NonLinear Models

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Here we explore the use of nonlinear models using some tools in R

```
require(ISLR)
```

```
## Loading required package: ISLR
```

```
attach(Wage)
```

Polynomials

First we will use polynomials, and focus on single predictor age:

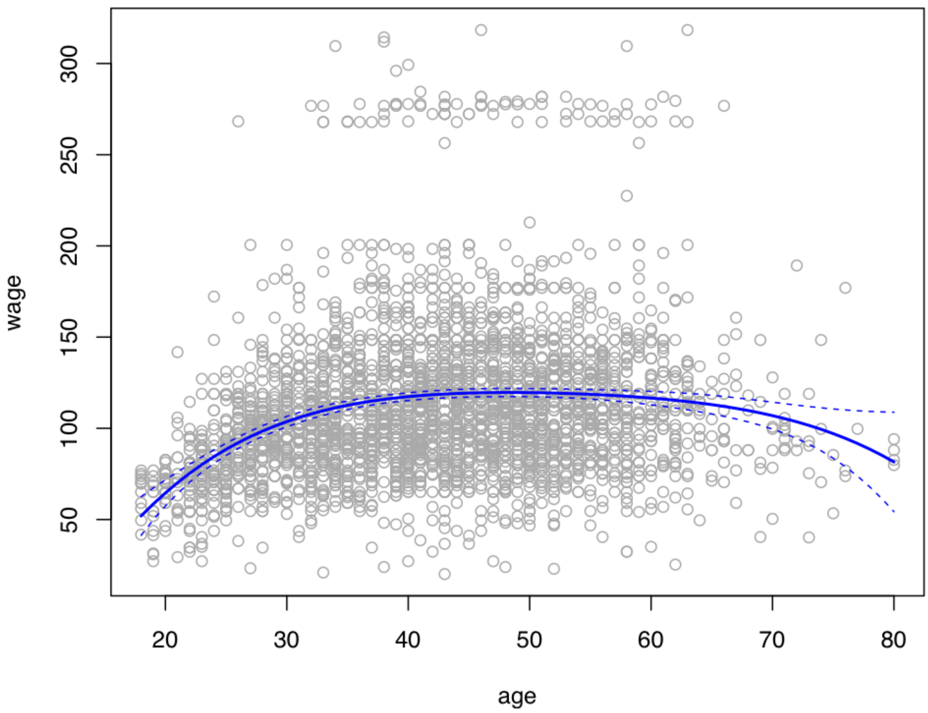
```
fit = lm(wage~poly(age,4),data=Wage)
summary(fit)
```

```
##
## Call:
## lm(formula = wage ~ poly(age, 4), data = Wage)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -98.707 -24.626  -4.993  15.217 203.693
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)    111.7036     0.7287 153.283  < 2e-16 ***
## poly(age, 4)1  447.0679    39.9148  11.201  < 2e-16 ***
## poly(age, 4)2 -478.3158    39.9148 -11.983  < 2e-16 ***
## poly(age, 4)3  125.5217    39.9148   3.145  0.00168 **
## poly(age, 4)4  -77.9112    39.9148  -1.952  0.05104 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 39.91 on 2995 degrees of freedom
## Multiple R-squared:  0.08626,    Adjusted R-squared:  0.08504
## F-statistic: 70.69 on 4 and 2995 DF,  p-value: < 2.2e-16
```

The `poly()` function generates a basis of *orthogonal Polynomials*. Lets make a plot of the fitted function along with the standard errors of the fit.

```
agelims = range(age)
age.grid= seq(from = agelims[1], to = agelims[2])
preds=predict(fit, newdata= list(age=age.grid), se = TRUE)
se.bands <- cbind(preds$fit+2*preds$se, preds$fit-2*preds$se)
```

```
plot(age,wage, col= "darkgrey")
lines(age.grid, preds$fit, lwd=2, col="blue")
matlines(age.grid, se.bands, col = "blue", lty=2)
```



There are other more direct ways of doing this in R. For example

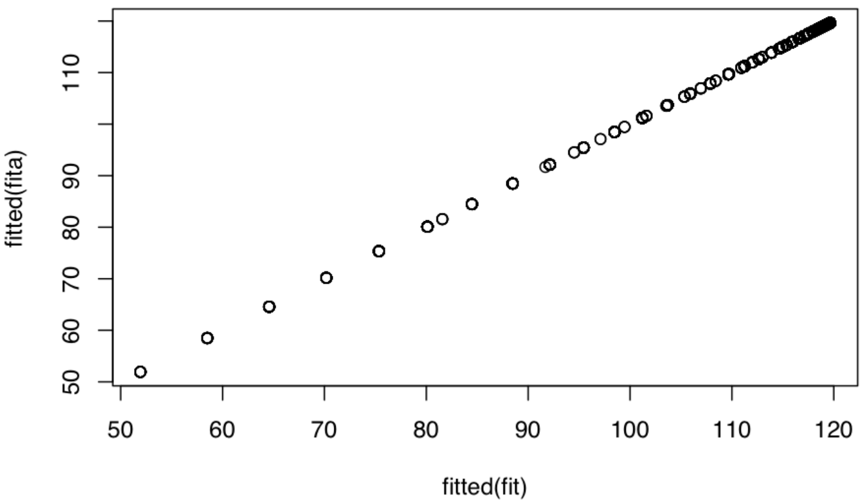
```
fita=lm(wage~age+I(age^2)+I(age^3)+I(age^4),data=Wage)
summary(fita)
```

```
##
## Call:
## lm(formula = wage ~ age + I(age^2) + I(age^3) + I(age^4), data = Wage)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -98.707 -24.626  -4.993  15.217 203.693
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.842e+02  6.004e+01  -3.067 0.002180 **
## age          2.125e+01  5.887e+00   3.609 0.000312 ***
## I(age^2)    -5.639e-01  2.061e-01  -2.736 0.006261 **
## I(age^3)     6.811e-03  3.066e-03   2.221 0.026398 *
```

```
## I(age^4)    -3.204e-05  1.641e-05  -1.952 0.051039 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 39.91 on 2995 degrees of freedom
## Multiple R-squared:  0.08626,	Adjusted R-squared:  0.08504
## F-statistic: 70.69 on 4 and 2995 DF,  p-value: < 2.2e-16
```

Here `I()` is a *wrapper* function; we need it because `age^2` means something to the formula language, while `I(age^2)` is protected. The coefficients are different to those we got before. Fits are the same:

```
plot(fitted(fit),fitted(fita))
```



This only works with linear regression, and if there is a single predictor. In general we would use `anova()` as this next example demonstrates.

```
fita=lm(wage~education,data=Wage)
fitb=lm(wage~education+age,data=Wage)
fitc=lm(wage~education+poly(age,2),data=Wage)
fitd=lm(wage~education+poly(age,3),data=Wage)
anova(fita,fitb,fitc,fitd)
```

```
## Analysis of Variance Table
##
## Model 1: wage ~ education
## Model 2: wage ~ education + age
## Model 3: wage ~ education + poly(age, 2)
## Model 4: wage ~ education + poly(age, 3)
##   Res.Df     RSS Df Sum of Sq        F Pr(>F)
## 1   2995 3995721
## 2   2994 3867992  1    127729 102.7378 <2e-16 ***
## 3   2993 3725395  1    142597 114.6969 <2e-16 ***
```

```
## 4   2992 3719809  1      5587   4.4936 0.0341 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Polynomial logistic Regression

Now we fit a logistic regression model to binary response variable, constructed from `wage`. We code this big earners (`>250K`) as 1, else 0.

```r
fit=glm(I(wage>250)~poly(age,3),data=Wage,family = binomial)
summary(fit)
```

```
##
## Call:
## glm(formula = I(wage > 250) ~ poly(age, 3), family = binomial,
##     data = Wage)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -0.2808  -0.2736  -0.2487  -0.1758   3.2868
##
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept)    -3.8486     0.1597 -24.100  < 2e-16 ***
## poly(age, 3)1  37.8846    11.4818   3.300 0.000968 ***
## poly(age, 3)2 -29.5129    10.5626  -2.794 0.005205 **
## poly(age, 3)3   9.7966     8.9990   1.089 0.276317
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 730.53  on 2999  degrees of freedom
## Residual deviance: 707.92  on 2996  degrees of freedom
## AIC: 715.92
##
## Number of Fisher Scoring iterations: 8
```

```r
preds=predict(fit,list(age=age.grid),se=T)
se.bands=preds$fit+cbind(fit=0,lower=-2*preds$se,upper=2*preds$se)
se.bands[1:5,]
```

```
##         fit      lower     upper
## 1 -7.664756 -10.759826 -4.569686
## 2 -7.324776 -10.106699 -4.542852
## 3 -7.001732  -9.492821 -4.510643
## 4 -6.695229  -8.917158 -4.473300
## 5 -6.404868  -8.378691 -4.431045
```

We have done the computations on the logit scale. To transform we need to apply the inverse logit mapping

$$p = \frac{e^{\eta}}{1 + e^{\eta}}.$$

(Here we have used the ability of mark Down to interpret TeX expressions.) We can do this simultaneously for all the three columns of `se.bands`

```
prob.bands =exp(se.bands)/(1+exp(se.bands))
matplot(age.grid,prob.bands,col="blue",lwd=c(2,1,1),lty=c(1,2,2),type="l",ylim=c(0,.1))
points(jitter(age),I(wage>250)/10,pch="|",cex=.5)
```