

Today's Agenda

- Binary to decimal
- Decimal to Binary
- Addition of Binary numbers
- Bitwise Operators
 - Properties
 - Single Number
 - Left Shift
 - Right Shift.

Today's Quote

It's not the will to win that matters,
everyone has that.

It's the will to prepare to win that matters.

Decimal Number System :-

↓

{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 } Base $\rightarrow 10$.

$$342 \rightarrow 300 + 40 + 2 = 3 \times 10^2 + 4 \times 10^1 + 2 \times 10^0$$

$$2569 \rightarrow 2000 + 500 + 60 + 9 \rightarrow 2 \times 10^3 + 5 \times 10^2 + 6 \times 10^1 + 9 \times 10^0$$

Binary Number System :- { 0, 1 } Base $\rightarrow 2$

$$\overset{2}{1} \overset{1}{1} \overset{0}{0} \rightarrow 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \rightarrow 4 + 2 = 6$$

$$1011 \rightarrow 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 11$$

0	10	20
1	11	21
2	12	22
3	13	23
⋮	⋮	⋮
9	19	29

0 $\rightarrow 0$	10 $\rightarrow 2$	100 $\rightarrow 4$	110 $\rightarrow 6$
1 $\rightarrow 1$	11 $\rightarrow 3$	101 $\rightarrow 5$	111 $\rightarrow 7$

Binary to decimal

$$(10110)_2 = (22)_{10}$$

$$\begin{array}{r} \overset{4}{1} \overset{3}{0} \overset{2}{1} \overset{1}{1} \overset{0}{0} \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 1 \times 2^4 = 16 \\ 0 \times 2^3 = 0 \\ 1 \times 2^2 = 4 \\ 1 \times 2^1 = 2 \\ 0 \times 2^0 = 0 \\ \hline 22 \end{array}$$

$$\begin{array}{cccccc} \overset{6}{1} \overset{5}{0} & \overset{4}{1} \overset{3}{1} & \overset{2}{0} & \overset{1}{1} & \overset{0}{0} & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array} \rightarrow 2^6 + 2^4 + 2^3 + 2^1 \rightarrow 64 + 16 + 8 + 2 \Rightarrow 90$$

$$(1011010)_2 \rightarrow (90)_{10}$$

^{5 4 3 2 1 0}
 $(1\ 0\ 2\ 0\ 1\ 0)_2 \rightarrow \text{Invalid.}$

Decimal to Binary

2	20	
2	10	0
2	5	0
2	2	1
2	1	0
	0	1

$$\begin{array}{r} 4\ 3\ 2\ 1\ 0 \\ \rightarrow 1\ 0\ 1\ 0\ 0 \\ \downarrow \quad \times \\ 2^4 \quad 2^2 \end{array}$$

$$\Rightarrow 16 + 4 = 20$$

$$(10100)_2 \rightarrow (20)_{10}$$

2	45	
2	22	1
2	11	0
2	5	1
2	2	1
2	1	0
	0	1

$$\begin{array}{r} 5\ 4\ 3\ 2\ 1\ 0 \\ (1\ 0\ 1\ 1\ 0\ 1) \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 2^5 \quad 2^3 \quad 2^2 \quad 2^0 \end{array}$$

$$2^5 + 2^3 + 2^2 + 2^0$$

$$\rightarrow 32 + 8 + 4 + 1$$

$$\rightarrow \underline{45}$$

$$(45)_{10} \rightarrow (101101)_2$$

Addition

$$\begin{array}{r} \textcircled{1} \quad \textcircled{1} \\ 9 \quad 6 \quad 8 \\ + \quad 4 \quad 5 \quad 9 \\ \hline 8 \quad 2 \quad 1 \end{array}$$

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = (10)_2 \rightarrow (2)_{10}$$

$$\begin{array}{r} 5 \rightarrow 1 \overset{1}{0} 1 \\ 9 \rightarrow 0 1 1 \\ \hline 1000 \rightarrow 8 \end{array}$$

$$\begin{array}{r} 1 \overset{1}{0} 1 \rightarrow 5 \\ 1 1 1 \rightarrow 7 \\ \hline 1100 \rightarrow 12 \end{array}$$

$$(9)_{10} \rightarrow (11)_2$$

$$\begin{array}{r} 1 \overset{1}{0} 1 1 0 \rightarrow 22 \\ + \quad 0 0 1 1 1 \rightarrow 7 \\ \hline 1 1 1 0 1 \rightarrow 29 \end{array}$$

Bitwise Operators

And, or, xor, not, left shift, Right shift
 $\&$ $|$ \wedge $!\sim$ $<<$ $>>$

$0 \rightarrow 1$
 $1 \rightarrow 0$

A	B	A & B	A B	A ^ B
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

{ same same
 puppy shame }

xor \rightarrow Addition without carry.

$$1 + 1 = \underline{0}$$

Bitwise operators on numbers.

$$\begin{array}{rcl} 5 & 2 & 6 \\ 5 & \rightarrow & 101 \\ 6 & \rightarrow & 110 \\ \hline 4 & \rightarrow & 100 \end{array}$$

$$\underline{5 \& 6 = 4}$$

$$\begin{array}{rcl} 20 & | & 45 \\ 20 & \rightarrow & 010100 \\ 45 & \rightarrow & 101101 \\ \hline & & 111101 \rightarrow 61 \end{array}$$

... ..

$$(20 \mid 45) \rightarrow 01$$

$$20 \wedge 45 = 57$$

$$20 \rightarrow 010100$$

$$45 \rightarrow 101101$$

$$\underline{111001} \rightarrow 57$$

$$41 \& 21$$

$$41 \rightarrow 101001$$

$$21 \rightarrow 010101$$

$$\underline{000001} \rightarrow 1$$

Properties :-

$$A \& 1 = ?$$

$$A = 10$$

$$1010$$

$$\underline{20001}$$

$$A \& 1$$

$$\underline{0000}$$

$$A = 9$$

$$1001$$

$$\underline{20001}$$

$$\underline{0001}$$

$A \& 1 \rightarrow 0$, if last bit is 0 $\Rightarrow A$ is even
 $A \& 1 \rightarrow 1$, if last bit is 1 $\Rightarrow A$ is odd.

0 \rightarrow unset

1 \rightarrow set.

$$543210$$

$$101101$$

Even

odd = odd

$$543210$$

$$101100$$

Even

Even = Even

$$2) A \oplus 0 = 0$$

$$\begin{array}{r} A \quad 1 \quad 0 \quad 1 \quad 1 \\ \oplus A \quad 1 \quad 0 \quad 1 \quad 1 \\ \hline 1 \quad 0 \quad 1 \quad 1 \end{array}$$

$$3) A \oplus A = A$$

$$4) A \vee 0 = A$$

$$5) A \wedge A = A$$

$$6) A \wedge 0 = A$$

$$\begin{array}{r} A \rightarrow 1 \quad 0 \quad 1 \\ 0 \rightarrow 0 \quad 0 \quad 0 \\ \hline 1 \quad 0 \quad 1 \end{array}$$

$$7) A \wedge A = 0$$

$$\begin{array}{r} A \rightarrow 1 \quad 0 \quad 1 \\ A \rightarrow 1 \quad 0 \quad 1 \\ \hline 0 \quad 0 \quad 0 \end{array}$$

10:00pm - 10:12pm.

8) Commutative property.

$$a \oplus b = b \oplus a$$

$$a \vee b = b \vee a$$

$$a \wedge b = b \wedge a$$

$$\frac{a \oplus b \oplus c}{2} = c \oplus \frac{a \oplus b}{n}$$

9) Associative property.

$$(a \oplus b) \oplus c = a \oplus (b \oplus c)$$

$$(a \vee b) \vee c = a \vee (b \vee c)$$

$$(a \wedge b) \wedge c = a \wedge (b \wedge c)$$

$$\rightarrow a \wedge b \wedge a \wedge d \wedge b \rightarrow \underbrace{a \wedge a}_0 \wedge \underbrace{b \wedge b}_0 \wedge d$$

$$\rightarrow 0 \wedge d$$

$$\rightarrow d$$

$$1 \wedge 3 \wedge 5 \wedge 3 \wedge 2 \wedge 1 \wedge 5 = 2$$

$$\underline{2} \nearrow 7 \nearrow 4 \nearrow 6 \nearrow 7 \nearrow 2$$

Ques) Given an integer array, where every element repeats twice except for one element which appears once, find that unique element.

$$A = [6, 9, 6, 10, 9] \quad \text{Ans} = \underline{10}.$$

$$A = [2, 3, 5, 6, 3, 6, 2] \quad \text{Ans} = \underline{5}$$

Ans \rightarrow Run a loop & take xor of all array elements.

$$\text{ans} = 0$$

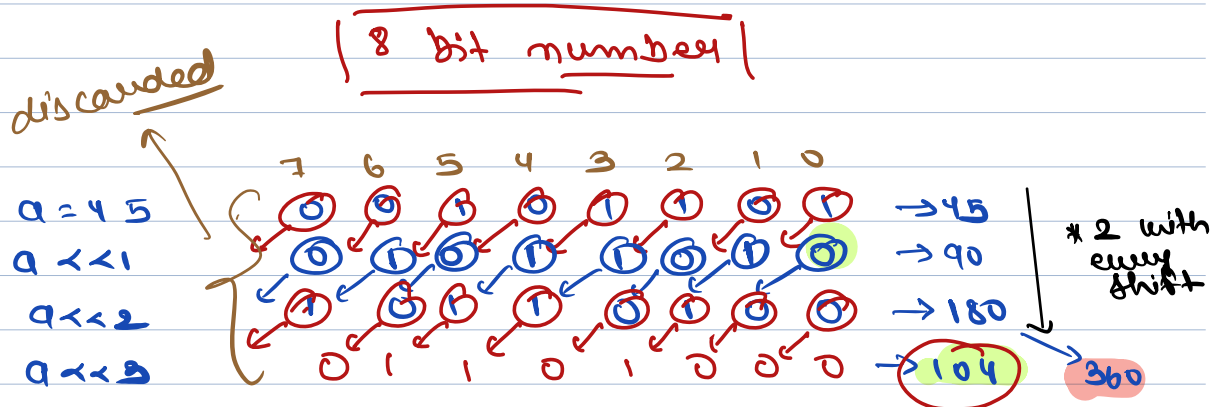
for $i \rightarrow 0$ to $(n-1)$

$$\text{ans} \wedge = A[i]$$

return ans

T.C $\rightarrow O(n)$
S.C $\rightarrow O(1)$.

Left Shift (<<) int \rightarrow 4 Bytes \rightarrow 32 bits.



Overflow. 360 is too large to store in 8 bits.

$$a << 1 = a * 2 \rightarrow a * 2^1$$

$$a << 2 = a * 4 \rightarrow a * 2^2$$

$$a << 3 = a * 8 \rightarrow a * 2^3$$

True when no overflow. \rightarrow

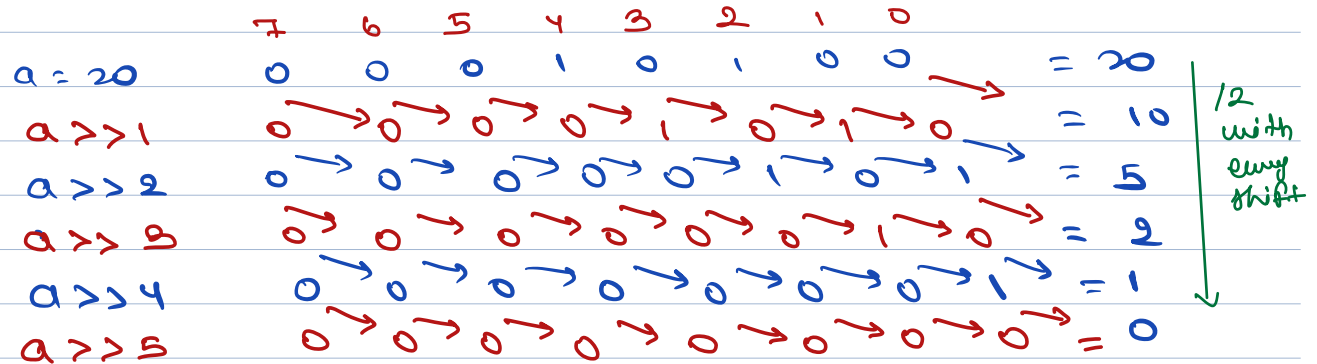
$$a << n = a * 2^n$$

$$1 << n = 2^n$$

8 bit

7	6	5	4	3	2	1	0	
1	1	1	1	1	1	1	1	
\downarrow	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow	
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	$= 255$

Right Shift (>>)



$$a >> 1 \Rightarrow \frac{a}{2}$$

$$a >> n = a / 2^n$$

$$a >> 2 \Rightarrow \frac{a}{2^2}$$

$$a >> 3 \Rightarrow \frac{a}{2^3}$$

→

~~18~~, ~~20~~, ~~15~~, ~~30~~, ~~10~~, ~~14~~

players 1

&

players → 2

18

20

14

15

30

10

1 2 3 4 5 6
18, 20, 15, 30, 10, 14

odd

$$\begin{array}{r} 18 \\ + 15 \\ + 10 \\ \hline 43 \end{array}$$

player 1

14

even

$$\begin{array}{r} 20 \\ + 30 \\ + 14 \\ \hline 64 \end{array}$$

player 2