6 February 2020

Note : Please get your own laptops to the class, pre-installed with compilers/interpreters corresponding to your coding language.

Question 1: Using a coin toss process to create an Uniform Random Number Generator

Assume that you have an unbiased coin with two outcomes. Use this coin to create a Uniform Random Number (URN) generator. Simulate the process using your favourite coding language.
Note : Almost all the scripting and statistical languages have a command to simulate a coin toss. For example in Python:

```
import numpy as np
np.random.choice(['H','T'])
np.random.choice([0,1])
```

Question 2: Uniform Distribution

Generate atleast 1000 URNs and prove that the numbers indeed obey a uniform distribution. Use the URN generator in your coding language and check the distribution obtained from it with that of yours.

Question 3: Calculating π value using Monte-Carlo scheme

Using Monte-Carlo scheme discussed in the class and your URN generator, estimate the value of π.
a) Plot the estimate obtained with increasing number of random points used in the scheme.
b) Plot the error observed (assuming the real value of π to be 3.14159) with increasing number of random points used in the scheme. Overlay the plot $1/\sqrt{N}$ on the obtained plot.

Question 4: Sampling from a different distribution

Transform the URN generator to sample random numbers from the following probability distribution in the interval $[0, \infty)$

$$P(x) = exp(-x)$$

Generate atleast 1000 such random numbers and show that they indeed obey the above distribution.

Question 5: A Boy and His Atom

In 2013, IBM Research division released a 1.5 minute stop-motion animated short film depicting the story of a boy and a wayward atom who meet and become friends. The individual frames of the movie were created by carefully placing carbon monoxide molecules on a copper surface and then visualizing them under a scanning tunneling microscope.

More details of the experiment: here
Check it out: www.youtube.com/watch?v=oSCX78-8-q0

Consider the surface of copper to be composed of N sites and the chamber filled with M CO molecules where N>>M. Calculate the possible number of configurations possible in each of the following scenarios:
a) Each site can accommodate only one molecule of CO.
b) Each site can accommodate more than one molecule of CO.

Now fill the chamber with M different gas molecules each having its own chemical identity
a) Each site can accommodate only one molecule.
b) Each site can accommodate more than one molecule .

Question 6: Making a Transition Matrix

One