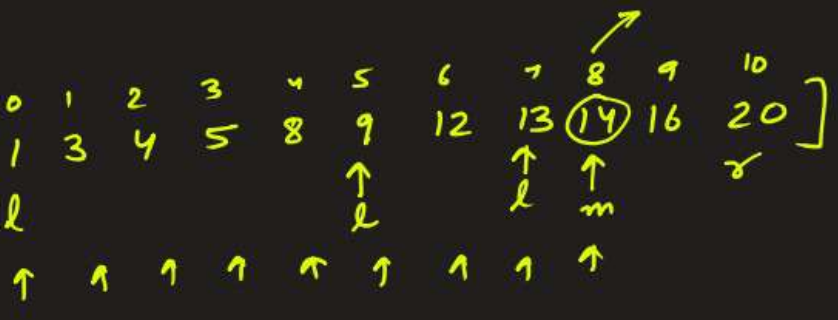
Arrays, Strings & Linked Lists Lecture 8

Wednesday, 31 July 2024 6:05 AM

Binary Search

Sorted array :- [1 3 4 5 8 9 12 13 14 16 20]

Find (14)



$T(n) = T(n/2) + O(1)$

$T(n) = O(\log n)$

Linear Search :- $O(n)$

$T(n/4) + 1 + 1 = T(n/4) + 2$

$T(n/8) + 1 + 1 + 1 = T(n/8) + 3$

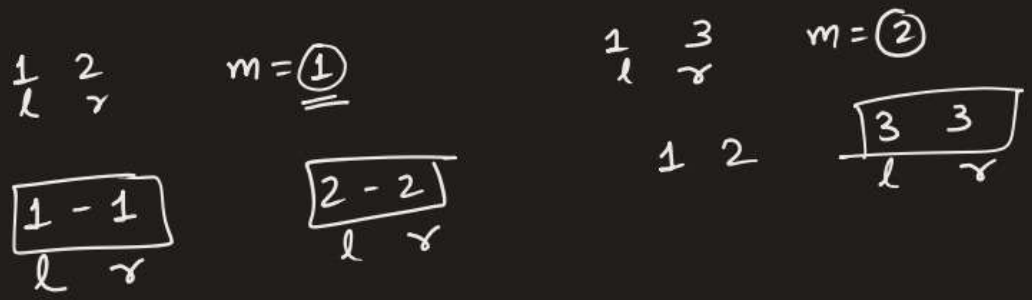
$= T(n/2^k) + k$

$= T(n/2^{\log n}) + \log n$

$= T(n/n) + \log n$

$= \log n$

https://leetcode.com/problems/binary-search/description/



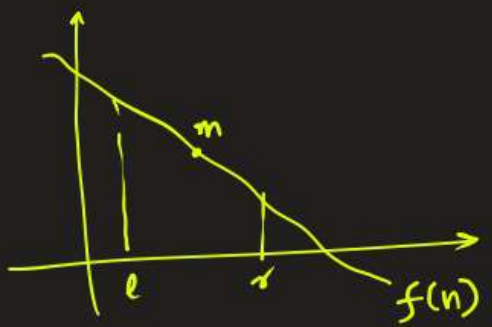

```python
def bs(nums, l, r, target):
    if l == r:
        if nums[l] == target:
            return l
        else:
            return -1
    m = (l+r)//2
    if nums[m] >= target:
        return bs(nums, l, m, target)
    else:
        return bs(nums, m+1, r, target)
class Solution:
    def search(self, nums: List[int], target: int) -> int:
        return bs(nums, 0, len(nums)-1, target)
```

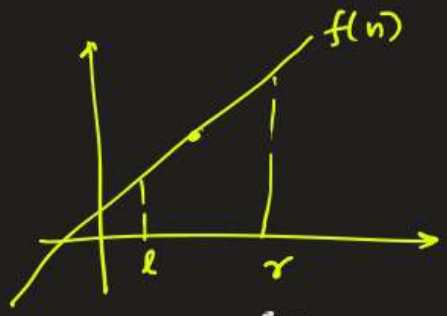
How to identify binary search problems?

① Input is sorted

② function (Input) is monotonous.

f(n)





Sorted array:-

Find (14)

all elem
≥14 T
<14 F

First occurrence of True

https://leetcode.com/problems/sqrtx

$n = 132$

$\sqrt{132} \approx 11.489$

$\lfloor \sqrt{132} \rfloor = 11$

1	2	3	4	5	6	7	8	9	10	11	12	13 ...	132
F	F	F	F	F	F	F	F	F	F	T	T	T	T

$\log n$ ✓

$n \to \log \log n$

$n \sim 10^{18}$

$\log n$

$18 \times \log 10$

$= 18 \times 3.1$

$\sim 50\text{-}60$

11 - 12

$\frac{133}{2} = 66$

1 - 66

1 - 33

1 - 17

8 - 17

8 - 12

10 - 12

11 - 12 → 11

l r

$66^2 > 132$

$33^2 > 132$

$17^2 > 132$

$8^2 < 132$

$12^2 > 132$

$10^2 < 132$

$11^2 < 132$

$i^2 < n$

$\sqrt{n}$ ✓

bs (l , m)
bs (m+1, r)

1 4 m = 2

1 2
3 4

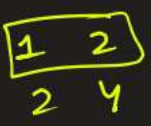
m = 1 → 1 1 ✓
2 2 ✓

1 3 m = 2

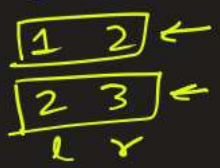
1 2 → 1 1
3 3 2 2

bs (l , m)
bs (m , r)

1 4 m = 2



1 3 m = 2

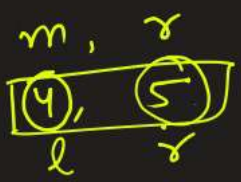


(18)

3 5

m = 4

m * m <= r



3 5 m = 4 (16)

```
class Solution {
public:
    int bs(long l, long r, long x) {
        if(l == r-1)
            return l;
        long m = (l+r)/2;
        if(m*m > x)
            return bs(l, m, x);
        else
            return bs(m, r, x);
    }
    int mySqrt(int x) {
        return bs(0, (long)x+1, x);
    }
};
```

https://www.geeksforgeeks.org/problems/koko-eating-bananas/1

n = 4 piles = [3, 6, 7, 11] (n) H = 8

S = 1 O(n) 3 + 6 + 7 + 11 = 27 hours ✓

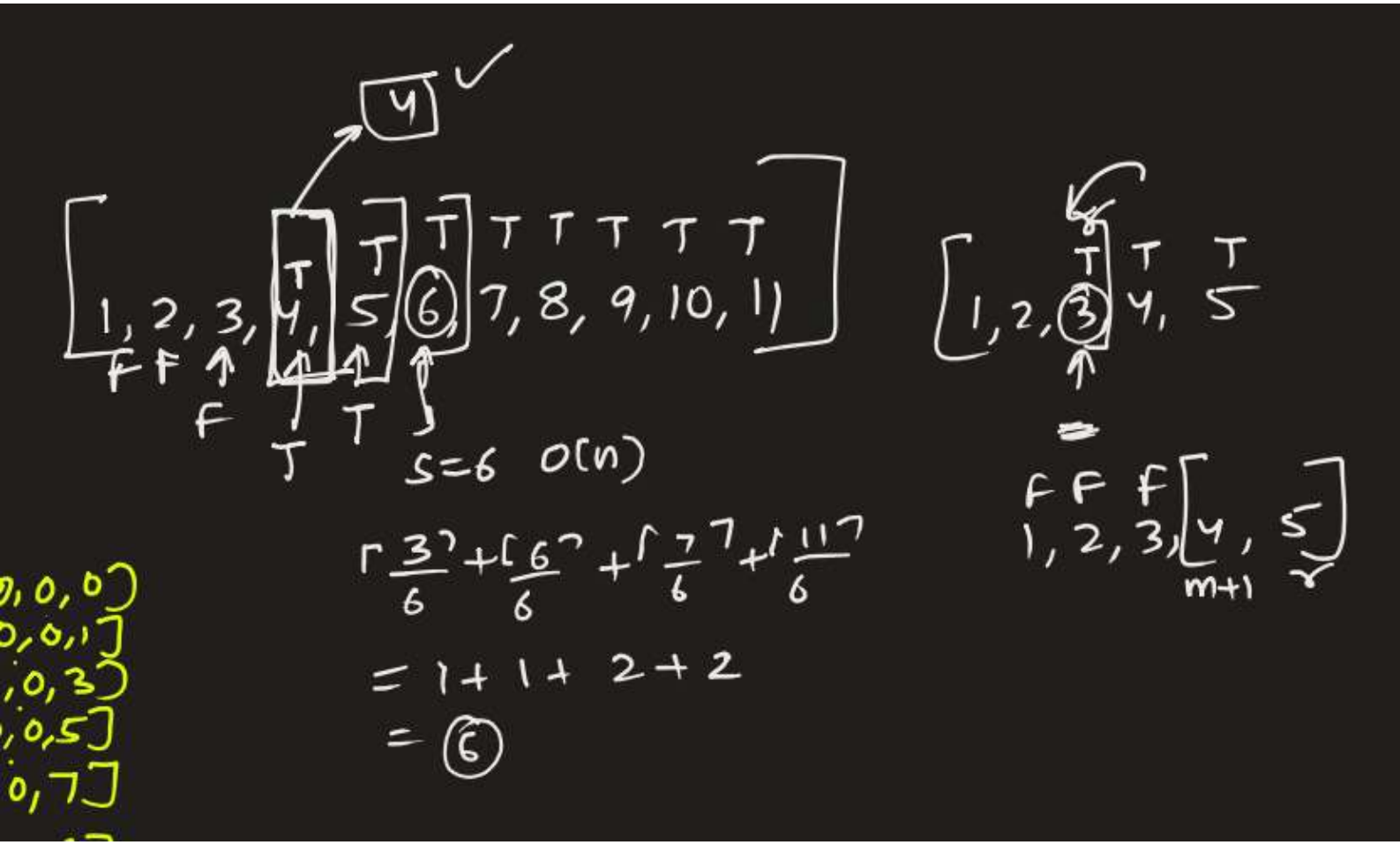
S = 2 O(n)

[3,6,7,11] → [1,6,7,11] → [0,6,7,11] → [0,4,7,11] → [0,2,7,11] → [0,0,7,11] ✓ → [0,0,5,11] → [0,0,3,11] → [0,0,1,11] → [0,0,0,11]

[0,0,0,0] [0,0,0,1] [0,0,0,3] [0,0,0,5] [0,0,0,7] [0,0,0,9]

15 hours

$\lceil \frac{3}{2} \rceil + \lceil \frac{6}{2} \rceil + \lceil \frac{7}{2} \rceil + \lceil \frac{11}{2} \rceil = 2 + 3 + 4 + 6 = 15$


4
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11
F F F F T T T T T T T T T
S=6 O(n)
⌈3/6⌉ + ⌈6/6⌉ + ⌈7/6⌉ + ⌈11/6⌉
= 1 + 1 + 2 + 2
= 6
1, 2, 3, 4, 5
T T T T T
F F f [4, 5]
m+1

O(n)

S=3 $\lceil \frac{3}{3} \rceil + \lceil \frac{6}{3} \rceil + \lceil \frac{7}{3} \rceil + \lceil \frac{11}{3} \rceil = 1+2+3+4 = 10$

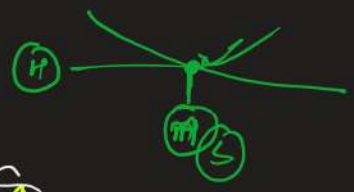
O(n)

S=4 $\lceil \frac{3}{4} \rceil + \lceil \frac{6}{4} \rceil + \lceil \frac{7}{4} \rceil + \lceil \frac{11}{4} \rceil = 1+2+2+3 = 8$

⋮

S=11 $\lceil \frac{3}{11} \rceil + \lceil \frac{6}{11} \rceil + \lceil \frac{7}{11} \rceil + \lceil \frac{11}{11} \rceil = 1+1+1+1 = 4$

S=12 ⋯ (4)



S= F F F T T T T

1, 2, 3, 4, 5, 6, 7 ⋯

T ~10^9 max(piles)

O(n) O(n) O(n)

h_1 h_2 h_3 h_4 h_5 h_m

$h_1 \geq h_2 \geq h_3 \geq h_4 \cdots$

$10^4 \sim n \rightarrow$ # piles

max piles $\rightarrow m \sim 10^9$

$O(n \cdot m)$

$O(10^4 \cdot 10^9)$

$O(n \cdot \log m) = O(10^4 \cdot (\log 10^9))$

$\sim 9 \times 10^4 \times 3$

$\sim \underline{10^6}$

T: $h \leq H$ ✓

F: $h > H$ ✓

if $h[m] \leq H$:

bs(l, m)

if $h[m] > H$:

bs(m+1, r)

```
def bs(piles, H, l, r):
    if l==r:
        return l
    m = (l+r)//2
    h = 0
    for p in piles:
        h += p//m
        if p%m != 0:
            h += 1
    if h <= H:
        return bs(piles, H, l, m)
    else:
        return bs(piles, H, m+1, r)

class Solution:
    def Solve(self, N, piles, H):
        return bs(piles, H, 1, max(piles))
```