

Start @ 9:07 pm

Subarray? Continuous part of an array.

$\rightarrow \frac{n(n+1)}{2}$

Subsequence

4 1 9 2 3 -1 6

no more

necessary to

be continuous.

Subarray

Subsequence

1 9 2

✓

✓

4 1 2 -1

X

✓

1 9 4

X

X

order is imp

Any sequence of elements which
can be created by deleting
0 or more elements.

[1 2 3 4 5]

→ 1 2 3 4 5 ✓

→ 4 ✓

→ 2 3 5 ✓

→ 5 4 3 X

e.g 3) 3 -1 0 6 2

3	0	6	✓
-1	6		✓
6	3	-1	✗
0, 2			✓

[4 -1 3] → [-1 3 4]

	Sum	Max	
{ 3	0	0	{ 3
{ 4 3	4	4	{ 4 3
{ -1 3	-1	-1	{ -1 3
{ 3 3	3	3	{ 3 3
{ 4, -1 3	3	4	{ -1, 4 3
{ 4, 3 3	7	4	{ 3, 4 3
{ -1, 3 3	2	3	{ -1, 3 3
{ 4, -1, 3 3	6	4	{ -1, 3, 4 3

Subset :- Collection of elements from array.

↳ Order / Continuity doesn't matter.

4 -1 3 9 2 6 14

			Subarray	Subsequence	Subset
4	-1	3	✓	✓	✓
-1	3	6	x	✓	✓
9	4	14	x	x	✓

Every subsequence is a subset.

Statement 1 : All subarrays are sub-sequences True

Statement 2 : All sub-sequences are subarrays false.

Distinct Elements

1 4 9 3 6 -1 2

$\{1, 4, 9, 3, 3\} \equiv \{1, 9, 3, 4, 3\} \equiv \{4, 1, 3, 9, 3\}$

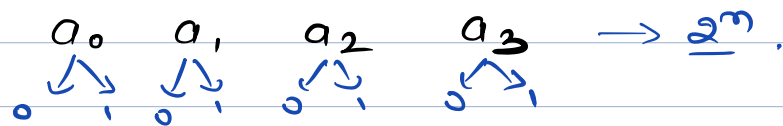
subseq.
subset

subset

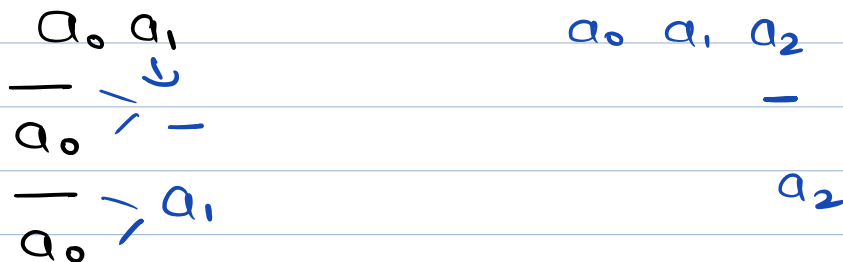
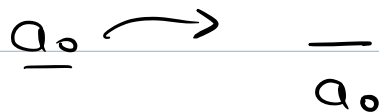
subset

no. of subsequences = no. of subsets.

Total count of subsequences / subsets .



a_0	a_1	a_2	a_3	—	a_1	a_2	a_3
a_0	a_1	a_2		—	a_1	a_2	
a_0	a_1		a_3	—	a_1		a_3
a_0	a_1			—	a_1		
a_0		a_2	a_3	—		a_2	a_3
a_0		a_2		—		a_2	
a_0			a_3	—			a_3
a_0				—			



n elements distinct = $2 * 2 * 2 * \dots$ ^{n times}
 $= 2^n$

continuity

order

count

Subarray

✓

✓

$n(n+1)/2$

Subsequences

x

✓

2^n

Subsets

x

x

2^n

4 1 9 -1 6 3 2

{ 9 1 4 3 }

{ 9 4 1 3 }

{ 1 4 9 3 }

{ 1 9 4 3 }

{ 4 1 9 3 }

{ 4 9 9 1 3 }

Ques) Given an array of size n , count no. of subsequences with $\text{sum} = x$.

-1 4 6 3 7 2 $x=10$

→ 4 6
→ 3 7
→ -1 4 7
→ -1 6 3 2

Soln :- Consider all subsequences,

-1 4 2 → $2^n \rightarrow 2^3 = 8$.

0-7

0	0	0	0	{ 3
0	0	1	1	{ 2 3
0	1	0	1	{ 4 3
0	1	1	1	{ 4, 2 3
1	0	0	1	{ -1 3
1	0	1	1	{ -1 2 3
1	1	0	1	{ -1, 4 3
1	1	1	1	{ -1, 4, 2 3

$a_0 \quad a_1 \quad a_2 \quad a_3 \rightarrow \underline{16}$

4 elements $\rightarrow \underline{16} \rightarrow 0 \text{ to } 15$

0 0 0 0 $\rightarrow 0$

0 0 0 1 $\rightarrow 1$

0 0 1 0 $\rightarrow 2$

0 0 1 1 $\rightarrow 3$

0 1 0 0

0 1 0 1

0 1 1 0

0 1 1 1

1 0 0 0 $\rightarrow 8$

1 0 0 1

1 0 1 0

1 0 1 1

1 1 0 0

1 1 0 1

1 1 1 0

1 1 1 1 $\rightarrow 15$

n, subseq $\rightarrow 2^n$.

$\{a_0, a_1, a_2\}$

Decimal $\rightarrow 0$ to $2^n - 1$

For ($i=0$; $i < 2^n$; $i++$) {

for ($j=0$; $j < n$; $j++$) {

if (checkBit(i, j)) {

Print (arr[j]);

}

Print (" ");

}

}

Go

in the number i ,
jth bit is
set or not.

2
↓

8 subsequences

Selection 0 to 7

	i	j
000	0	0
001	1	0
010	2	1
011	3	1
100	4	2
101	5	2
110	6	3
111	7	3

Ques)

Given an array of distinct elements.

Find sum of all subsequences

sum.

-1 4 2

$$\begin{array}{r} 4 \times 4 \\ + \\ -1 \times 4 \\ + \\ 2 \times 4 \end{array}$$

{ 3

{ -1 3

{ 4 3

{ 2 3

{ -1, 4 3

{ -1, 2 3

{ 4, 2 3

{ -1, 4, 2 3

0

1

4

2

3

1

6

5

20

$1 \leq N \leq 10^5$

for every element

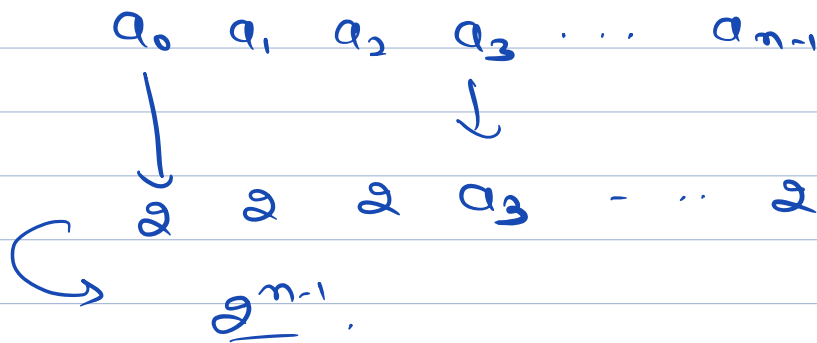
↳ In how many

subsequence a particular

element is present.

-1 4 2
0 1 0 1 0 1 → $2 \times 2 \times 2 = 8$

2 4 2 → 4
 $2 \times 2 = 4$



$$\sum_{i=0}^{n-1} a[i] * 2^{n-1}$$

$$a \quad \underline{b \quad c \quad d} \rightarrow \underline{2^3}$$

$$a * 2^3 + b * 2^3 + c * 2^3 + d * 2^3$$

$$\Rightarrow (a+b+c+d) * 2^3$$

$$\text{ans} = 2^{n-1} * \text{sum of array elements}$$

(*) array of size n find (sum of all subseq.)
sum
 2^n

$$\rightarrow \frac{2^{n-1} * \text{sum of array}}{2^n}$$

Q

Sum of array
2

Ques) find sum of all subsequence
max.

{ 3, -1, 43 }

	<u>Max</u>
{ 3 }	0
{ 33 }	3
{ -13 }	-1
{ 43 }	4
{ 3, -13 }	3
{ 3, 43 }	4
{ -1, 43 }	4
{ 3, -1, 43 }	4
<hr/>	
+ -> <u>21</u>	

