Assignment1 : Kernel Ridge Regression

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Introduction

In machine learning, kernel methods are a class of algorithms for pattern analysis, whose best known member is the support vector machine (SVM). The general task of pattern analysis is to find and study general types of relations (for example clusters, rankings, principal components, correlations, classifications) in datasets.

For many algorithms that solve these tasks, the data in raw representation have to be explicitly transformed into feature vector representations via a user-specified feature map: in contrast, kernel methods require only a user-specified kernel, i.e., a similarity function over pairs of data points in raw representation.

Problem Formulation

We need to generate a dataset with `1052` observations and targets for the regression are obtained by the function `t` as stated in our assignment. Code used to achive this is given below:

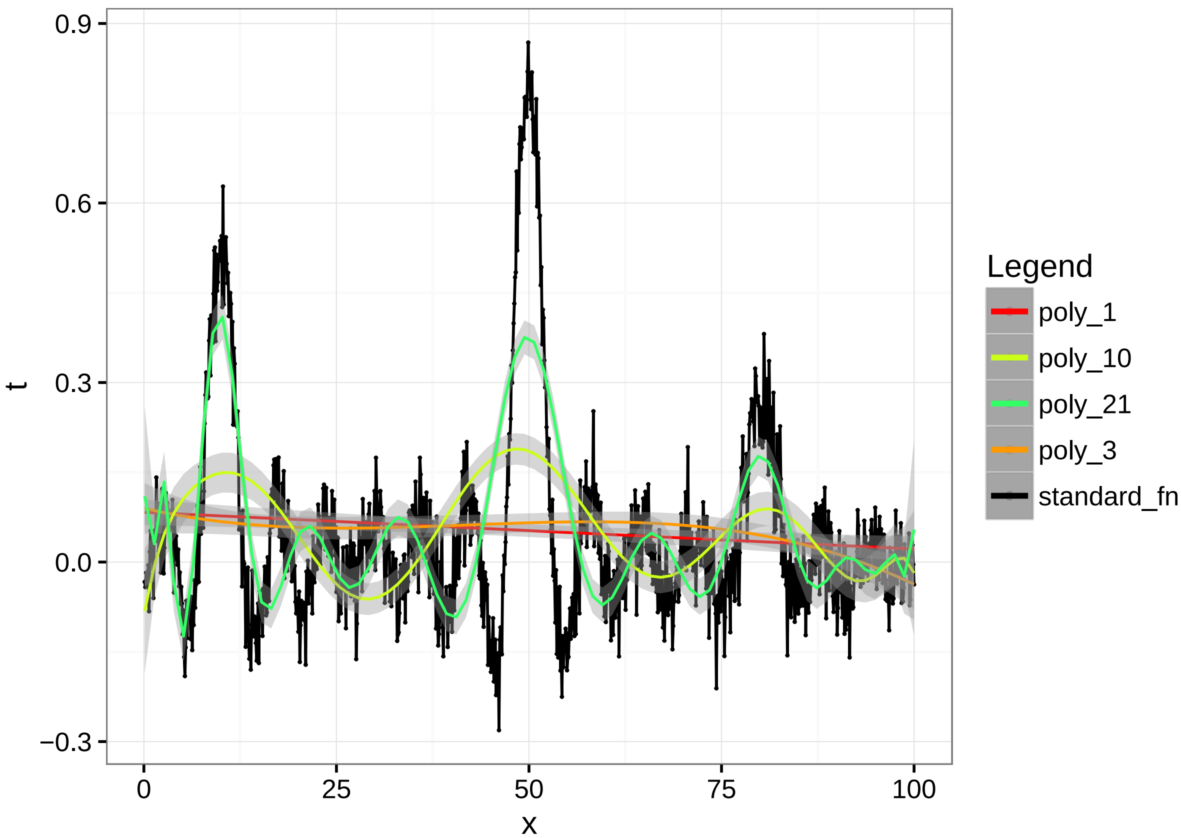
```
library(ggplot2)
# Set the seed to make the expriment reproduceable
set.seed(6046)

# As per the assignment generate data
A <- 1052
x <- seq(0.1, 100, length.out = A)
a = 10
b = 50
c = 80

# Function t
t = 0.5 * sin(x - a)/(x - a) +
  0.8 * sin(x - b)/(x - b) +
  0.3 * sin(x - c)/(x - c) + rnorm(x, sd=0.05)
```

First six observations from our dataframe looks like below

```
##           x            t
## 1 0.1000000 -0.033113515
## 2 0.1950523 -0.041954810
## 3 0.2901047 -0.017648850
## 4 0.3851570 -0.036504437
## 5 0.4802093 -0.020698630
## 6 0.5752617 -0.005138883
```

Polynomial Regression

We fit standard polynomial regression with degrees `1, 3, 10, 21` and observe that polynomial regression with degree `1` has a strong bias does not fit well conversely polymomial regresssion with degree `21` has high variance seems like its overfitting.

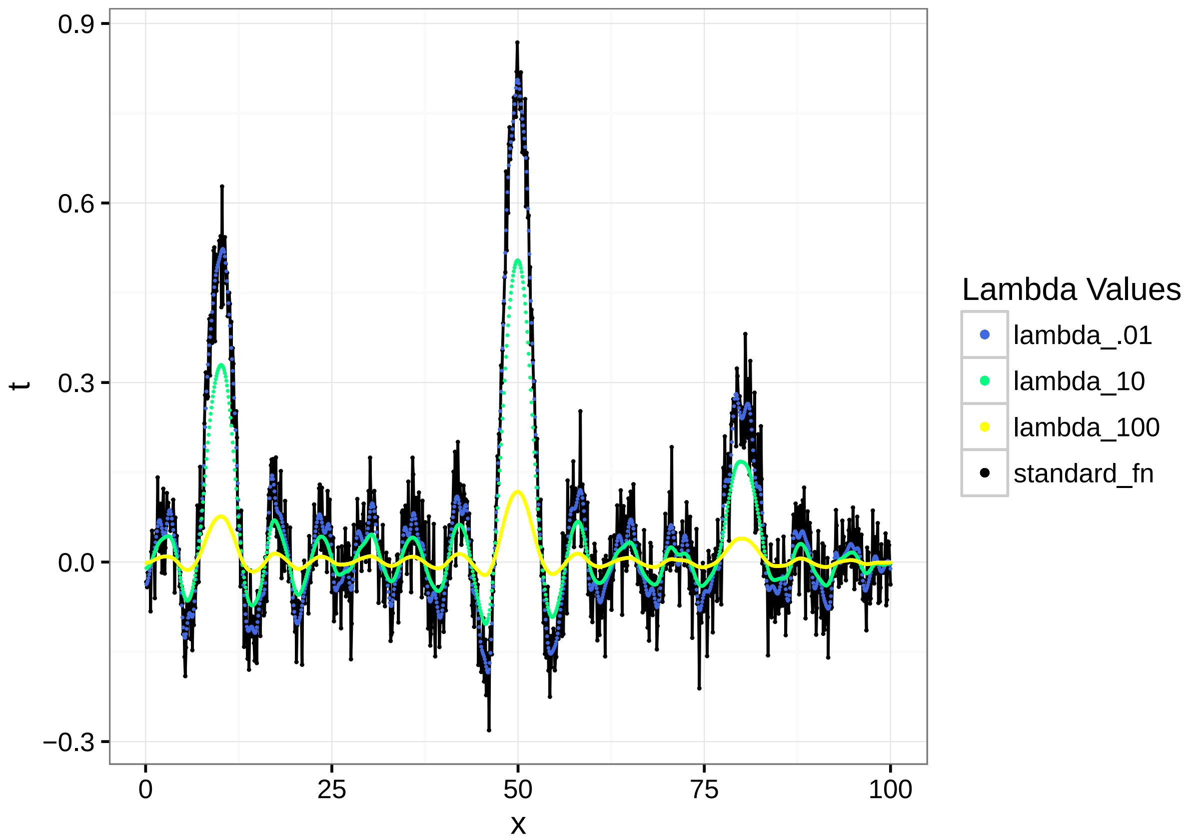


RBF Kernel

In machine learning, the (Gaussian) radial basis function kernel, or RBF kernel, is a popular kernel function used in various kernelized learning algorithms. We will conduct some experiments below sto estimate the best fit by varing the paramater `sigma` and `lambda`.

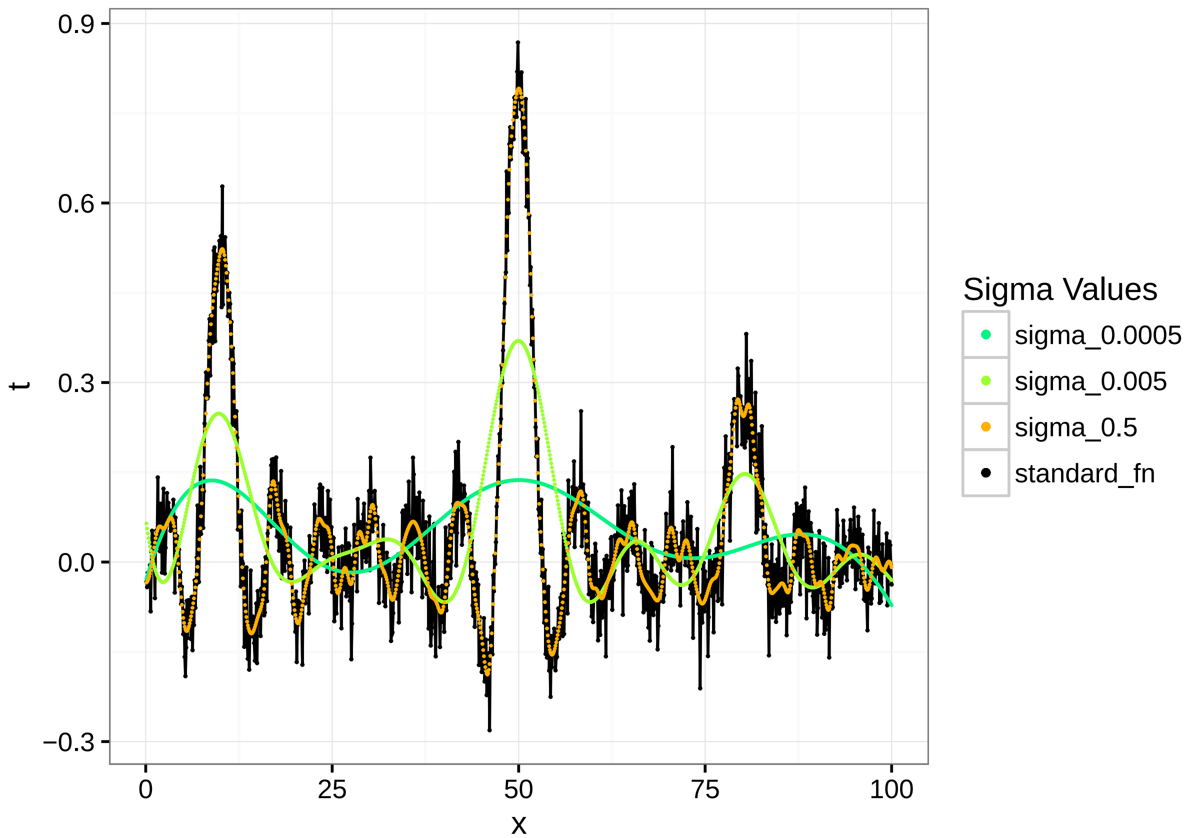
Constant Sigma

Keeping the value of `sigma` as a constant `1` we vary the value of lambda as `0.01, 10, 100`. First thing we observe is that the RBF kernel fits better than the polynomial regression. On increaseing the value parameter `lambda`, We observe that the model bias also increases. This will be useful when we want to prevent over-fitting or genralize better by tuning `lambda`.



Constant Lambda

On keeping the value of `lambda` constant `0.01` and varying the value of sigma as `0.0005, 0.005, 0.05`. We observe that higher the value of sigma, stronger the bias in the model. `Sigma` with value `0.05` fits the data best. This paramter can again be used to tune for bias variance tradeoff.



Conclusion

After conducting these experiments, We observe that the best value of `sigma` would be around `0.5` and best value for `lambda` would be around `0.01` (We could also conduct more experiments to search around the `values` obtained to tune the model even more). Here I assume that we dont overfit the data. Only way to be sure is by the use of k-fold cross validation. The values of `sigma` and `lambda` in the RBF kernel can easily be tuned to gain the best accuracy from our model.