Exempler- 11.9.2.12

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Question:
The ratio of sums of m and n terms of an A.P. is $m^2 : n^2$.Show
that the ratio of mth and n^{th} term is (2m-1):(2n-1).

Solution:
The general equation for the sum of n terms of an A.P. is given by

$$S(n) = \frac{n}{2}(2a + (n-1)d) \tag{1}$$

The general equation for the nth term of an A.P is given by

$$T(n) = a + (n-1)d \tag{2}$$

where a is the first term and d is the common difference of A.P.

Given,

$$\frac{S(n)}{S(m)} = \frac{m^2}{n^2} \tag{3}$$

now substituting S(m) and S(n) into the (2) we get,

$$\frac{\frac{n}{2}(2a + (n-1)d)}{\frac{m}{2}(2a + (n-1)d)} = \frac{m^2}{n^2} \tag{4}$$

By cross multiplying and solving we get a relation between a and d that is

$$2a = d \tag{5}$$

The ratio between m^{th} and n^{th} terms is given by

$$\frac{T(m)}{T(n)} = \frac{a + (m-1)d}{a + (n-1)d} \tag{6}$$

Now substituting (5) in (6) we get **T(m):T(n)=(2m-1):(2n-1)**.