Day 06

Base rates, Conditional probabilities

- Prosecutor's fallacy
- Base rate fallacy
- Conditional probability – definitions & properties
- Posing the right conditional statement
- Revisiting hypothesis testing
- Why is FPR confused with FDR?

Prosecutor's fallacy

Prosecutor: We have a perfect match between the DNA of your client and the DNA on the murder weapon.

You: But surely the match could have occurred by chance?

Prosecutor: No. There is a one-in-a-million chance that a random person's DNA would match the sample from the weapon. Clearly he is guilty beyond a reasonable doubt.

You: Hold on a minute! Let's make a little table considering that there are ~2 mil people in Central MI...

	Match	No match
Guilty	1	0
Innocent	1E–6 * 2 mil	2 mil

You: What you have is P(Match | Innocent) but what we want to know is P(Guilty | Match), which is equal to 33%!

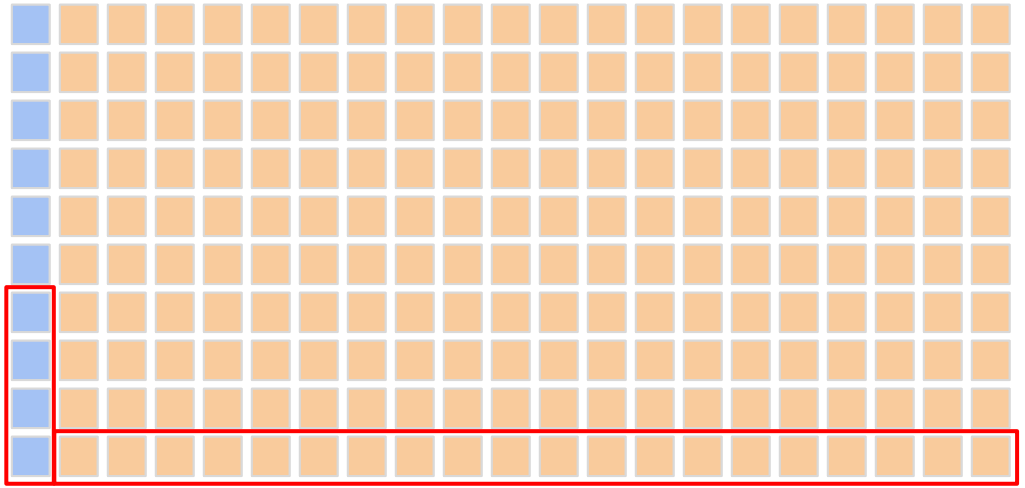
Base rate fallacy

Jordan is very shy and withdrawn, invariably helpful but with very little interest in people or in the world of reality. A meek and tidy soul, he has a need for order and structure, and a passion for detail.

Which of the following do you feel more likely?

1. Jordan is a librarian.
2. Jordan is a farmer.

Base rate of **Librarians : Farmers** :: **1 : 20**



40 %

10 %

A librarian is 4 times more likely as a farmer to fit the description!

There are way more farmers than librarians!

Kahneman & Tversky

Base rate fallacy

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New evidence does not completely *determine* your believes in a vacuum.

It should *update* prior believes.

Kahneman & Tversky

Probability and Conditional probability

$P(H)$: Probability of H

$P(H, E)$: Probability of H **and** E happening
(Probability of H and E being true)

$P(E \mid H)$: Probability of E **given** H has happened
(Probability of E if H is true)

$$= \frac{P(H, E)}{P(H)} \qquad \Rightarrow \quad P(H, E) = P(H) * P(E \mid H)$$

$$\neq \quad P(H \mid E)$$

Bayes theorem

$$P(H \mid E) * P(E) = P(H, E) = P(H) * P(E \mid H)$$

$$P(H \mid E) = \frac{P(E \mid H) * P(H)}{P(E)}$$

$$P(H \mid E) = \frac{P(E \mid H) * P(H)}{P(E \mid H) * P(H) \; + \; P(E \mid \neg H) * P(\neg H)}$$

Bayes theorem – the basis of scientific discovery

$P(H)$	Probability a hypothesis is true (before *any* evidence)
$P(E \mid H)$	Probability of seeing the evidence *if* the hypothesis is true
$P(E)$	Probability of seeing the evidence
$P(H \mid E)$	Probability a hypothesis is true given some evidence

$$P(H \mid E) = \frac{P(E \mid H) * P(H)}{P(E)}$$

Evidence should not *determine* believes. It should *update* them.

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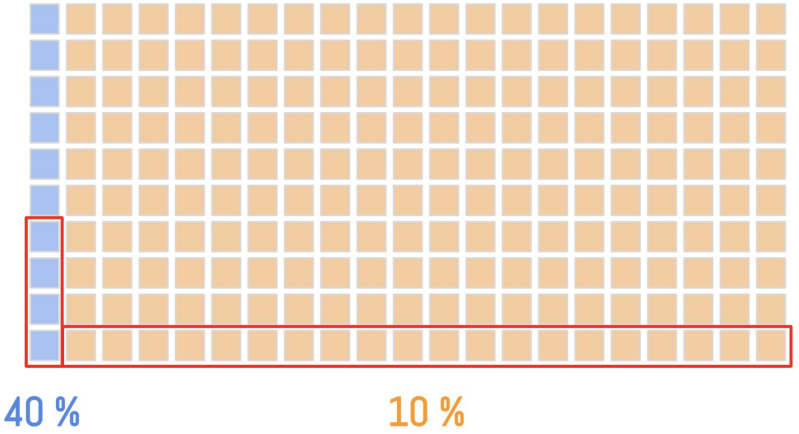
Base rate of **Librarians : Farmers** :: **1 : 20**



Evidence should not *determine* believes. It should *update* them.

Base rate fallacy

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student she was deeply concerned with the issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which of the following do you feel more likely?

1. Linda is a bank teller.
2. Linda is a bank teller and is active in the feminist movement.

Kahneman & Tversky

Base rate fallacy

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student she was deeply concerned with the issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

There are 100 people who fit this description.
How many are:

1. Bank tellers? _____ of 100
2. Bank teller and is active in the feminist movement? _____ of 100

Kahneman & Tversky

Interpreting diagnostic tests

A diagnostic test for a particular disease is 99% reliable:

- 99% of people who are sick $\longrightarrow$ test positive
- 99% of people who are healthy $\longrightarrow$ test negative

If a patient tests positive, what is the probability that the patient is sick?

Interpreting diagnostic tests

0.8% of women who get mammograms have breast cancer.

90% of women with breast cancer → mammogram result is positive.

7% of women without breast cancer → mammogram result is positive.

If a woman gets a positive mammogram result, what are the chances that the woman has breast cancer?

COVID

Although less than 10% of the adult population are unvaccinated, they account for 50% of Covid patients in hospital, intensive care.

COVID

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Peston: Covid infections higher in doubled vaxxed aged 40-79 than for non-vaxxed | ITV News

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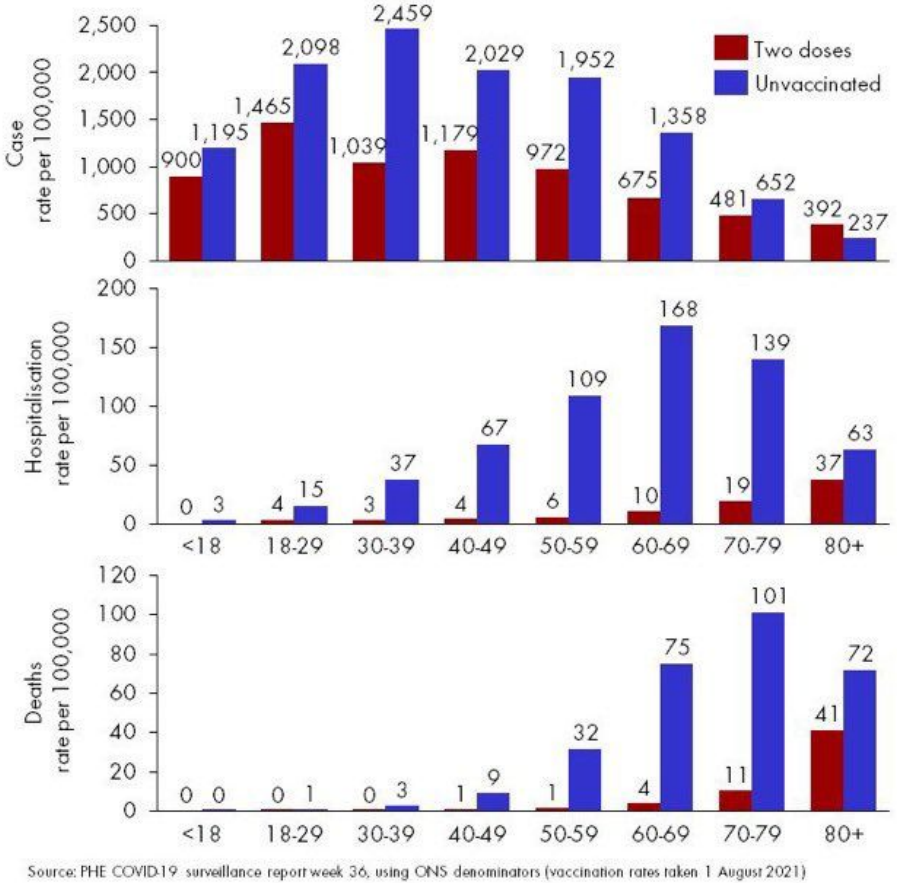


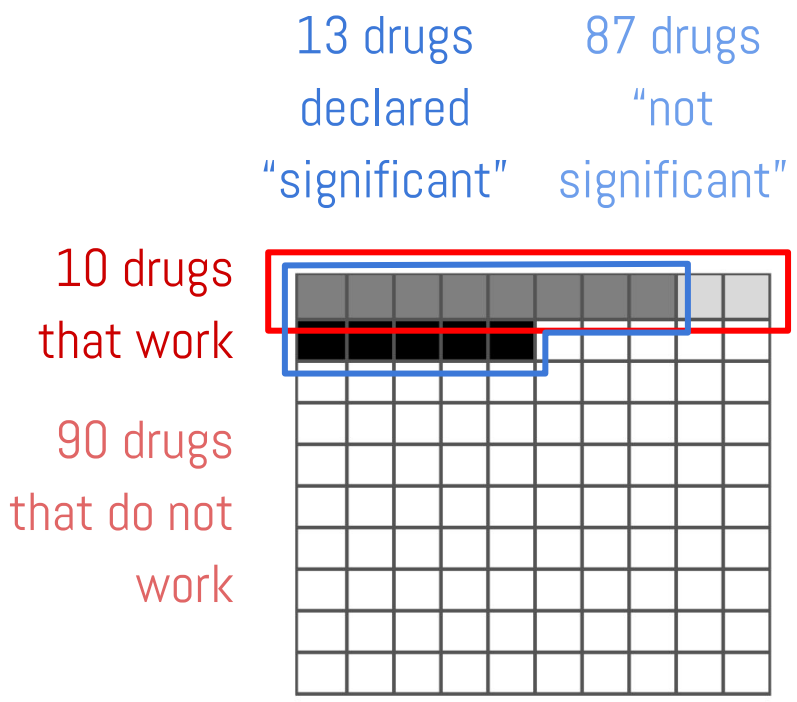
Source: PHE COVID-19 surveillance report week 36, using ONS denominators (vaccination rates taken 1 August 2021)

Interpreting hypothesis testing

100 potential drugs; 10 of them actually work.

- Statistical test for each.
  P-value < 0.05; Assume power = 0.8.

- What do you think about the following?
  - "There is a 1 in 20 chance that the drugs picked by the trial are ineffective."



Interpreting hypothesis testing

		Evidence-based hypothesis test		
		Reject null (significant)	Accept null (not-significant)	*Total*
Ground truth	Null is false (real effect)	**TP**	**FN**	P
	Null is true (no effect)	**FP**	**TN**	N
	Total	Num. significant results	Num. non-significant results	P + N = all results

What is the **False Positive Rate** (FPR) and the **False Discovery Rate** (FDR)?

What are we interested from the conclusions of a study?

		Statistical hypothesis test: Reject null (significant)	Statistical hypothesis test: Accept null (not-significant)	Total
Ground truth	Null is false (real effect)	**TP**	**FN**	P
	Null is true (no effect)	**FP**	**TN**	N
Total		Num. significant results	Num. non-significant results	P + N = all results

FPR = P(significant | no effect)

FDR = P(no effect | significant)

"Evidence should not *determine* believes. It should *update* them."

- When we read the conclusions from a paper, FDR is what we are interested.
- But this is not reported because we typically don't know what numbers to put in the top row (H0 is false). For this, we need to know how likely H1 is to be true before we do the experiment.
  - For crazy hypotheses, most numbers will be in the bottom row.
  - For more likely hypotheses, equal representation in both rows.
- So, what we believe actually turns out to depend on what we thought was true at the start.
- In science, all the time, we communicate what the prosecutor was trying to trick you with not what the defense attorney was correctly presenting.