Assignment 4

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Introduction

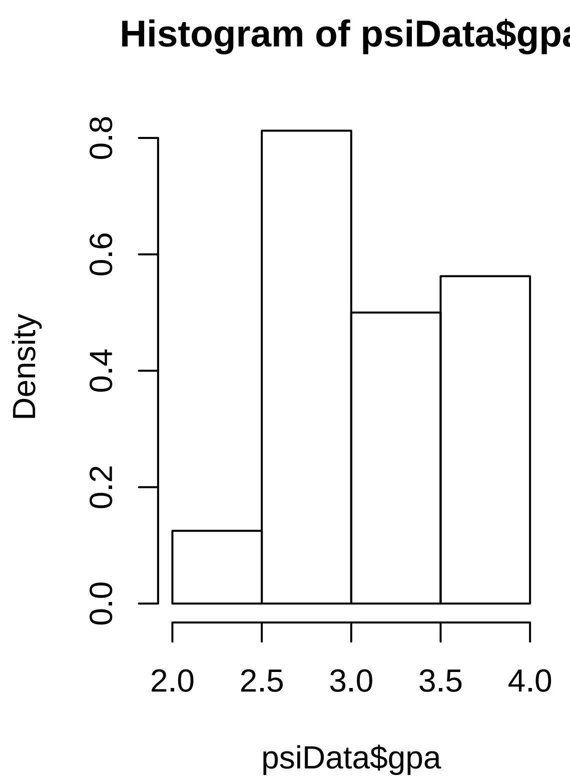
This document describes the solutions found and implemented for the exercises of assignment 3. Exercises can be found in their corresponding sections. This document is created by Rmd, and figure captions are omitted since it changes the structure of the document in a bad way that makes it hard to follow

Question 1

Question 2

Section 1

Given data set for this question consists of one binary response and two explanatory variables which one of them is also a binary variable. The numeric variable, gpa seems to be from a standart normal distribution and its histogram and QQ-Plot can be seen below. As a first step, binary variables are converted into factors.



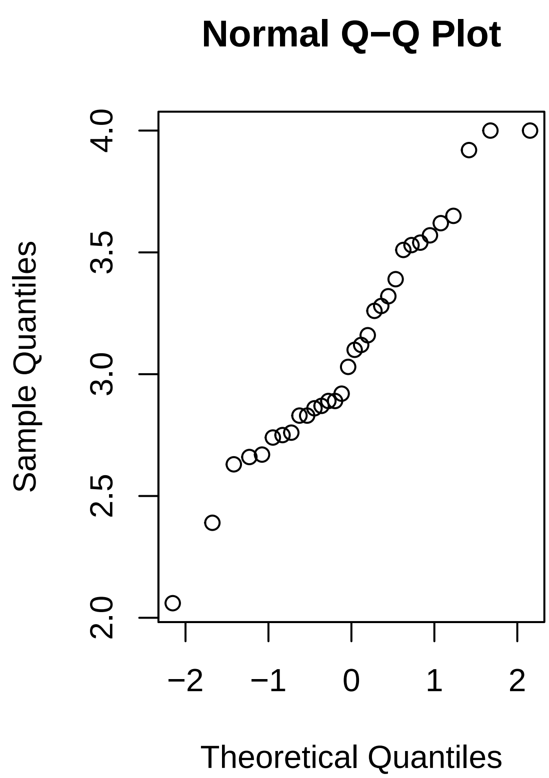


Table of combination of two binary variables can be seen in the table below. From this table we can say that psi is looking promising since more students have passed upon receiving psi.

```r
str(psiData)
```

```
## 'data.frame':    32 obs. of  3 variables:
##  $ passed: Factor w/ 2 levels "Fail","Pass": 2 2 2 2 1 2 2 2 2 1 ...
##  $ psi   : Factor w/ 2 levels "No","Yes": 2 2 2 2 2 2 2 2 2 2 ...
##  $ gpa   : num  2.66 2.89 3.28 2.92 4 2.86 2.76 2.87 3.03 3.92 ...
```

```r
xtabs(~passed + psi, data = psiData)
```

```
##       psi
## passed No Yes
##   Fail  8   3
##   Pass  6  15
```

Section 2

The output of the basic logistic regression model fitted with glm command using both numeric and binary variables can be seen below. The model is trained on training data set and validated on test data set as can be seen below. Test data set uses 20% of the whole data set without replacement.

```r
## 80% of the size
smpSize = floor(0.8 * nrow(psiData))

## Seed for reproduction
set.seed(12345)
train_ind = sample(seq_len(nrow(psiData)), size = smpSize)

# Training and Test sets
train = psiData[train_ind, ]
test = psiData[-train_ind, ]

# Fit the model
logRegModel = glm(passed ~ psi + gpa, data = train, family = "binomial")
logSummary = summary(logRegModel)
logSummary
```

```
##
## Call:
## glm(formula = passed ~ psi + gpa, family = "binomial", data = train)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.9794  -0.5233   0.2693   0.5267   1.8826
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)    9.047      4.289   2.109   0.0349 *
## psiYes         2.915      1.337   2.180   0.0292 *
## gpa           -3.029      1.392  -2.176   0.0295 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 31.343  on 24  degrees of freedom
## Residual deviance: 19.506  on 22  degrees of freedom
## AIC: 25.506
##
## Number of Fisher Scoring iterations: 5
```

From the output, this model corresponds to the equation given below.

$P(Y) = \Psi(9.0469091 + (2.9154076) * psi + (-3.0292819) * gpa)$

Validation of the given model can be seen below with the test data set.

```r
test
```

```
##    passed psi  gpa
## 2    Pass Yes 2.89
## 11   Pass Yes 2.63
## 14   Fail Yes 3.26
## 17   Pass Yes 2.75
## 19   Pass  No 3.12
## 27   Fail  No 3.39
## 29   Fail  No 3.65
```

```r
predict(logRegModel, test, type="response")
```

```
##         2        11        14        17        19        27        29
## 0.9611227 0.9819307 0.8896192 0.9742138 0.4002436 0.2275220 0.1181601
```

Section 3

From the table given in Section 1, we can calculate the probability of a student passing the assignment given he or she received psi is $P(Passed = TRUE|PSI = TRUE) = 0.7142857$. From the predictions made with the model given in Section 2, we see higher probabilities for students which received psi, therefore we are safe to assume that psi works.

Section 4

Section 5

Section 6

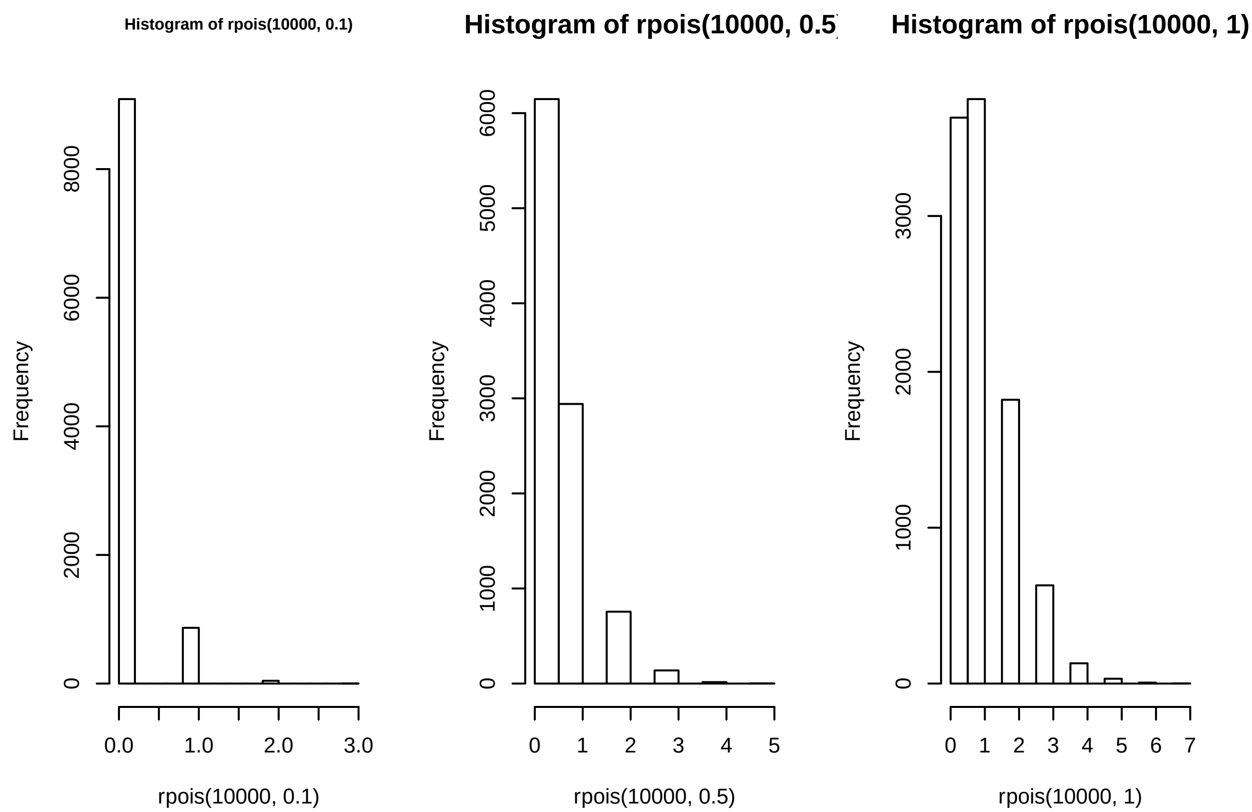
Section 7

Section 8

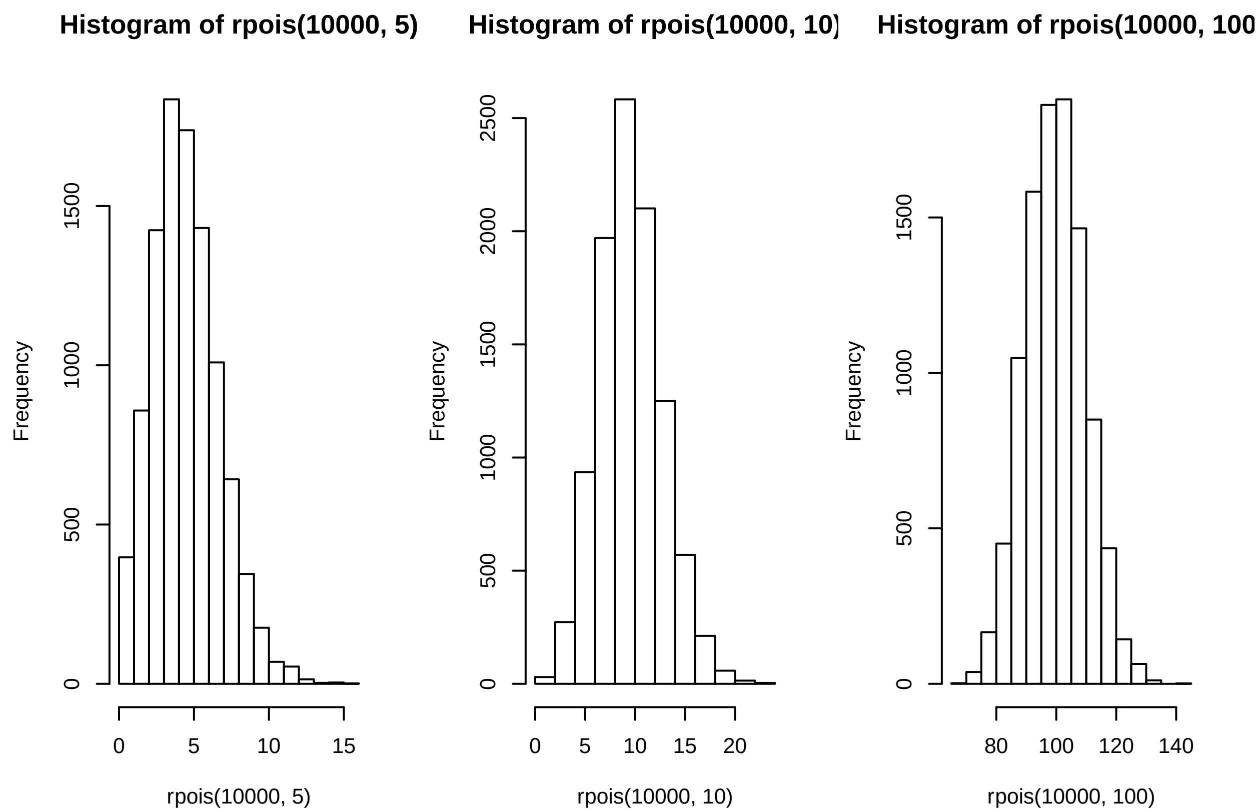
Question 3

Section 1

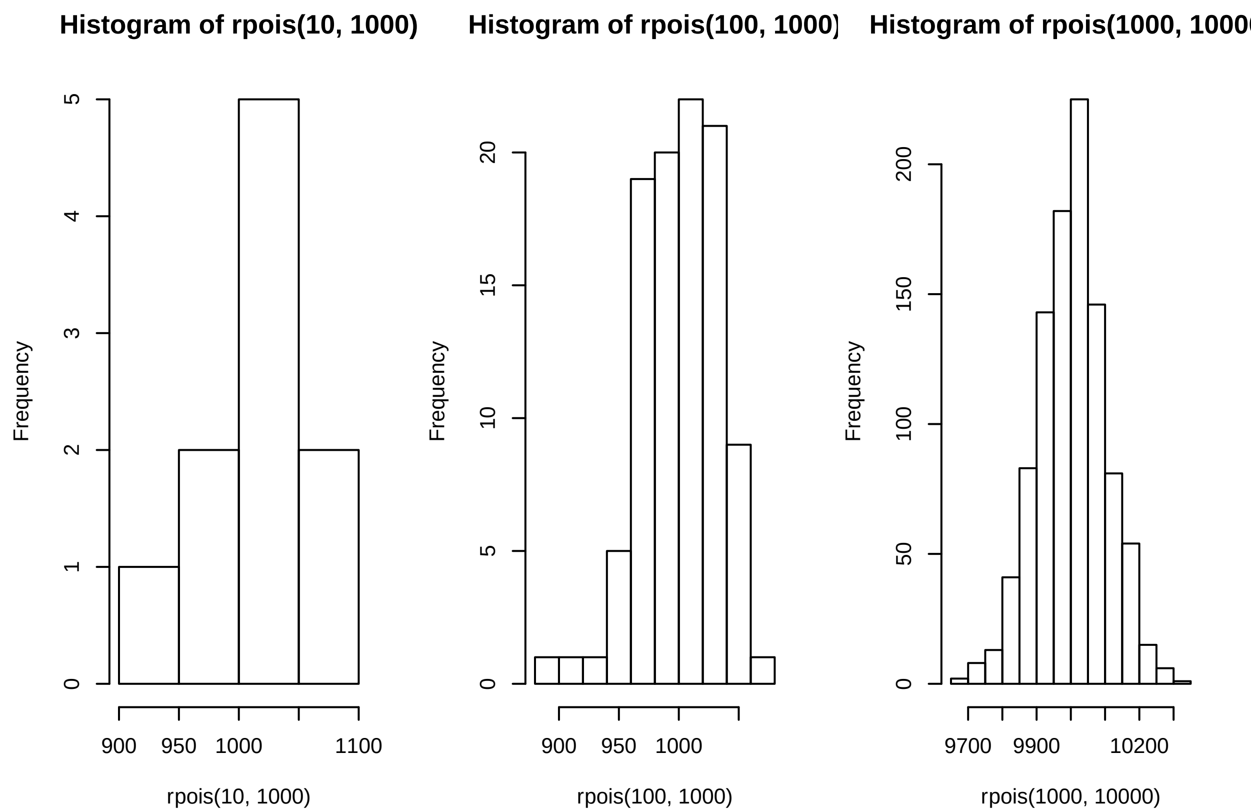
```r
par(mfrow=c(1,3))
hist(rpois(10000,.1), cex.main=.8); hist(rpois(10000,.5)); hist(rpois(10000,1))
```




```r
hist(rpois(10000,5)); hist(rpois(10000,10)); hist(rpois(10000,100))
```




```
hist(rpois(10,1000)); hist(rpois(100,1000)); hist(rpois(1000,10000))
```



For larger values of λ, the distribution is similar to a normal distribution with the mean and variance both equal to λ. Parameter n is of limited influence - it merely determines the amount of values to be sampled from the Poisson distribution. So long as a reasonable amount of points are sampled, the same distribution should emerge for equal λ.

Section 2

In order for the distribution of a randomly distributed variable Y to be in a location-scale family as a given random variable X, Y must have the same distribution as $a + bX$ for some parameters a and b (in other words, $Y \stackrel{d}{=} a + bX$, where $Y \stackrel{d}{=}$ means 'equal in distribution'.

In the case of the Poisson distribution, the distribution is both scaled by parameter λ, since the mean and variance are both equal to λ. Thus, it can be said that, given a variable Y and a variable X that follow a Poisson distribution, $Y \stackrel{d}{=} \lambda X$, which satisfies the above condition for location-scale families.

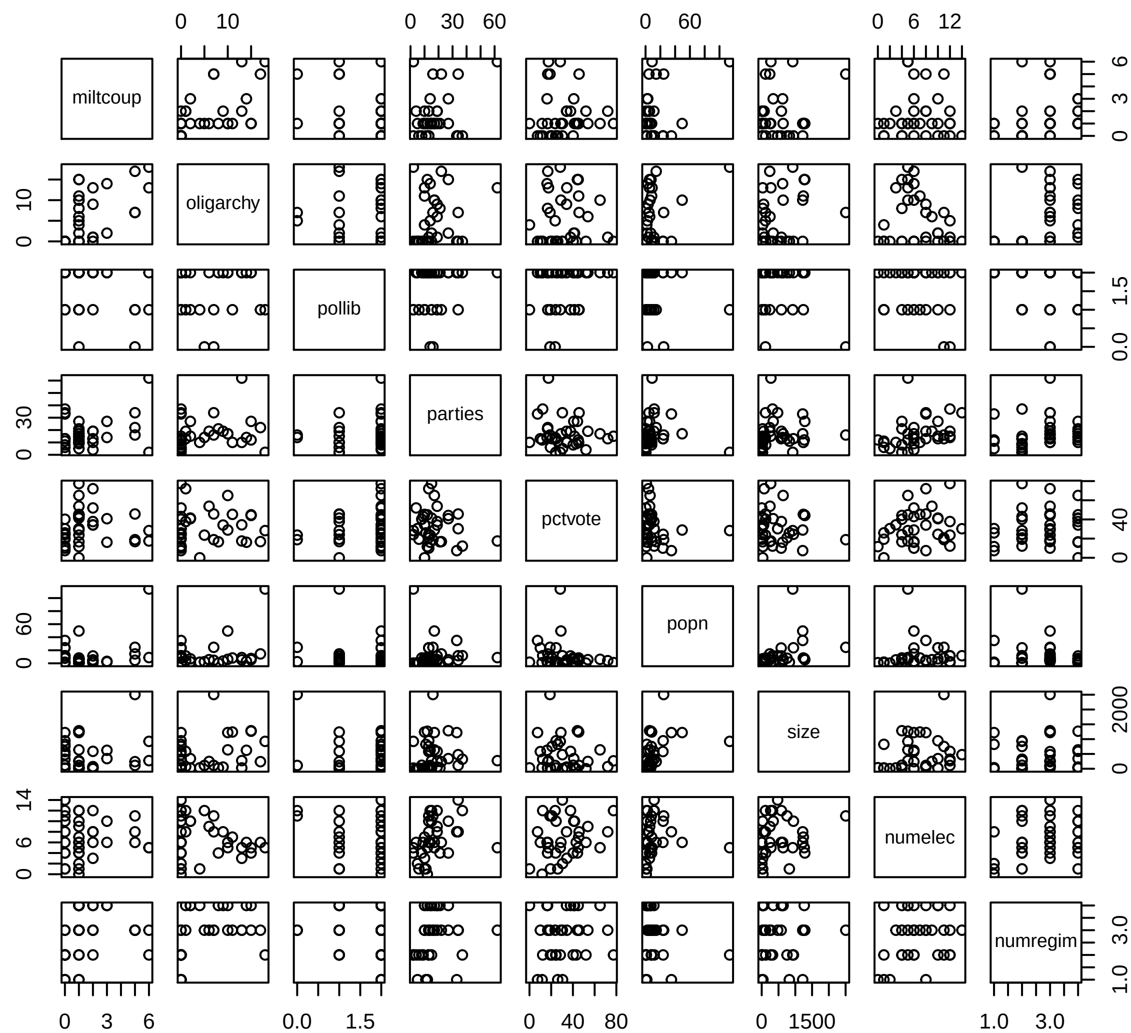
However, for very small values of lambda ($\lambda < 1$),where the distribution looks less similar to a normal distribution, it may prove difficult to produce Poisson distributions with larger λ values via a linear transformation, as a scaling transformation may not be able to fit a normal distribution.

Section 3

```
africa = read.table("data/africa.txt",header=TRUE)
africaglm=glm(miltcoup~oligarchy+pollib+parties+pctvote+popn+size+numelec+numregim,
```

```r
             family=poisson,data=africa)

plot(africa)
```




```r
summary(africaglm)
```

```
##
## Call:
## glm(formula = miltcoup ~ oligarchy + pollib + parties + pctvote +
##     popn + size + numelec + numregim, family = poisson, data = africa)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.3443  -0.9542  -0.2587   0.3905   1.6953
##
```

```
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept) -0.5102693  0.9053301  -0.564  0.57301
## oligarchy    0.0730814  0.0345958   2.112  0.03465 *
## pollib      -0.7129779  0.2725635  -2.616  0.00890 **
## parties      0.0307739  0.0111873   2.751  0.00595 **
## pctvote      0.0138722  0.0097526   1.422  0.15491
## popn         0.0093429  0.0065950   1.417  0.15658
## size        -0.0001900  0.0002485  -0.765  0.44447
## numelec     -0.0160783  0.0654842  -0.246  0.80605
## numregim     0.1917349  0.2292890   0.836  0.40303
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 65.945  on 35  degrees of freedom
## Residual deviance: 28.668  on 27  degrees of freedom
## AIC: 111.48
##
## Number of Fisher Scoring iterations: 6
```

```
confint(africaglm)
```

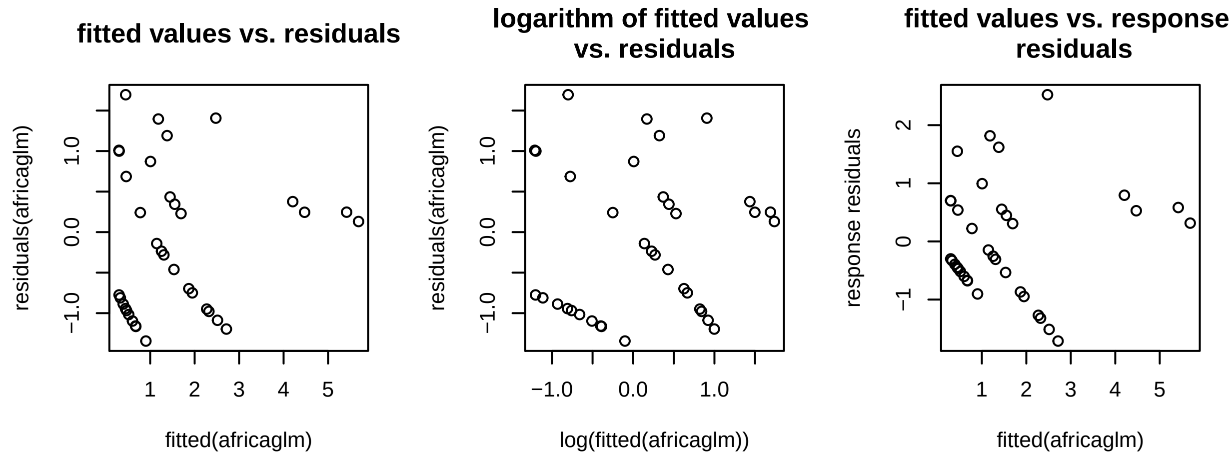
```
## Waiting for profiling to be done...
```

```
##                     2.5 %       97.5 %
## (Intercept) -2.4335049109  1.148089620
## oligarchy    0.0045915288  0.141483576
## pollib      -1.2570629668 -0.182012570
## parties      0.0080568606  0.052321186
## pctvote     -0.0054171503  0.032940743
## popn        -0.0038404317  0.022244262
## size        -0.0007146351  0.000272539
## numelec     -0.1438197483  0.114689702
## numregim    -0.2632334399  0.643070807
```

```
coef(africaglm)
```

```
##   (Intercept)     oligarchy        pollib       parties       pctvote
## -0.5102692854  0.0730813725 -0.7129778804  0.0307739289  0.0138722128
##          popn          size       numelec      numregim
##  0.0093429334 -0.0001899975 -0.0160783349  0.1917349158
```

```
# Assumption checks:
par(mfrow=c(1,3))
plot(fitted(africaglm),residuals(africaglm), main='fitted values vs. residuals')
plot(log(fitted(africaglm)),residuals(africaglm), main='logarithm of fitted values \nvs. residuals')
plot(fitted(africaglm),residuals(africaglm,type="response"), main='fitted values vs. response \n residua
```



Performing visual checks on the residuals of the model shows some odd relationships between the relationships and the fitted values, as the variance of the residuals doesn't seem to increase for higher fitted values. This is expected under a Poisson distribution, as higher fitted values correspond to higher variances as lambda is modeled differently for each observation. The first plot also shows some collinearity between variables such as `popn` and `pollib`.

Section 4

```
summary(glm(miltcoup~oligarchy+pollib+parties+pctvote+popn+size+numelec+numregim,
            family=poisson,data=africa))
```

```
##
## Call:
## glm(formula = miltcoup ~ oligarchy + pollib + parties + pctvote +
##     popn + size + numelec + numregim, family = poisson, data = africa)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.3443  -0.9542  -0.2587   0.3905   1.6953
##
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept) -0.5102693  0.9053301  -0.564  0.57301
## oligarchy    0.0730814  0.0345958   2.112  0.03465 *
## pollib      -0.7129779  0.2725635  -2.616  0.00890 **
## parties      0.0307739  0.0111873   2.751  0.00595 **
## pctvote      0.0138722  0.0097526   1.422  0.15491
## popn         0.0093429  0.0065950   1.417  0.15658
## size        -0.0001900  0.0002485  -0.765  0.44447
## numelec     -0.0160783  0.0654842  -0.246  0.80605
## numregim     0.1917349  0.2292890   0.836  0.40303
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
```

```
##
##     Null deviance: 65.945  on 35  degrees of freedom
## Residual deviance: 28.668  on 27  degrees of freedom
## AIC: 111.48
##
## Number of Fisher Scoring iterations: 6
```

```r
# `numelec` has the highest p-value, and is removed.
summary(glm(miltcoup~oligarchy+pollib+parties+pctvote+popn+size+numregim,
            family=poisson,data=africa))
```

```
##
## Call:
## glm(formula = miltcoup ~ oligarchy + pollib + parties + pctvote +
##     popn + size + numregim, family = poisson, data = africa)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.3997  -0.9381  -0.2666   0.4220   1.6998
##
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept) -0.6078028  0.8239267  -0.738  0.46070
## oligarchy    0.0781368  0.0277656   2.814  0.00489 **
## pollib      -0.6773897  0.2290130  -2.958  0.00310 **
## parties      0.0296786  0.0102888   2.885  0.00392 **
## pctvote      0.0131290  0.0092895   1.413  0.15756
## popn         0.0089313  0.0063746   1.401  0.16120
## size        -0.0002021  0.0002436  -0.830  0.40682
## numregim     0.1758198  0.2210498   0.795  0.42639
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 65.945  on 35  degrees of freedom
## Residual deviance: 28.728  on 28  degrees of freedom
## AIC: 109.54
##
## Number of Fisher Scoring iterations: 5
```

```r
# `numregim` is removed next.
summary(glm(miltcoup~oligarchy+pollib+parties+pctvote+popn+size,
            family=poisson,data=africa))
```

```
##
## Call:
## glm(formula = miltcoup ~ oligarchy + pollib + parties + pctvote +
##     popn + size, family = poisson, data = africa)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.3522  -0.9651  -0.1945   0.4833   1.6179
```

```
##
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept) -0.1126871  0.5163030  -0.218 0.827228
## oligarchy    0.0859620  0.0259100   3.318 0.000908 ***
## pollib      -0.6894029  0.2278572  -3.026 0.002481 **
## parties      0.0291944  0.0101954   2.863 0.004190 **
## pctvote      0.0141588  0.0091980   1.539 0.123723
## popn         0.0062736  0.0053994   1.162 0.245272
## size        -0.0001950  0.0002425  -0.804 0.421378
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 65.945  on 35  degrees of freedom
## Residual deviance: 29.363  on 29  degrees of freedom
## AIC: 108.17
##
## Number of Fisher Scoring iterations: 5
```

```r
# removing `size`
summary(glm(miltcoup~oligarchy+pollib+parties+pctvote+popn,
            family=poisson,data=africa))
```

```
##
## Call:
## glm(formula = miltcoup ~ oligarchy + pollib + parties + pctvote +
##     popn, family = poisson, data = africa)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.4109  -0.9943  -0.1399   0.5516   1.6125
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -0.244466   0.495708  -0.493  0.62190
## oligarchy    0.083168   0.025437   3.270  0.00108 **
## pollib      -0.652830   0.221234  -2.951  0.00317 **
## parties      0.029800   0.010294   2.895  0.00379 **
## pctvote      0.013842   0.009282   1.491  0.13591
## popn         0.005587   0.005378   1.039  0.29883
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 65.945  on 35  degrees of freedom
## Residual deviance: 30.044  on 30  degrees of freedom
## AIC: 106.85
##
## Number of Fisher Scoring iterations: 5
```

```r
# removing `popn`
summary(glm(miltcoup~oligarchy+pollib+parties+pctvote,
            family=poisson,data=africa))
```

```
##
## Call:
## glm(formula = miltcoup ~ oligarchy + pollib + parties + pctvote,
##     family = poisson, data = africa)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.5456  -0.9841  -0.1881   0.5948   1.6705
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -0.093657   0.463279  -0.202  0.83979
## oligarchy    0.095358   0.022421   4.253 2.11e-05 ***
## pollib      -0.666615   0.217564  -3.064  0.00218 **
## parties      0.025630   0.009502   2.697  0.00699 **
## pctvote      0.012134   0.009056   1.340  0.18031
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 65.945  on 35  degrees of freedom
## Residual deviance: 31.081  on 31  degrees of freedom
## AIC: 105.89
##
## Number of Fisher Scoring iterations: 5
```

```r
# removing `pctvote`
summary(glm(miltcoup~oligarchy+pollib+parties,
            family=poisson,data=africa))
```

```
##
## Call:
## glm(formula = miltcoup ~ oligarchy + pollib + parties, family = poisson,
##     data = africa)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.3583  -1.0424  -0.2863   0.6278   1.7517
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  0.251377   0.372689   0.674  0.50000
## oligarchy    0.092622   0.021779   4.253 2.11e-05 ***
## pollib      -0.574103   0.204383  -2.809  0.00497 **
## parties      0.022059   0.008955   2.463  0.01377 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 65.945  on 35  degrees of freedom
## Residual deviance: 32.856  on 32  degrees of freedom
## AIC: 105.66
##
## Number of Fisher Scoring iterations: 5
```

The remaining parameters appear significant, as their p-value is lower than 0.05. By examining the collinearity of the remaining variables using the plot below, it appears that none of the remaining variables are excessively collinear.

```
plot(africa[,1:4])
```