

Subarray Content

→ Subarray Basics

→ Printing Subarray

→ Generating All Subarrays

→ Printing all subarray sums.

- Approach 1 • Approach 2

- Max Subarray sum

- Sum of all subarray sums

Doubt Session

Assignment → Bulbs question.

Homework → Even Subarrays.



Subarray Basics :-

- Continuous part of an array.
- Single element is a subarray
- Complete array is subarray.

Ex:-

	0	1	2	3	4	5	6	7	8	9
arr[10] =	-2	4	6	3	8	1	4	3	2	-10

indices: {3, 4, 5, 7, 8} ✗

indices: {4, 5, 6, 7, 8} ✓

indices: {2, 6} ✗

— Print Sub(arr[], s, e) {

```
    for (i = s; i <= e; i++) {  
        |  
        | print (arr[i])  
        |  
    }  
    |  
    |  
    3  
3
```

T.C → O(N)

S.C → O(1)

// How many Subarrays.

Ex1: $arr[4] = \overset{0}{-1} \overset{1}{3} \overset{2}{2} \overset{3}{3} \rightarrow \overset{2}{(4)} \frac{(4+1)}{2} \Rightarrow \underline{10}$

0-0 : -1

1-1 : 3

2-2 : 2

0-1 : -1 3

1-2 : 3 2

2-3 : 2 3

0-2 : -1 3 2

1-3 : 3 2 3

3-3 : 3

0-3 : -1 3 2 3

$\Rightarrow 10$ Subarrays.

// Given N elements, How many subarrays?

$$\text{arr}[N] = \{ 0, 1, 2, 3, 4, \dots, i, i+1, \dots, N-2, N-1 \}$$

Start $\rightarrow 0$	Start $\rightarrow 1$	Start $\rightarrow 2$	Start at $N-2$	Starts at $N-1$
0-0	1-1	2-2	$(N-2) - (N-2)$	$(N-1) - (N-1)$
0-1	1-2	2-3	$(N-2) - (N-1)$	
0-2	1-3	2-4		
0-3	\vdots	\vdots	\vdots	
\vdots				
0-N-1	1-N-1	2-N-1		
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
N	$N-1$	$N-2$	2	1

$$N + N-1 + N-2 + \dots + 2 + 1 \Rightarrow \frac{n(n+1)}{2}$$

no. of subarrays in a given arr

// Printing all subarrays

```

for (s=0; s<n; s++) {
  for (e=s; e<n; e++) {
    for (i=s; i<=e; i++) {
      print(arr[i]);
    }
  }
}

```

0 1 2 3
2 4 5 6

s e %

0 0
0 1
0 2
0 3

1 1

1 2

1 3

2 2

2 3

3 3

T.C $\rightarrow O(N^3)$

S.C $\rightarrow O(1)$

2

2, 4

2, 4, 5

2, 4, 5, 6

4

4, 5

4, 5, 6

5

5 6

6

Max Subarray Sum :-

$$\text{arr}[4] = \begin{matrix} 0 & 1 & 2 & 3 \\ [8 & 2 & 9 & 10] \end{matrix}$$

$$[0-0] = \{8\} = 8$$

$$[0-1] = \{8, 2\} = 10$$

$$[0-2] = \{8, 2, 9\} = 19$$

$$[0-3] = \{8, 2, 9, 10\} = 29$$

$$[1-1] = \{2\} = 2$$

$$[1-2] = \{2, 9\} = 11$$

$$[1-3] = \{2, 9, 10\} = 21$$

$$[2-2] = \{9\} = 9$$

$$[2-3] = \{9, 10\} = 19$$

$$[3-3] = \{10\} = 10$$

29

// Print all subarray sum

int maxSum = -∞ → Integer.MIN
-VALUE

```
for (s = 0; s < n; s++) {  
    for (e = s; e < n; e++) {  
        int sum = 0  
        for (i = s; i <= e; i++) {  
            sum += arr[i];  
            print sum  
        }  
    }  
}
```

```
if (sum > maxSum)  
{ maxSum = sum  
}
```

T.C → $O(N^3)$

S.C → $O(1)$

prefix sum (s, e) → if s == 0 pf[s]
→ else pf[s] - pf[s-1]

T.C → $O(N^2)$

S.C → $O(1)$

Kadane's Algorithm :- T.C → $O(N)$
S.C → $O(1)$



Advance first lecture.

arr[4] = ⁰8 ¹2 ²9 ³10
 ↑ ↑

sum = 8, 10, 19, 29, 2, 11, 21, 9, 19, 10

```
for (s = 0; s < n; s++) {
```

```
    int sum = 0;
```

```
    for (e = s; e < n; e++)
```

```
        sum += arr[e];
```

```
    print sum)
```

3

3

T.C $\rightarrow O(n^2)$

S.C $\rightarrow O(1)$

// Printing all subarrays sum starting at index 2.

Ex:- arr[7]: ⁰7 ¹3 ²2 ³-1 ⁴6 ⁵8 ⁶2 ⁷3

[2-2] → 2

[2-3] → 1

[2-4] → 7

[2-5] → 15

[2-6] → 17

[2-7] → 20

sum = 0;

for(j = 2; j < n; j++) {
 sum = sum + arr[j]
 print(sum)

}

Print all subarray sums starting
at index i.

sum = 0;

for(j = i; j < n; j++) {

 sum = sum + arr[j]

 print(sum)

}

// Printing all subarray sums using
carry forward.

```
for (i=0; i<n; i++) {
```

```
    sum = 0;
```

```
    for (j=i; j<n; j++) {
```

```
        sum = sum + arr[j]
```

```
        print (sum)
```

```
    }
```

T.C $\rightarrow O(n^2)$

S.C $\rightarrow O(1)$

Break

10:00 pm - 10:10 pm.

$(i+1) * (n-i)$

Sum of Subarray Sums :-

arr[4] = [8, 2, 9, 10]

0-0 $\rightarrow \{8\}$

8

0-1 $\rightarrow \{8, 2\}$

10

0-2 $\rightarrow \{8, 2, 9\}$

19

0-3 $\rightarrow \{8, 2, 9, 10\}$

29

1-1 $\rightarrow \{2\}$

2

1-2 $\rightarrow \{2, 9\}$

11

1-3 $\rightarrow \{2, 9, 10\}$

21

2-2 $\rightarrow \{9\}$

9

2-3 $\rightarrow \{9, 10\}$

19

3-3 $\rightarrow \{10\}$

10

138

totalSum = 0;

for (i = 0; i < n; i++) {

sum = 0;

for (j = 0; j < n; j++) {

sum = sum + arr[j]

totalSum += sum;

}

}

T.C $\rightarrow O(n^2)$

S.C $\rightarrow O(1)$

Ques) In how many subarray index 3 is present?

arr = ⁰ 3 ¹ -2 ² 4 ³ -1 ⁴ 2 ⁵ 6

s	e
0	3
1	4
2	5
3	

$$\Rightarrow 4 * 3 = \underline{12}$$

↓

Possible subarrays.

$$(i+1) * (n-i) \Rightarrow 4 * 3 = \underline{12}$$

Ques) In how many subarrays index 1 is present?

arr = ⁰3 ¹-2 ²4 ³-1 ⁴2 ⁵6

3
0
-
5

e
-
2
3
5

$$\Rightarrow 2 + 8 = 10$$

Ques) In how many subarrays index 0 is present?

arr = ⁰3 ¹-2 ²4 ³-1 ⁴2 ⁵6

3
0
-
5

e
0
-
2
3
5

$$\Rightarrow 1 \times 6 = 6$$

Generalize

0, 1, 2, 3, ..., i, ..., n-1

s	e
0	i
1	i+1
2	i+2
3	⋮
⋮	⋮
i	n-1

$\Rightarrow (i+1)(n-i)$

i+1 n-i

Total subarrays in which i th index
would be present $\rightarrow (i+1)(n-i)$

// Sum of all subarray sums :-

```
sum = 0;
for (i = 0; i < n; i++) {
    // freq of i
    freq = (i+1) * (n-i)
    sum = sum + freq * arr[i]
```

|₃

return sum

T.C $\rightarrow O(n)$

S.C $\rightarrow O(1)$

⑧ Even Subarrays :-

$A \rightarrow \{2, 4, 6, 8\} \quad 2, 4, \underline{6}, 8,$

\rightarrow One or more subarrays of even length.

first & last of all subarrays should be even.

$A = \{0, 1, 2, 3, \dots, n-1\}$

odd
 \uparrow

\downarrow
even.

\rightarrow $n-1$ should be even.

\rightarrow 0^{th} should be even.

\rightarrow arr. leng should be even.

-: Bulbs :-

0, 1, 0, 1
↑
→ 1, 0, 1, 0
↑
→ 1, 1, 0, 1
↑
→ 1, 1, 1, 0 → 1 1 1 1
↑

0, 0, 0, 0 → 1

0 1 0 1 → sum

Bulb = 1 ← on
0 ← off

0 0 1 1 0 0 1 0 1
— — — — —
0 1 0 1 0 1

0 \rightarrow not worst case

worst case $\rightarrow O(n^2)$

$O(1)$

for ($i=0$; $i \leq n$; $i++$) {

if ($arr[i] == k$)

return $True$

}

}