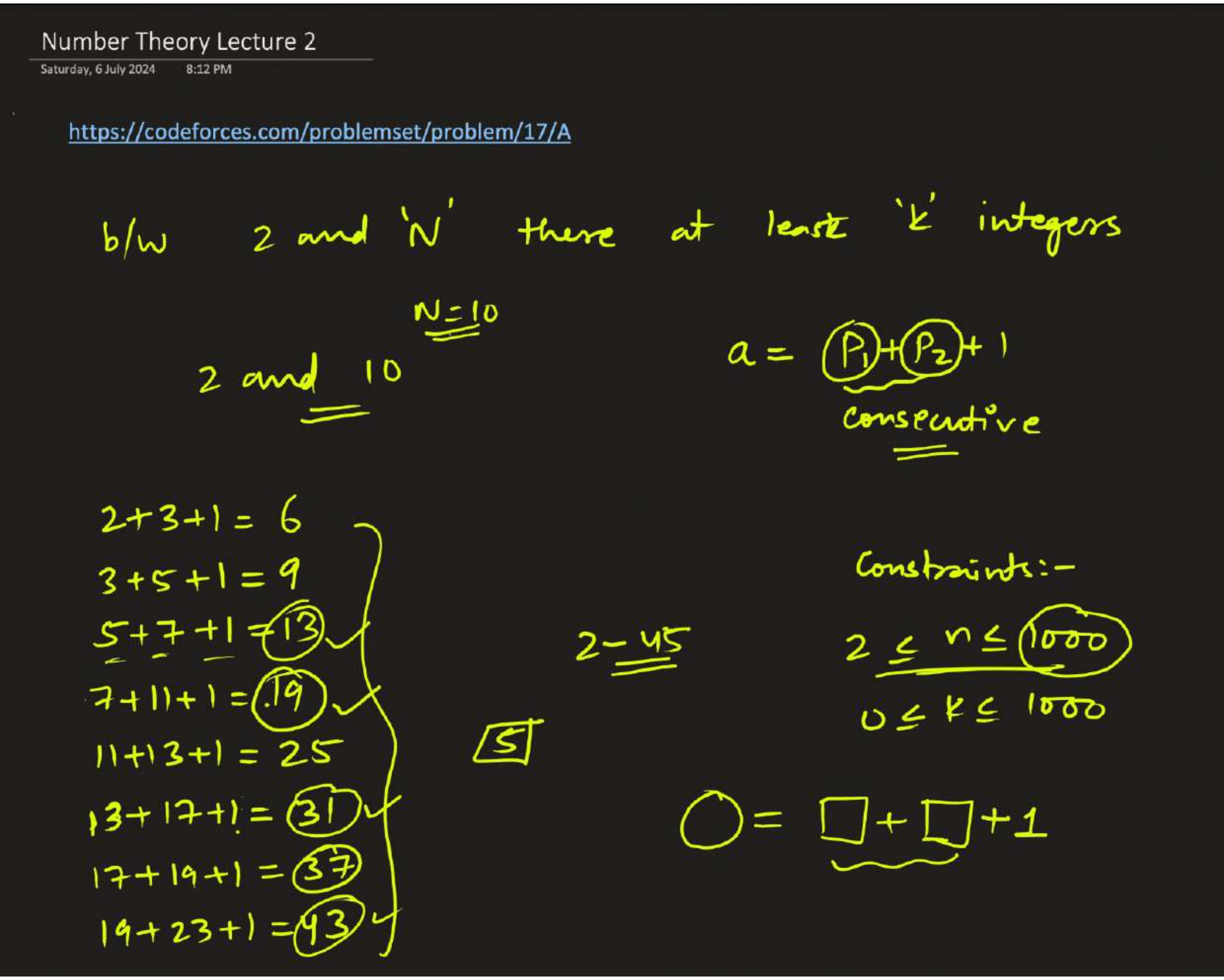


Number Theory Lecture 2

Saturday, 6 July 2024 8:12 PM

https://codeforces.com/problemset/problem/17/A

b/w 2 and 'N' there at least 'k' integers

$\underline{\underline{N=10}}$

2 $\underline{\underline{\text{and}}}$ 10

$a = \underbrace{p_1 + p_2}_{} + 1$

$\underline{\underline{\text{consecutive}}}$

$2+3+1 = 6$

$3+5+1 = 9$

$5+7+1 = 13$ ✓

$7+11+1 = 19$ ✓

$11+13+1 = 25$

$13+17+1 = 31$ ✓

$17+19+1 = 37$

$19+23+1 = 43$ ✓

$2 - \underline{\underline{45}}$

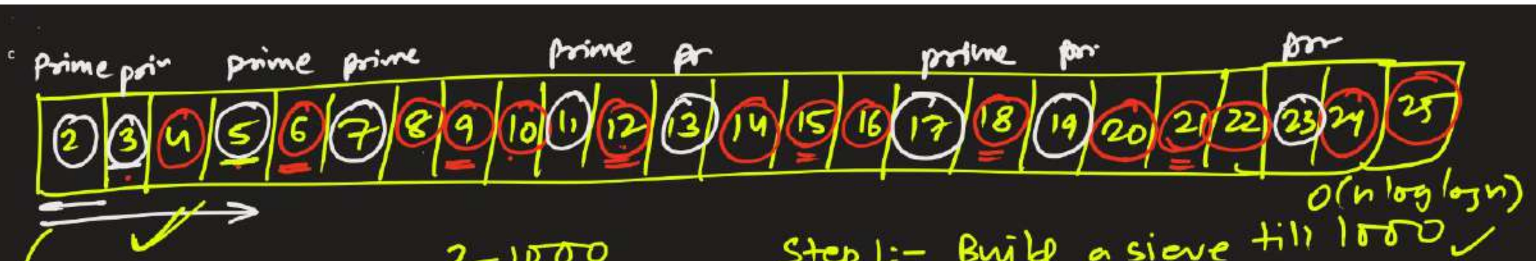
$\boxed{5}$

Constraints:-

$2 \le n \le 1000$

$0 \le k \le 1000$

$\bigcirc = \underbrace{\square + \square}_{} + 1$



2–1000

$\boxed{n}$

2 – n

Step 1:- Build a sieve till 1000 ✓

Step2:- Build a vector of a primes till 1000 ✓ O(n)

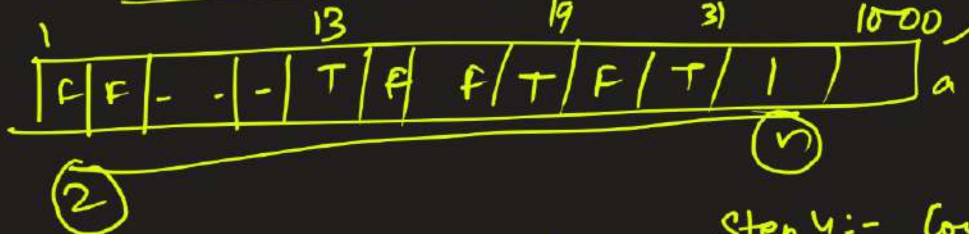
Step3: Iterate over the prime array and take cons- primes O(n)

a[$P_1 + P_2 + 1$] = True

$P_1 + P_2 + 1$

O(n)

Pr = 2, 3, 5, 7, 11, 13, 17, ... O(n)



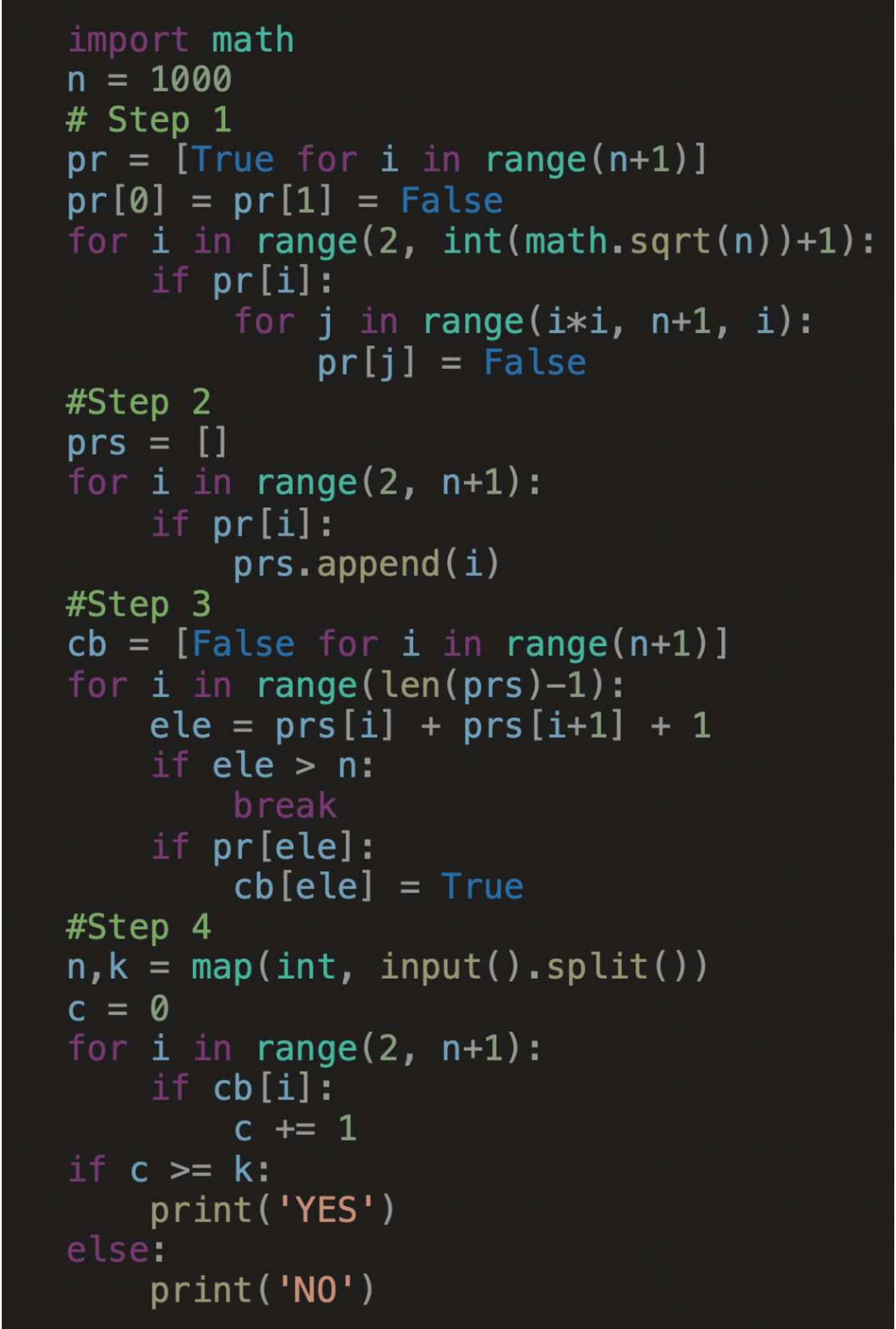
Step 4:- Count the number of elements b/w 2 and n which are True in this array

↳ ⓒ ≥ k → True/Yes

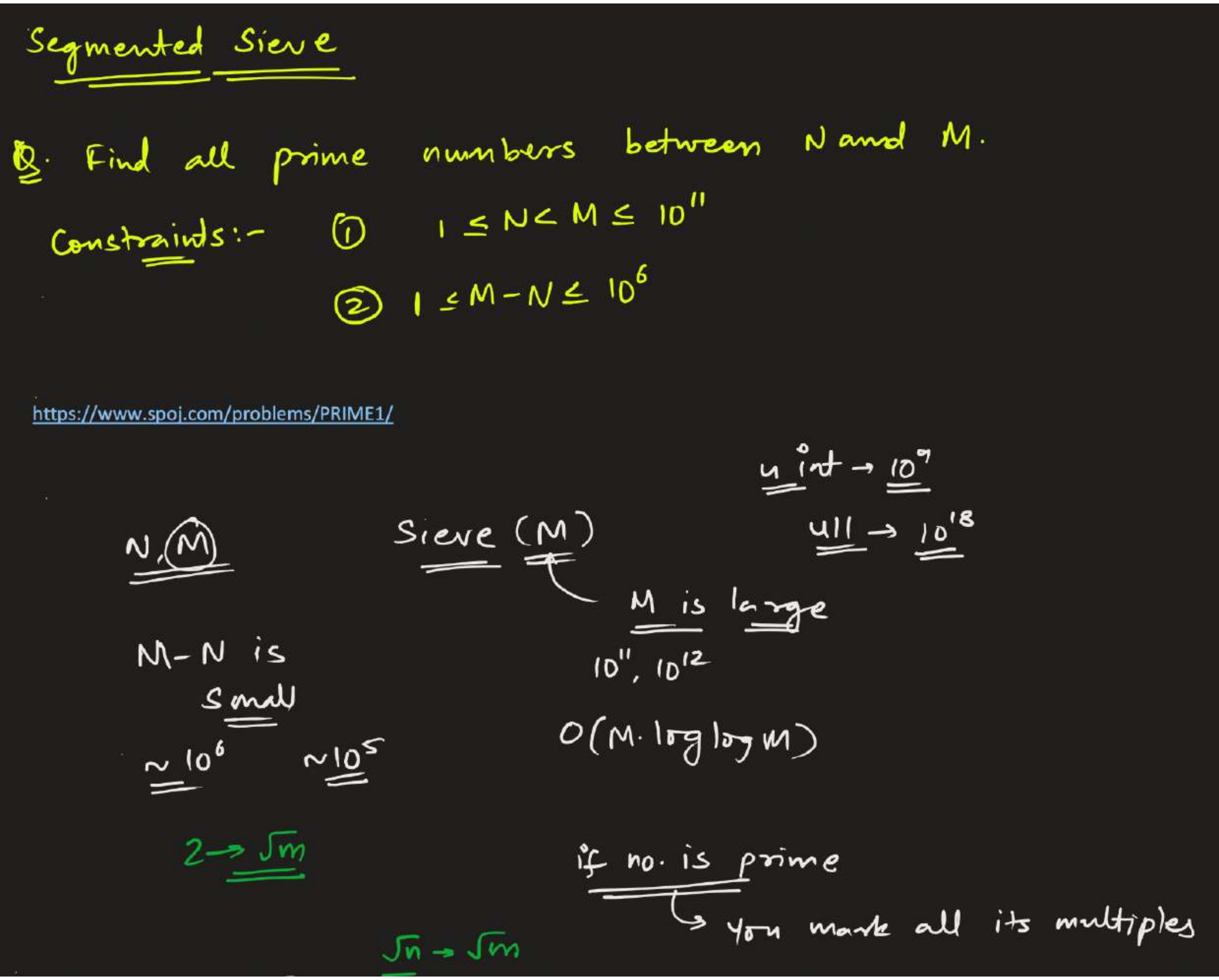
other → No

2, 3, 5, 7, 11 ⑧

i i+1 i i+1



```
import math
n = 1000
# Step 1
pr = [True for i in range(n+1)]
pr[0] = pr[1] = False
for i in range(2, int(math.sqrt(n))+1):
    if pr[i]:
        for j in range(i*i, n+1, i):
            pr[j] = False
#Step 2
prs = []
for i in range(2, n+1):
    if pr[i]:
        prs.append(i)
#Step 3
cb = [False for i in range(n+1)]
for i in range(len(prs)-1):
    ele = prs[i] + prs[i+1] + 1
    if ele > n:
        break
    if pr[ele]:
        cb[ele] = True
#Step 4
n,k = map(int, input().split())
c = 0
for i in range(2, n+1):
    if cb[i]:
        c += 1
if c >= k:
    print('YES')
else:
    print('NO')
```



Segmented Sieve

Q. Find all prime numbers between N and M.

Constraints:- ① $1 \le N < M \le 10^{11}$

② $1 \le M - N \le 10^{6}$

https://www.spoj.com/problems/PRIME1/

u int → 10^9

ull → 10^{18}

$N, \boxed{M}$

Sieve (M)

M is large

$10^{11}, 10^{12}$

$O(M \cdot \log\log M)$

M−N is small

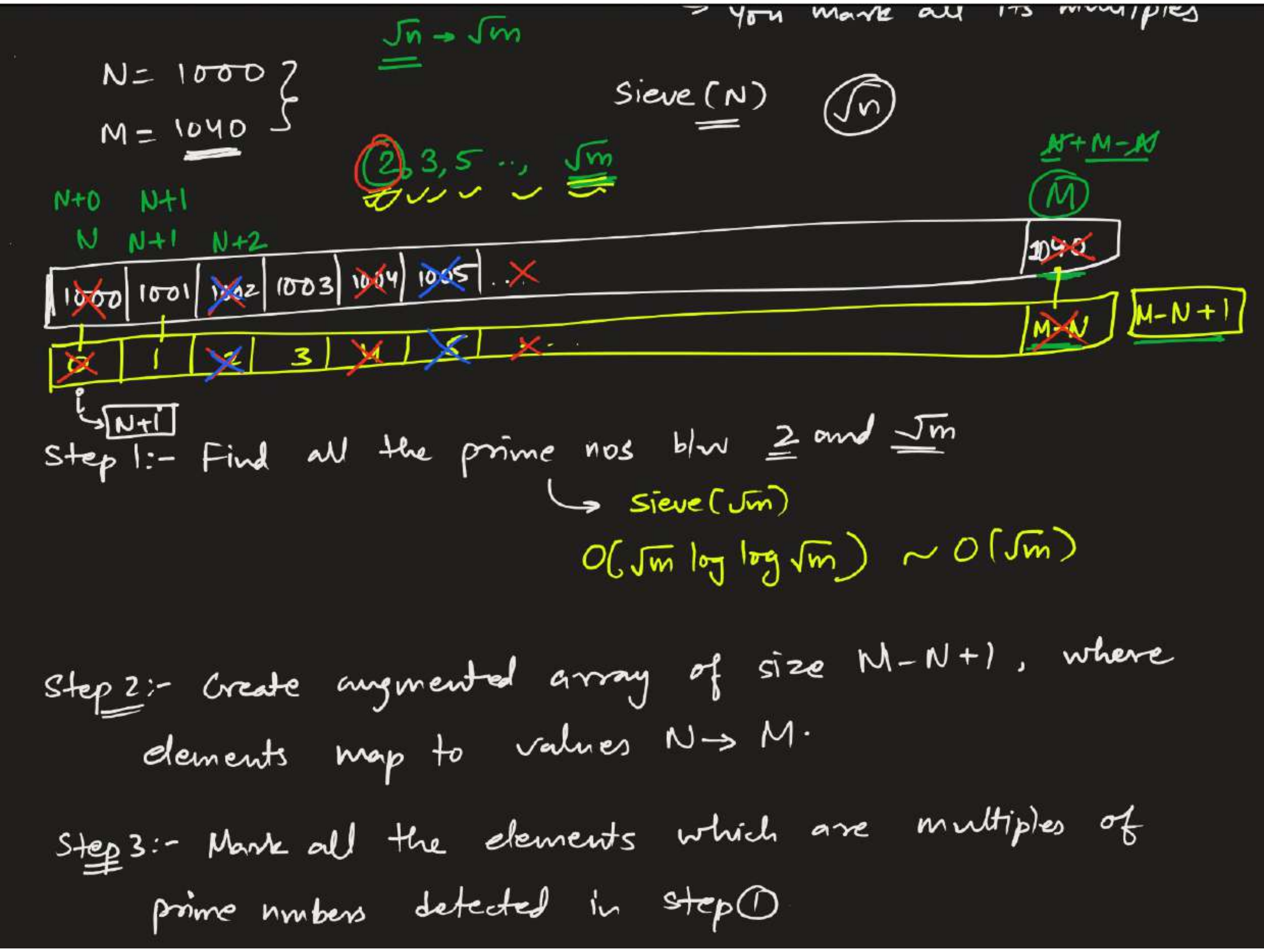
$\sim 10^6$ $\sim 10^5$

$2 \rightarrow \sqrt{m}$

if no. is prime

↳ You mark all its multiples

$\sqrt{n} \rightarrow \sqrt{m}$



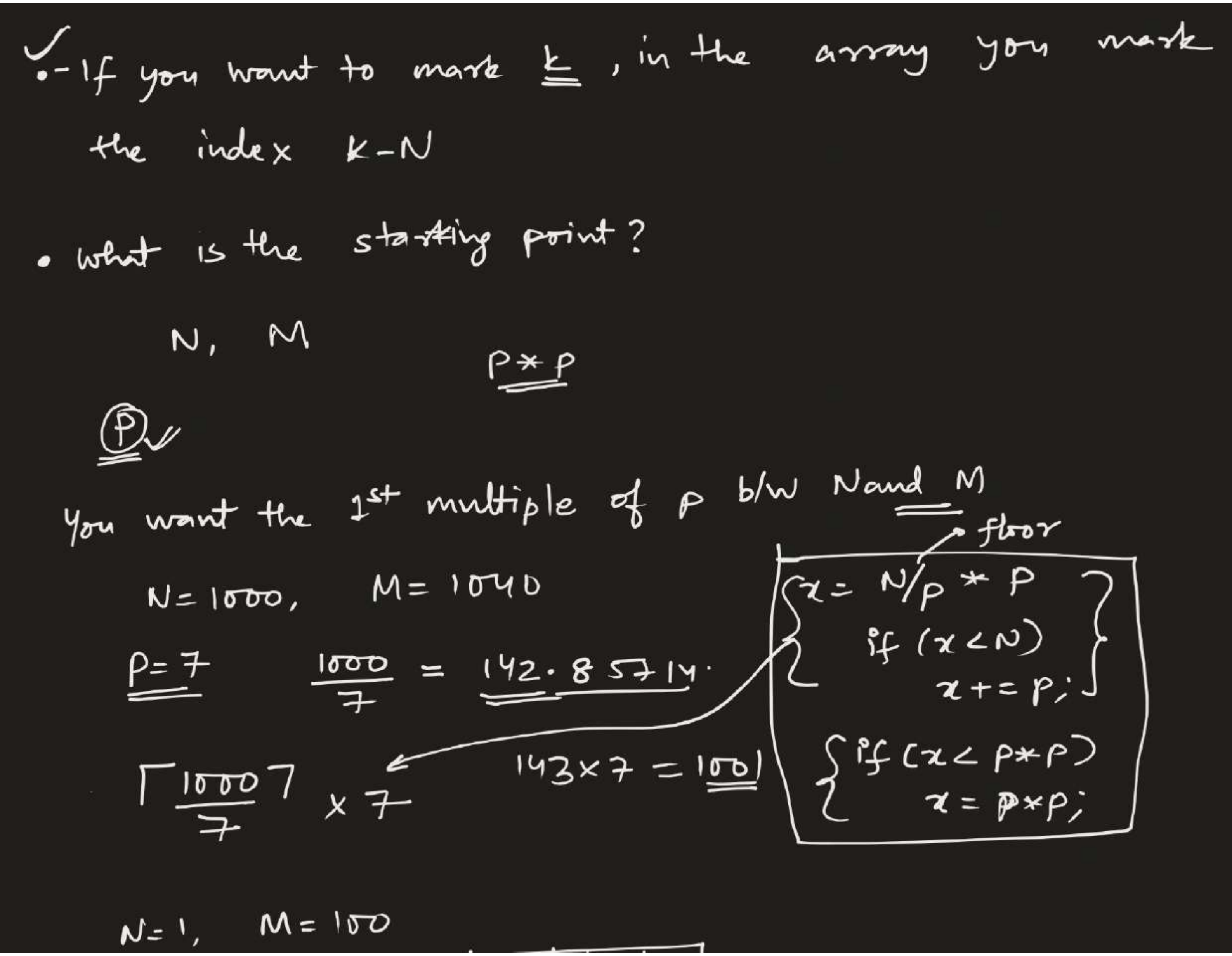
Step 1:- Find all the prime nos b/w 2 and $\sqrt{m}$

→ sieve($\sqrt{m}$)

$O(\sqrt{m} \log\log\sqrt{m}) \sim O(\sqrt{m})$

Step 2:- Create augmented array of size M−N+1, where elements map to values N→M.

Step 3:- Mark all the elements which are multiples of prime numbers detected in step ①



✓ •- If you want to mark $\underline{\underline{k}}$, in the array you mark the index $k-N$

- what is the starting point?

$N, \quad M$ $\qquad\qquad \underline{\underline{P*P}}$

$\underline{\underline{\boxed{P}}}$ ✓

You want the 1st multiple of p b/w N and $\underline{\underline{M}}$

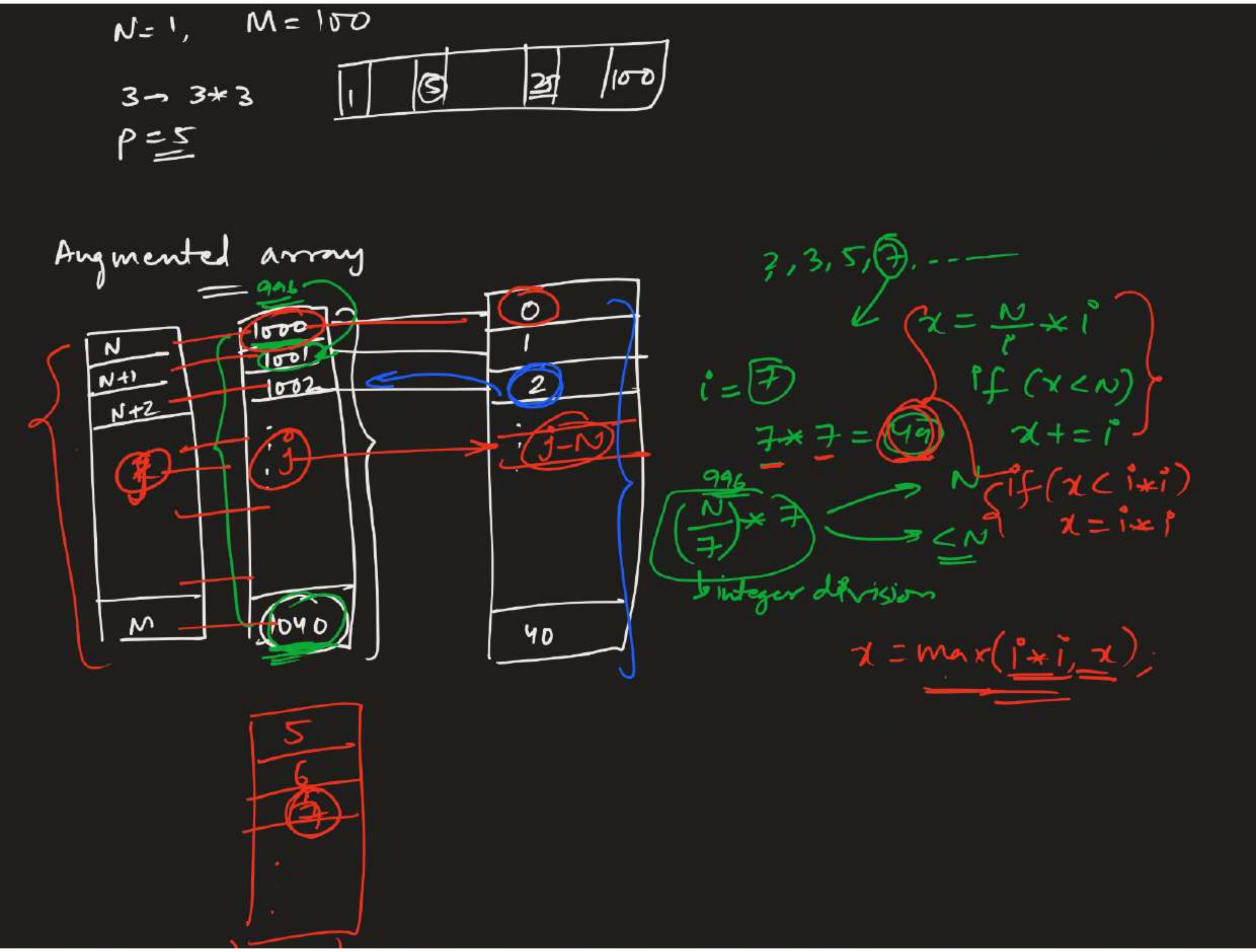
$N = 1000, \qquad M = 1040$

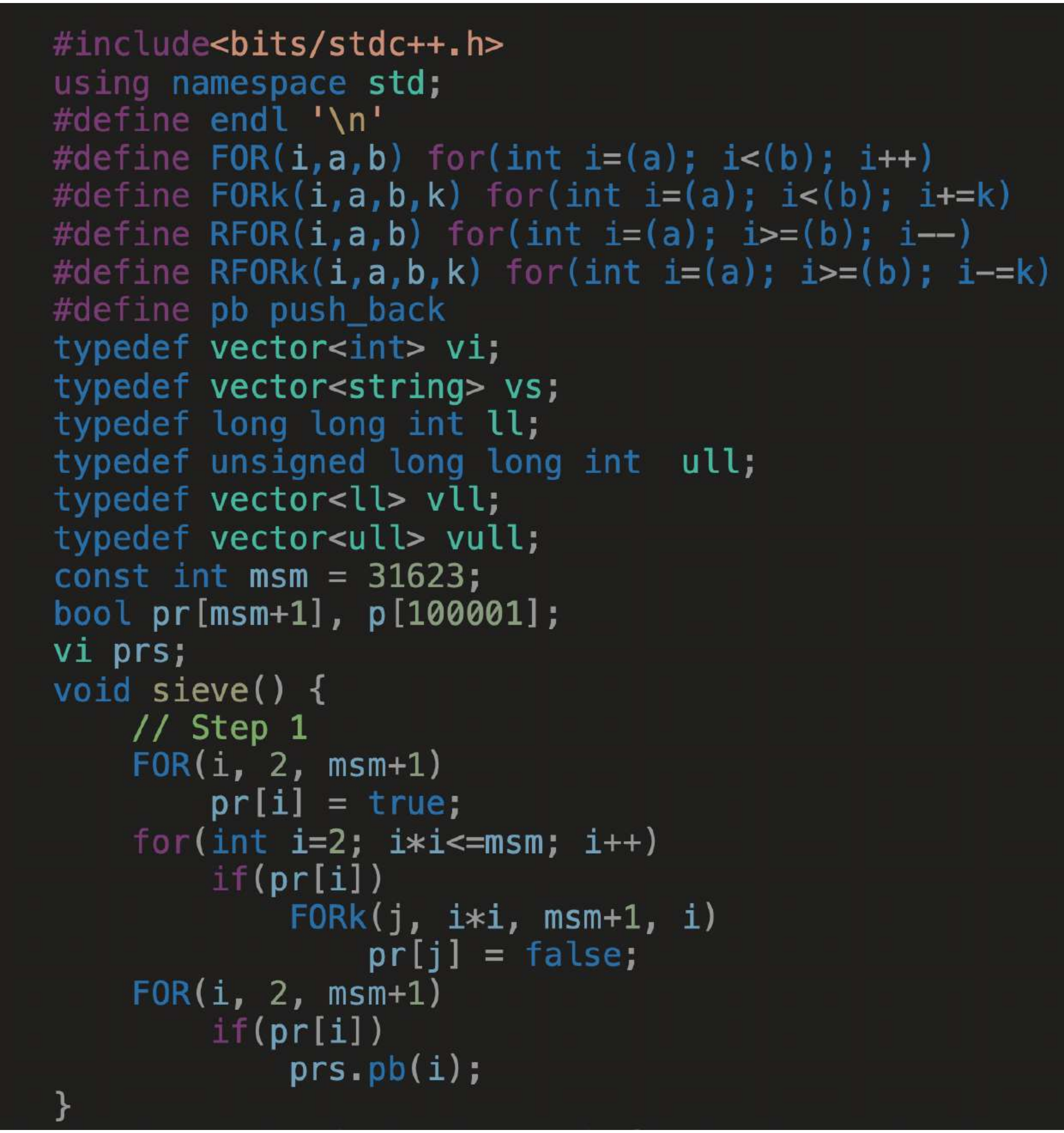
$\underline{\underline{p=7}} \qquad \frac{1000}{7} = \underline{\underline{142.857}}\underline{14\cdot}$

$\left\lceil \frac{1000}{7} \right\rceil \times 7 \qquad 143 \times 7 = \underline{\underline{1001}}$

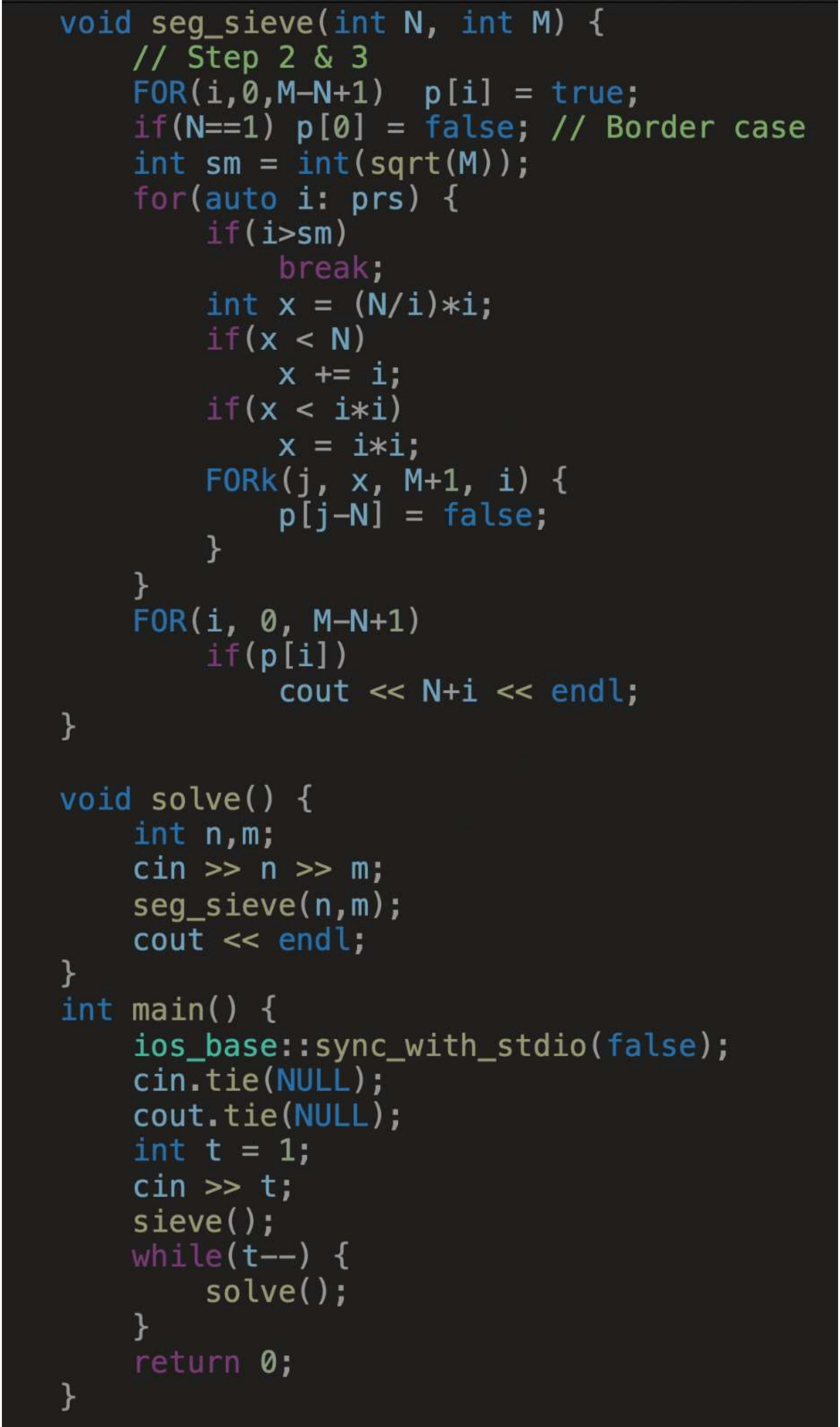
```
                floor
{ x = N/p * p
{     if (x < N)        }
          x += p;       }

{ if (x < p*p)
{       x = p*p;
```

$N = 1, \quad M = 100$


N = 1,   M = 100
3 → 3*3
p = 5
1 | 5 | 25 | 100
Augmented array
996
N
N+1
N+2
M
1000
1001
1002
j
1040
0
1
2
j−N
40
5
6
7
2, 3, 5, 7, ...
i = 7
7 * 7 = 49
996
(N/7) * 7
≤ N
integer division
x = N/i * i
if (x < N)
x += i
if (x < i*i)
x = i*i
x = max(i*i, x);



```
#include<bits/stdc++.h>
using namespace std;
#define endl '\n'
#define FOR(i,a,b) for(int i=(a); i<(b); i++)
#define FORk(i,a,b,k) for(int i=(a); i<(b); i+=k)
#define RFOR(i,a,b) for(int i=(a); i>=(b); i--)
#define RFORk(i,a,b,k) for(int i=(a); i>=(b); i-=k)
#define pb push_back
typedef vector<int> vi;
typedef vector<string> vs;
typedef long long int ll;
typedef unsigned long long int  ull;
typedef vector<ll> vll;
typedef vector<ull> vull;
const int msm = 31623;
bool pr[msm+1], p[100001];
vi prs;
void sieve() {
    // Step 1
    FOR(i, 2, msm+1)
        pr[i] = true;
    for(int i=2; i*i<=msm; i++)
        if(pr[i])
            FORk(j, i*i, msm+1, i)
                pr[j] = false;
    FOR(i, 2, msm+1)
        if(pr[i])
            prs.pb(i);
}
```



```
void seg_sieve(int N, int M) {
    // Step 2 & 3
    FOR(i,0,M-N+1)  p[i] = true;
    if(N==1) p[0] = false; // Border case
    int sm = int(sqrt(M));
    for(auto i: prs) {
        if(i>sm)
            break;
        int x = (N/i)*i;
        if(x < N)
            x += i;
        if(x < i*i)
            x = i*i;
        FORk(j, x, M+1, i) {
            p[j-N] = false;
        }
    }
    FOR(i, 0, M-N+1)
        if(p[i])
            cout << N+i << endl;
}

void solve() {
    int n,m;
    cin >> n >> m;
    seg_sieve(n,m);
    cout << endl;
}
int main() {
    ios_base::sync_with_stdio(false);
    cin.tie(NULL);
    cout.tie(NULL);
    int t = 1;
    cin >> t;
    sieve();
    while(t--) {
        solve();
    }
    return 0;
}
```