Exempler- 11.9.2.12

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Question:

The ratio of sums of m and n terms of an A.P. is $m^2 : n^2$.Show that the ratio of m^{th} and n^{th} term is (2m-1):(2n-1).

Solution:

nth term of an A.P. is given by

$$y(n) = y(0) + nd \tag{1}$$

sum of n terms is given by

$$x(n) = \sum_{k=0}^{n} y(k) \tag{2}$$

$$x(n) = \frac{n+1}{2}(2x(0) + nd) \tag{3}$$

Given,

$$\frac{x(m)}{x(n)} = \frac{m^2}{n^2} \tag{4}$$

now substituting x(m) and x(n) into the (4) we get,

$$\frac{\frac{m+1}{2}(2x(0) + md)}{\frac{n+1}{2}(2x(0) + nd)} = \frac{m^2}{n^2} \tag{5}$$

$$\frac{nm^2 + m^2}{2x(0) + nd} = \frac{n^2m + n^2}{2x(0) + md} \tag{6}$$

$$2x(0)nm^2 + 2x(0)m^2 + m^2nd = 2x(0)n^2m + n^2md + 2x(0)n^2 \tag{7}$$

$$2x(0)(m^2 - n^2 + nm^2 - mn^2) = d(n^2m - m^2n) \tag{8}$$

$$2x(0)(m + n + mn) = -d(mn) \tag{9}$$

$$2x(0) = \frac{-mnd}{m + n + mn} \tag{10}$$

The ratio between m^{th} and n^{th} terms is given by

$$\frac{y(m)}{y(n)} = \frac{y(0) + md}{y(0) + nd} \tag{11}$$

$$\frac{y(m)}{y(n)} = \frac{m(2m + n + 2mn)}{n(2n + m + 2mn)} \tag{12}$$

Parameter	Description	Value
n	an integer	1,2,3...
$y(n)$	general term of an A.P.	$(y(0) + nd) \cdot u(n)$
$x(n)$	sum of n terms of an A.P.	$\frac{n+1}{2}(2x(0) + nd) \cdot u(n)$
$x(m) : x(n)$	ratio of mth term is to nth term	$m^2 : n^2$

TABLE 1: Parameter Table

1) Analysis of equation for sum of n terms of an A.P:

   By the differentiation property :

$$nx(n) \overset{\mathcal{Z}}{\longleftrightarrow} (-z)\frac{dX(z)}{dz} \tag{13}$$

$$\implies nu(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}}{(1 - z^{-1})^2}, \{z \in \mathbb{C} : |z| > 1\} \tag{14}$$

$$\implies n^2u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}\left(z^{-1} + 1\right)}{(1 - z^{-1})^3}, \{z \in \mathbb{C} : |z| > 1\} \tag{15}$$

$$n^2 \cdot u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3}, \{\ z \in \mathbb{C} : |z| > 1\} \tag{16}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n} \tag{17}$$

$$X(z) = \sum_{n=0}^{\infty} \frac{n+1}{2}(2x(0) + nd) \cdot u(n) \cdot z^{-n} \tag{18}$$

$$X(z) = 2x(0)U(z) - 2x(0)z\frac{dU(z}{dz} + dz^2\frac{d^2U(z)}{dz^2} \tag{19}$$

$$\therefore X(z) = \frac{z^{-1}(d - x(0)) + x(0)}{2((1 - z^{-1})^3)}, \{\ z \in \mathbb{C} : |z| > 1\} \tag{20}$$

2) Analysis equation for the n^{th}term of an A.P.:

$$y(n) = y(0) + nd \tag{21}$$

$$n \cdot u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-}1}{(1 - z^{-1})^{-2}}, \{\ z \in \mathbb{C} : |z| > 1\} \tag{22}$$

$$Y\,(z) = \sum_{n=-\infty}^{\infty} y\,(n) \cdot z^{-n} \tag{23}$$

$$Y\,(z) = \sum_{n=0}^{\infty} (y(0) + nd) \cdot u(n) \cdot z^{-n} \tag{24}$$

$$Y(z) = y(0)U(z) - zd\frac{dU(z)}{dz} \tag{25}$$

$$\therefore Y\,(z) = \frac{y(0) + (d - y(0))\,z^{-1}}{(1 - z^{-1})^2}, \{\ z \in \mathbb{C} : |z| > 1\} \tag{26}$$