stable.

The system is marginally stable if $\alpha < 0$

To find out the BIBO stability consider, $C(SI-A)^{-1}B$

$$(sI-A)^{-1} = \begin{bmatrix} s & 0 & 0 \\ 0 & s+2 & 1 \\ 0 & 0 & s-\alpha \end{bmatrix}^{-1} = \frac{1}{s(s+2)(s-\alpha)} \begin{bmatrix} s^2-\alpha s+2s-2\alpha & 0 & 0 \\ 0 & s^2-\alpha s & 0 \\ 0 & -s & s^2+2s \end{bmatrix}$$

$$= \frac{1}{s(s+2)(s-\alpha)} \begin{bmatrix} s^2-\alpha s+2s-2\alpha & 0 & 0 \\ 0 & s^2-\alpha s & -s \\ 0 & 0 & s^2+2s \end{bmatrix} = \frac{1}{s(s+2)(s-\alpha)} [1 \ -1 \ 2] \begin{bmatrix} s^2-\alpha s+2s-2\alpha & 0 & 0 \\ 0 & s^2-\alpha s & -s \\ 0 & 0 & s^2+2s \end{bmatrix}$$

$$\Rightarrow \frac{1}{s(s+2)(s-\alpha)} \begin{bmatrix} s^2-\alpha s+2s-2\alpha & -s^2+\alpha s & s+2s^2+4s \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$\Rightarrow \frac{1}{s(s+2)(s-\alpha)} \left[s^2-\alpha s+2s-2\alpha-\cancel{2s^2}+2\alpha s+s+\cancel{2s^2}+4s \right] = \frac{1}{s(s+2)(s-\alpha)} \left[s^2+7s+\alpha s-2\alpha \right]$$

$\Rightarrow$ substitute $s=0$ in the numerator, $s^2+7s+\alpha s-2\alpha=0$; $2\alpha=0$; $\alpha=0$

then, the transfer function reduces to.

$$= \frac{1}{s(s+2)(s)} [s^2+7s]$$

$$= \frac{1}{s^{\cancel{2}}(s+2)} \cancel{s}(s+7) = \frac{s+7}{s(s+2)}$$

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here the poles are '0' and -2 therefore it is not BIBO stable

Substitute $s=2$ then,

$$s^2+7s+\alpha s-2\alpha=0$$

$$4-14-2\alpha-2\alpha=0$$

$$-10-4\alpha=0$$

$$4\alpha=-10$$

$$\alpha=-5/2$$

The the transfer function is

$$=\frac{1}{s(s+2)(s-\alpha)}\left[s^2+7s+\alpha s-2\alpha\right]$$

$$=\frac{1}{s(s+2)(s+5/2)}\left[s^2+7s-\frac{5s}{2}+5\right]$$

$$=\frac{2s^2+14s-5s+10}{2s(s+2)(s+5/2)}=\frac{2s^2+9s+10}{2s(s+2)(s+5/2)}$$

Here the poles are $s=0$ & $s=-2$, $s=-5/2$
it is not BIBO stable as one pole lies on the origin.

Substitute $s=\alpha$ then

$$s^2+7s+\alpha s-2\alpha=0$$

$$\alpha^2+7\alpha+\alpha^2-2\alpha=0$$

$$2\alpha^2+5\alpha=0$$

$$\alpha(2\alpha+5)=0$$

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$\alpha = 0$, $\alpha = -5/2$

The System is not a BIBO Stable, as one of the poles is on the origin.

A.S. $\rightarrow$ NO

M.S $\rightarrow$ Yes if $\alpha < 0$

No if $\alpha > 0$

B.I.B.O $\rightarrow$ No.

Problem 4: System Zeros

Please find the System Zeros, Tx Zeros, i/p decoupling Zeros and the output decoupling Zeros for the following systems.

1. System dynamics in the State Space form.

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 2 & 0 \end{bmatrix} x(t).$$

System zeros can be found by using the determinant of the Rosenbrock System Matrix

$$P(s) = \begin{bmatrix} sI-A & B \\ -C & D \end{bmatrix}$$ ~~to~~ ~~Zeros~~

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$$\left|\begin{bmatrix} s & -1 & 1 \\ 2 & s+3 & 1 \\ -2 & 0 & 0 \end{bmatrix}\right|$$

$$\Rightarrow \cancel{s(s)}\ s\cancel{\left[(s+3)(0)-(1)(0)\right]}^{0} + 1\left[(2)(0)-(1)(-2)\right] + 1\left[(2)(0)-(2)(s+3)\right] = 0$$

$$\Rightarrow 2 + 2s + 6 = 0$$

$$\Rightarrow 2s + 8 = 0$$

$$\Rightarrow \underline{s = -4}$$

$$|sI - A| = 0$$

$$\begin{vmatrix} s & -1 \\ 2 & s+3 \end{vmatrix} = 0$$

$$(s)(s+3) + 2 = 0$$

$$s^2 + 3s + 2 = 0$$

$$s^2 + 2s + s + 2 = 0$$

$$s(s+2) + 1(s+2) = 0$$

$$(s+1)(s+2) = 0 \Rightarrow s = -1, -2.$$

O/p decoupling zeros :

$$[sI - A \quad B]$$

$$\begin{bmatrix} s & -1 & 1 \\ 2 & s+3 & 1 \end{bmatrix} \text{ Substitute } s = -1 \text{ then } \begin{bmatrix} -1 & -1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$$

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The rank remains the same $^{6}2^{2}$

Substitute $S = -2$ then.

$$\begin{bmatrix} -2 & -1 & 1 \\ 2 & 1 & 1 \end{bmatrix} \Rightarrow$$ The rank is the same.

There are no i/p decoupling zeros

O/p decoupling zeros:

$$\begin{bmatrix} SI - A \\ -C \end{bmatrix} = \begin{bmatrix} S & -1 \\ 2 & S+3 \\ -2 & 0 \end{bmatrix}$$

Substitute $S = -1$ then. $\begin{bmatrix} -1 & -1 \\ 2 & 2 \\ -2 & 0 \end{bmatrix}$; here there are no identical columns, it doesn't lose its rank

Substitute $S = -2$ then $\begin{bmatrix} -2 & -1 \\ 2 & 1 \\ -2 & 0 \end{bmatrix}$ no identical coloumns, therefore it doesn't lose its rank.

Hence no o/p decoupling zeros

To find Tx zeros. $H(S) = C(SI - A)^{-1} B$

$$\begin{bmatrix} 2 & 0 \end{bmatrix} \begin{bmatrix} S & -1 \\ 2 & S+3 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{S^2+3S+2} \begin{bmatrix} 2 & 0 \end{bmatrix} \begin{bmatrix} S+3 & 1 \\ -2 & S \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{S^2+3S+2} \begin{bmatrix} 2S+6 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow \frac{1}{S^2+3S+2} \begin{bmatrix} 2S+8 \end{bmatrix} = \frac{2(S+4)}{S^2+3S+2}$$

$$= \frac{2(S+4)}{(S+1)(S+2)}$$ here after pole-zero cancellation, the zeros of the T.F are Tx zeros.

$S = -4$ is the Tx zero.

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2. System dynamics in the State Space Form.

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 2 & -2 \end{bmatrix} x(t).$$

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System dynamics, System zeros can be found by using determinant

$$|P(s)| = \left| \begin{bmatrix} sI - A & B \\ -C & D \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} s & -1 & 1 \\ 2 & s+3 & -1 \\ -2 & +2 & 0 \end{bmatrix} \right| = 0$$

$$s\left[(s+3)(0) - (-1)(+2)\right] - (-1)\left[(2)(0) - (-2)(-1)\right] + 1\left[(2)(2) - (-2)(s+3)\right] = 0$$

$$2s - 2 + 4 + 2s + 6 = 0$$

$$4s + 8 = 0$$

$$s = -2$$

for input decoupling zero.

$$|[sI - A]| = 0.$$

$$\Rightarrow \left| \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix} \right| = 0.$$

$$s(s+3) - (-1)(2) = 0$$

$$s^2 + 3s + 2 = 0 \Rightarrow \cancel{s(s+3)+1(}$$

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$s^2+2s+s+2=0$

$s(s+2)+1(s+2)=0$

$(s+1)(s+2)=0$

$s=-1,-2.$

$$[sI-A \quad B] = \begin{bmatrix} s & -1 & 1 \\ 2 & s+3 & -1 \end{bmatrix}$$

at $s=-1$ then

$$= \begin{bmatrix} -1 & -1 & 1 \\ 2 & +2 & -1 \end{bmatrix}$$

Rank = 2

at $s=-2$ then.

$$\Rightarrow \begin{bmatrix} -2 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix} \qquad \begin{bmatrix} -2 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank = 1. Loses the Rank and hence. i/p decoupling

Zero = -2

o/p decoupling.

$$\rho\left\{\begin{bmatrix} sI-A \\ C \end{bmatrix}\right\} = 2 \quad \Rightarrow \rho\left\{\begin{bmatrix} s & -1 \\ 2 & s+3 \\ 2 & -2 \end{bmatrix}\right\} \cancel{=0}. \qquad \begin{bmatrix} -1 & -1 \\ 0 & 0 \\ 2 & -2 \end{bmatrix}$$

$s=-1$ then

$$\rho\left\{\begin{bmatrix} -1 & -1 \\ 2 & 2 \\ 2 & -2 \end{bmatrix}\right\} \Rightarrow \begin{bmatrix} -1 & -1 \\ 1 & 1 \\ 2 & -2 \end{bmatrix}$$

Rank = 2

Loss the Rank and hence $s=-1$ is o/p decoupling Zero.

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at $s = -2$

then

$$\begin{bmatrix} -2 & -1 \\ 2 & 1 \\ 2 & -2 \end{bmatrix}$$

The Rank of the above matrix is 2

and hence $S = -2$ isn't o/p decoupling zero.

To find Tx Zeros

consider

$$C(SI - A)^{-1} B$$

$$= \frac{1}{s^2+3s+2} \begin{bmatrix} 2 & -2 \end{bmatrix} \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \frac{1}{s^2+3s+2} \begin{bmatrix} 2s+10 & 2-2s \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{s^2+3s+2} \begin{bmatrix} 2s+10-2+2s \end{bmatrix}$$

$$= \frac{1}{(s+1)(s+2)} \begin{bmatrix} 4s+8 \end{bmatrix} = \frac{4}{s+1}$$

After pole Zero cancellation there aren't any zeros hence Tx Zeros = '0'

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