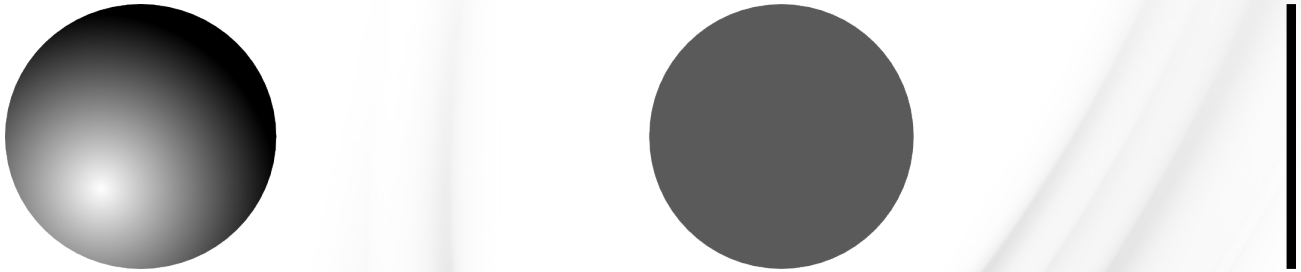


Dimensionality reduction and coordinated movement control

How nature alleviates the "*Curse of Dimensionality*"

Naveen Kuppuswamy

Lecture at the Hamburg University,
Department of Computer Science

ai lab

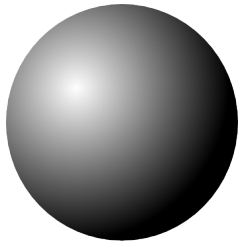
University of Zurich[UZH]

Department of Informatics

“Less is More”

- Ludwig Mies van der Rohe, Architect
(1886 – 1969)

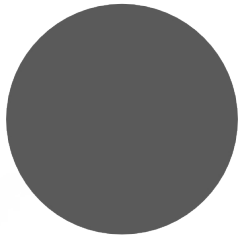
Outline



About Me

I. Introduction and Background

- Dimensionality and movement control : the boon and the curse
- Robot control / machine learning and dimensionality
- Motor neuroscience and dimensionality

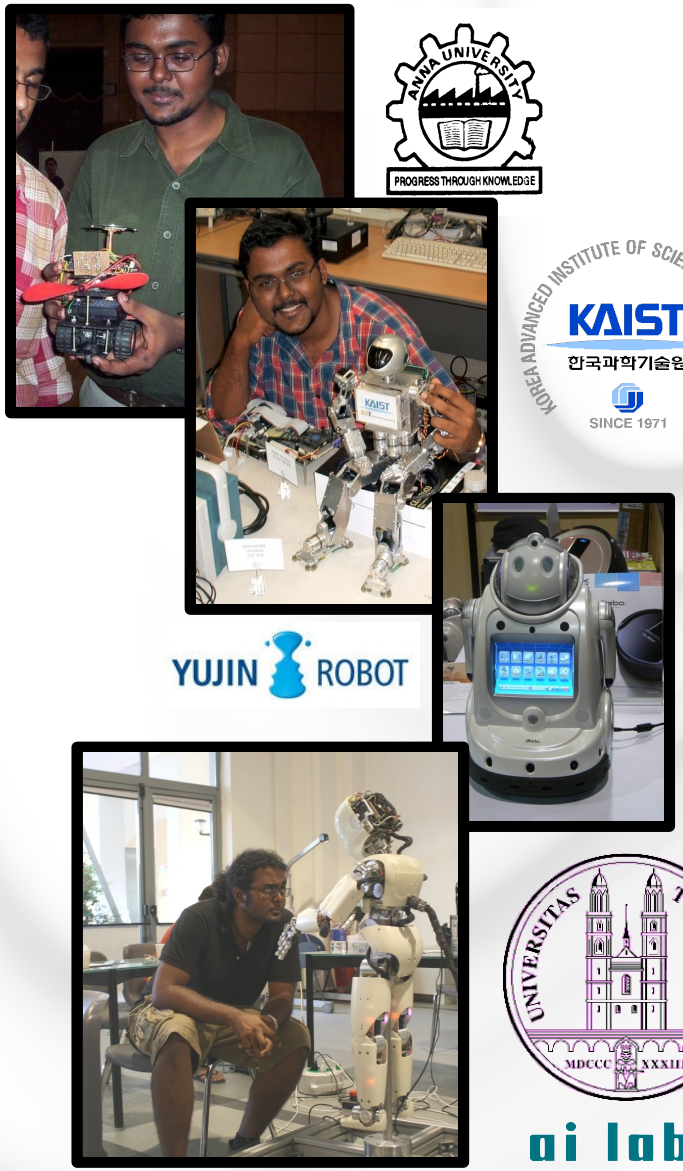


II. Alleviating the Curse of Dimensionality

- Neuroscientfic evidence
- Mathematical framework and Analysis
- Principle of reduced dimensionality

III. Concluding remarks

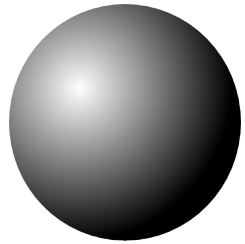
About Me



- B.E. in Instrumentation and Control, Anna University, Chennai, India (2001-2005).
- M.S. in Electrical Engg. and Comp. Sci. KAIST, Daejeon, South Korea (2006-2007).
  - R.I.T Lab, Prof. Jong-Hwan Kim

    Master Thesis : "*Nonlinear Inverse Dynamic Control of an Omnidirectional Mobile Robot using Slip Rolling Modes*"
- Engineer, R&D, Yujin Robot Co. Ltd. (2008).
- Ph.D. in Artifical Intelligence, University of Zürich, Switzerland (2009-2013).
  - A.I Lab, Prof. Rolf Pfeifer

    PhD Thesis : "*Exploiting Reduced Dimensionality in the Design and Control of Embodied Systems*"



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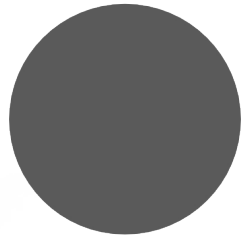

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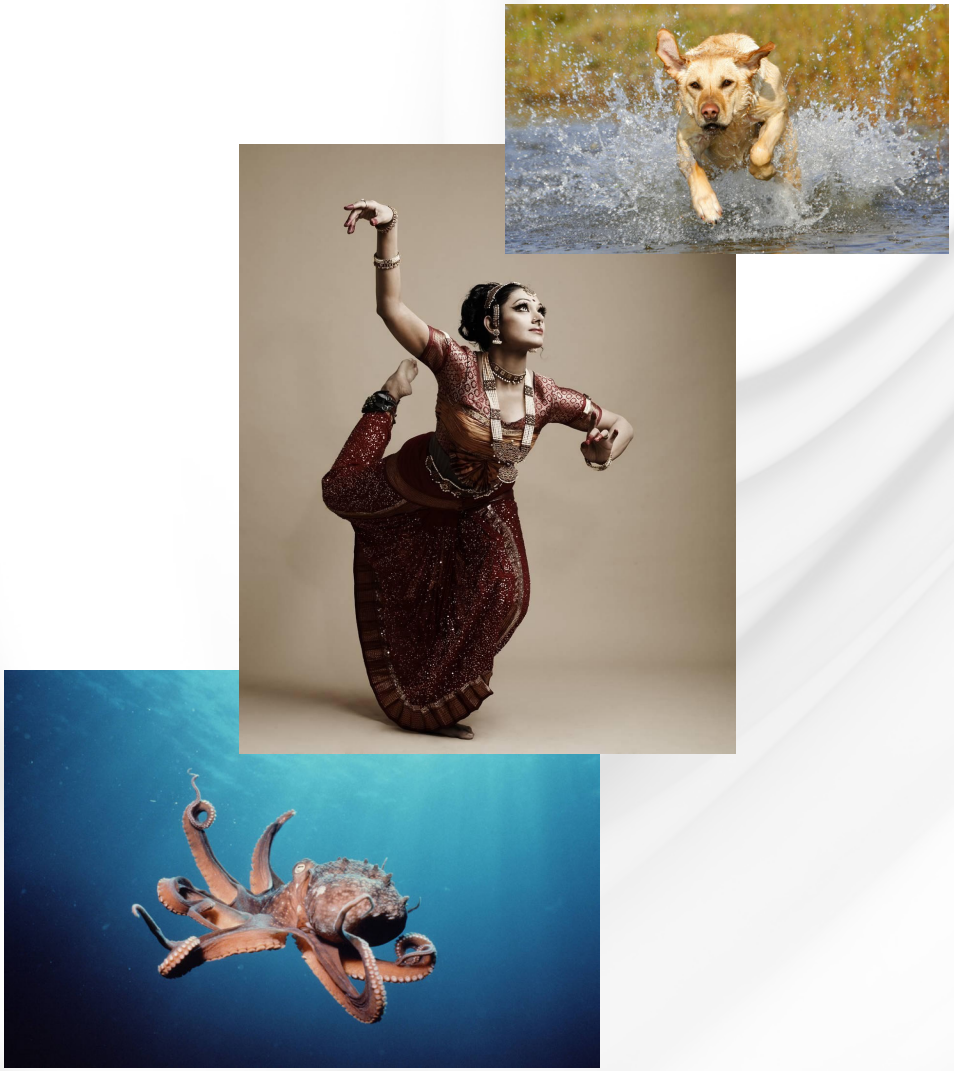
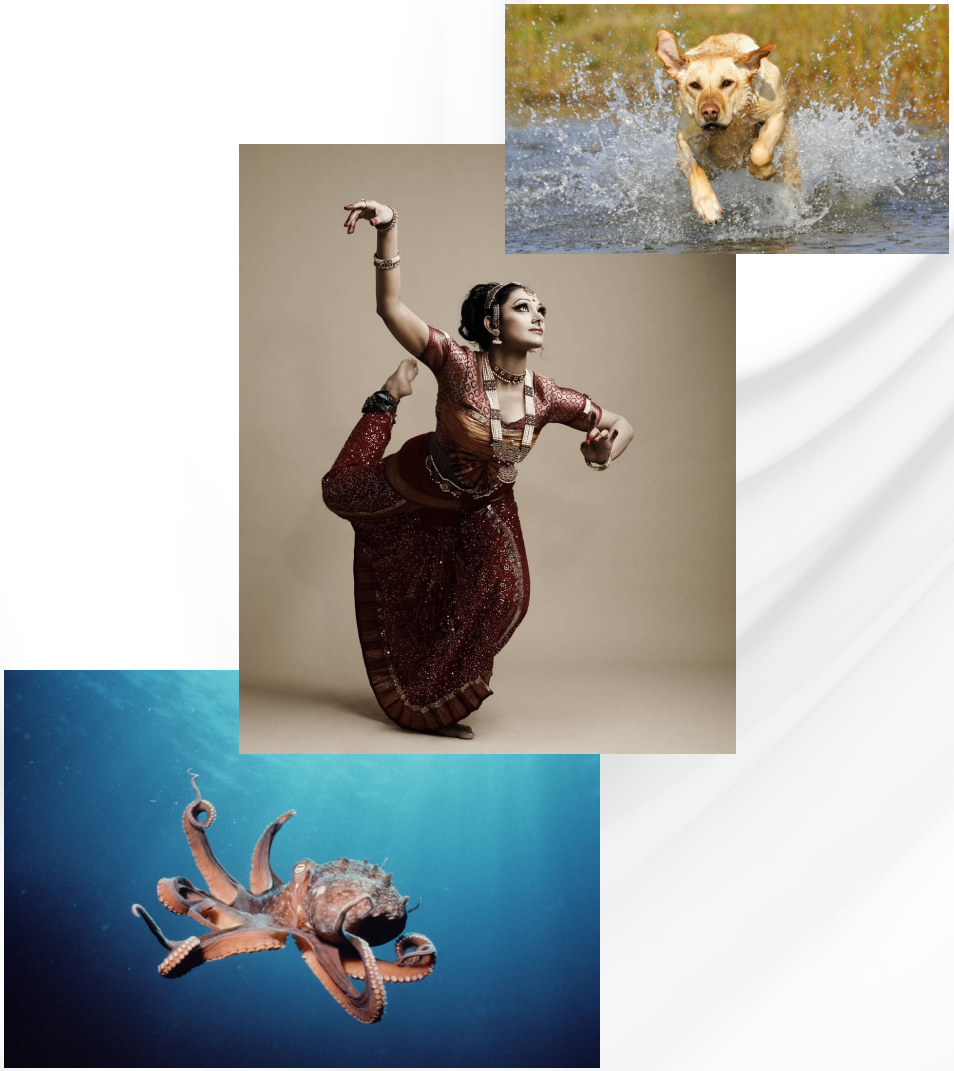


Natural Systems

Immense adaptability and diversity of behaviour

Introduction



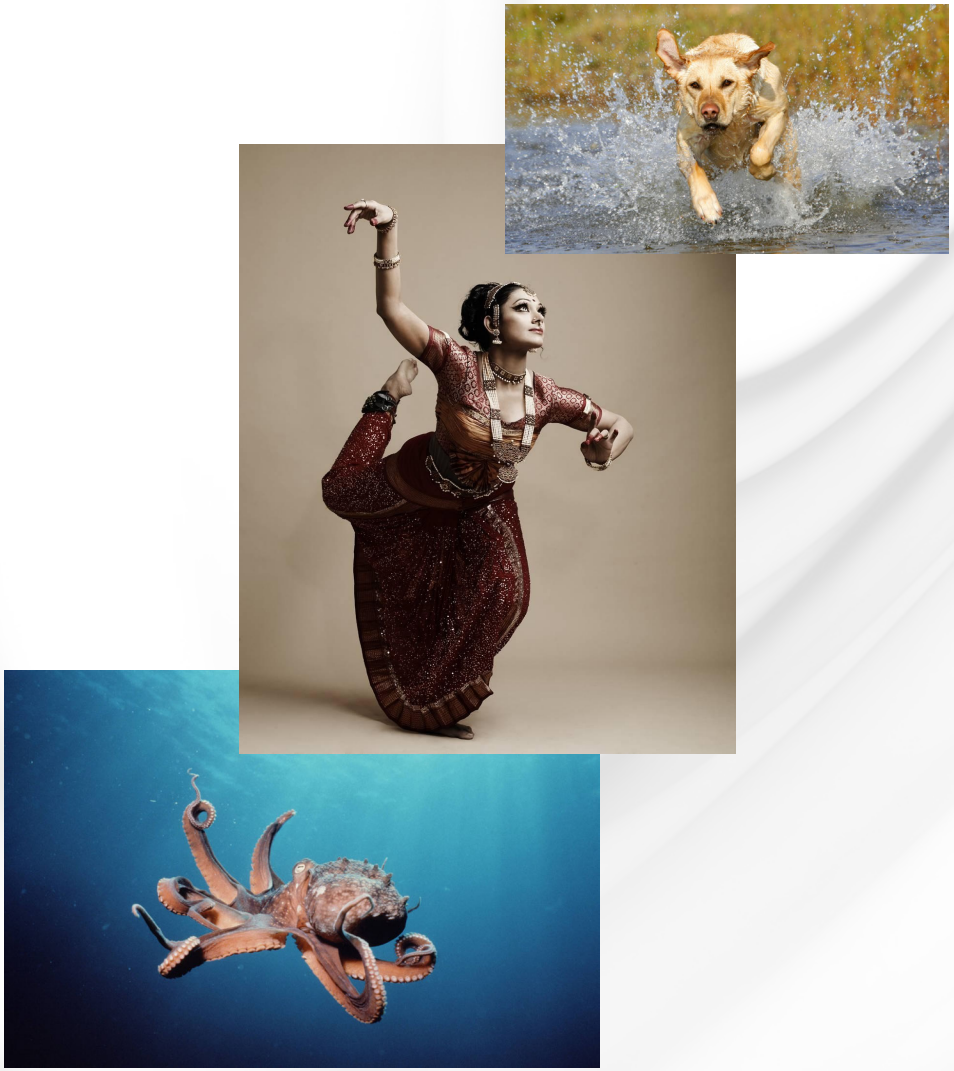
Natural Systems

Immense adaptability and diversity of behaviour in coping with the natural environment

Why?

Evolutionary advantages in coping with adverse conditions

Introduction



Natural Systems

Immense adaptability and diversity of behaviour in coping with the natural environment

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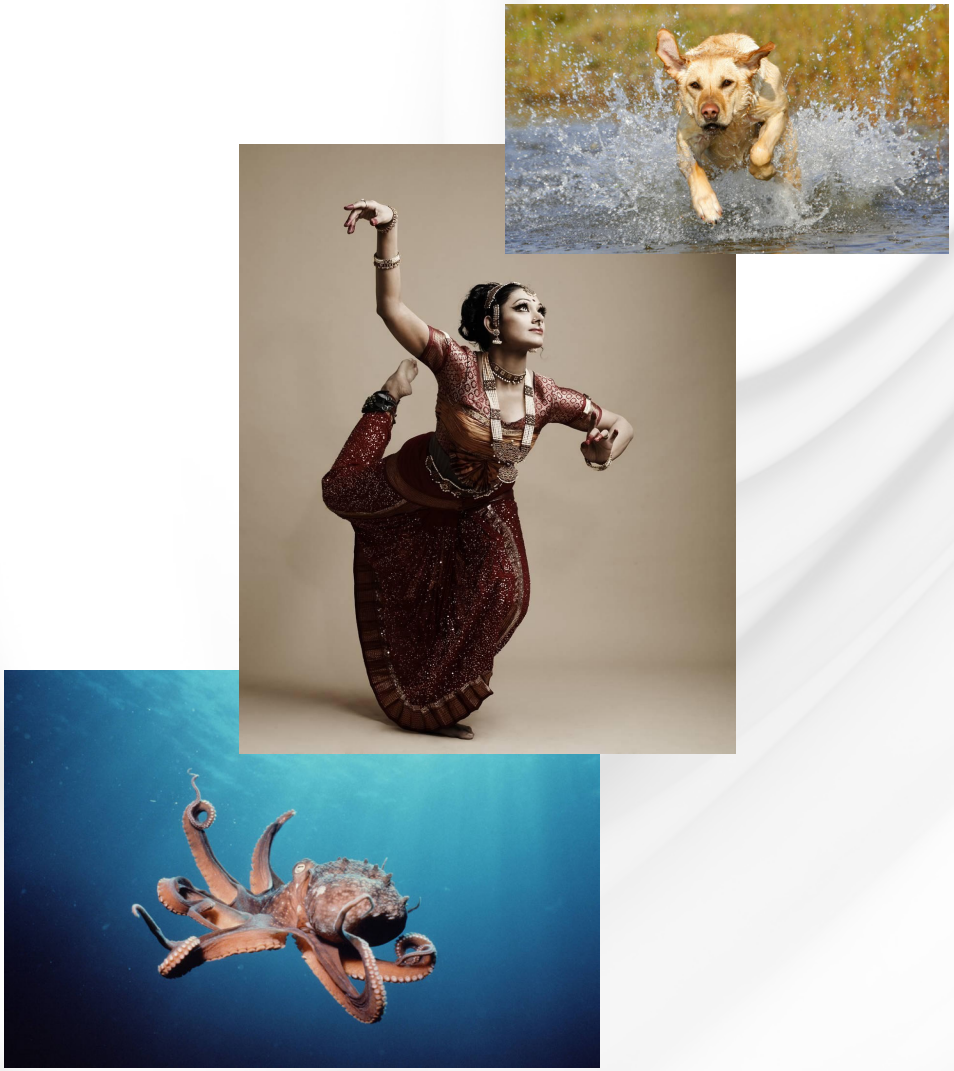
How?

Large neuro-mechanical dimensionality

Introduction



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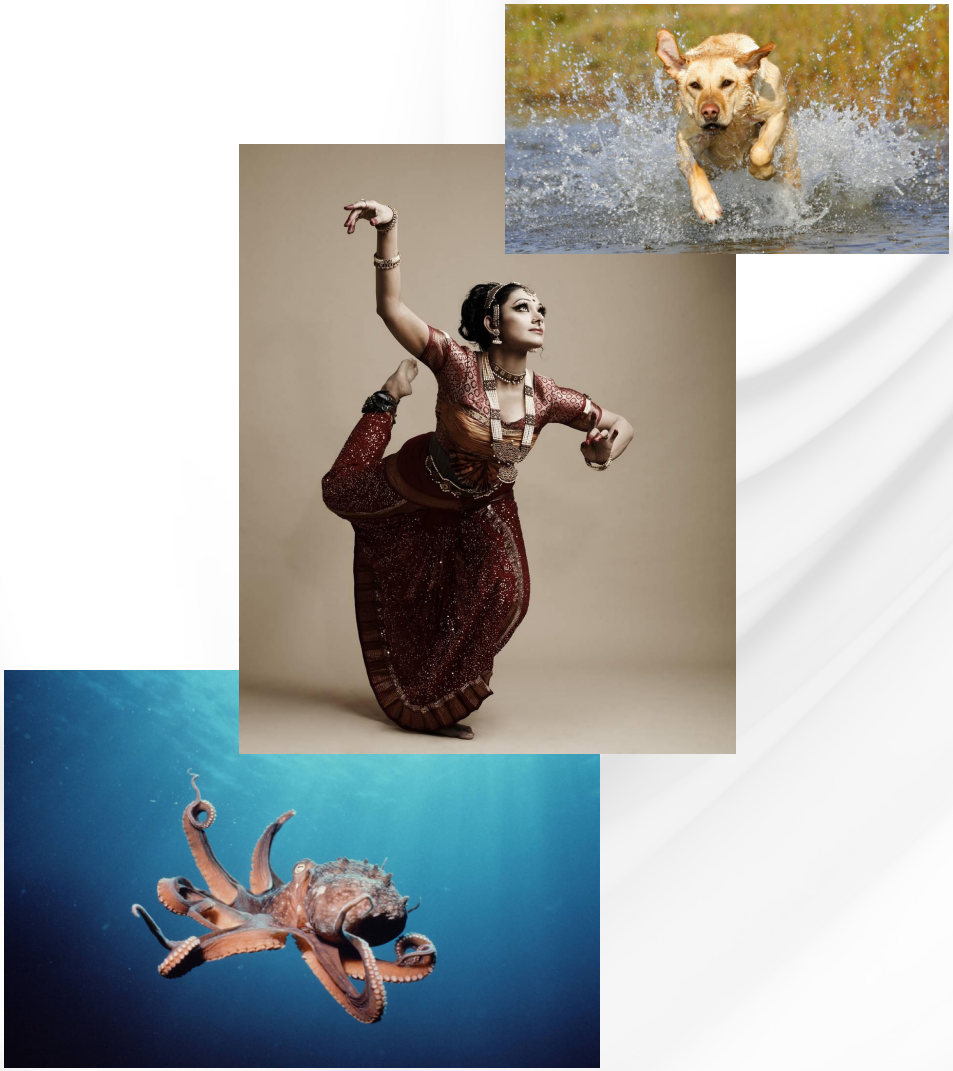
Large neuro-mechanical dimensionality

Why is dimensionality useful?



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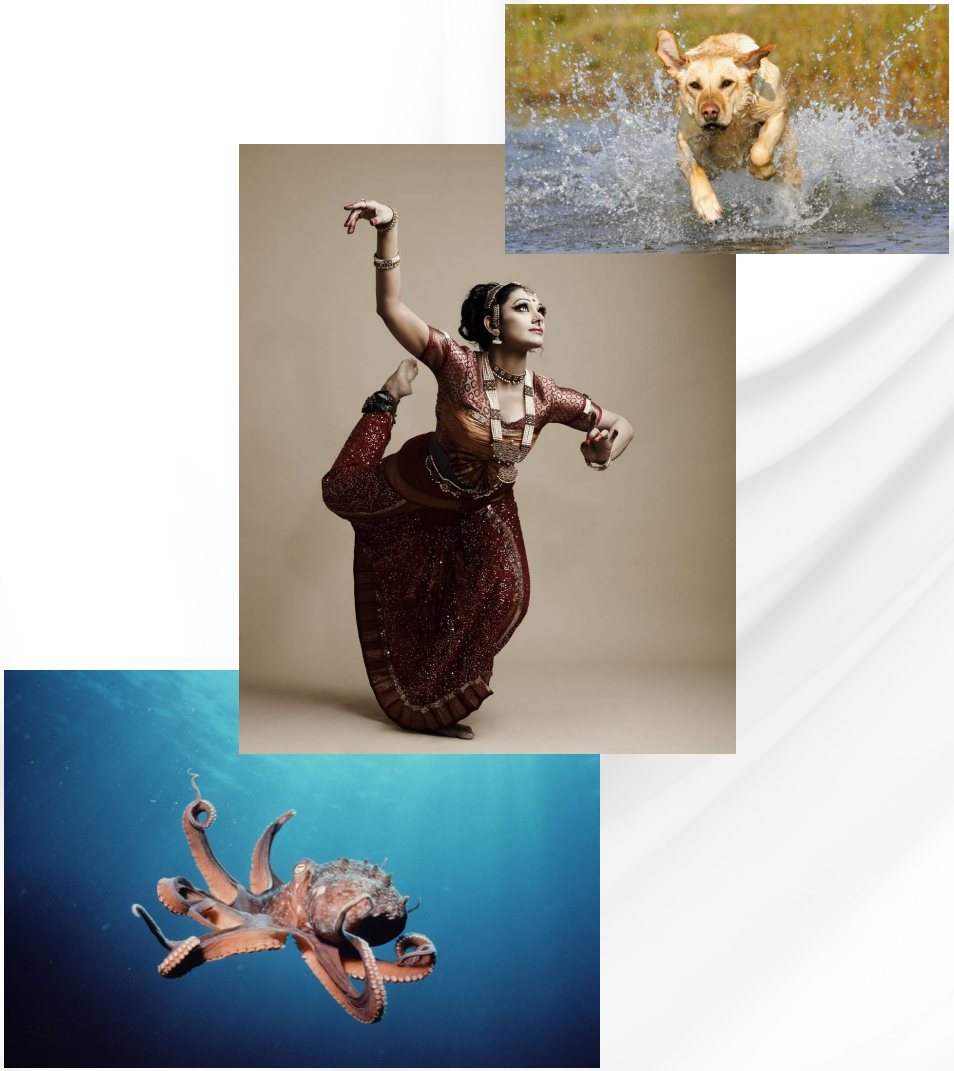
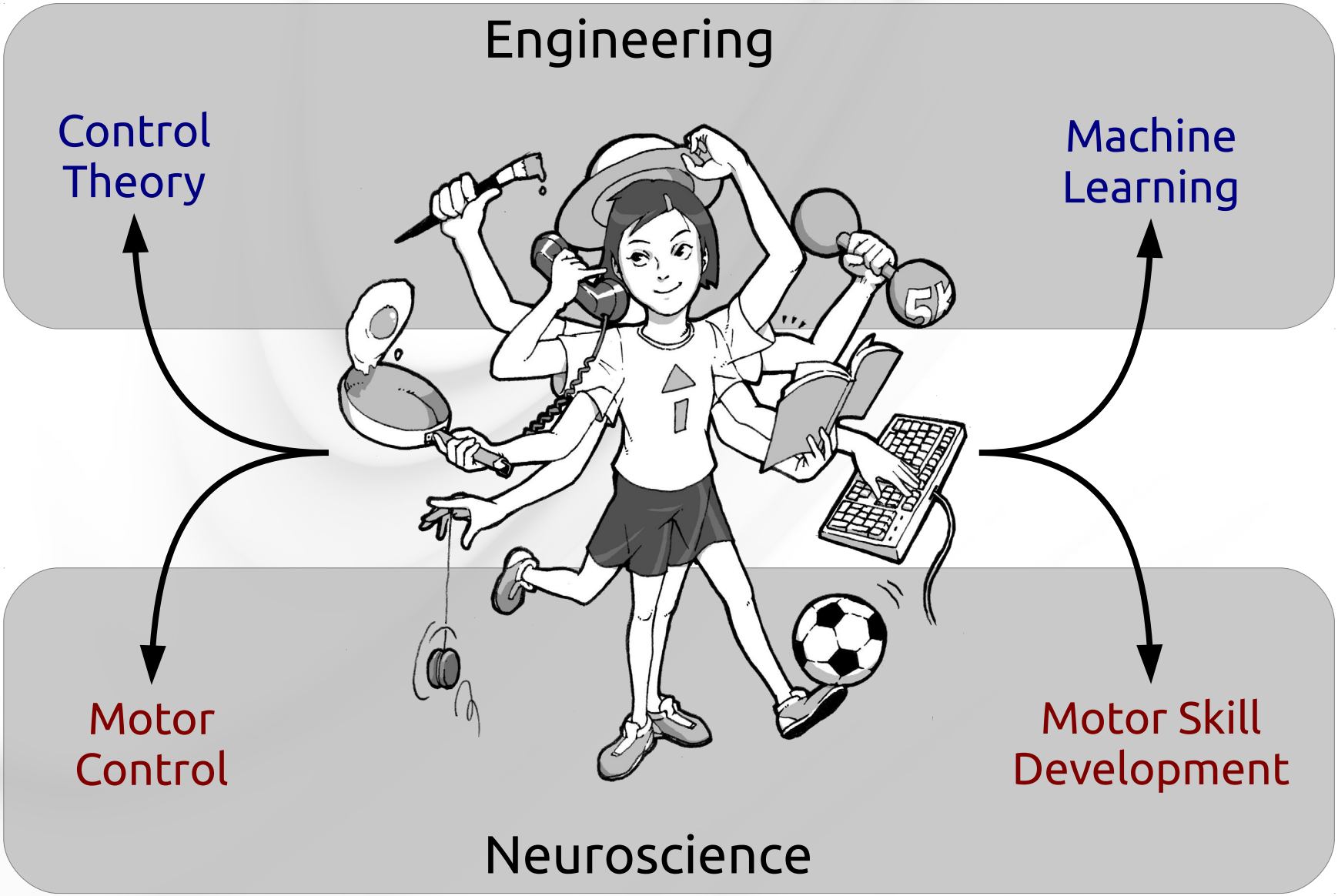


It gives options!

The dimensionality problem

How is the voluntary control of coordinated behaviour achieved despite this large dimensionality?

Perspectives on the dimensionality problem



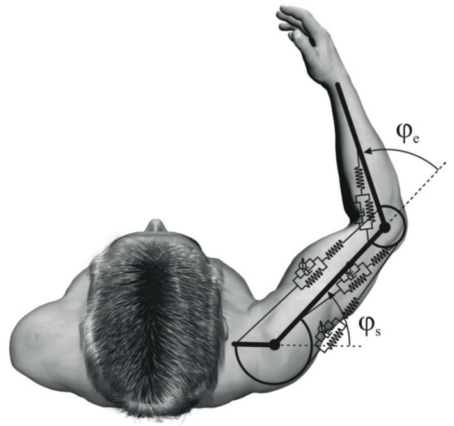
Why is large dimensionality a problem?

- Engineering viewpoint
  - Real-time Control
  - Optimisation and Machine Learning

"Curse of Dimensionality"
- Richard Bellman, *1961*

- Neuroscientific viewpoint
  - Redundancy Resolution
  - Development of motor skill

"Degree of Freedom Problem"
- Nikolai Bernstein, *1967*



The *Curse of Dimensionality*

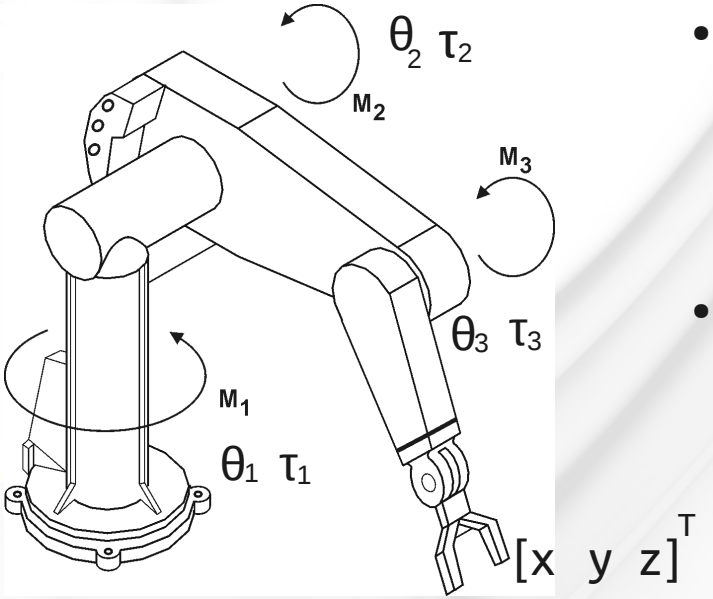
"*In view of all that we have said in the forgoing sections, the many obstacles we appear to have surmounted, what casts the pall over our victory celebration? It is* ***the curse of dimensionality****, a malediction that has plagued the scientist from the earliest days*".

Adaptive Control Processes : A Guided Tour(1961)

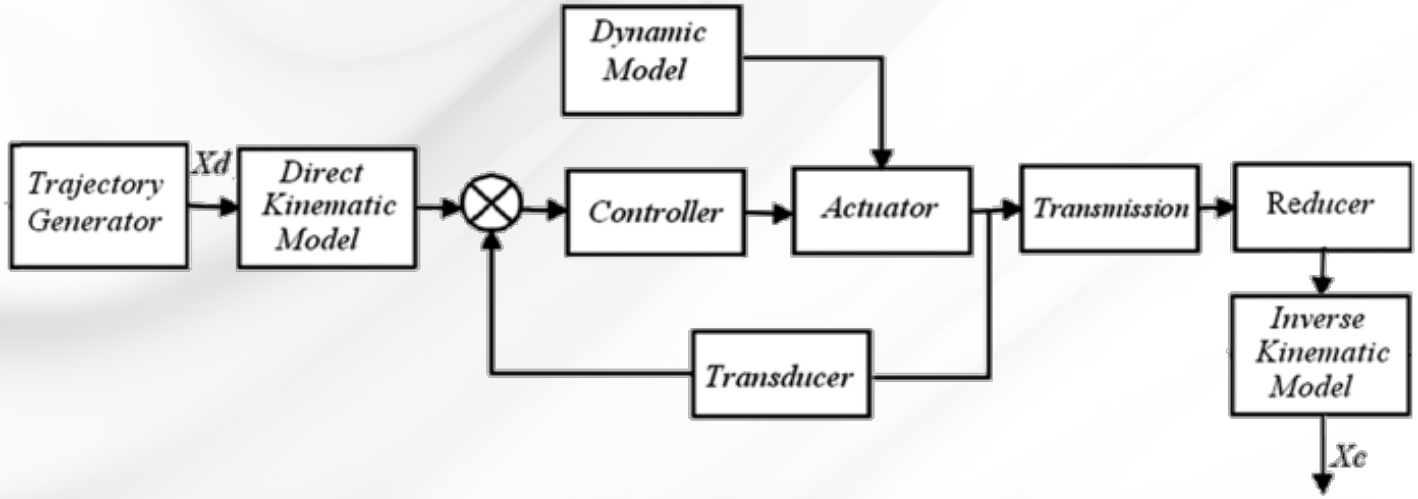
Bellman's observation : the intractability of optimizing a function of many variables by a brute force search on a discrete multidimensional grid. The number of grids points increases exponentially with dimensionality, i.e., with the number of variables.

Richard Ernest Bellman
(1896 – 1966)

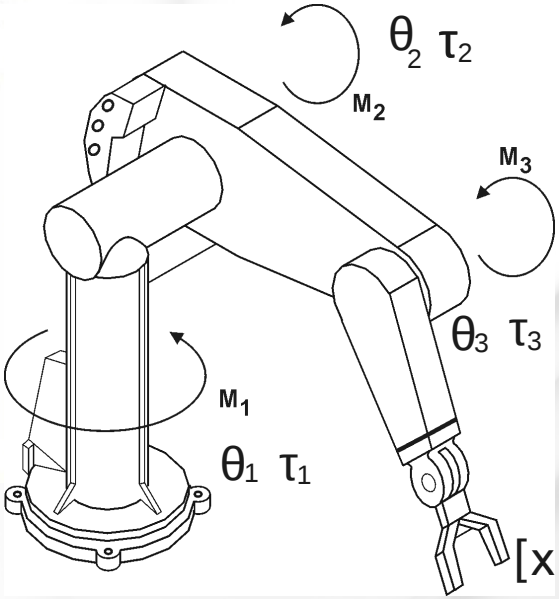
Engineering perspective : Robot Control



- Traditional (GOFAI) control techniques are designed for Rigid robots and use model based control
- Equations of motion are derived using Kinematics (positions and velocities) and Dynamics (accelerations, torques and forces)

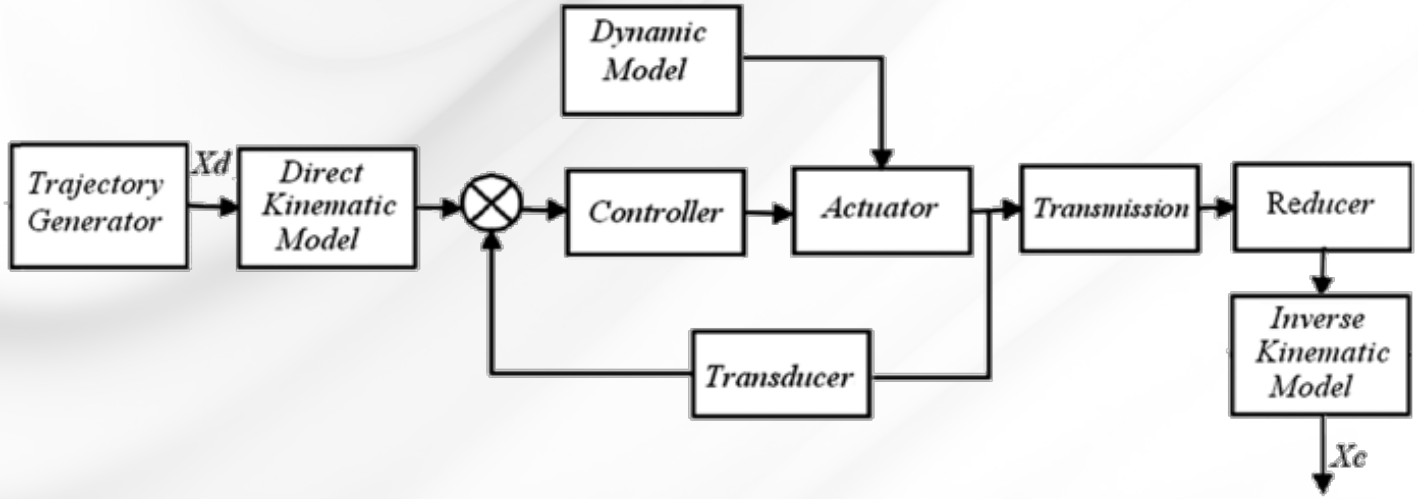


Engineering perspective : Robot Control

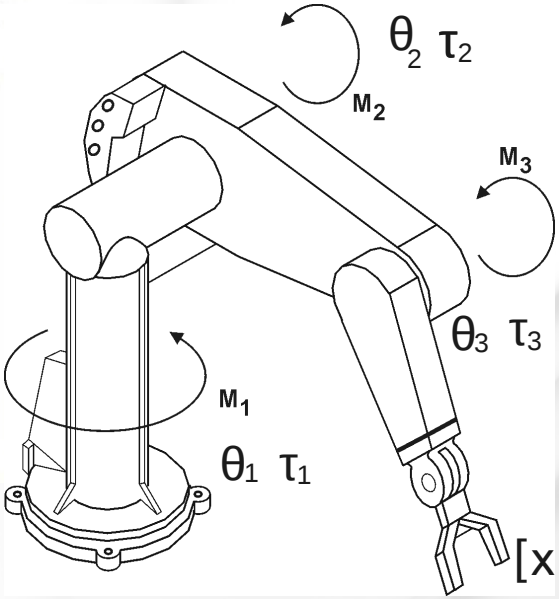


The difficulty of the problem depends on 2 important factors :

- Number of Degrees of Freedom (DoF).
- Complexity of the dynamics



Engineering perspective : Robot Control



The difficulty of the problem depends on 2 important factors :

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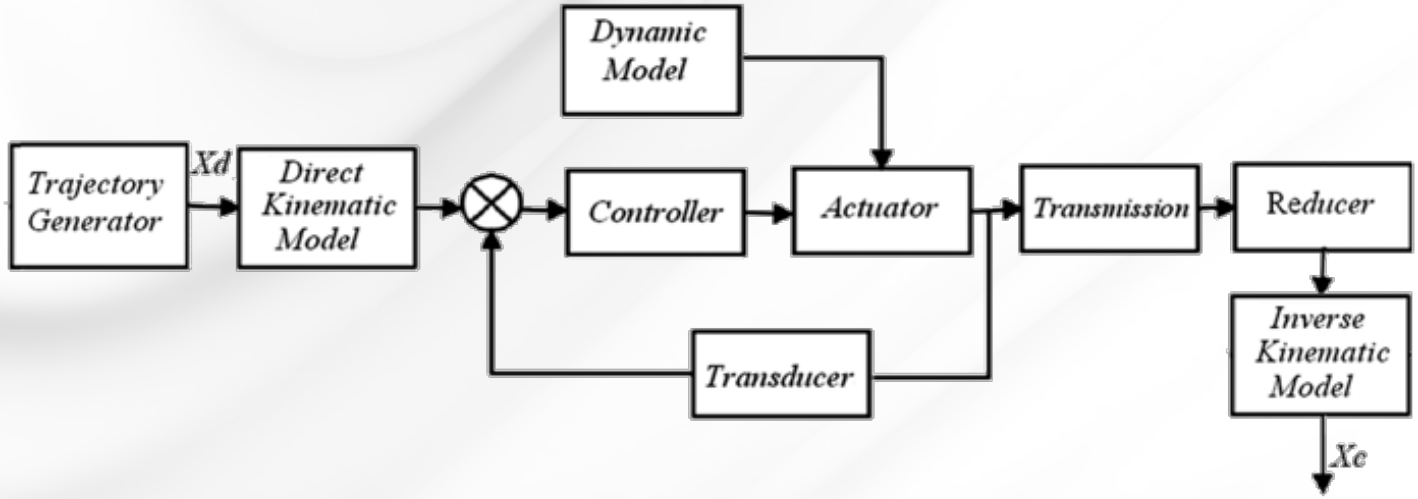
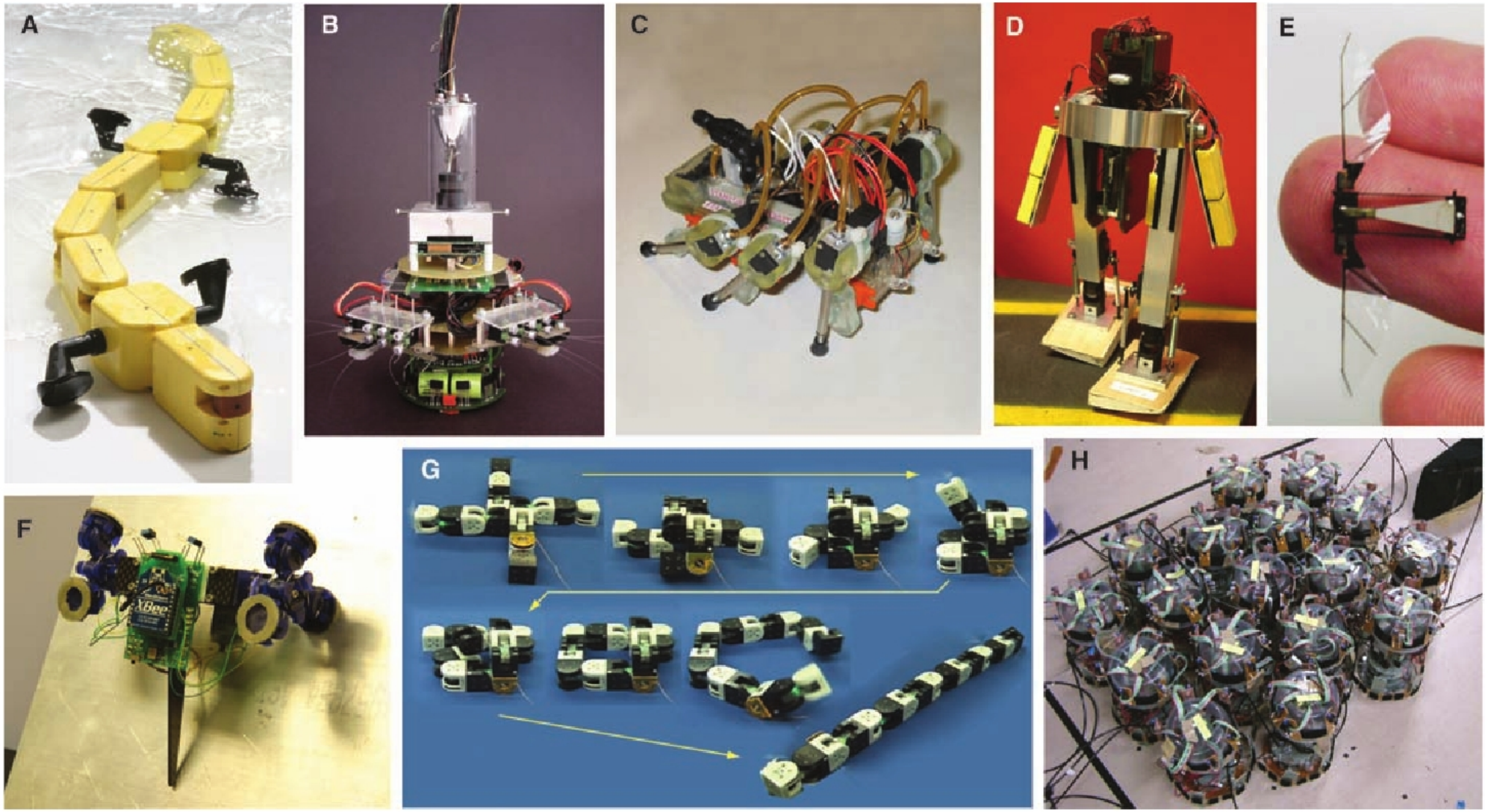
- Complexity of the dynamics

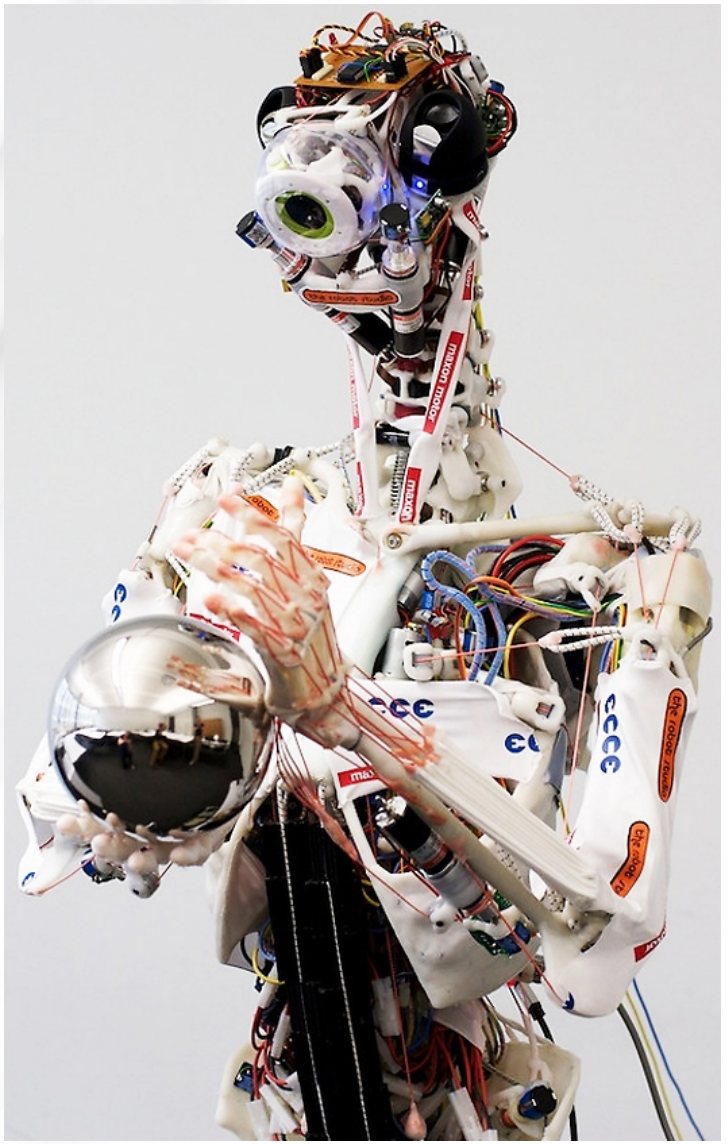
How scalable are these methods?



Biomimetic Robots

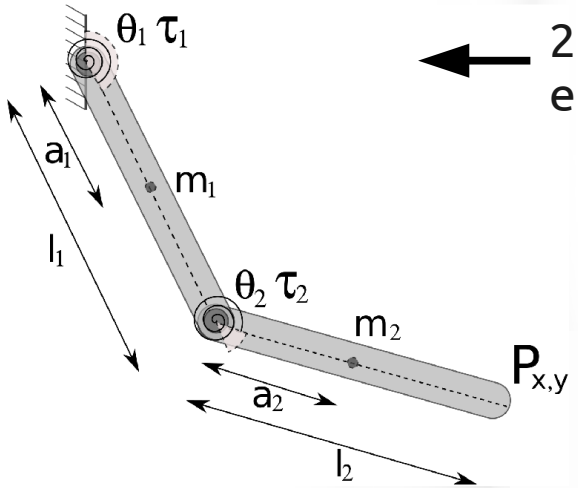


Biomimetic Robots



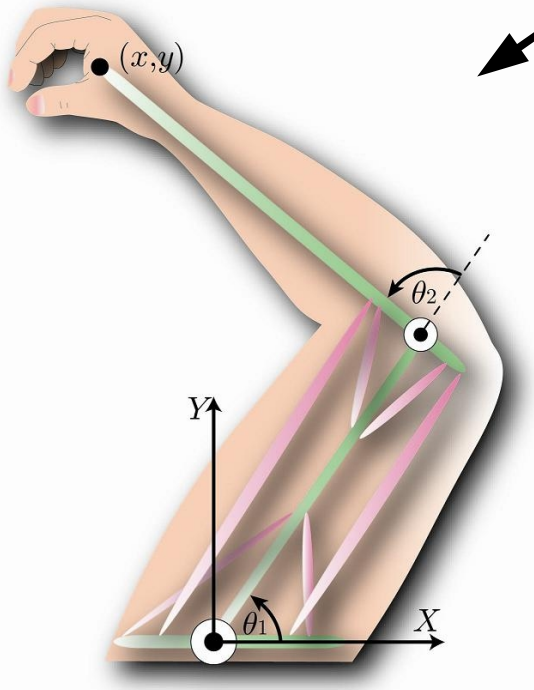
Methods do not scale up. WHY?

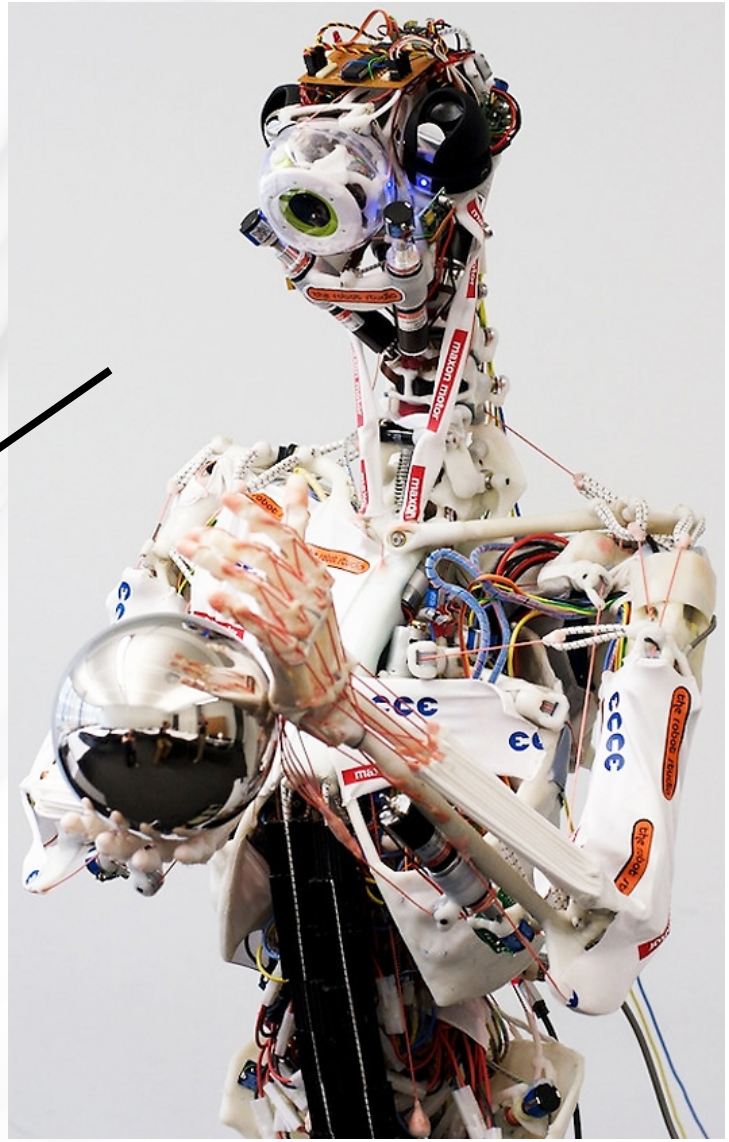
Biomimetic Robots



2 joints, 2 inputs,
endpoint : 2d

2 joints, 6+ inputs,
endpoint : 2d

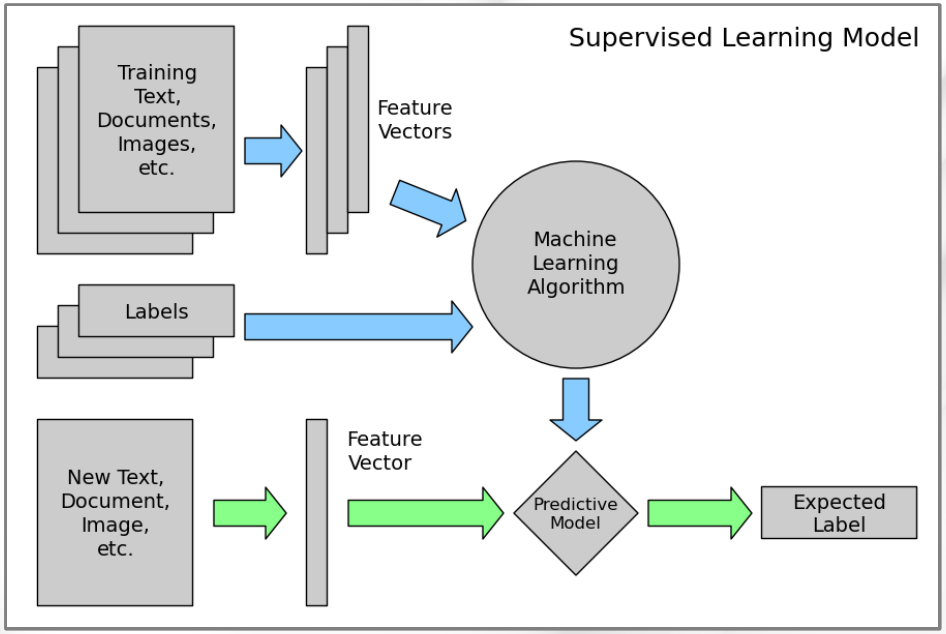


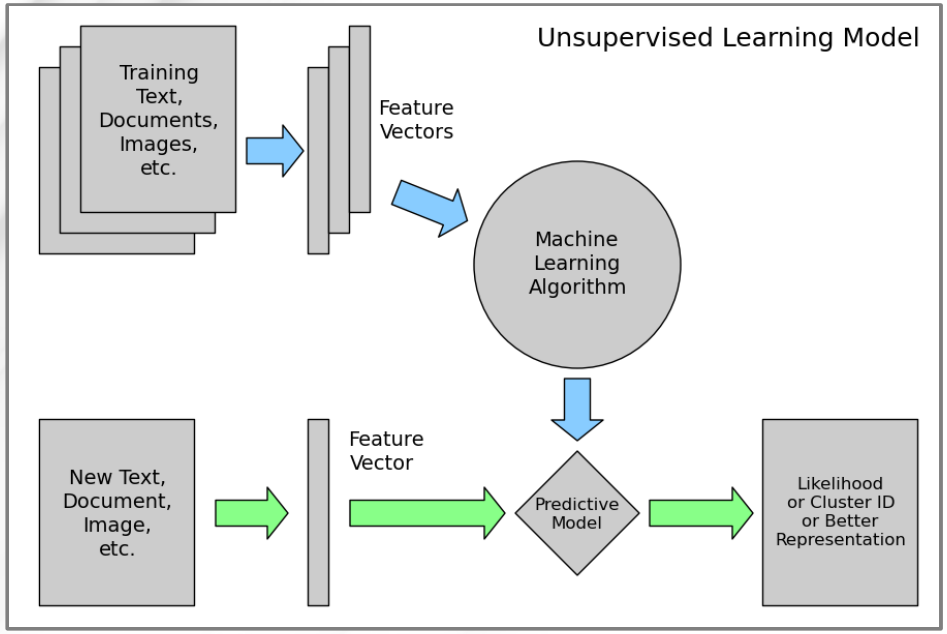


The dimensionality is clearly a source of complexity for real-time control

Engineering Perspective : Machine Learning

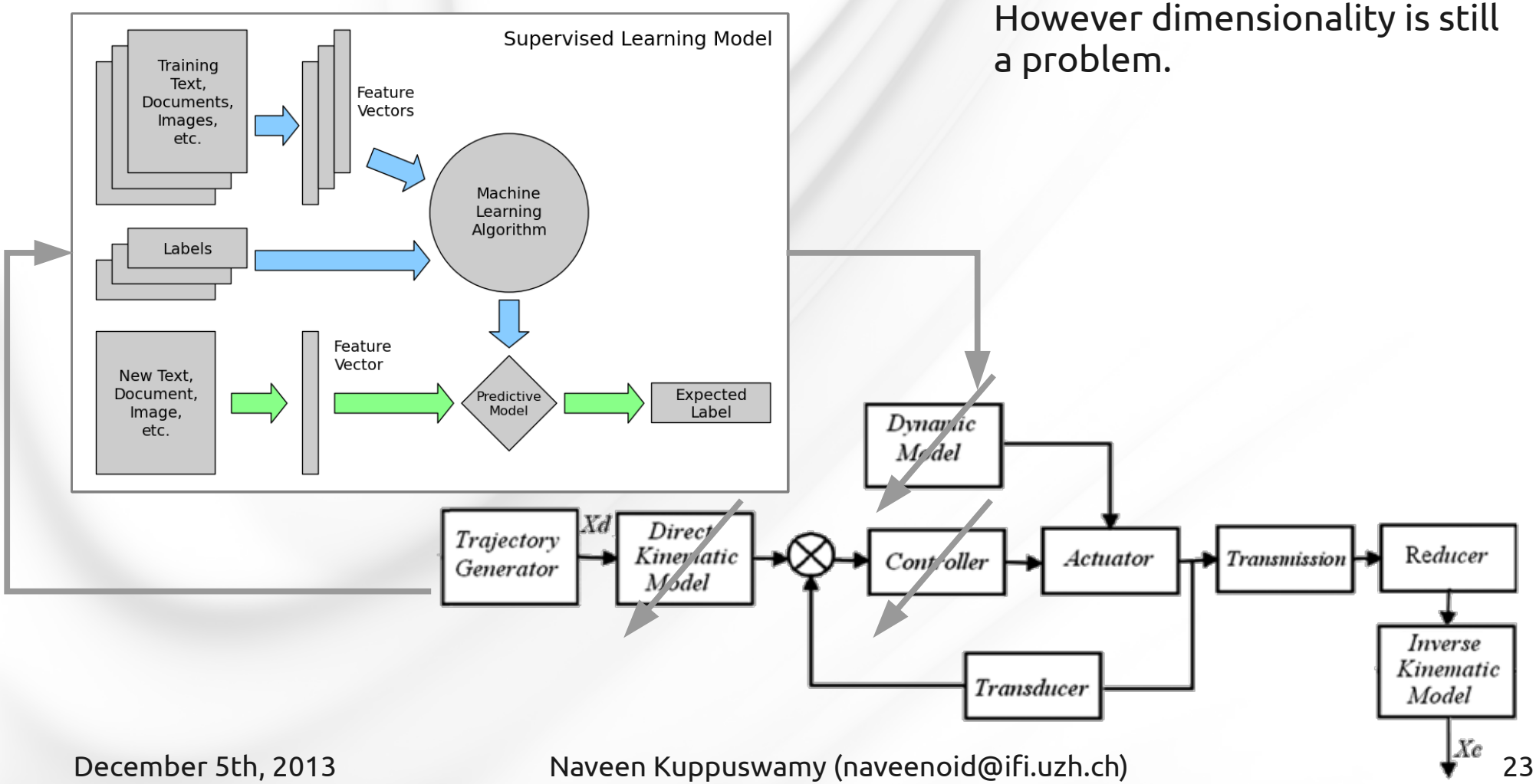
Machine learning methods aim to acquire meaningful information from data.
Two fundamental paradigms :





Engineering Perspective : Machine Learning

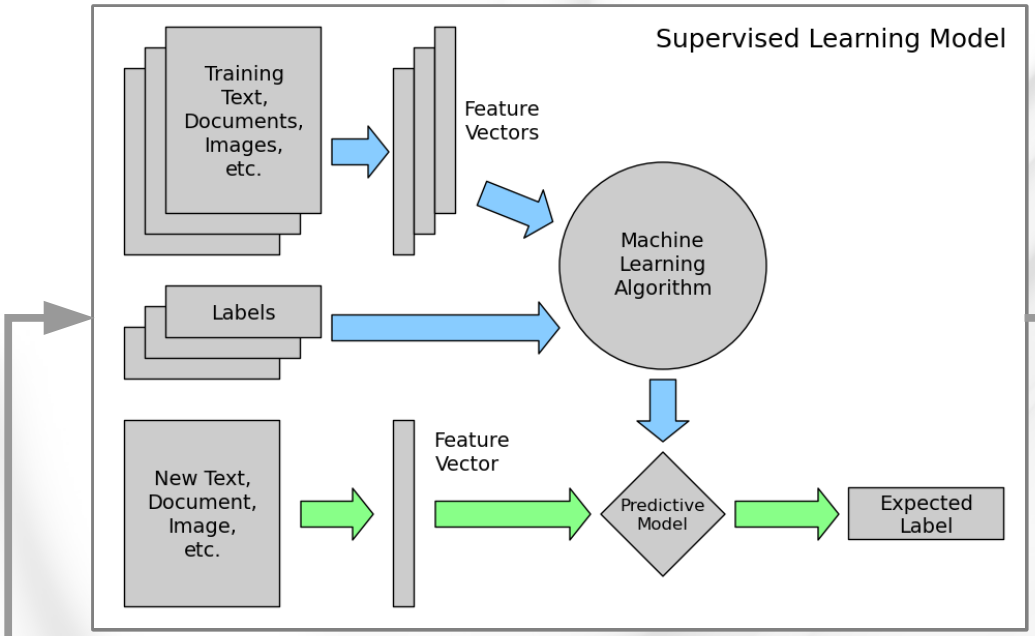
Machine learning methods can be used for learning control



However dimensionality is still a problem.

Engineering Perspective : Machine Learning

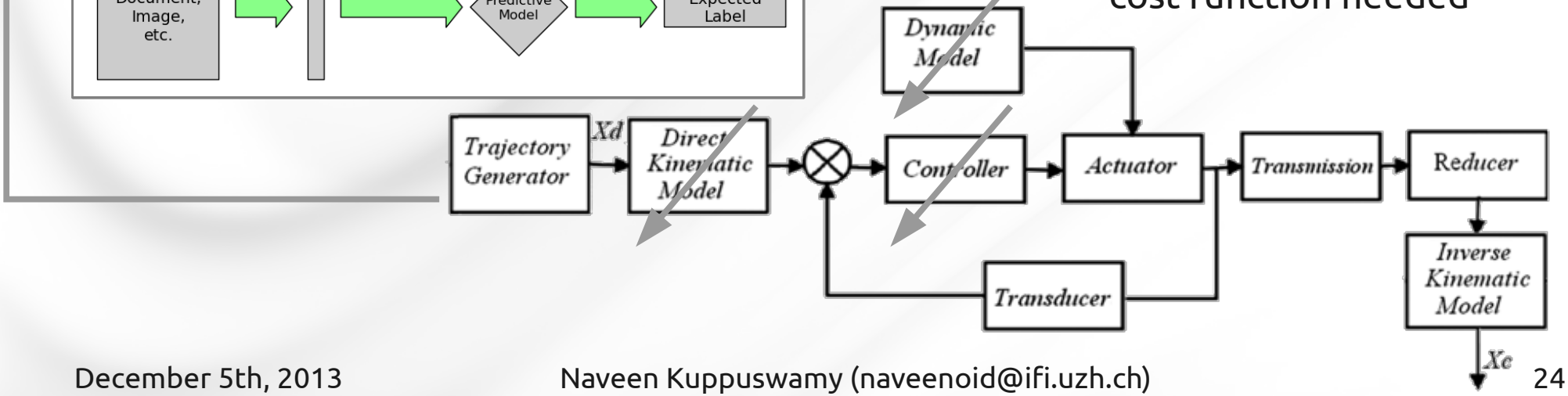
Machine learning methods can be used for learning control



However dimensionality is still a problem.

Some Effects :

- Sparsity of training data
- Insufficiency of training data
- Intractability of learning
- Number of computations of cost function needed



Engineering Perspective : Machine Learning

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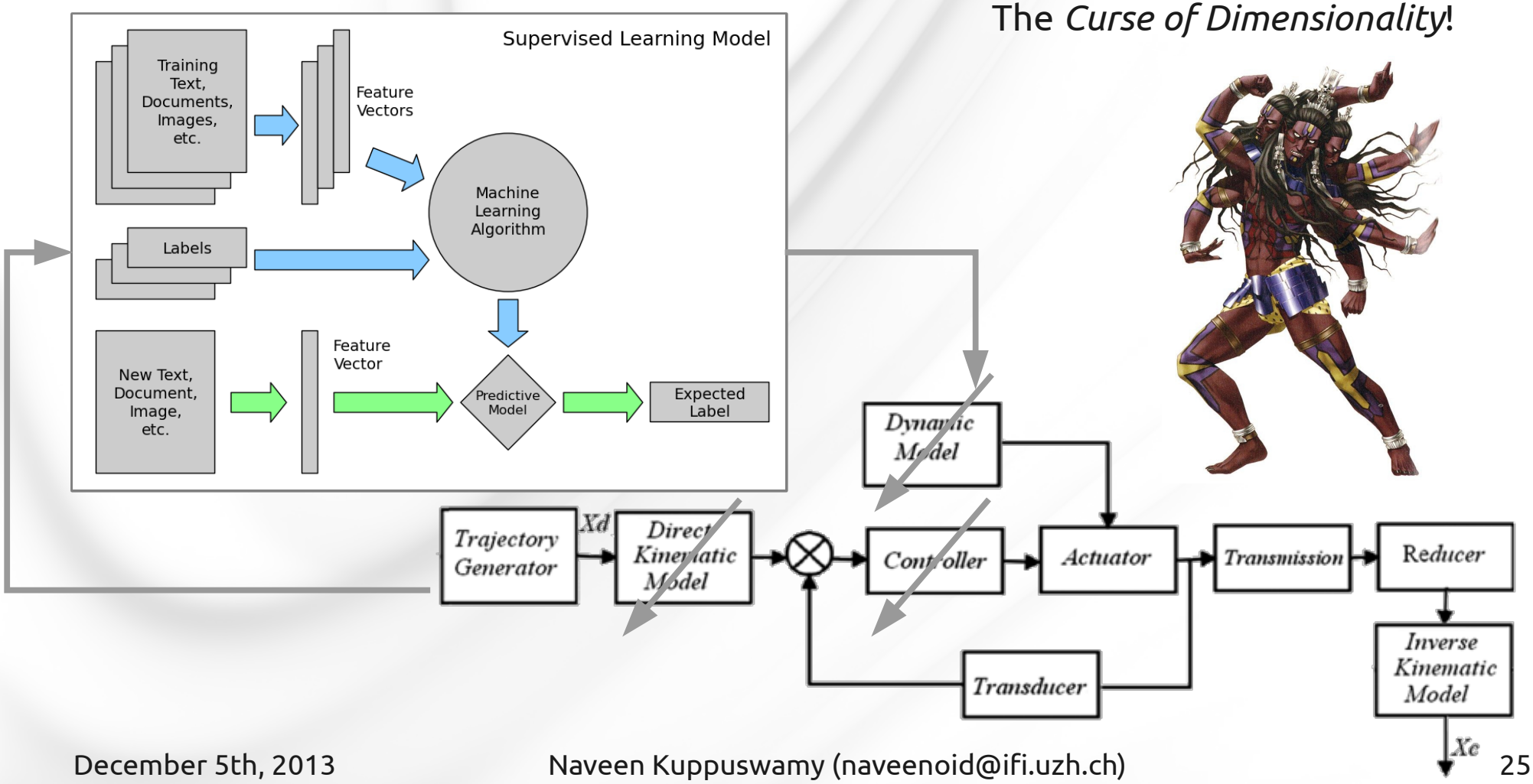

The *Curse of Dimensionality*!

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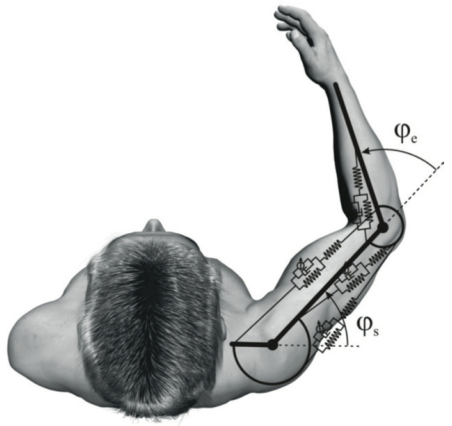


The *Degree-of-Freedom* problem

"It is clear that the basic difficulties for coordination consist precisely in the extreme abundance of degrees of freedom, with which the [nervous] center is not at first in a position to deal"

The Coordination and Regulation of Movement (1961)

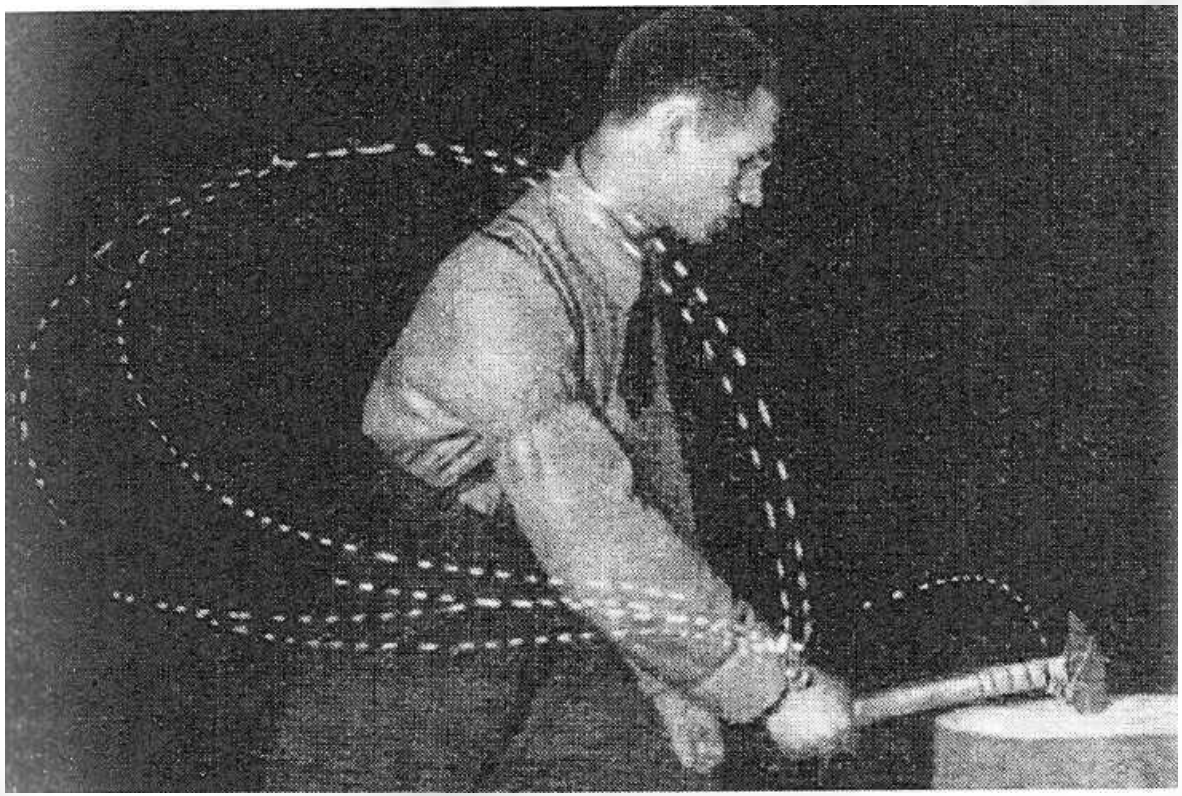
There is a non-univocal relationship between observed movements and the control signals that generate them

Joint Level :
Inverse Kinematics Problem

Neromuscular Level :
Inverse Dynamics Problem

Nikolai Aleksandrovich Bernstein
(1896 – 1966)

The *Degree-of-Freedom* problem



Nikolai Aleksandrovich Bernstein
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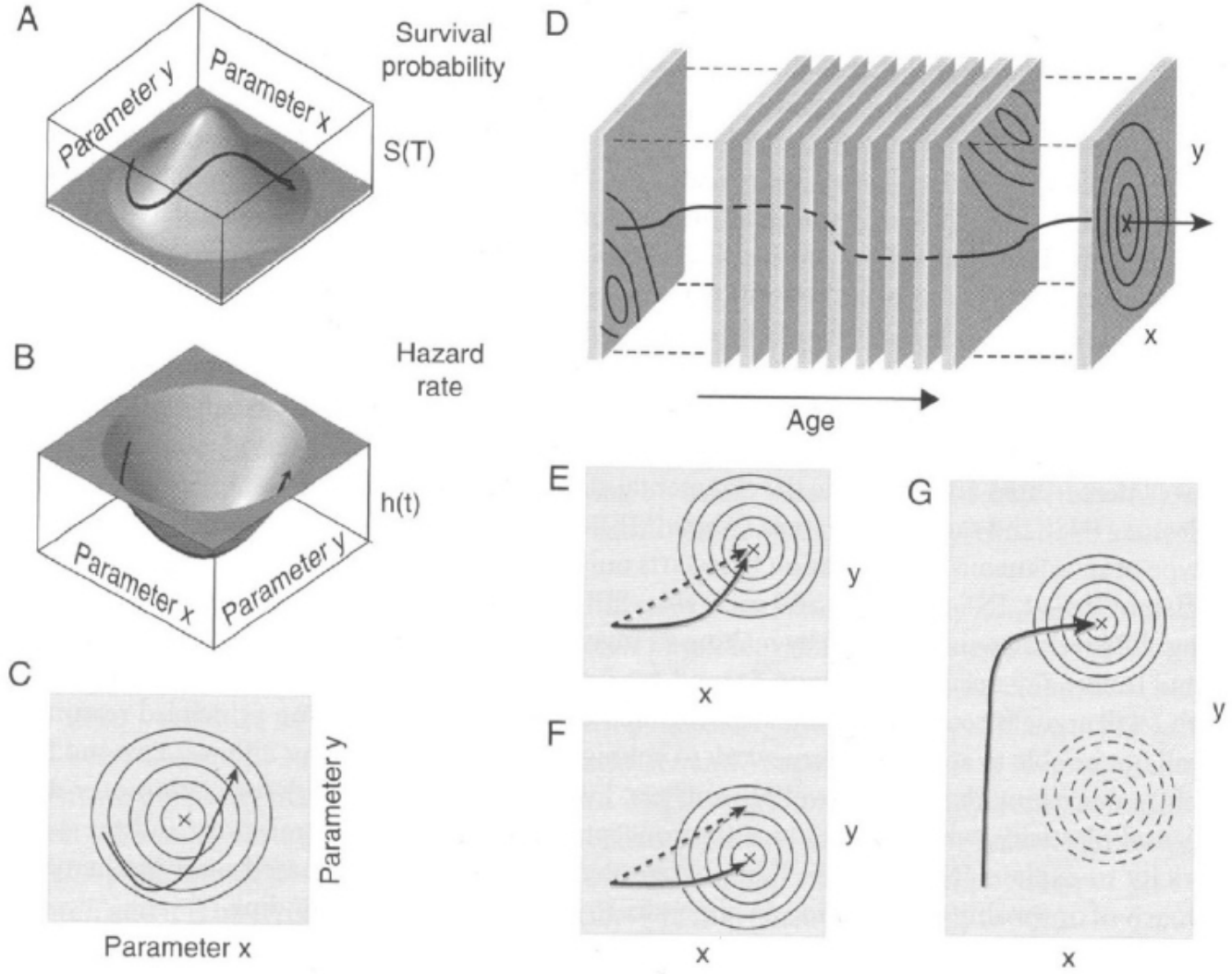
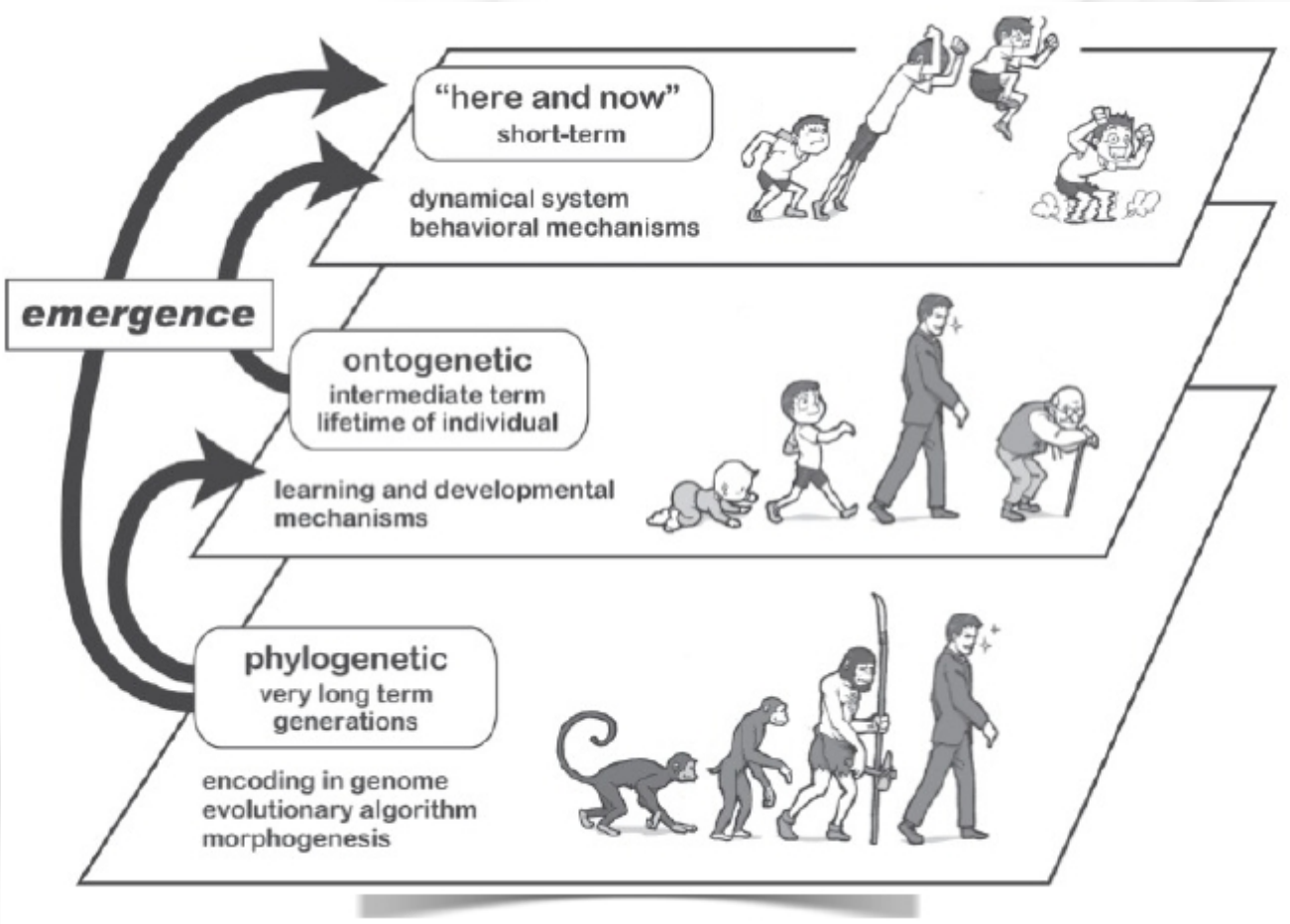
Development and the DoF Problem

Synaptic Plasticity



Reproduced from C.M. Harris (2011) Oculomotor Developmental Pathology : an evo-devo perspective

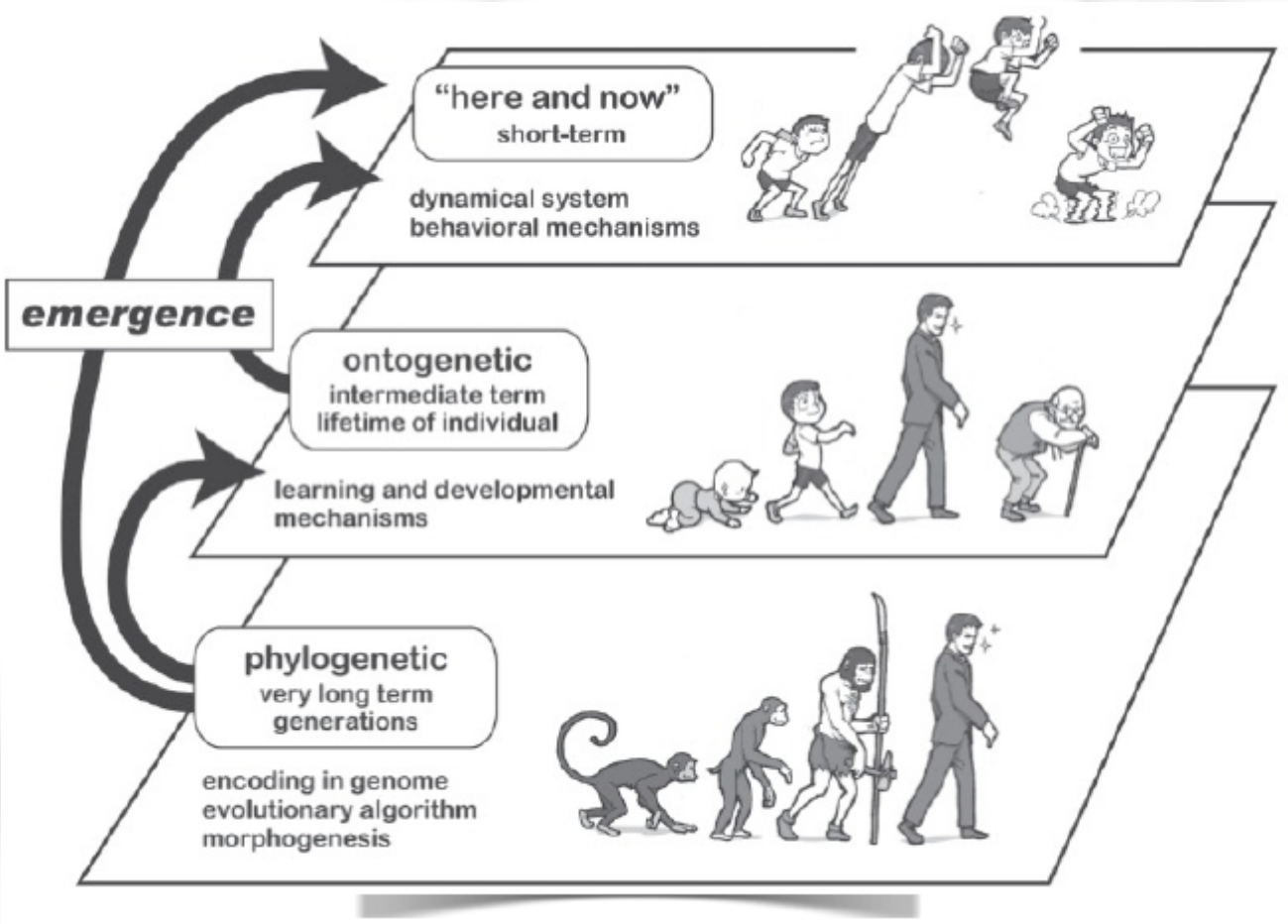
Three time scales perspective



Real-time redundancy resolution in the CNS : The cost of various solutions

Developmental acquisition of motor skills : circumventing curse of dimensionality

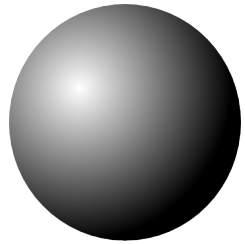
Three time scales perspective



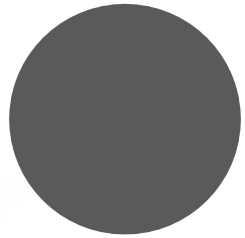
Real-time redundancy resolution in the CNS : The cost of various solutions

Developmental acquisition of motor skills : circumventing curse of dimensionality

Impact on Evolutionary fitness of an organism



II. Alleviating the *Curse of Dimensionality*



The dimensionality problem

How does nature circumvent the apparent difficulties of learning and control in high dimensions in achieving coordinated movement control?

The dimensionality problem

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CLUE :

Neurobiological evidence for dimensionality reduction happening in behaviours (*stereotypy*) and in their neural control (*modularity*)

The dimensionality problem

How does nature circumvent the apparent difficulties of learning and control in high dimensions in achieving coordinated movement control?

CLUE :

Neurobiological evidence for dimensionality reduction happening in behaviours (*stereotypy*) and in their neural control (*modularity*)

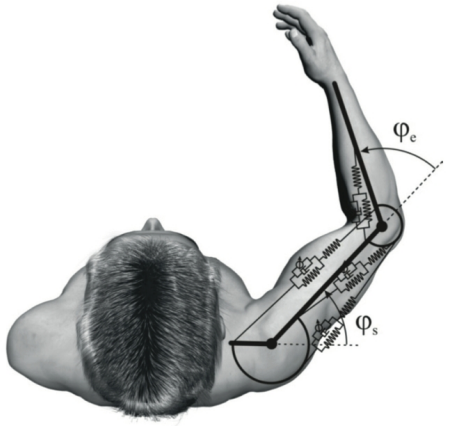
Reduced Dimensionality principles seem to exist in the design and control

Review of Neuroscientific Evidence

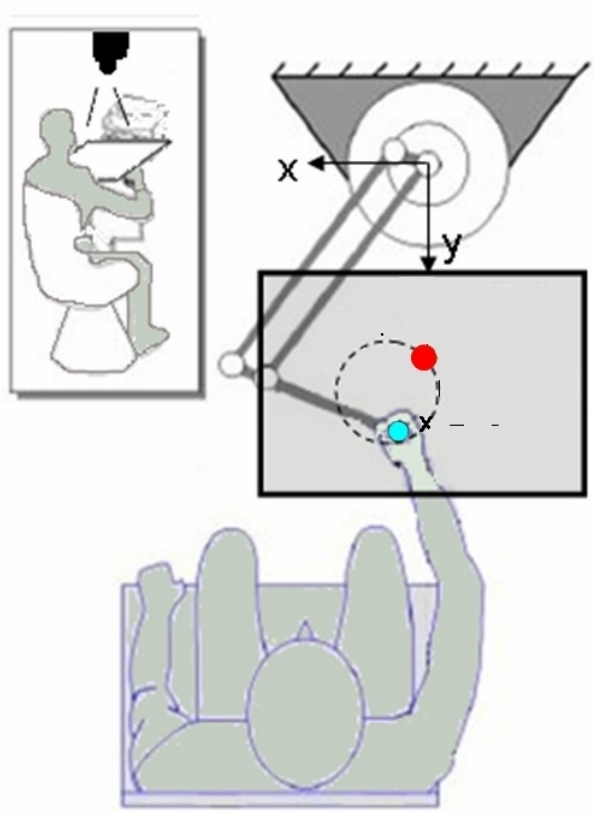
Optimal Motor Control

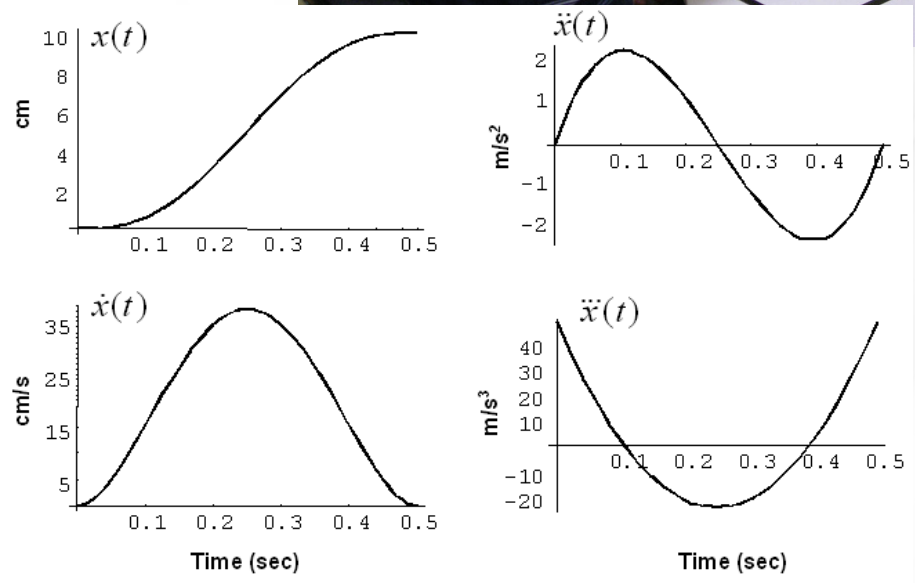
Modularisation of Control

Developmental Strategies



Invariants in human movements

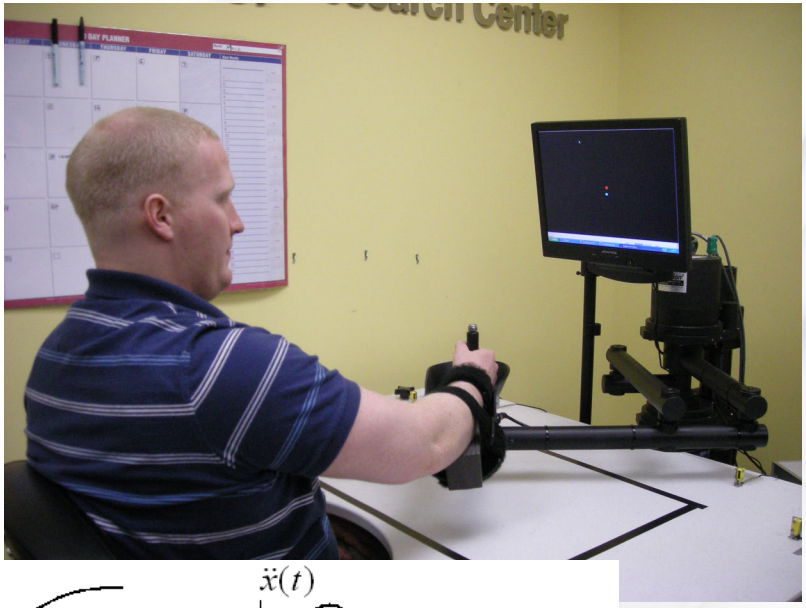


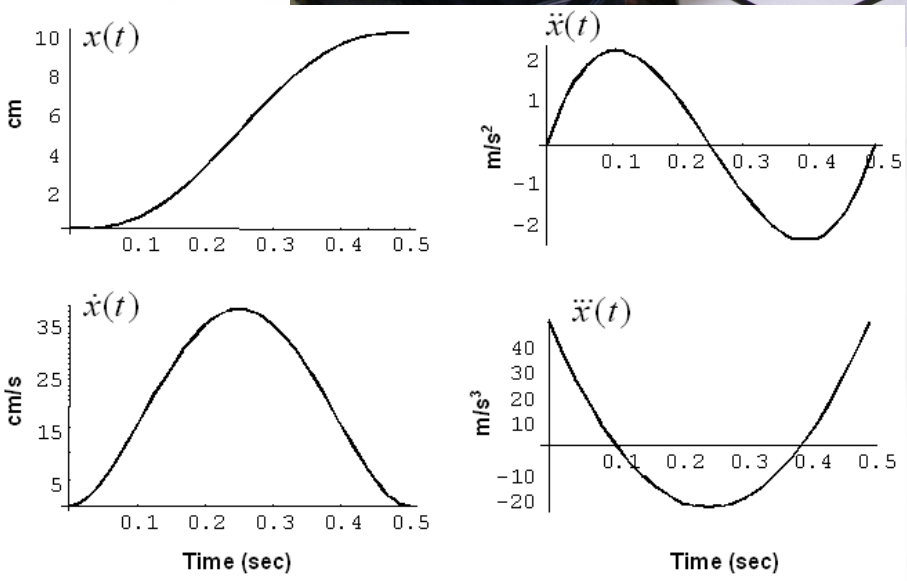


Human movements are characterised by invariant kinematic profiles :

- Eg. Bell-shaped velocity profiles in reaching
- Two-thirds power law in cyclic motions

Optimal Motor Control

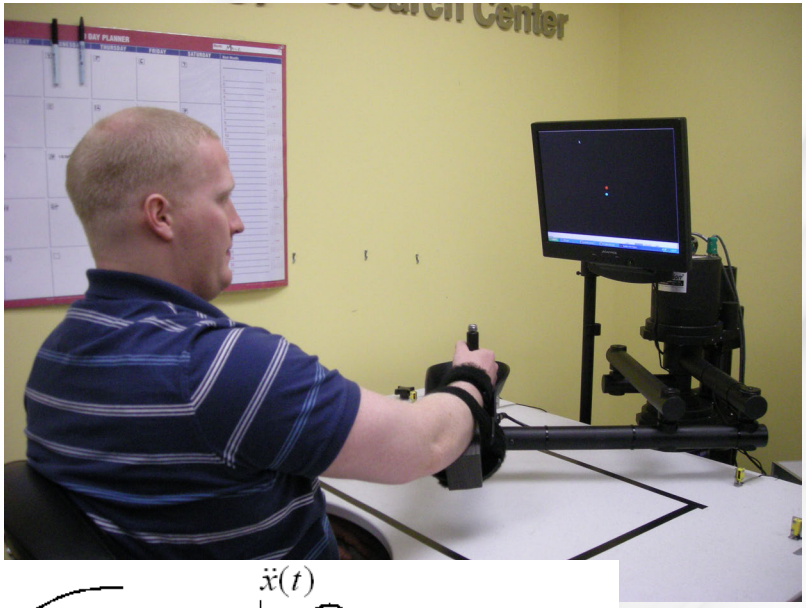


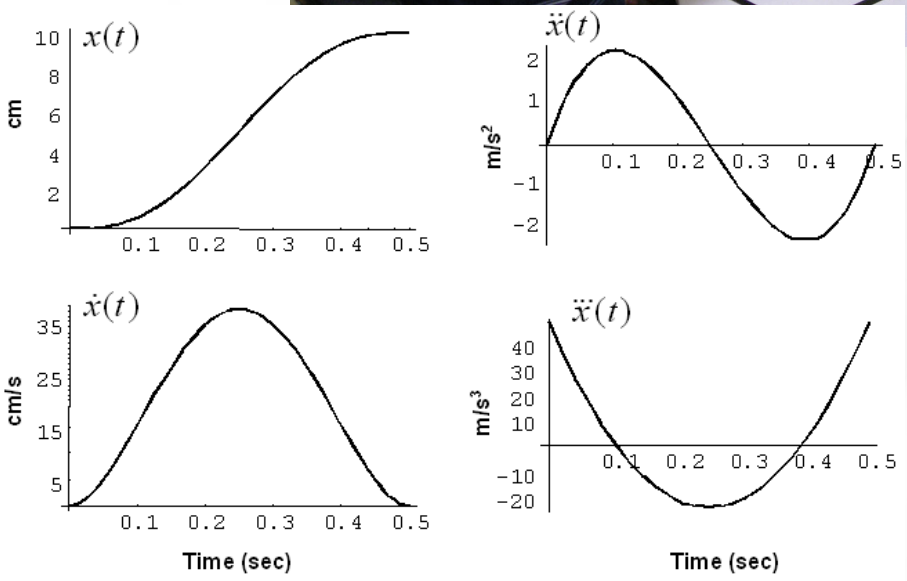


- Optimal Control Theory (OCT) predicts that invariants are a result of the CNS optimising some form of Performance Index (PI).
- Some predicted PI (context dependent) :
  - Minimising Jerk (rate-of-change of acceleration)
  - Minimising torque-change
  - Minimising energy
  - Minimising endpoint variance (movement accuracy)

Harris (1998), Journal of Neuroscience Methods : On the optimal control of behaviour - A stochastic perspective

Optimal Motor Control





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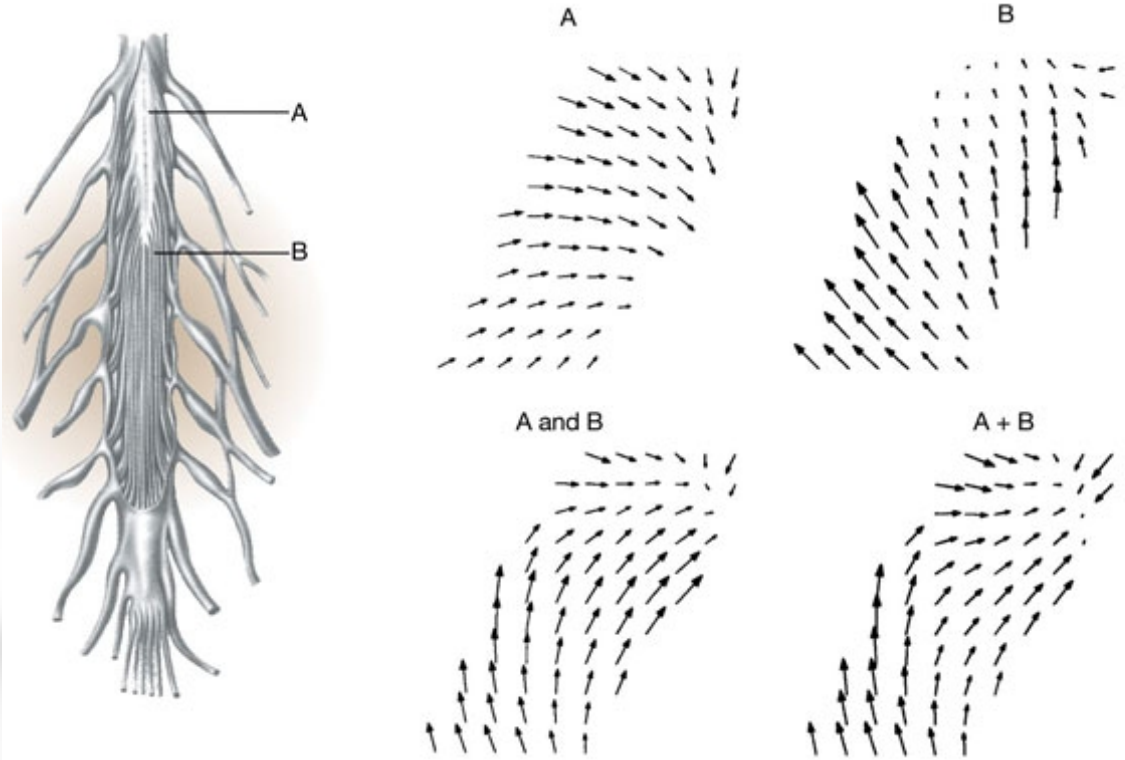
The optimal solution seems to exist in a lower dimensional space : low temporal dimensionality is evidence.

Harris (1998), Journal of Neuroscience Methods : On the optimal control of behaviour - A stochastic perspective

Modularisation : Spinal Force Fields

Mussa Ivaldi's Spinal Force Field : Natural evidence for reduced dimensionality at the spinal level.

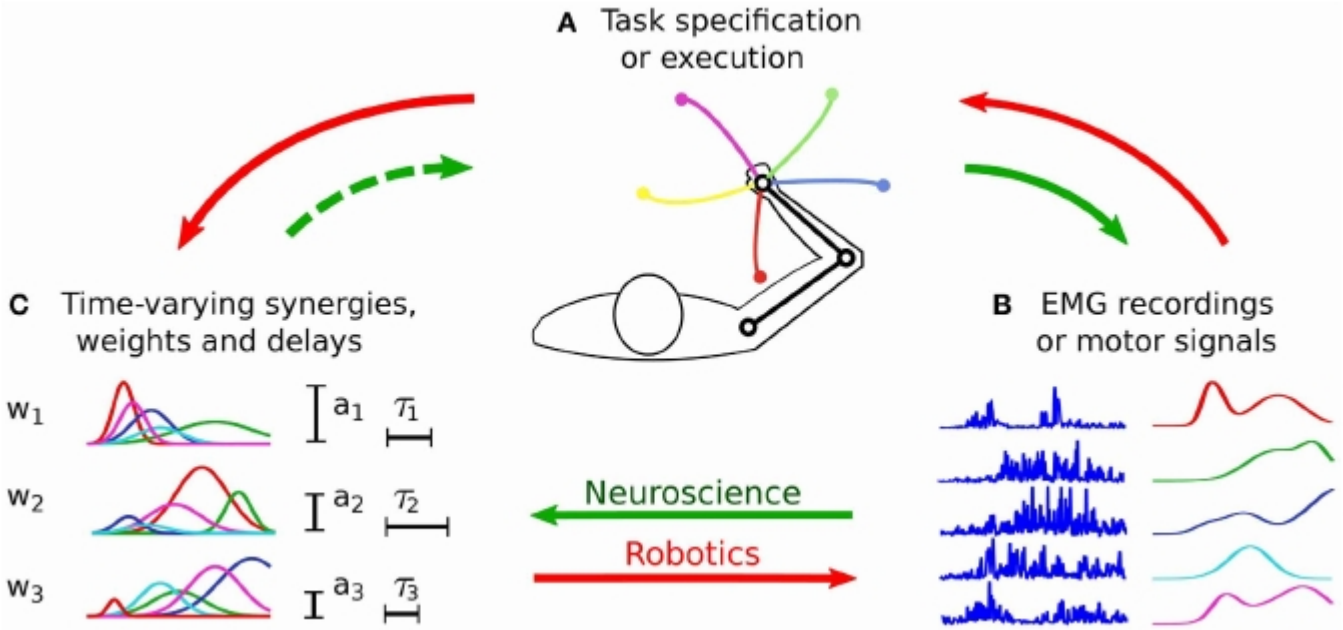
- Stimulation leads to a force field at end point with a unique equilibrium point
- Co-stimulation leads to vector summation for force fields thus "linearizing" the system



Microstimulation of any of 4 specific sites on the spine resulted in coordinated action among 13 muscles in the leg of a frog

E Bizzi, FA Mussa-Ivaldi, and S Giszter, Science 19 July 1991, Computations underlying the execution of movement: a biological perspective

Muscle Synergies



Coordinated activation of muscles in performing tasks.

Evidence :

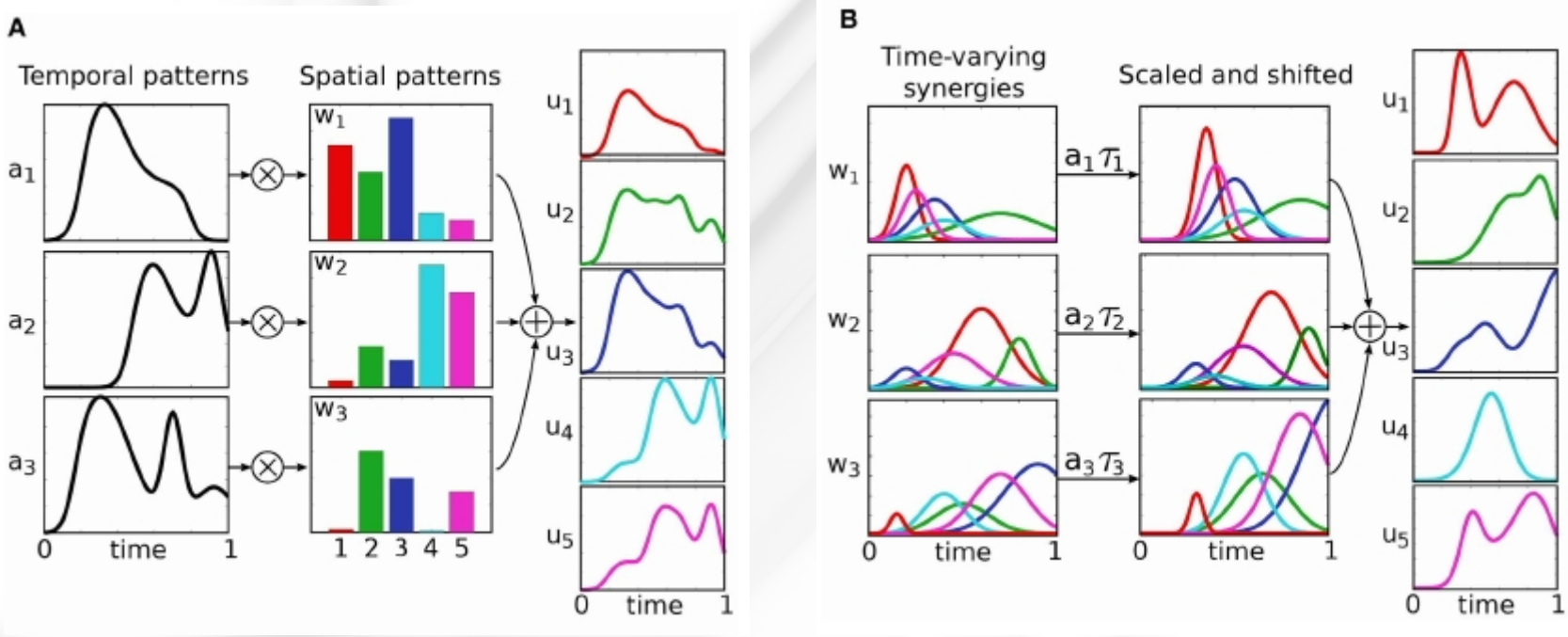
spatio-temporal regularities in kinematics (motion tracking) + muscle activation (EMG) data

Alessandro (2013), Frontiers in Computational Neuroscience :

Muscle synergies in neuroscience and robotics - from input-space to task-space perspectives

Muscle Synergies

While SFFs are a time-invariant synergy, other models include :



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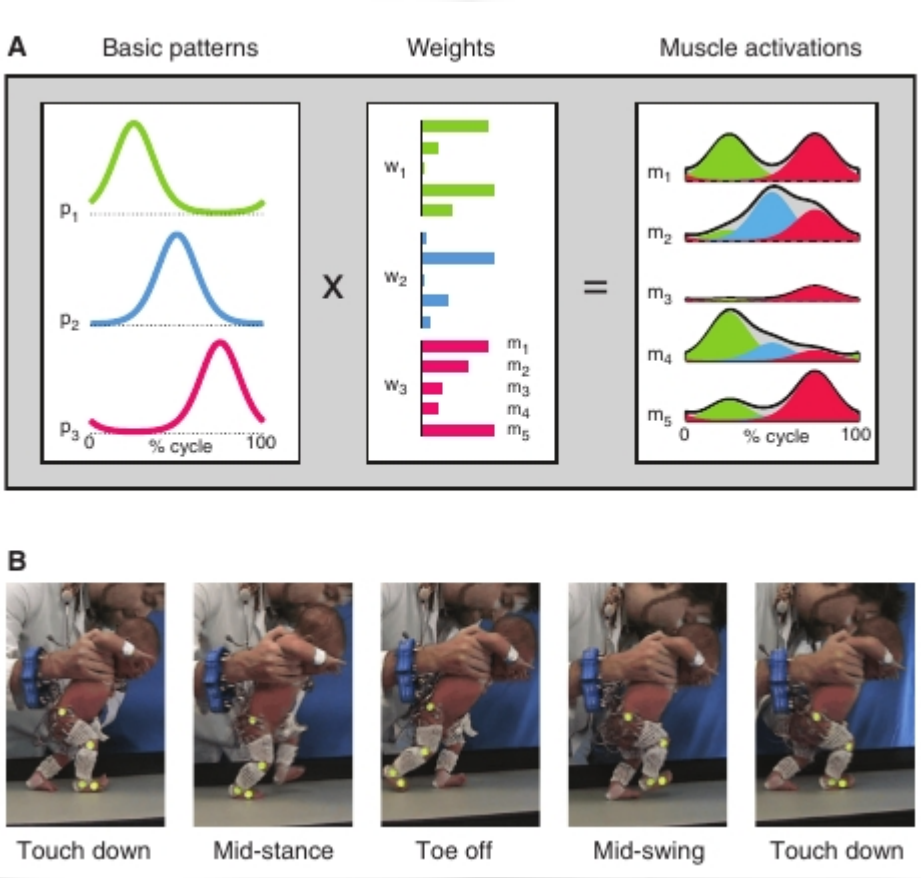
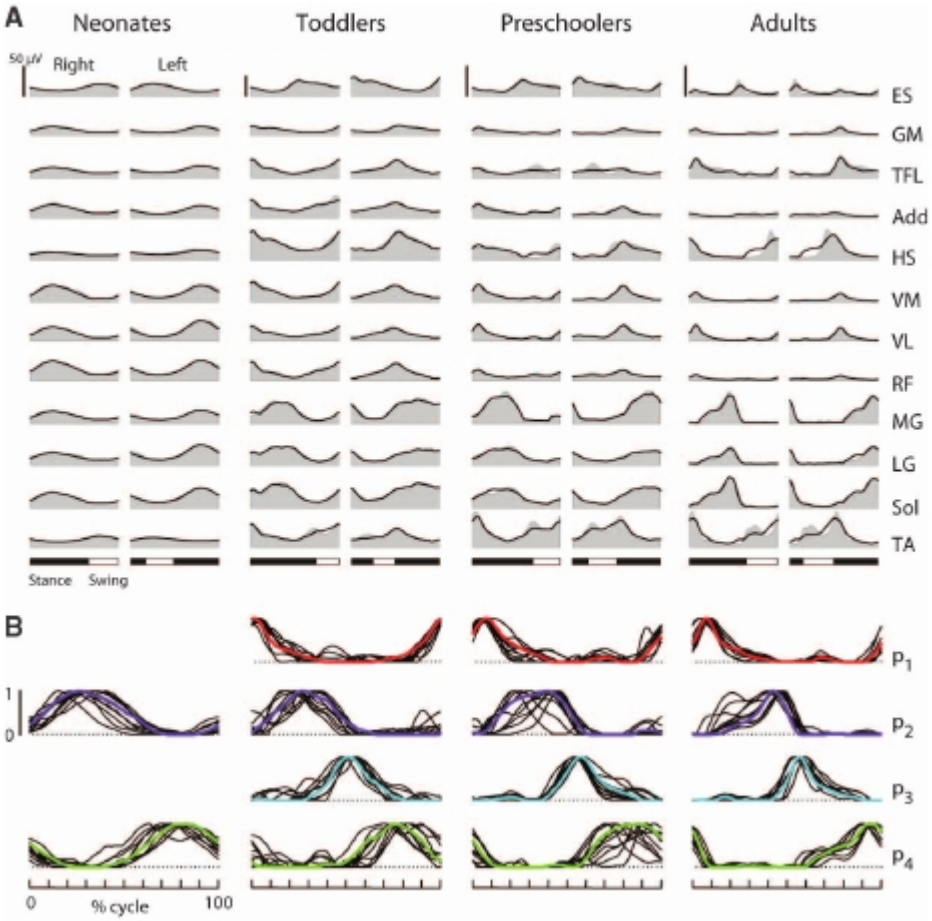
Muscle Synergies

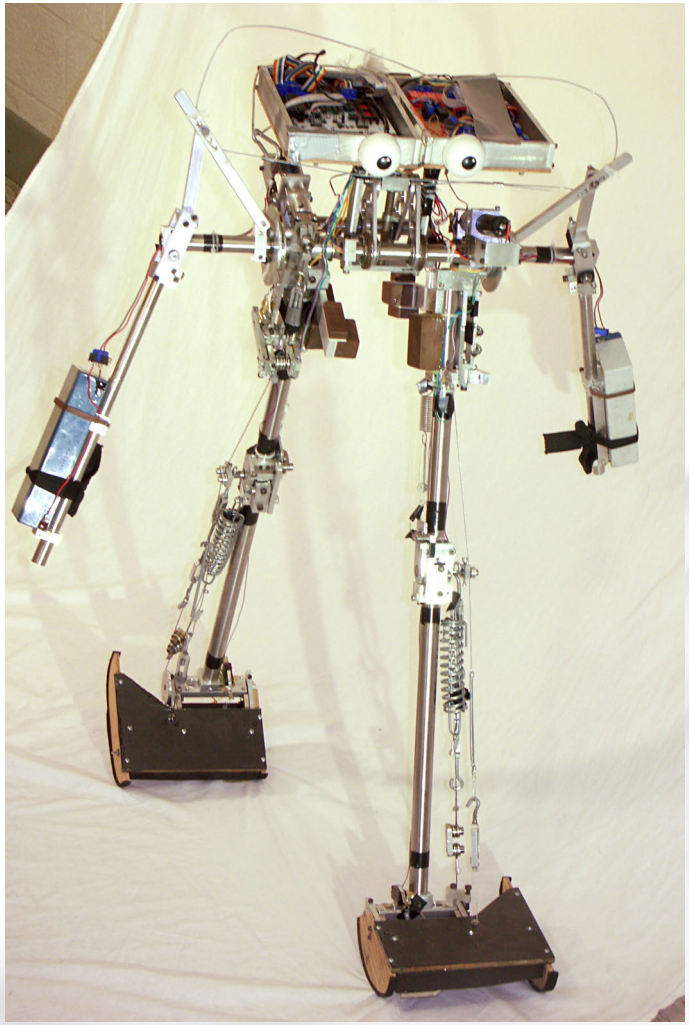




Developmental Implications : More synergies = increasing behaviour complexity

Dominici et al (2011), Science : Locomotor Primitives in Newborn Babies and Their Development

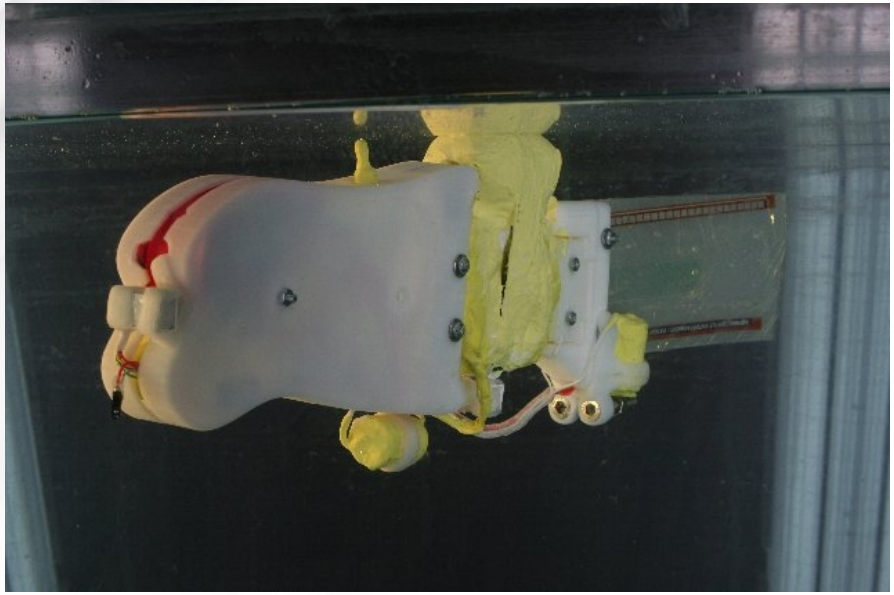
Robotic case : Underactuated robots



Cornell walker

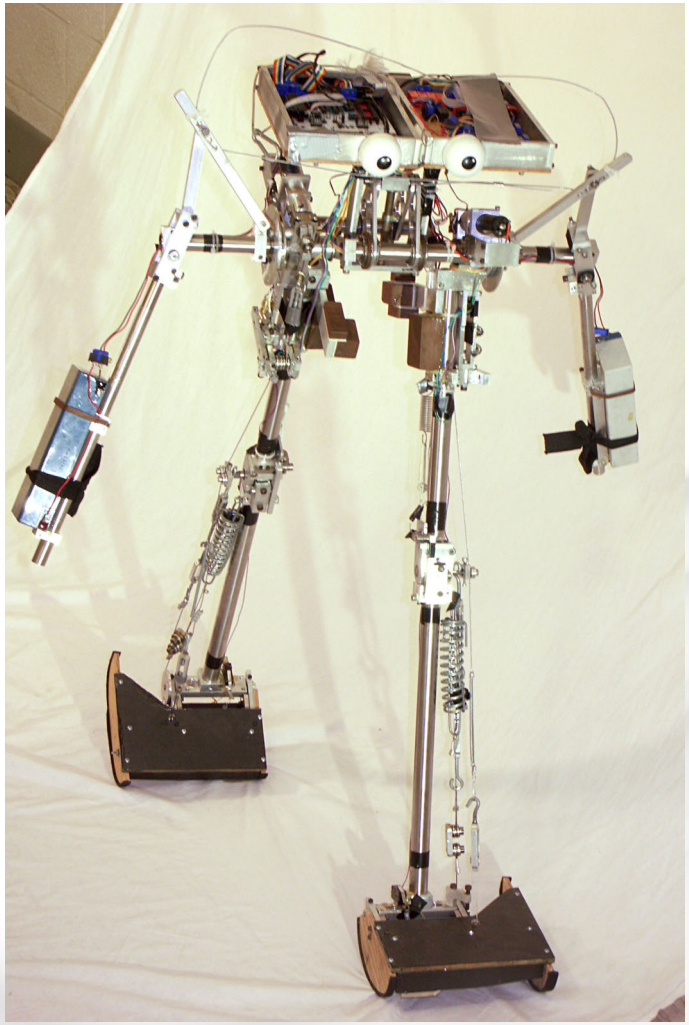
Underactuated robots :

- Reduce the number of actuated DoF
- Exploit morphological properties in control



Wanda the Fish (UZH)

Robotic case : Underactuated robots



Cornell walker

Underactuated robots :

- Reduce the number of actuated DoF
- Exploit morphological properties in control

Although a very good idea in principle, robots built by such methods are optimised only for a narrow range of tasks.

How to increase range of tasks?

Research Question

What is reduced dimensionality and how can it be exploited for the design and control of embodied systems?

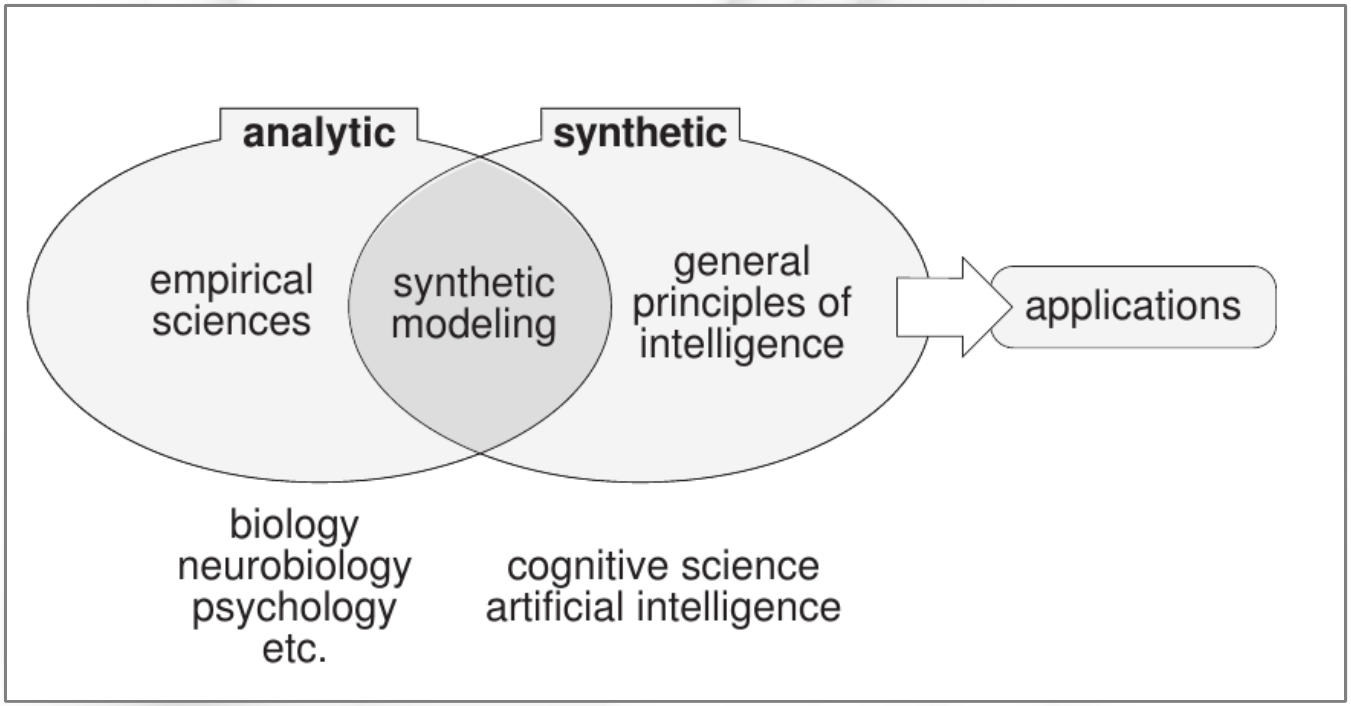
Research Question

What is reduced dimensionality and how can it be exploited for the design and control of embodied systems?

Mathematical Quantification of Reduced Dimensionality

Systematic exploration of the factors affecting Reduced Dimensionality

The synthetic methodology



What is Reduced Dimensionality?

Reduced Dimensionality ≠ Dimensionality Reduction

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Broad definition for the scope of this thesis :

> *The properties of a system that enable its behaviour to be represented as being composed of a lower number of dimensions than it actually is.*

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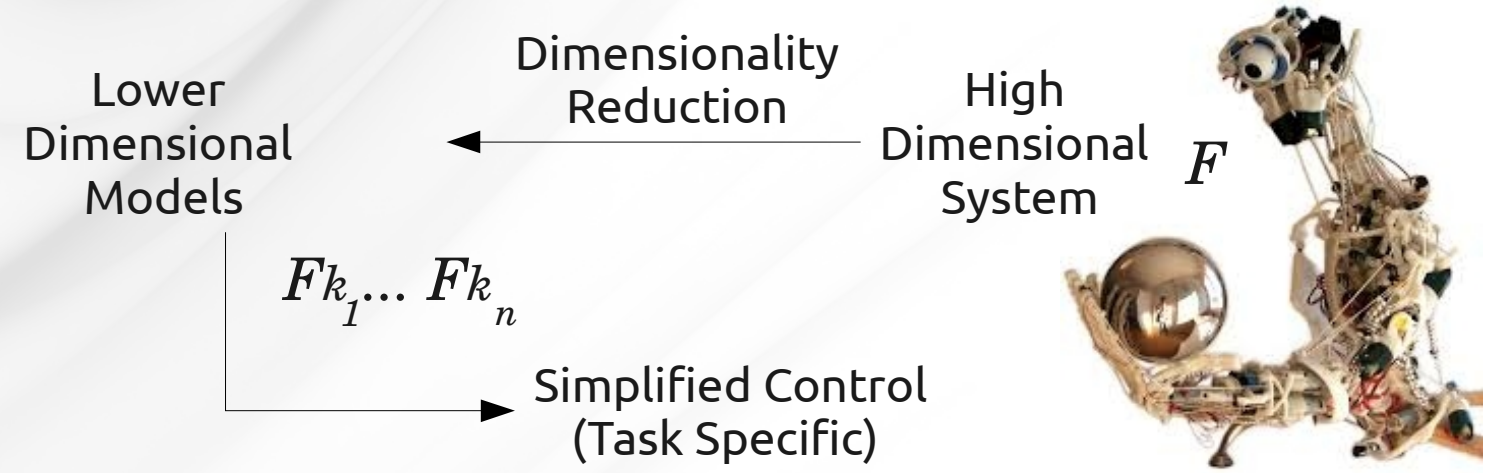

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> *The properties of a system that enable its behaviour to be represented as being composed of a lower number of dimensions than it actually is.*

Facilitated by the Reduced Dimensionality of F

Lower Dimensional Models

Dimensionality Reduction

High Dimensional System F

$Fk_1 \ldots Fk_n$

Simplified Control (Task Specific)

Research Question

What is reduced dimensionality and how can it be exploited for the design and control of embodied systems?

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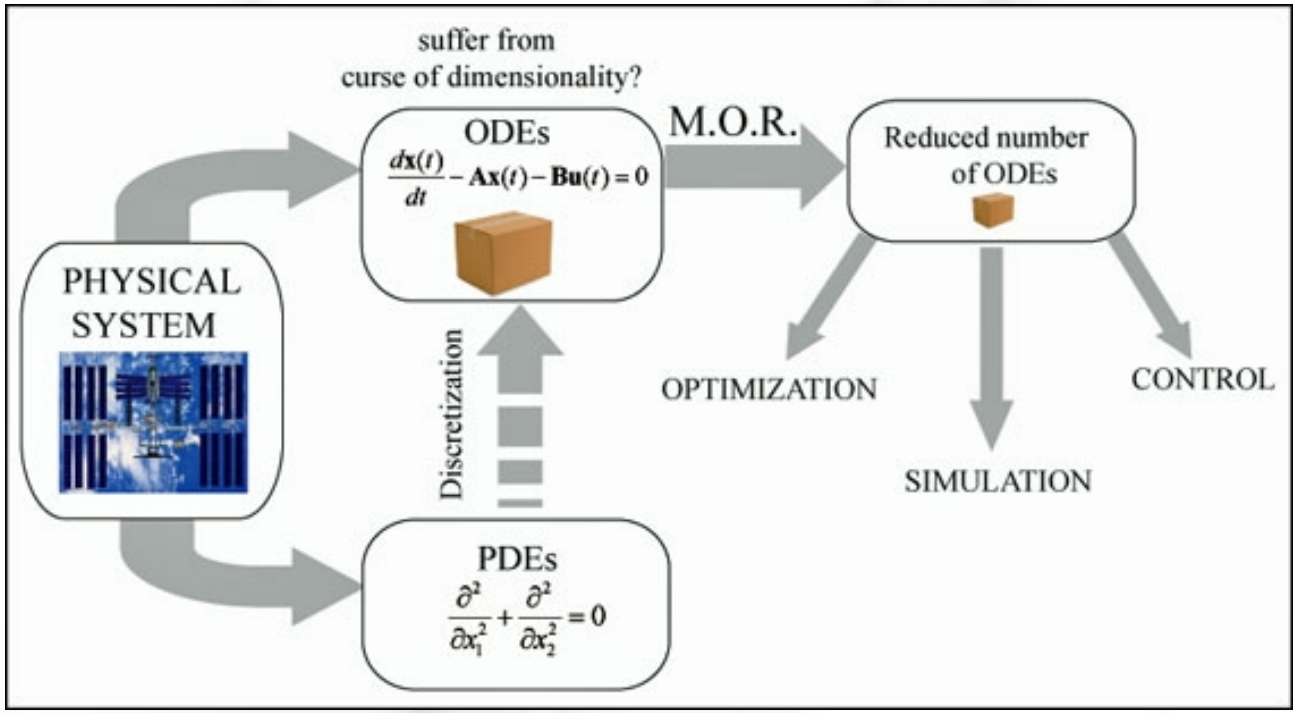
→ Mathematical Quantification of Reduced Dimensionality

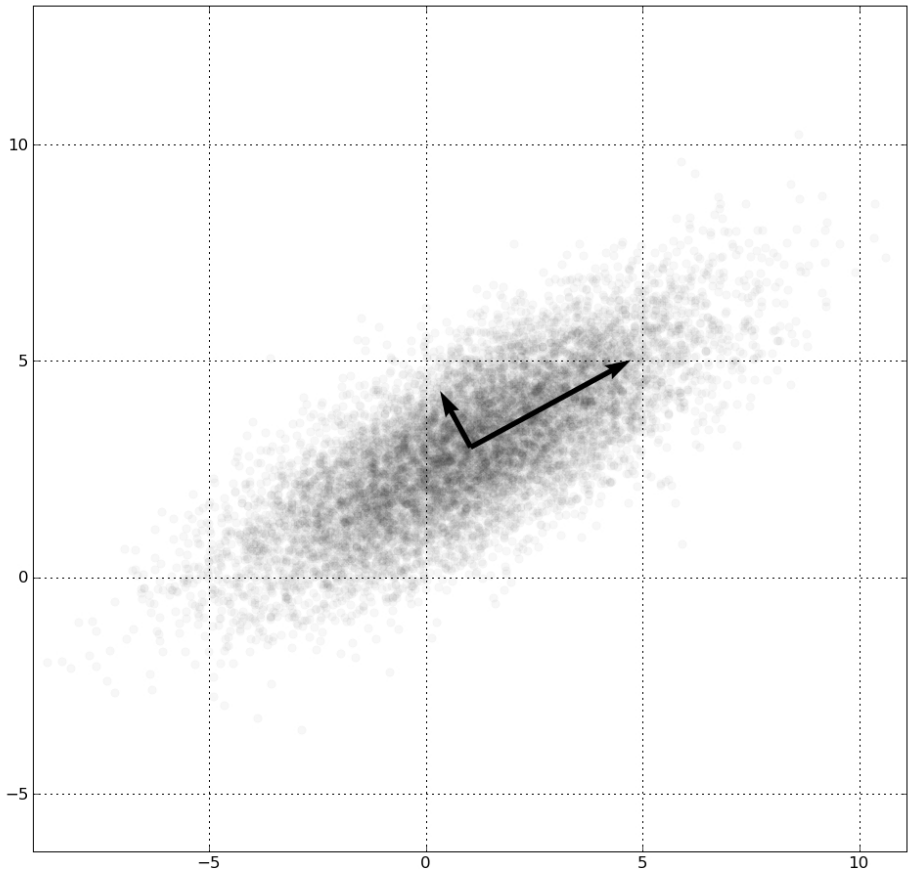
Model Order Reduction



- Control-theoretic perspective of dimensionality
- Based on state-space dynamical description of behaviour.
- MOR algorithms perform dimensionality reduction of dynamics
- MOR methods used : POD, BT

POD = PCA

Proper Orthogonal Decomposition is Principal Component Analysis in the phase space



Mathematical Formalism

What is Dimensionality?

Consider the ODE : $$x^{(n)} = \mathrm{F}\left(t, x, \dot{x}, \ddot{x}, \cdots x^{(n-1)}\right)$$

Set of 1st order ODEs : $$\dot{\mathbf{x}} = \mathbf{F}(t, \mathbf{x})$$

$$\mathbf{x} \in \mathbb{R}^n$$

Mathematically, dimensionality is simply the order of a set of ODEs, i.e. *n*

Mathematical Formalism

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Mathematically, dimensionality is simply the order of a set of ODEs, i.e. *n*

Control Theory Perspective :
State-space dynamical system representation

$$\mathbf{y}(t) = h(\mathbf{x}, t), \quad \dot{\mathbf{x}} = f(\mathbf{x}, t) + g(\mathbf{x}, \mathbf{u}, t)$$

$N \in \mathbb{Z}^+$

$\mathbf{y}(t) \in \mathbb{R}^{N_o}$ Output

$\mathbf{x}(t) \in \mathbb{R}^{N}$ State

$\mathbf{u}(t) \in \mathbb{R}^{N_i}$ Input

Research Question

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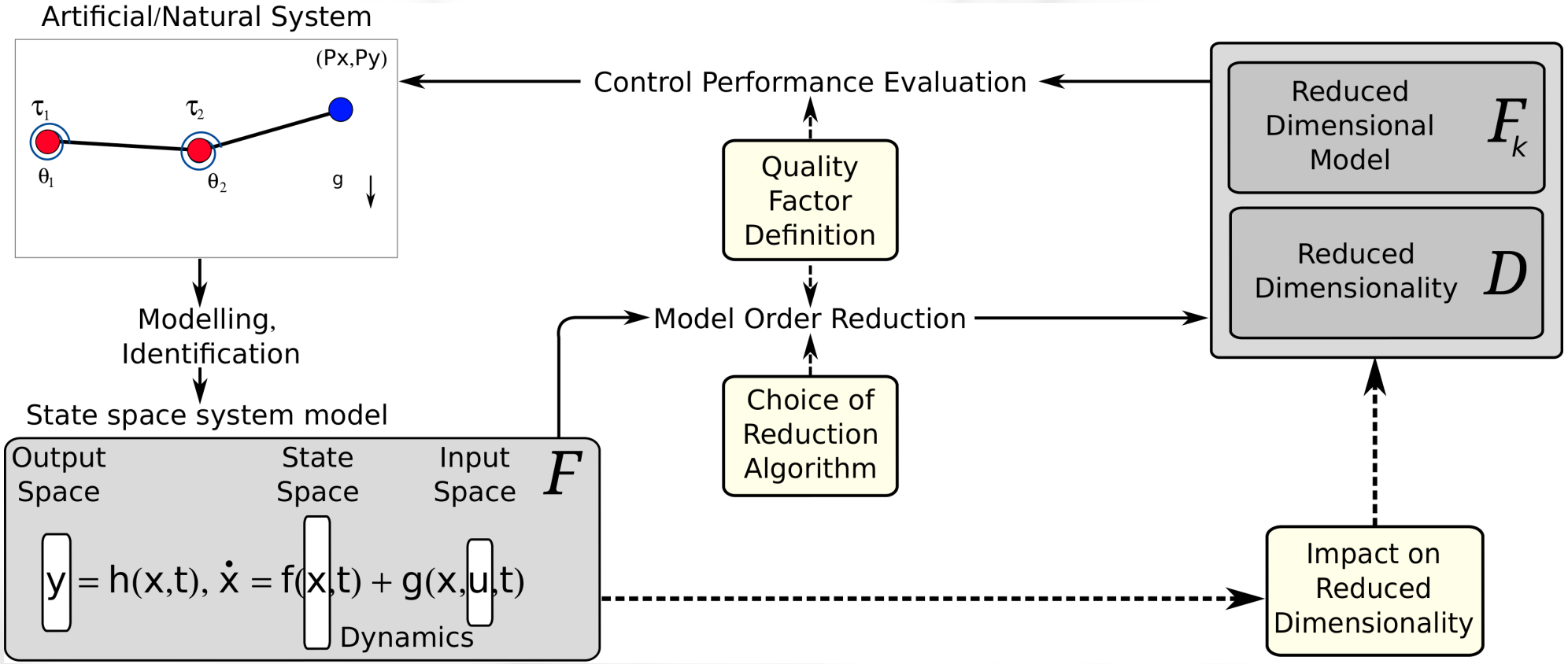
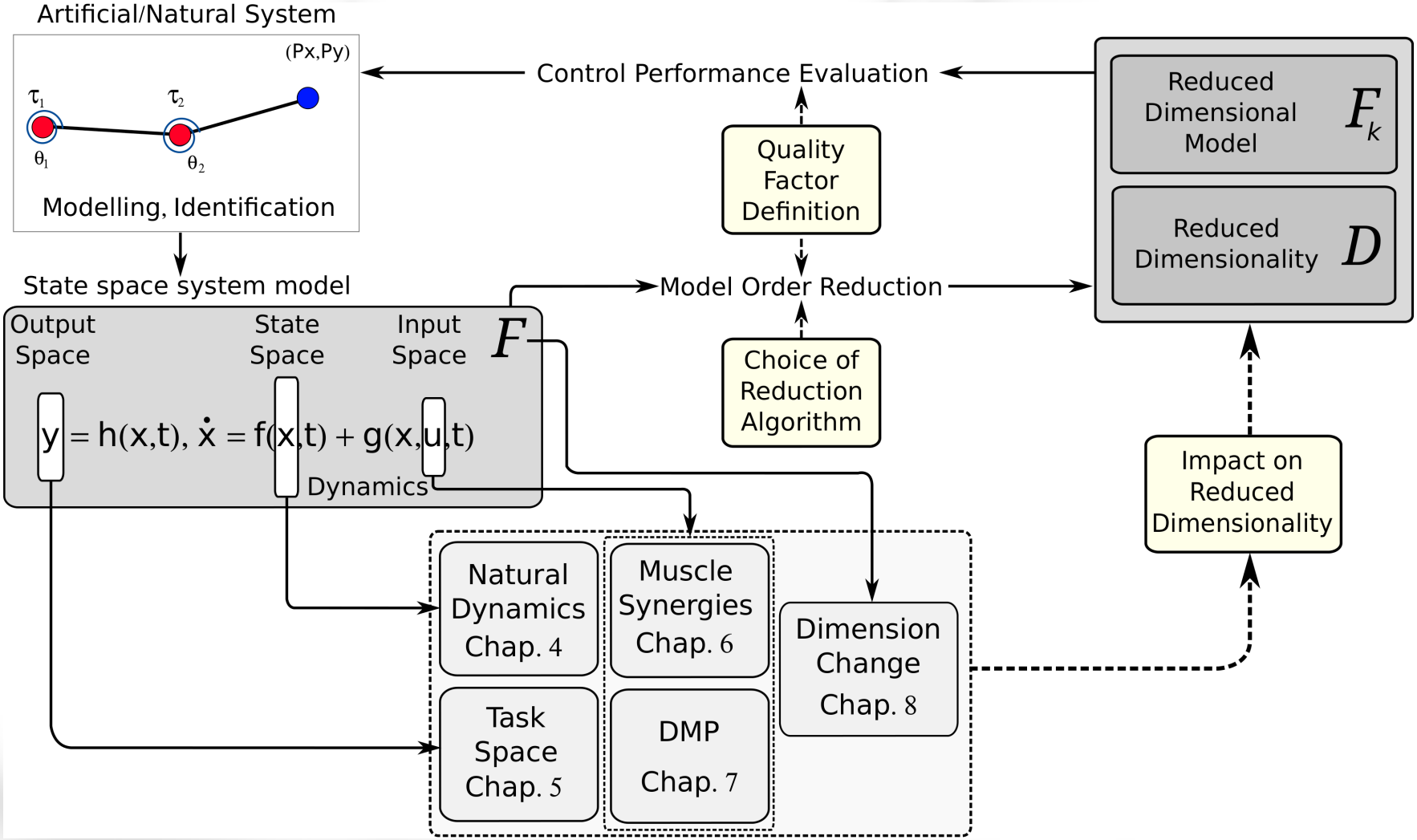
Systematic exploration of the factors affecting Reduced Dimensionality

Reduced Dimensionality Analysis



Exploiting Reduced Dimensionality

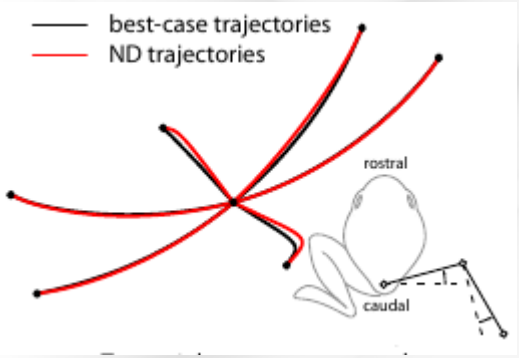


Natural Dynamics

Kuppuswamy, Marques, and Hauser (2012) "Synthesising a Motor-Primitive Inspired Control Architecture for Redundant Compliant Robots", From Animals to Animats – proc. Of the International Conference on Simulation of Adaptive Behaviour, Odense, Denmark, 2012.

Motor Primitives and reduced dimensionality

- Regulating reduced dimensionality → regulating control learning implicitly
- Can reduced dimensionality explicity simplify Control?
- i.e. Can expoitation of natural dynamics allow control simplification?



Berniker et al (2009)

Motor Primitives : Input constrained into a linear combination of *k* primitives

$$u = U^* C.$$

$$U^* = u^*_{1 \cdots k}$$

Reduced Dimensionality

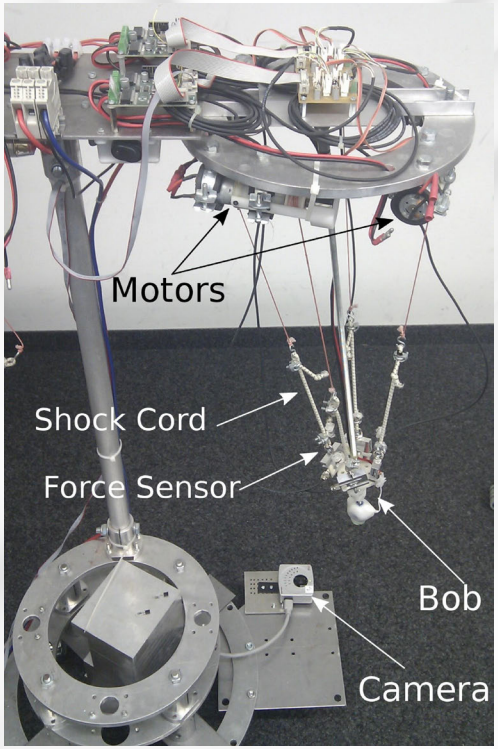
Natural Dynamics

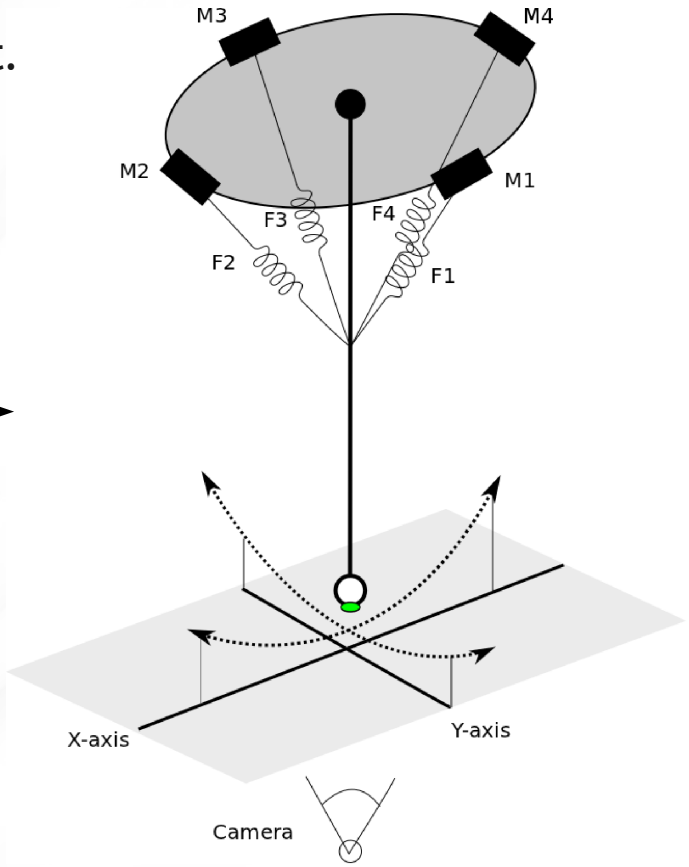
Kuppuswamy, Marques, and Hauser (2012) "Synthesising a Motor-Primitive Inspired Control Architecture for Redundant Compliant Robots", From Animals to Animats – proc. Of the International Conference on Simulation of Adaptive Behaviour, Odense, Denmark, 2012.

Motor Primitives and reduced dimensionality

Primary Contribution :

- Developmental synthesis technique for a controller inspired by motor primitives for a compliant robot system.
- Tested on a simulated linear model of pendulum robot.





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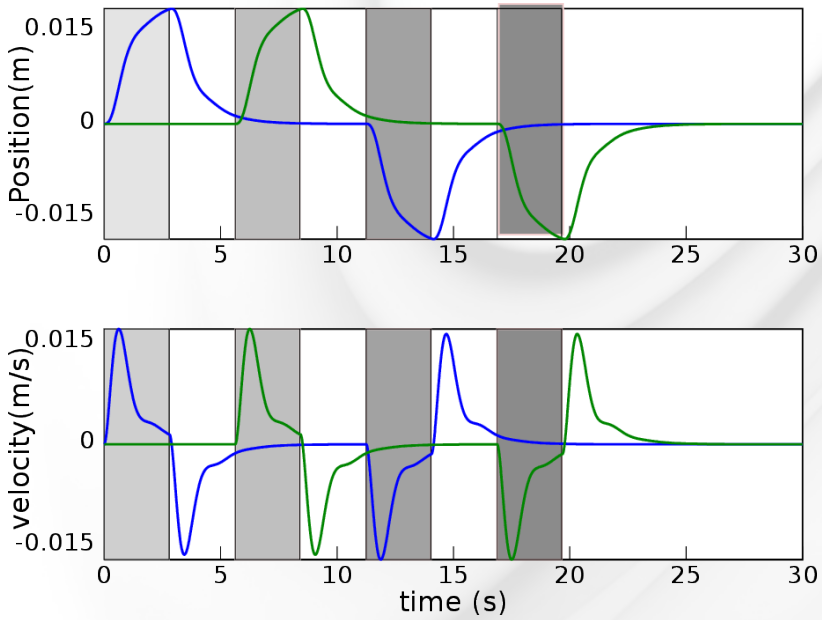
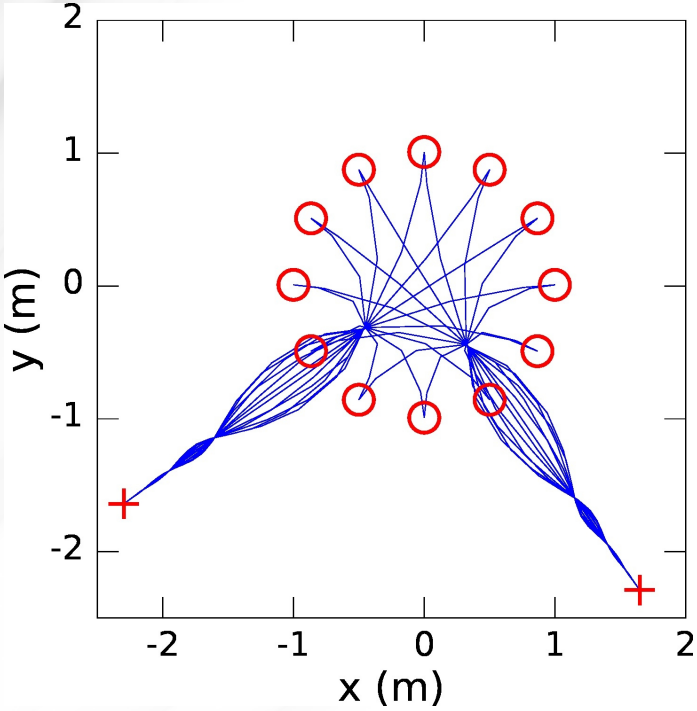
Motor Primitives and reduced dimensionality



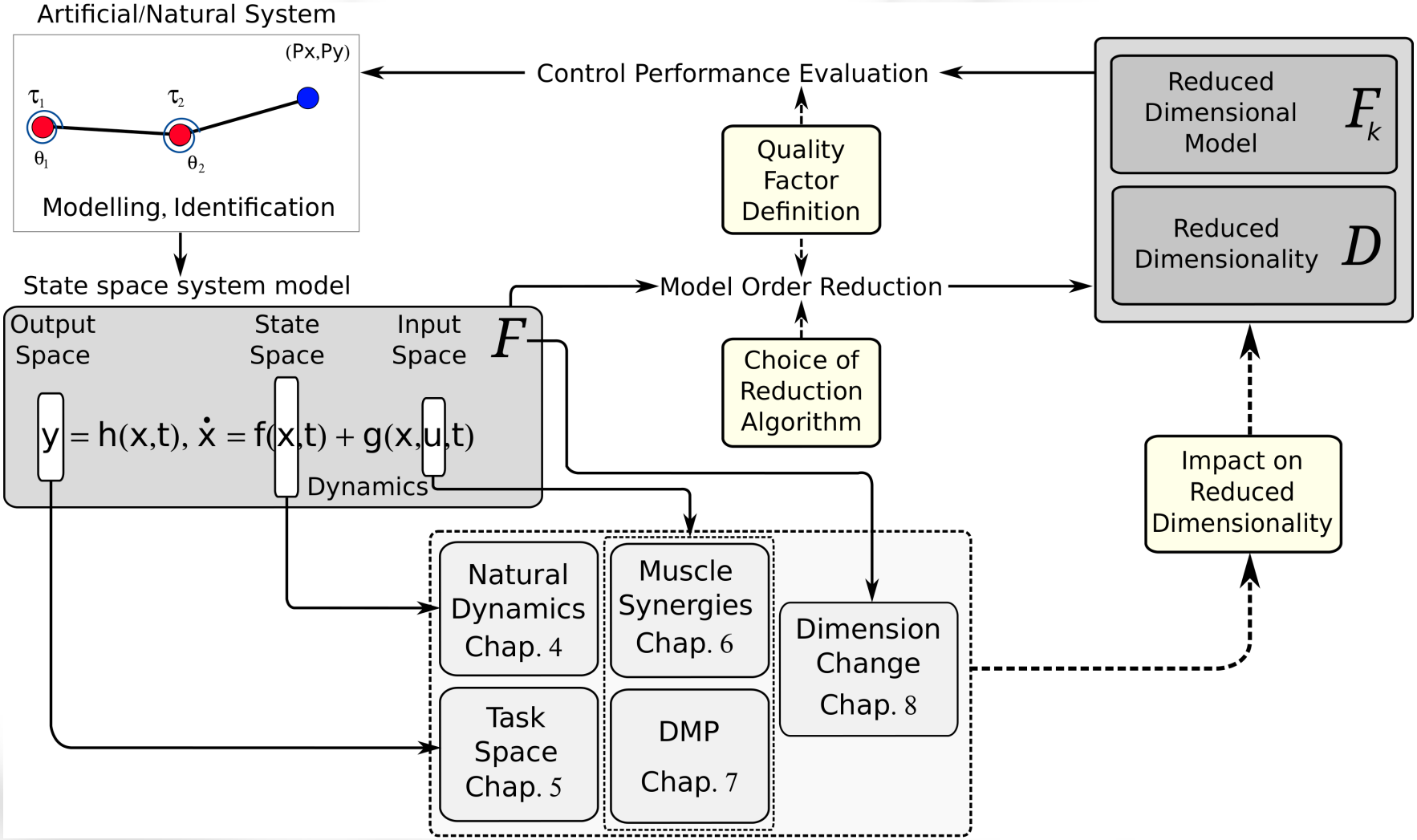


Proposed Synthesis Technique

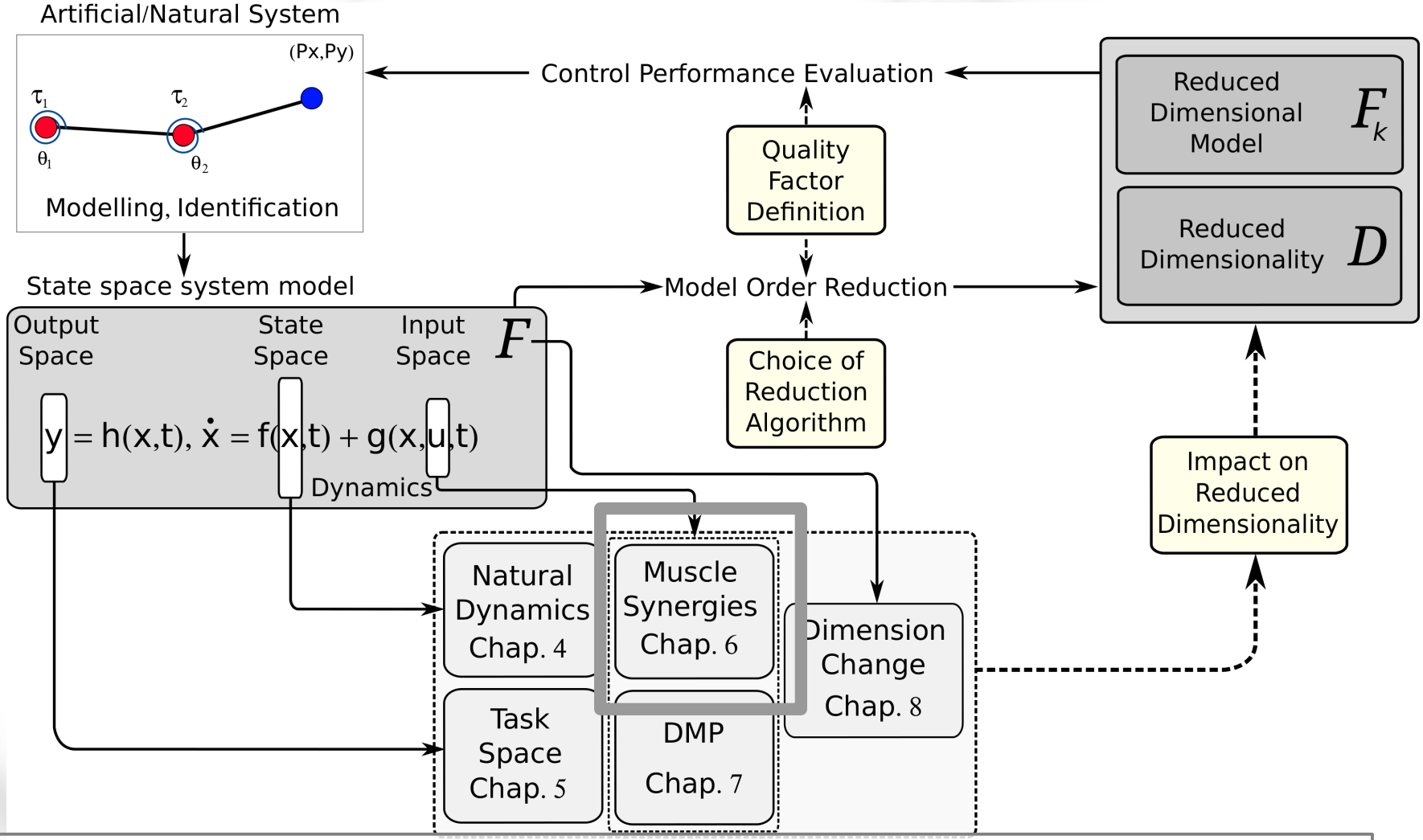
$\mathcal{K}$= 2 primitive system

Spontaneous Motor Activity → System Identification → Reduction (POD) → $\mathcal{K}$= 2 primitive system

Exploiting Reduced Dimensionality



Muscle Synergies



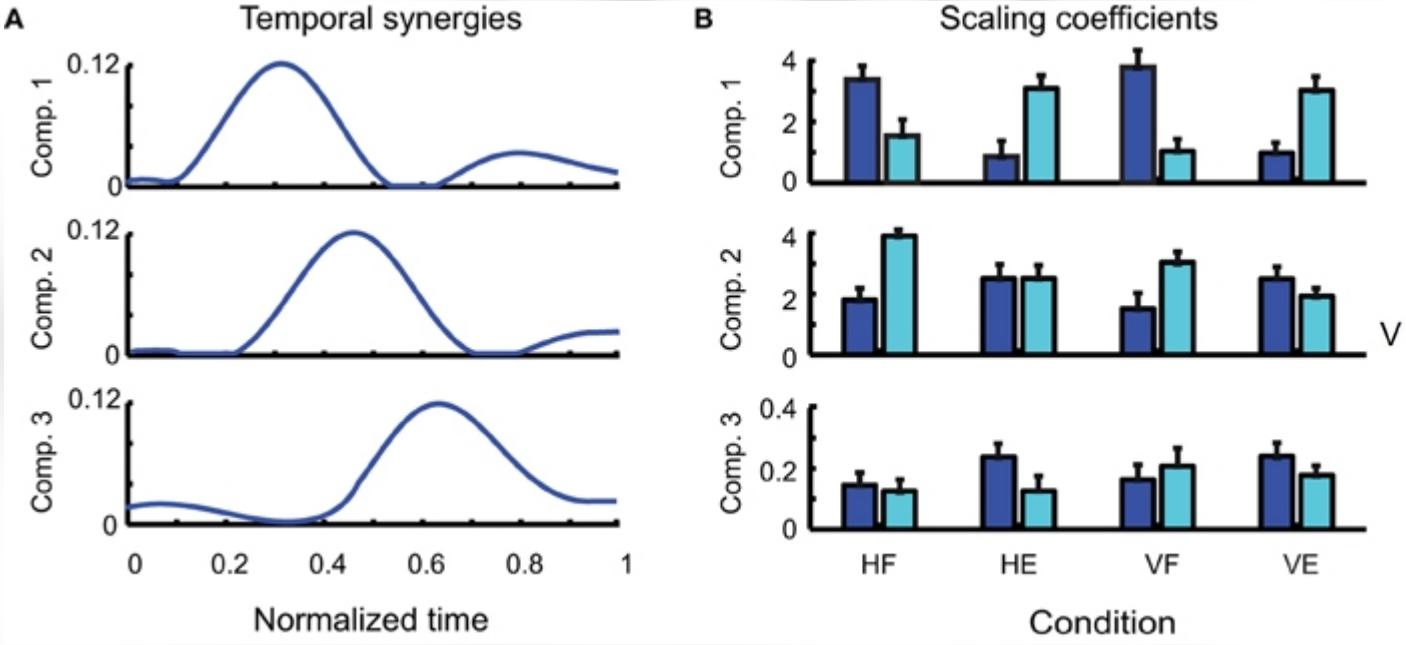
Research Question : *Does control using Muscle Synergies facilitate reduced dimensionality?*

Muscle Synergies

Kuppuswamy, and Harris (2013) "Do muscle synergies reduce the dimensionality of behaviour?", Frontiers in Computational Neuroscience (submitted).

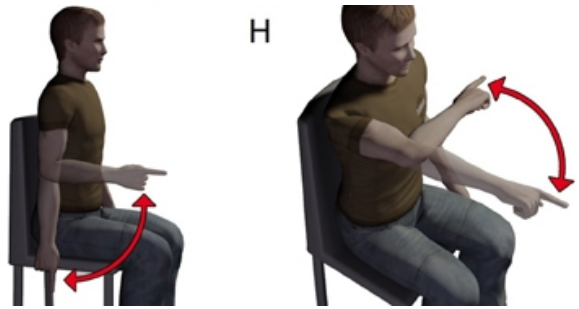
Muscle Synergies – Coordinated activations of groups of muscles

- Input Dimensionality reduction through modularity.
- Decouple task-dependent coefficients from a task-independent set of modules.



Temporal Synergy Model : Input is a weighted linear combination of *S* synergies

$$\mathbf{u}(t) = \sum_{i=1}^{S} w_i \psi_i(t)$$

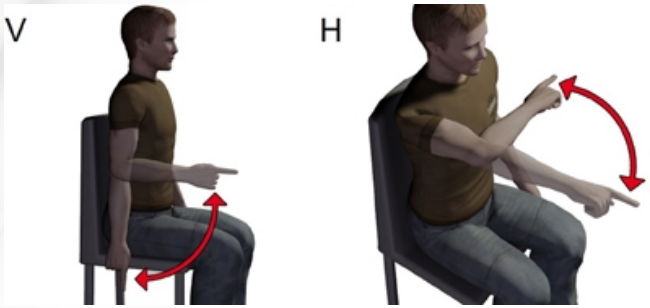


From Chiovetto et al(2013)

Muscle Synergies

Kuppuswamy, and Harris (2013) "Do muscle synergies reduce the dimensionality of behaviour?", Frontiers in Computational Neuroscience (submitted).

- Many open questions remain on the validation of the muscle synergy hypothesis.
- Reduction in input-space might ease up control, but optimal behaviour is in the task space.
- Possible validation approach → How does the synergy control affect task dimensionality?



$$\mathbf{u}(t) = \sum_{i=1}^{S} w_i \psi_i(t) = \hat{W}\Psi(t)$$

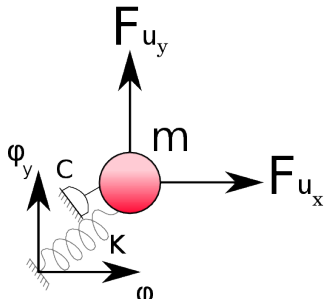
Key Insight : Under synergy control, behaviour is described by an equivalent dynamical system parameterised by the weights

PROPOSAL :

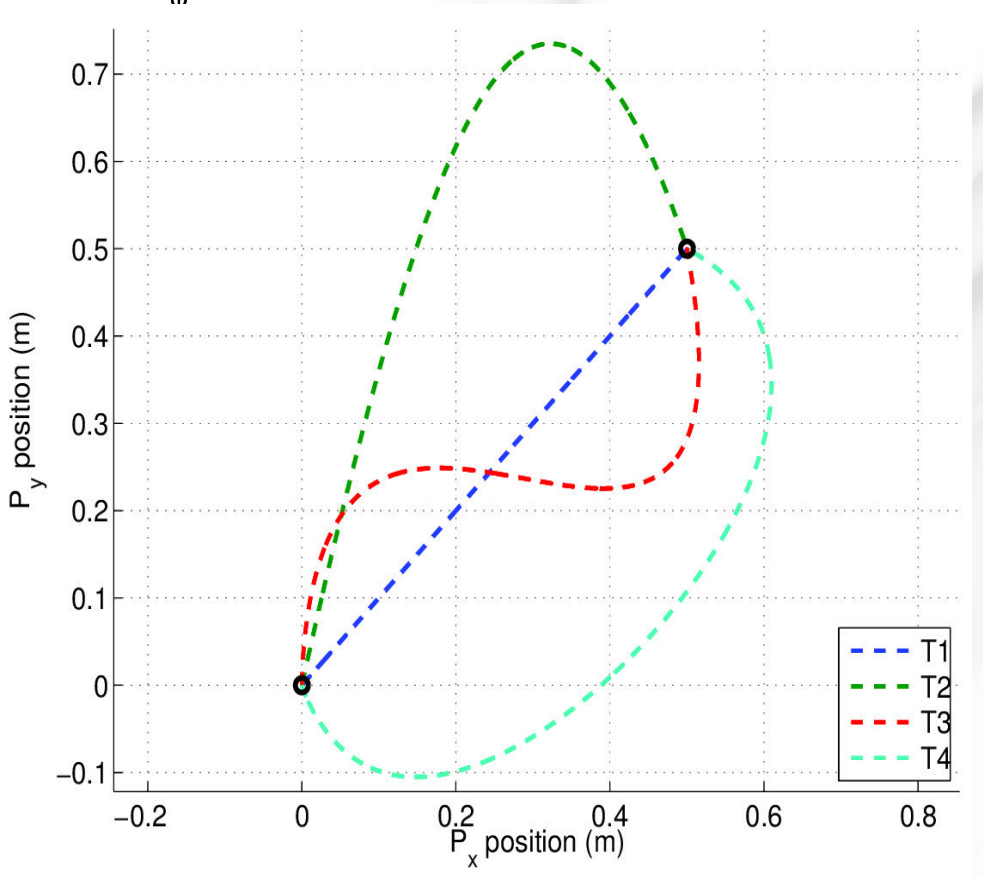
1. **TSDA** : Trajectory Specific Dimensionality Analysis
2. **MDC** : Minimum Dimensional Control

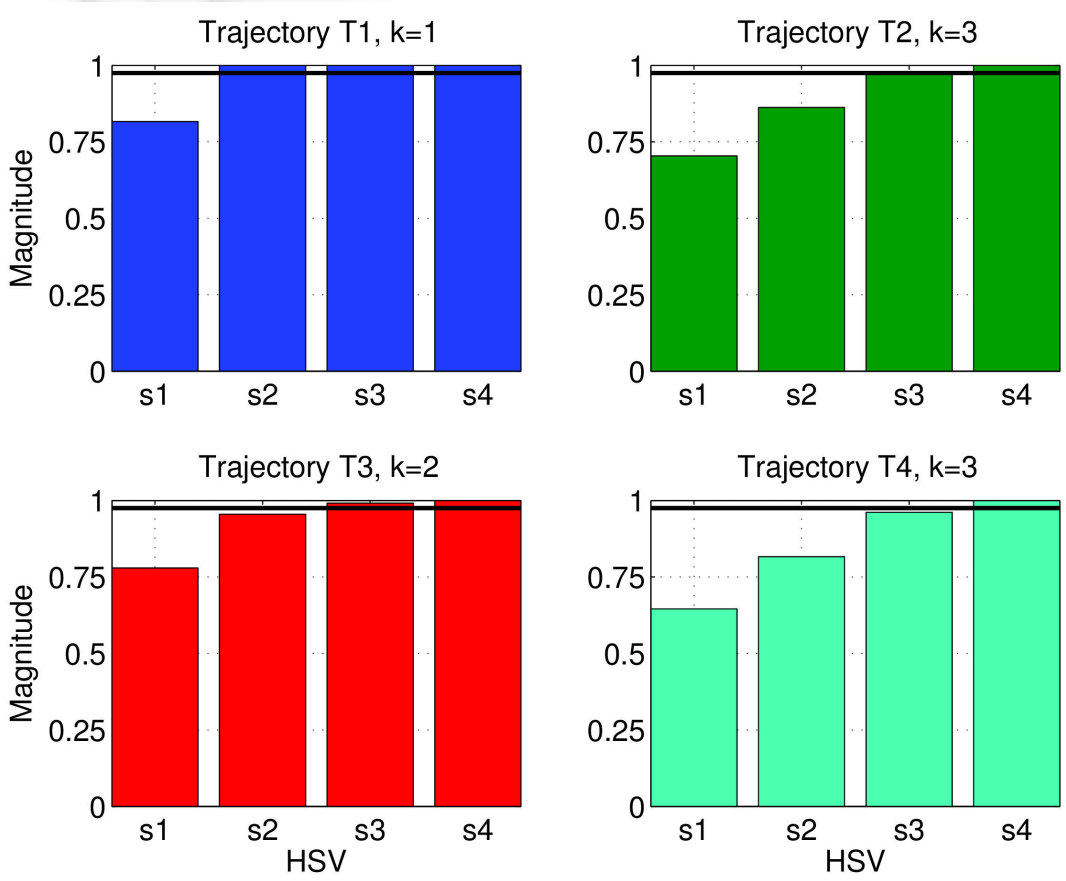
Muscle Synergies

Kuppuswamy, and Harris (2013) "Do muscle synergies reduce the dimensionality of behaviour?", Frontiers in Computational Neuroscience (submitted).



TSDA on a linear system : 2D *tethered* mass.
Synergies : Normalised polynomials, Fourier bases





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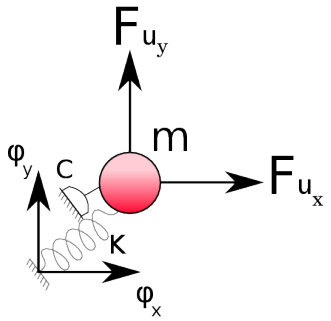
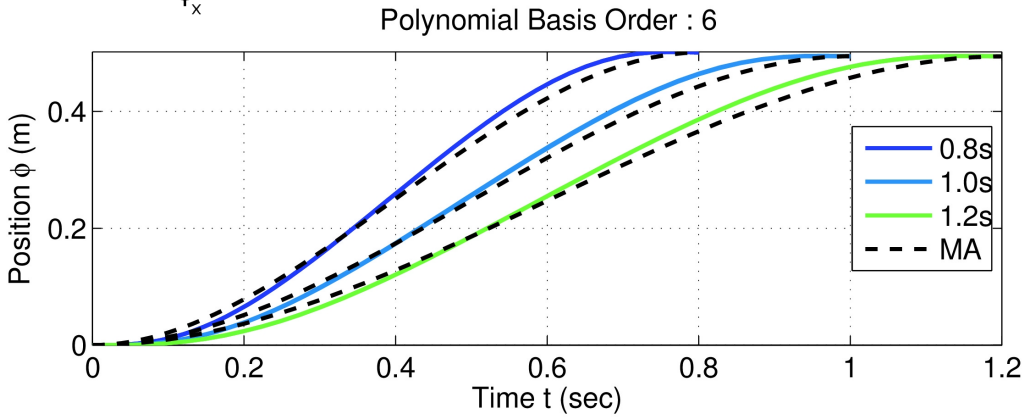
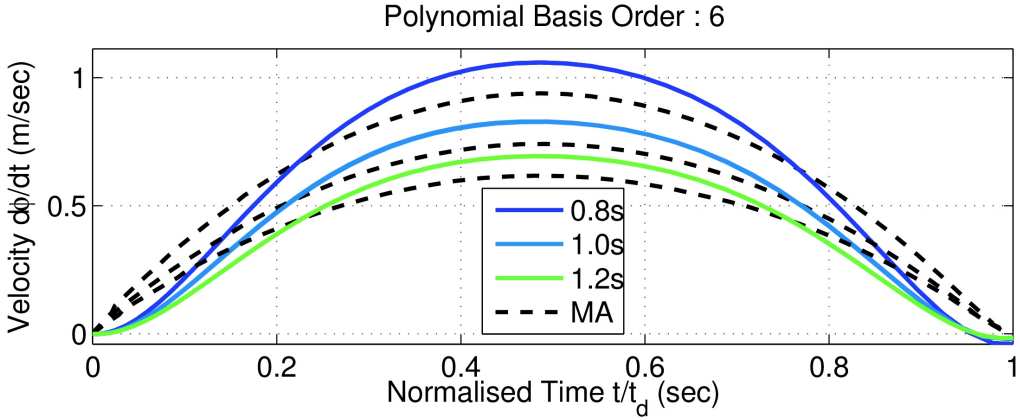
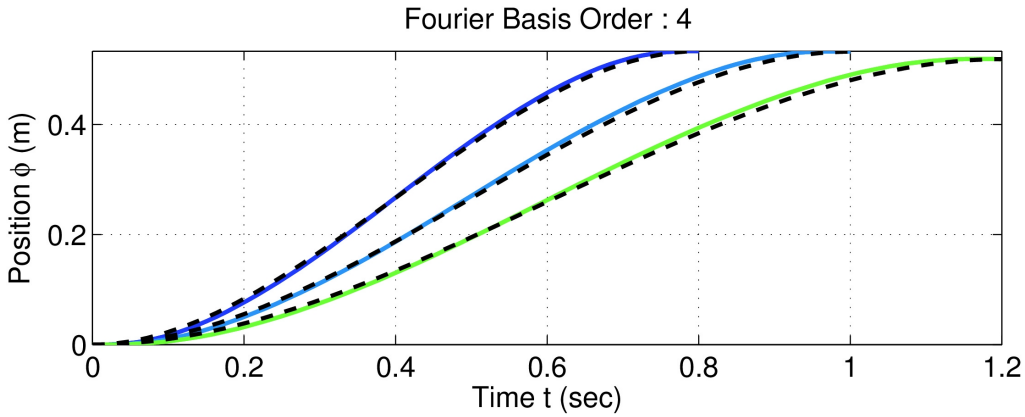
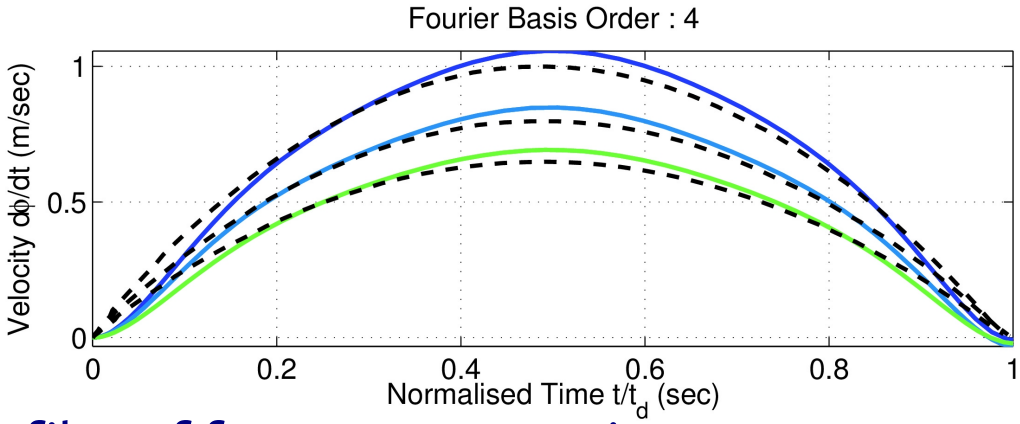


MDC on a linear system : Point-to-point *reaching* task.
Synergies : Normalised polynomials, Fourier bases







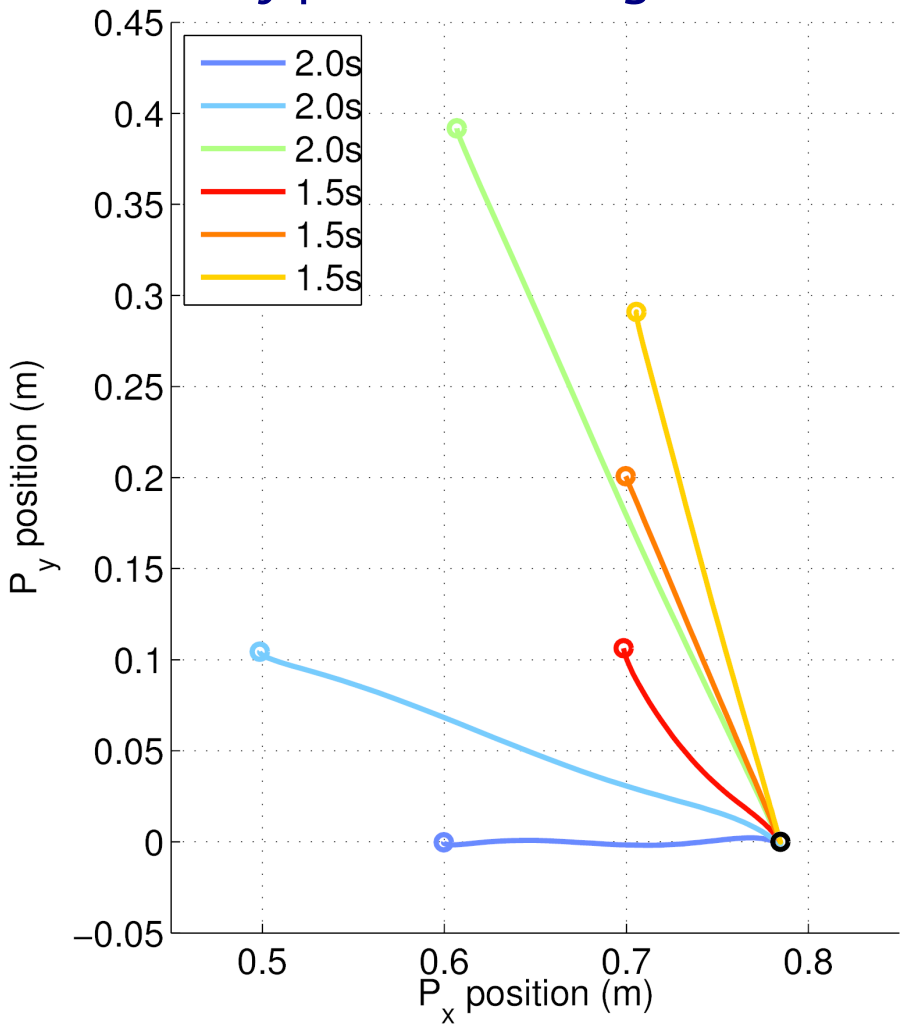


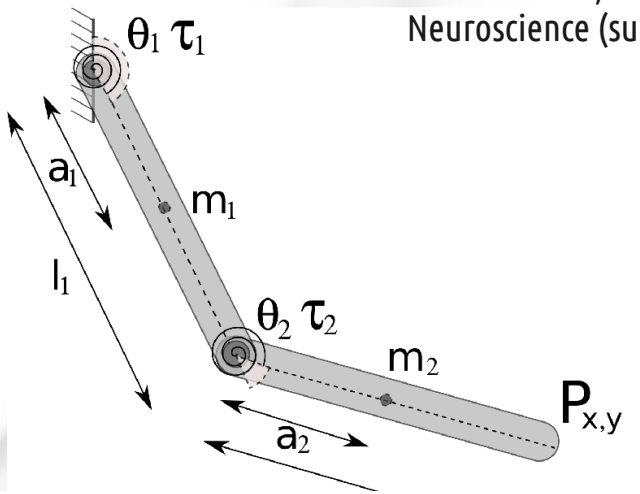
Very similar to the velocity profiles of fast movements in nature!

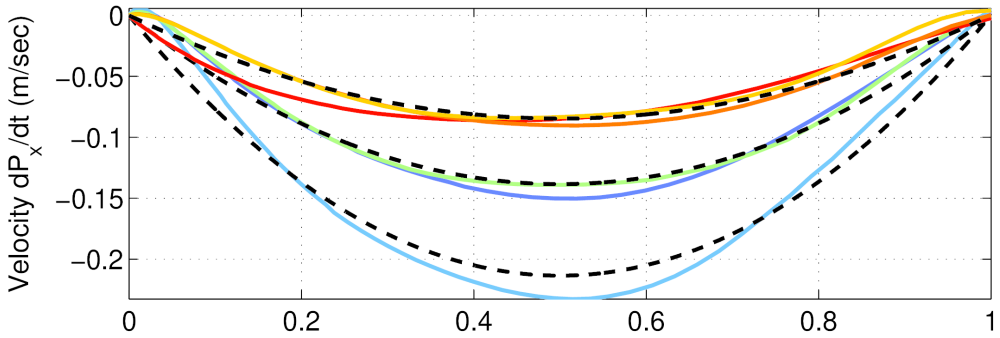
Muscle Synergies

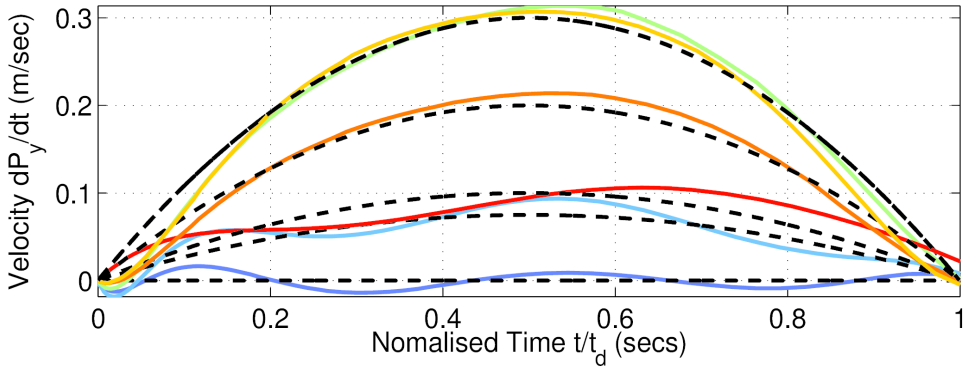
Kuppuswamy, and Harris (2013) "Do muscle synergies reduce the dimensionality of behaviour?", Frontiers in Computational Neuroscience (submitted).

MDC on compliant kinematic chain :
Smooth Straight lines with similar velocity profiles emerge!









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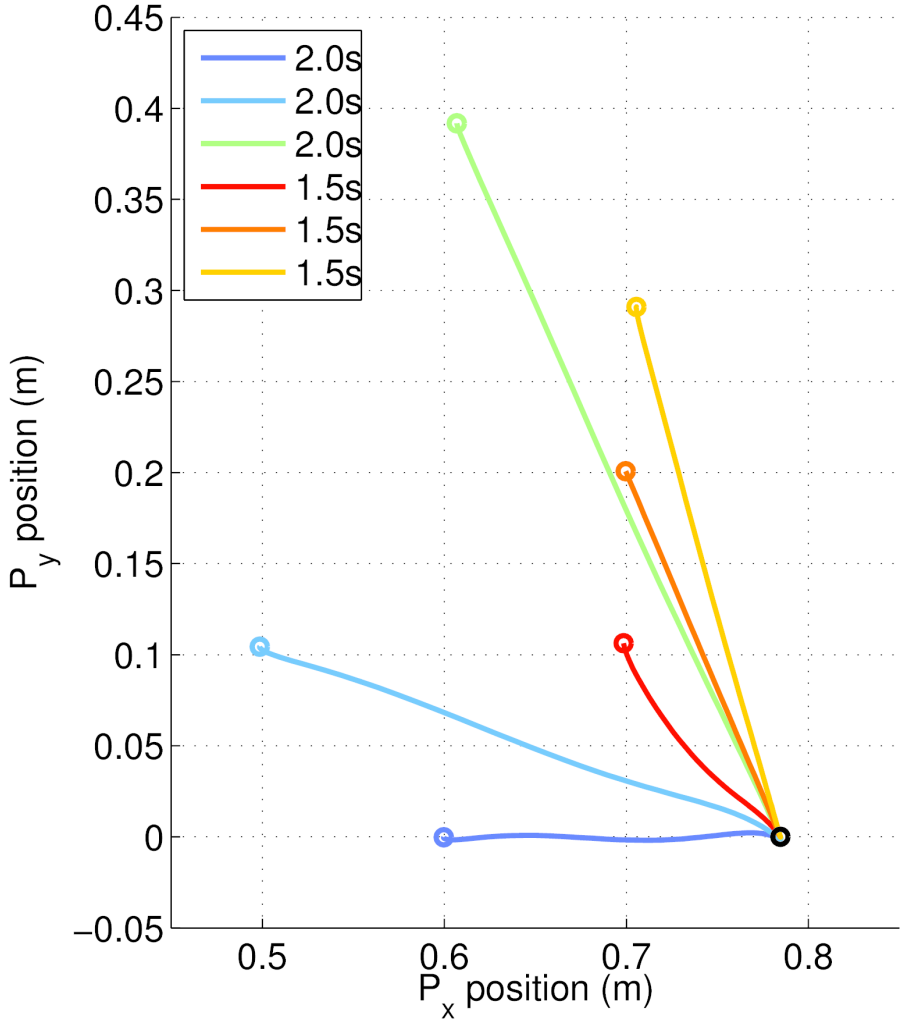
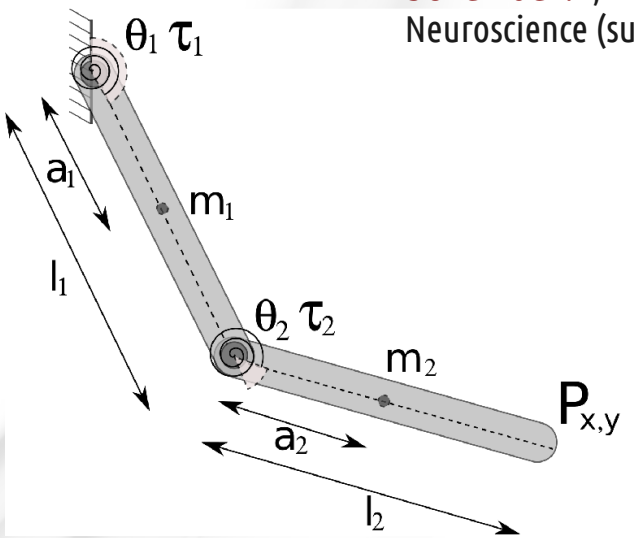
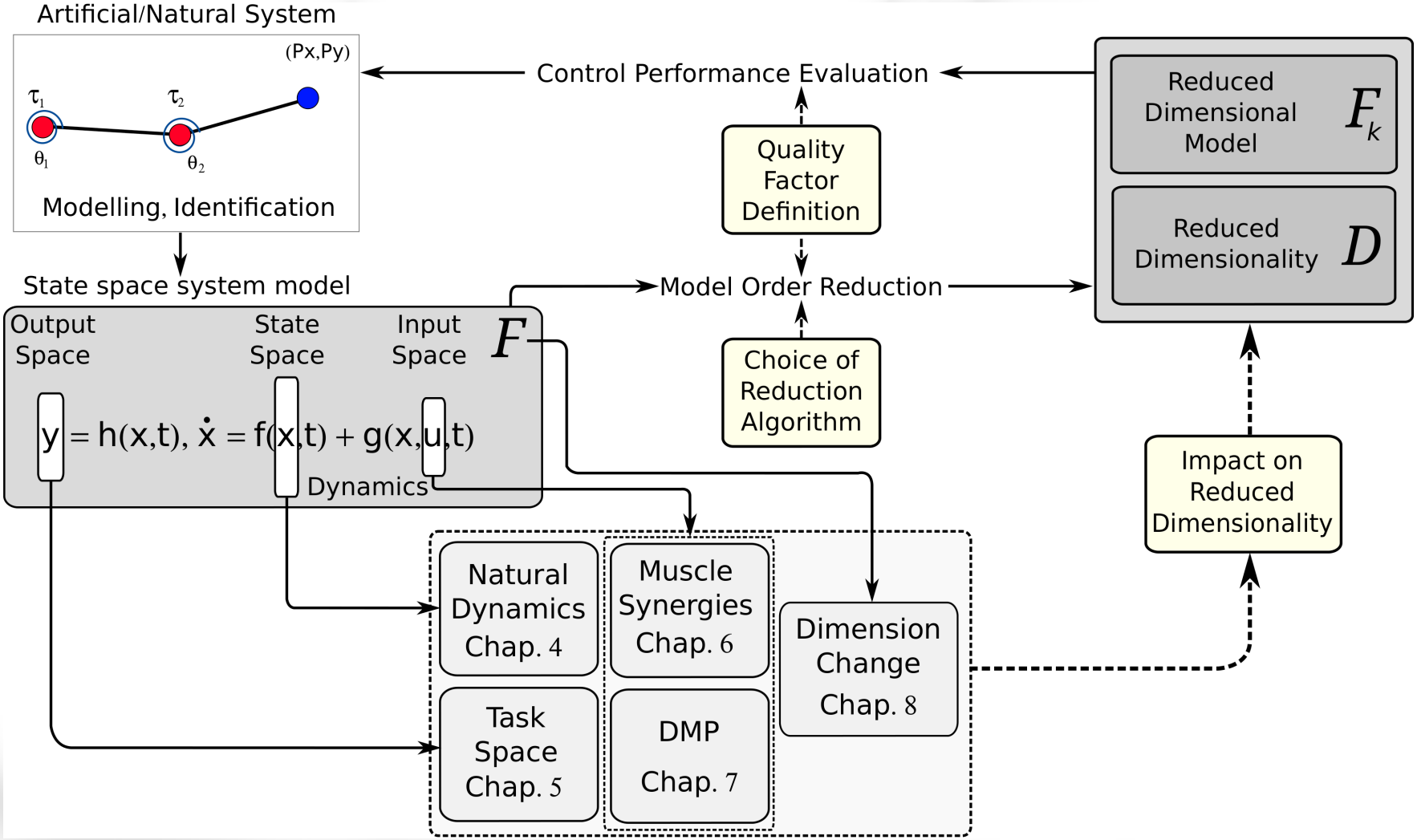




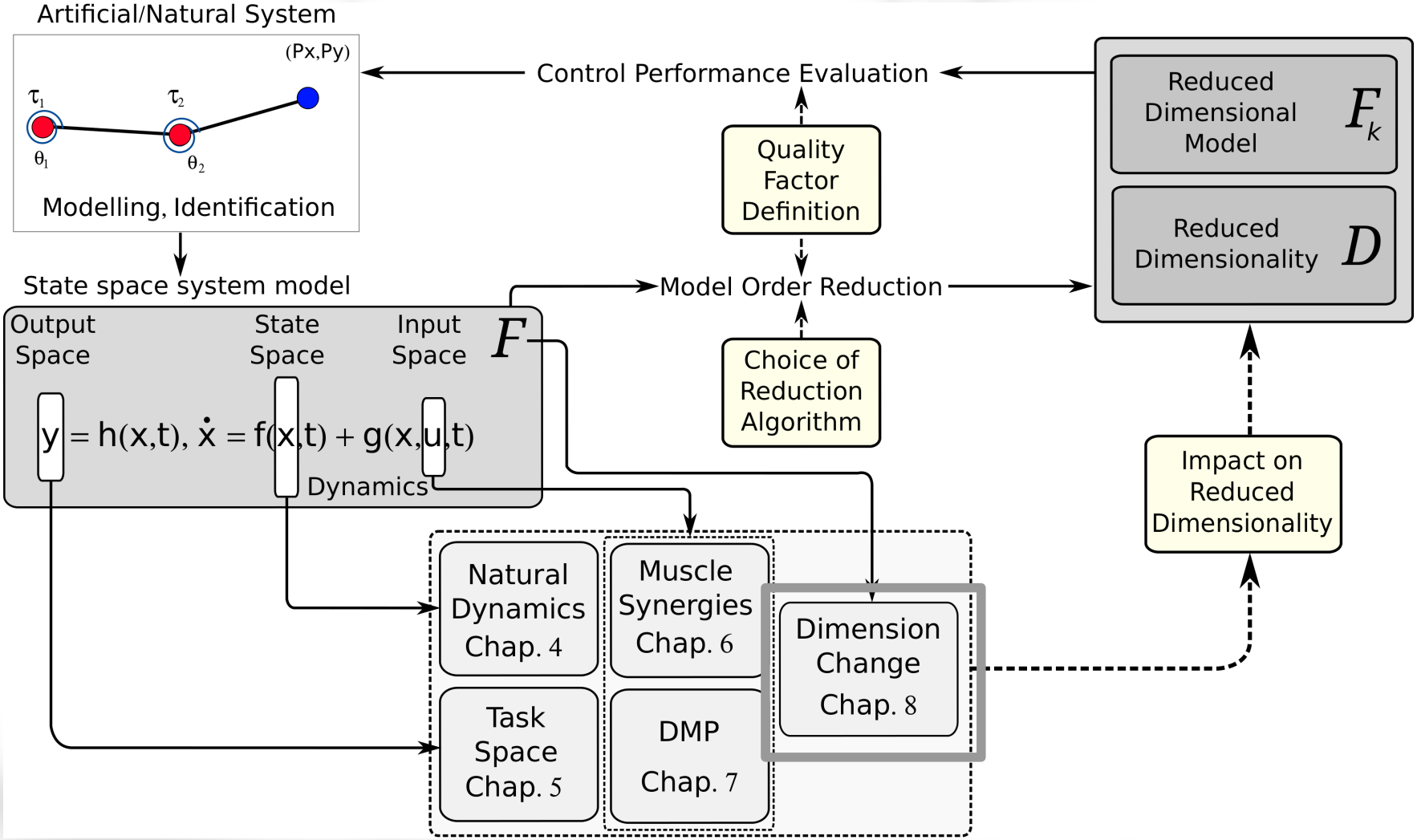
Implications of the Result :

- Method is a testable approach for quantifying task dimensionality when using synergies.
- Can be used for validation of the muscle synergy hypothesis
- MDC might be an underlying principle for biological movements → rationale is the circumvention of *curse of dimensionality* during development.

Exploiting Reduced Dimensionality



Dimension Change with Development

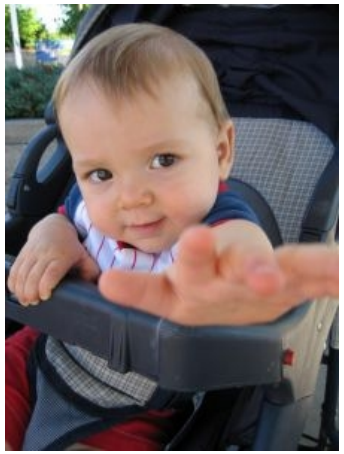


Kuppuswamy, Oesinghaus, and Harris (2013)
"Development and Dimensionality Reduction",
(to be submitted).

Dimensional Change

- Learnability → important constraint on development of motor skills
- Affects evolutionary fitness
- Reduced dimensionality can circumvent this problem.
- What is the underlying mechanism?
- What is the direction of Dimensional Change?

Proximodistal reaching evidence in infants



Bernstein's 3 stage model

Reduce Peripheral DoF to minimum – freeze the DoF

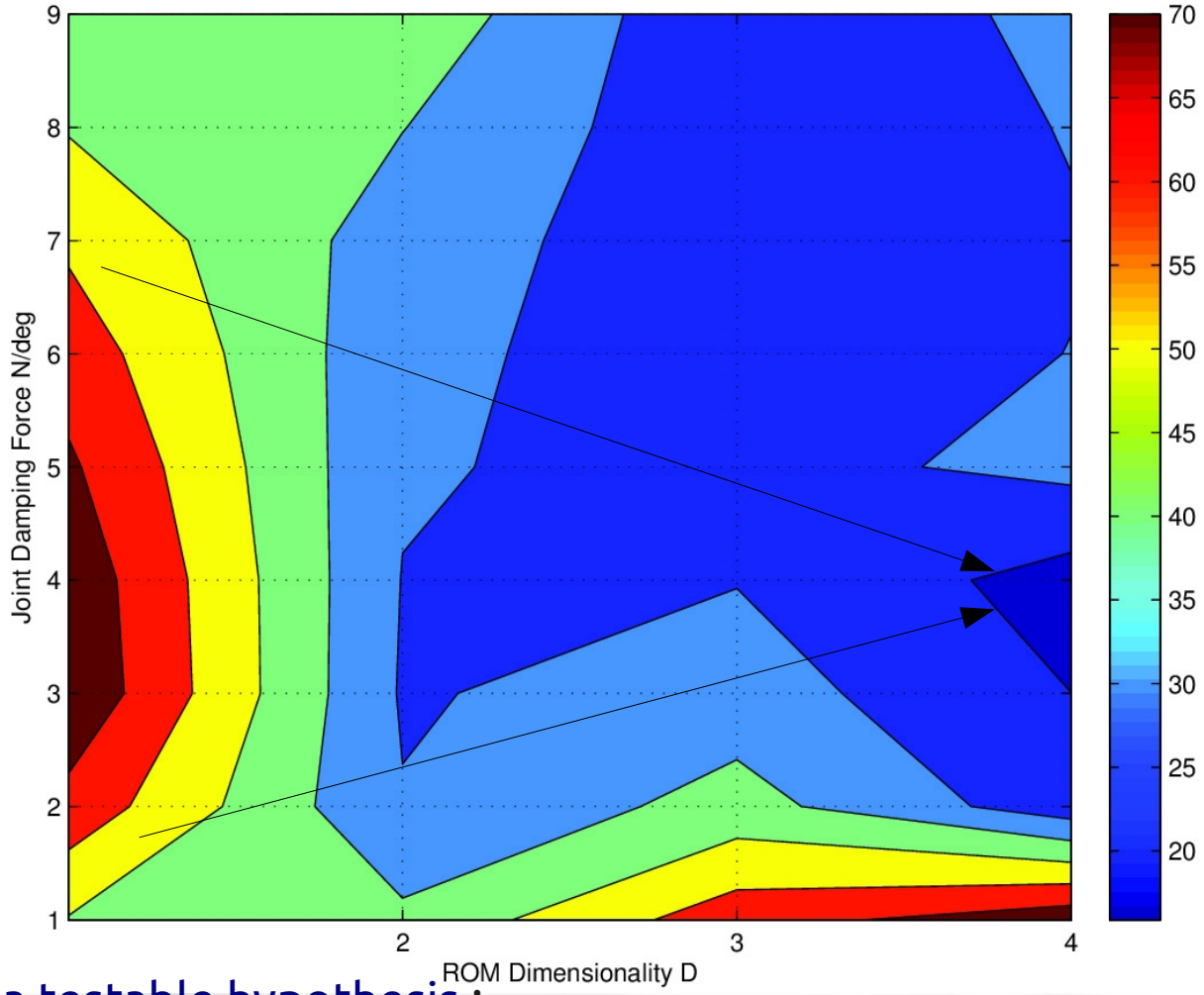
↓

Progressively release restrictions on DoF – Unfreeze DoF

↓

Explore and exploit reactive phenomena in movement control

Dimension Change with Development



Effect of Joint Damping on reaching error – progressive increase in controller dimensionality

Towards a testable hypothesis :

- Dimension change due to passive properties → Optimal trajectories in parametric space facilitate progressive improvements in quality of behaviour.
- Methodology has developmental robotics applications.

Thesis Research Question

What is reduced dimensionality and how can it be exploited for the design and control of embodied systems?

Mathematical Quantification of Reduced Dimensionality

Systematic exploration of the factors affecting Reduced Dimensionality

Conclusions : Principle of Reduced Dimensionality

- Reduced Dimensionality is an important principle that must be exploited in autonomously learning embodied systems.
- Task-specific Reduced Dimensionality enables the dimensionality problem to be overcome without losing out on behavioural diversity

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> **Principle of Reduced Dimensionality**
> Systems undergoing autonomous development of abilities must be built in a manner that the reduced dimensionality can be regulated in a task-specific manner

Conclusions

- Outlook for Robotics
  - Tractability of model based control methods for complex robots
  - Extending movement primitive ideas into compliant platforms
  - Developmental systems must consider Reduced Dimensionality principles
- Outlook for Neuroscience
- Towards testable hypothesis for validation of :
  - Muscle Synergy Hypothesis
  - Developmental acquisition of motor skills

Thanks for all the fish...

RobotDoC

Robotics for Development of Cognition

AMARSi

Questions?