Exempler- 11.9.2.12

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Question:
The ratio of sums of m and n terms of an A.P. is $m^2 : n^2$.Show that the ratio of m^{th} and n^{th} term is (2m-1):(2n-1).
Solution:
The general equation for the sum of n terms of an A.P. is given by

$$S(n) = \frac{n}{2}(2a + (n-1)d) \tag{1}$$

The general equation for the n^{th} term of an A.P is given by

$$T(n) = a + (n-1)d \tag{2}$$

where a is the first term and d is the common difference of A.P.

Given,

$$\frac{S(m)}{S(n)} = \frac{m^2}{n^2} \tag{3}$$

now substituting S(m) and S(n) into the (2) we get,

$$\frac{\frac{m}{2}(2a + (m-1)d)}{\frac{n}{2}(2a + (n-1)d)} = \frac{m^2}{n^2} \tag{4}$$

By cross multiplying we get

$$\frac{n}{2a + (m-1)d} = \frac{m}{2a + (n-1)d} \tag{5}$$

solving the above equation we get

$$2a = d \tag{6}$$

The ratio between m^{th} and n^{th} terms is given by

$$\frac{T(m)}{T(n)} = \frac{a + (m-1)d}{a + (n-1)d} \tag{7}$$

Now substituting (5) in (6) we get

$$\frac{T(m)}{T(n)} = \frac{a + (m-1)2a}{a + (n-1)2a} \tag{8}$$

By cancelling a in both the denominator and numerator we get **T(m):T(n)=(2m-1):(2n-1)**. if we consider that the A.P is starting from n=0 we will have sum of n terms and nth term as

$$S(n) = \frac{n+1}{2}(2a + nd) \tag{9}$$

$$x(n) = \frac{n+1}{2}(2x(0) + nd) \tag{10}$$

$$T(n) = y(0) + nd \tag{11}$$

$$y(n) = y(0) + nd \tag{12}$$

Given,

$$\frac{x(m)}{x(n)} = \frac{m^2}{n^2} \tag{13}$$

now substituting x(m) and x(n) into the (9) we get,

$$\frac{\frac{m+1}{2}(2x(0) + md)}{\frac{n+1}{2}(2x(0) + nd)} = \frac{m^2}{n^2} \tag{14}$$

$$\frac{nm^2 + m^2}{2x(0) + nd} = \frac{n^2m + n^2}{2x(0) + md} \tag{15}$$

$$2x(0)nm^2 + 2x(0)m^2 + m^2nd = 2x(0)n^2m + n^2md + 2x(0)n^2 \tag{16}$$

$$2x(0)(m^2 - n^2 + nm^2 - mn^2) = d(n^2m - m^2n) \tag{17}$$

$$2x(0)(m + n + mn) = -d(mn) \tag{18}$$

$$2x(0) = \frac{-mnd}{m + n + mn} \tag{19}$$

The ratio between m^{th} and n^{th} terms is given by

$$\frac{y(m)}{y(n)} = \frac{y(0) + md}{y(0) + nd} \tag{20}$$

Now substituting y(0)(same as x(0))in (20) we get

$$\frac{y(m)}{y(n)} = \frac{m(2m + n + 2mn)}{n(2n + m + 2mn)} \tag{21}$$