Solve $ax + by = c$

$$0 \leq x \leq 15$$
$$-300 \leq y \leq 150$$
How many such solutions exists?

Ext. Euclidian $\rightarrow (x_1, y_1)$

$a = 15, \quad b = 10, \quad c = 20$

$15x + 10y = 20$

$x = 4, \quad y = -4, \quad gcd = \boxed{5}$

$5(3x+2y) = 20$

$\Rightarrow 3\underline{\underline{x}} + 2\underline{\underline{y}} = 4$ ✓

$(x, y_1) \equiv (4, -4)$

$3\underline{\underline{x_1}} + 2\underline{\underline{y_1}} = 4$ ✓

$x, x_1, y, y_1 \in \mathbb{Z}$

$3(\underline{x-x_1}) + 2(\underline{y-y_1}) = 0$
$\quad\quad\downarrow \quad\quad\quad\quad \downarrow$
$\quad\quad u \quad\quad\quad\quad v$

$\Rightarrow x-x_1, y-y_1 \in \mathbb{Z}$

$\Rightarrow u, v \in \mathbb{Z}$

$\Rightarrow \boxed{3u + 2v = 0}$

$\hookrightarrow \underline{\underline{u}} = \underline{2k}, \quad \underline{\underline{v}} = \underline{-3k}, \quad k \in \mathbb{Z}$

$k = 0, \Rightarrow u = 0, v = 0$

$k = 1 \Rightarrow u = 2, v = -3$

$k = -1 \Rightarrow u = -2, v = +3$

$$x - x_1 = 2k, \qquad y - y_1 = -3k$$

$$\Rightarrow x = \underline{x_1} + 2k, \qquad y = \underline{y_1} - 3k$$

$$\Rightarrow (x, y) \equiv (\underline{x_1 + 2k}, \underline{y_1 - 3k}), \qquad k \in \mathbb{Z}$$

$$15x + 10y = 20$$

$$\Rightarrow \underline{3x + 2y = 4} \qquad\qquad x_1 = \underline{4}, \quad y_1 = -\underline{4}$$

$$(x_1 + 2k, y_1 - 3k)$$

$$\equiv (\underline{4 + 2k, -4 - 3k}) \;, \; k \in \mathbb{Z} \qquad\qquad L \le x \le R$$

$$k = 0 \qquad (4, -4) \checkmark \qquad\qquad\qquad L \le \underline{4 + 2k} \le R$$

$$k = 1 \qquad (\underline{6, -7}) \checkmark \qquad\qquad\qquad 4 + 2k \ge L$$

$$k = -1 \qquad (2, -1) \checkmark \qquad\qquad\qquad k \ge \left(\frac{L-4}{2}\right)$$

$$k = -2 \qquad (0, 2) \checkmark \qquad\qquad\qquad \Rightarrow k \ge \text{ceil}\left(\frac{L-4}{2}\right)$$

$$Ax + By = C \qquad , \gcd(A, B) = g$$

$$g\left(\frac{A}{g}x + \frac{B}{g}y\right) = C$$

$$\boxed{(x, y) \equiv \left(x_1 + \frac{B}{g}k, \quad y_1 - \frac{A}{g}k\right)} \checkmark$$

$$Ax + By = C$$

$$ex-gcd \rightarrow \quad \underline{x_1, y_1, g}$$

$$\gcd(-15, -20) = 4$$

$$(x, y) \equiv \left(x_1 + \frac{B}{g}k, \quad \underline{y_1 - \frac{A}{g}k}\right)$$

$$A, \underline{B}, C > 0 \quad , \qquad x, y > 0$$

$$\boxed{x_1 + \frac{B}{g}k > 0}$$

$$\Rightarrow \frac{B}{g}k > -x_1$$

$$\Rightarrow k > \frac{-x_1}{(B/g)}$$

lower limit.

$$k > \left(\frac{12}{3}\right) \Rightarrow k > 4 \Rightarrow k \geq \boxed{5}$$

$$k > \frac{13}{3} \Rightarrow k > 4.33 \Rightarrow k \geq \boxed{5}$$

$$\frac{14}{3} \rightarrow \boxed{5}$$

$$\frac{15}{3} \rightarrow \boxed{6}$$

$$\frac{-x_1}{B/g}$$

$$\boxed{\text{llmit} = \quad -x_1 // (B//g) + 1}$$

$$k \geq \text{llmit}$$

$$y_1 - \frac{A}{g}k > 0$$

$$\Rightarrow \frac{A}{g}k < y_1$$

$$\Rightarrow k < \frac{y_1}{(A/g)} \qquad \left(\because \begin{array}{c} A, B > 0 \\ g > 0 \end{array} \right)$$

upper limit

$$k < \left(\frac{12}{3}\right) \Rightarrow k < 4 \Rightarrow k \leq \boxed{3}$$

$$k < \left(\frac{13}{3}\right) \Rightarrow k < 4.33 \Rightarrow k \leq \boxed{4}$$

$$\left\{ \frac{14}{3} \rightarrow \boxed{4} \right.$$

$$\left\{ \frac{15}{3} \rightarrow \underline{\boxed{4}} \right. \qquad \text{ceil}(\underline{\quad}) - 1$$

$$\boxed{\begin{array}{l} \text{ulimit} = y_1 // (A//g) \\[4pt] \text{if}\,(y_1 \% (A//g) == 0) \\[4pt] \qquad \checkmark \text{ulimit} -= 1 \end{array}}$$

$$k \leq \text{ulimit}$$

$$\text{\# Solutions} = ulimt - llimit + 1$$

```
for k in range (llimit, ulimt+1):
    print ( x_1 + B//g * k , y_1 - A//g * k)
```

```python
def f_gcd(a, b):
    # Returns x, y, g
    if b==0:
        return 1, 0, a
    x0, y0, g = f_gcd(b, a%b)
    return y0, x0-(a//b)*y0, g

def solve(i):
    a,b,c = map(int, input().split())
    if a>b:
        x,y,g = f_gcd(a,b)
    else:
        y,x,g = f_gcd(b,a)
    if c%g == 0:
        # print(f'Case {i}: x = {(c//g)*x}, y = {(c//g)*y}')
        x1 = (c//g)*x
        y1 = (c//g)*y
        llmit = -x1//(b//g)+1
        ulimit = y1//(a//g)
        if y1%(a//g) == 0:
            ulimit -= 1
        print(ulimit-llmit+1)
        for k in range(llmit, ulimit+1):
            print(f'{x1+b//g*k} {y1-a//g*k}')
    else:
        print(0)

t = int(input())
for i in range(1, t+1):
    solve(i)
```

ABABAB | AB   ABCD∤AB ?

ABA∤AB

                               gcd(a,b)
                                  |
(ABAB)AB% (ABAB)   gcd(b, a%b)
   = AB   ⋮

                                              a%b < b

A A A A (A)% AA = A   ABCDE % (AB)
                                  = CDE

   gcd(ABABAB, ABAB)   ‼

   = gcd(ABAB, AB)

      = gcd(AB, ")

         = AB

```python
def mods(a, b):
    while len(a) >= len(b) and a[:len(b)] == b:
        a = a[len(b):]
    return a

def f_gcd(a, b):
    if b == '':
        return a
    md = mods(a, b)
    if len(md) >= len(b):
        return ''
    return f_gcd(b, md)

class Solution:
    def gcdOfStrings(self, str1: str, str2: str) -> str:
        if(len(str1) > len(str2)):
            return f_gcd(str1, str2)
        return f_gcd(str2, str1)
```

— ✗ —

Least Common Multiple (LCM)

$a = 12,$ $\qquad b = 18$

$a = 12 = 2^2 \times 3^1$

$b = 18 = 2 \times 3^2$

$= 2 \times 2 \times 3$ ✓

$= 2 \times 3 \times 3$ ✓

$gcd = \underline{2 \times 3} = \underline{\underline{6}}$

$lcm = \underline{2 \times 3 \times 2 \times 3} = \underline{\underline{36}}$

$a = \boxed{2} \times \boxed{2} \times \boxed{3} \rightarrow$ lcm, gcd

$b = \boxed{2} \times \boxed{3} \times \boxed{3} \rightarrow$ lcm

$\rightarrow$ lcm

gcd one time

$\rightarrow$ lcm one time

$a * b =$

$|a * b| = \underline{gcd} * \underline{lcm}$

$\longrightarrow O(\log b)$

smaller number

— ✳ —

GCD and LCM for an array of integers?

* $gcd(a,b,c) = gcd(gcd(a,b), c)$ ✓
* $lcm(a,b,c) = lcm(lcm(a,b), c)$ ✓

array of size n

$\quad\quad\hookrightarrow gcd \equiv O(n \log k)$

$\quad\quad\quad lcm \equiv O(n \log k)$

$k = min(A)$

= ✳ =