Topic 10 - Diagnostics

STAT 525 - Fall 2013

Outline

- Scatterplots
- Correlation Matrix
- Residual Plots
- Tests
- Remedies
- Example

Diagnostics

- Diagnostics play a key role in both the **development and assessment** of multiple regression models
- Most of the previous diagnostics carry over to multiple regression
- However, given more than one predictor, must also consider relationship between predictors
- Specialized diagnostics discussed later in Chpts 9 and 10

Scatterplots

- Scatterplot matrix summarizes bivariate relationships between Y and X_j as well as between X_j and X_k $(j, k = 1, 2, ..., p-1)$
  - Nature of bivariate relationships
  - Strength of bivariate relationships
  - Detection of outliers
  - Range spanned by X's
- Scatterplot matrix cmbines many scatterplots
- Examples presented later in this topic

Correlation Matrix

- Complementary summary
- Displays all pairwise correlations
- When interpreting, be wary of
  - Nonlinear relationships
  - Outliers
  - Influential observations

Residual Plots

- Used for similar assessment of assumptions
  - Model is "correct"
  - Errors are Normally distributed
  - Errors have constant variance
  - Errors are independent
- Plot e vs $\hat{Y}$ (overall)
- Plot e vs X_j (with respect to X_j)
- Plot e vs non-included variable (e.g., $X_j X_k$)

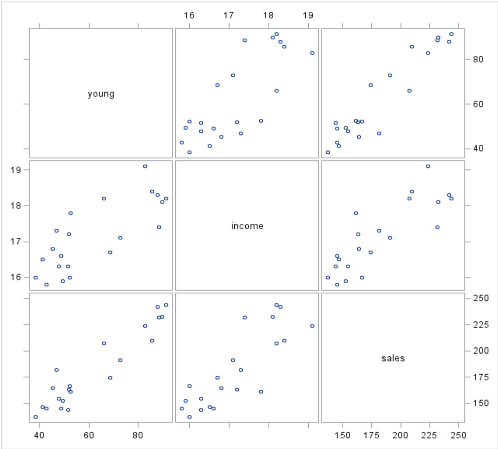
Tests

- Univariate graphical summaries of e still preferred
- NORMAL option in UNIVARIATE test normality
- Modified Levene's and Breusch-Pagan for constant variance
- Lack of fit test : But need repeat observations where all X fixed at same levels or can be comfortably grouped together....this hinders its applicability

Example I - Dwaine Studios (pg 236)

- Company that specializes in portraits of children. It has studios in 21 medium-sized cities nationwide and is considering expansion into other cities.
- Goal: To investigate whether sales are associated with certain characteristics of the city. If so, this could help in determining where to expand.
- Variables:
  - Annual sales (Y) - expressed in thousands of $
  - Persons aged 16 and younger (X_1) - expressed in thousands
  - Per capita disposable income (X_2) - expressed in thousands of $

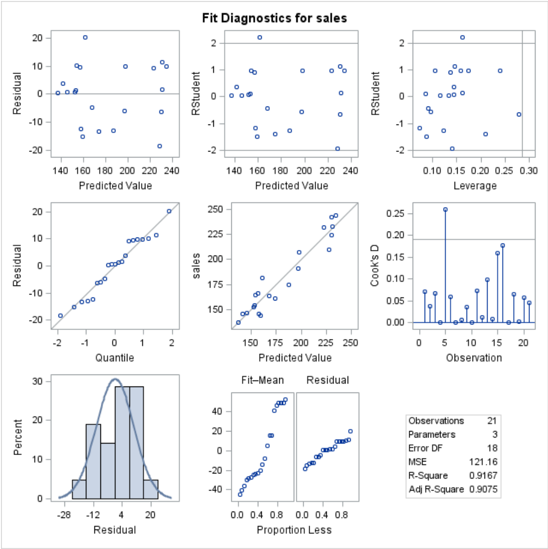
Scatterplot Matrix



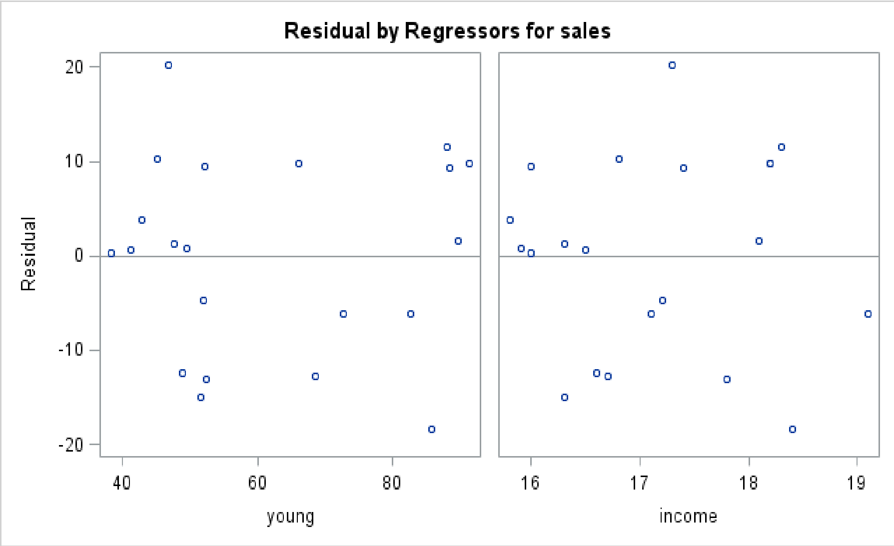
Correlations

```
proc corr data=a1;
var young income sales;

     Pearson Correlation Coefficients, N = 21
             Prob > |r| under H0: Rho=0
                 young     income     sales
   young      1.00000    0.78130   0.94455
                          <.0001    <.0001
  income      0.78130    1.00000   0.83580
               <.0001               <.0001
   sales      0.94455    0.83580   1.00000
               <.0001     <.0001
```


Model Fit Diagnostics



Residual Plot Panel



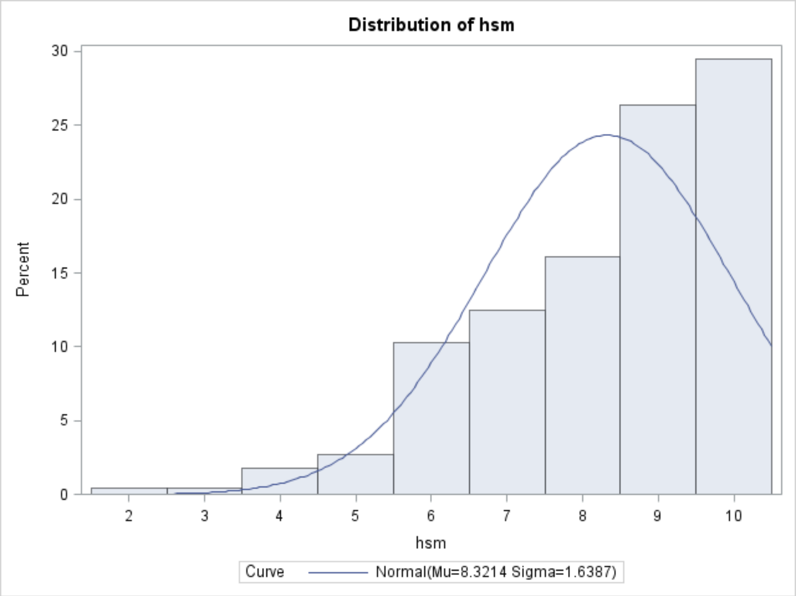
Example II - Predict Success?

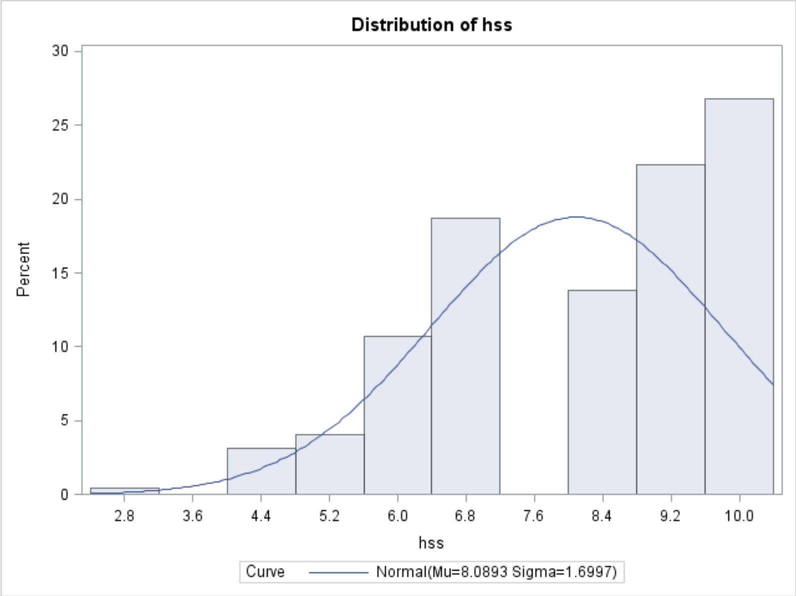
- Goal: To find entry-level predictors of academic success
- Define academic success as high GPA after 3 semesters
- Predictors include
  - GPA after three semesters
  - HS math grades
  - HS science grades
  - HS english grades
  - SAT Math
  - SAT Verbal
- Data available on $n = 224$ students

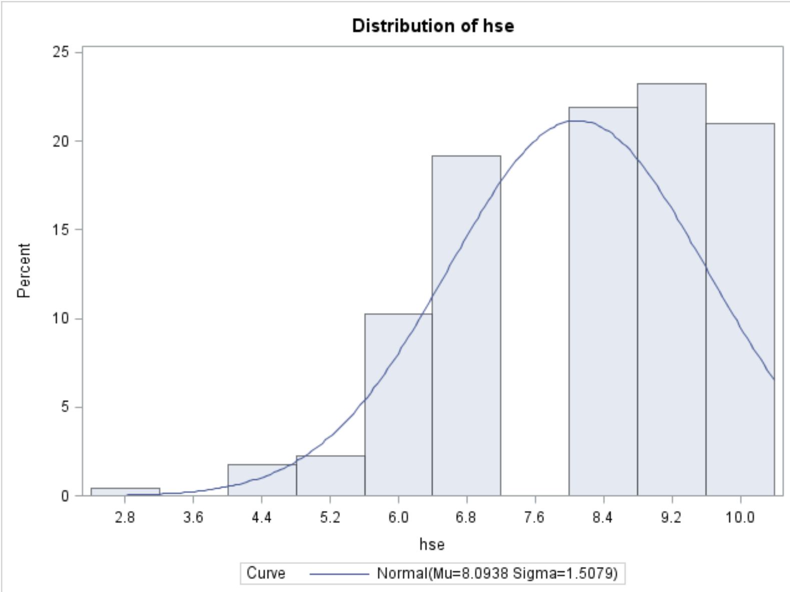
Descriptive Statistics

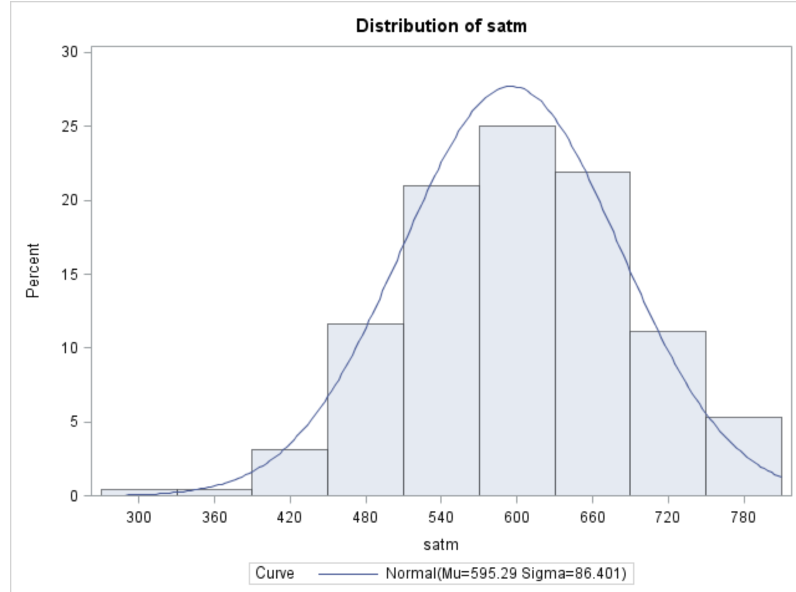
- Using Proc MEANS or Proc UNIVARIATE

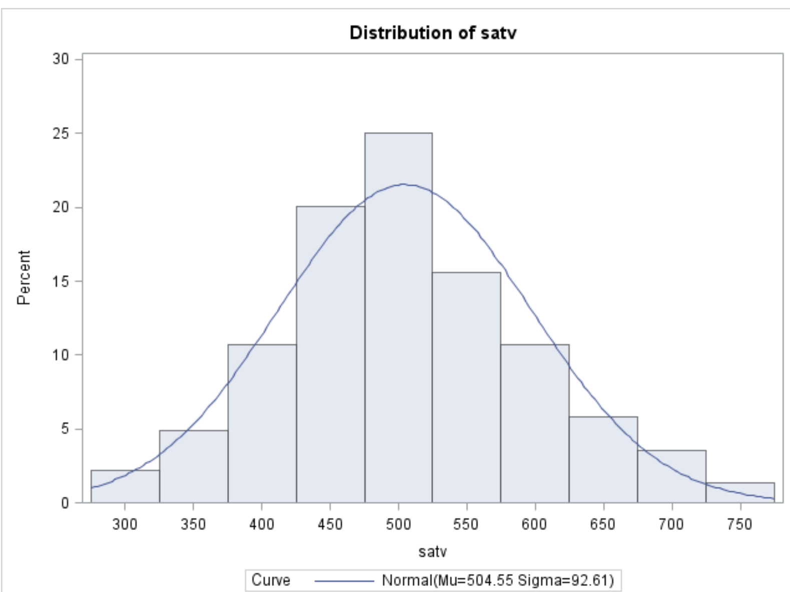
Var	N	Mean	Std Dev
gpa	224	2.64	0.78
hsm	224	8.32	1.64
hss	224	8.09	1.70
hse	224	8.09	1.51
satm	224	595.29	86.40
satv	224	504.55	92.61

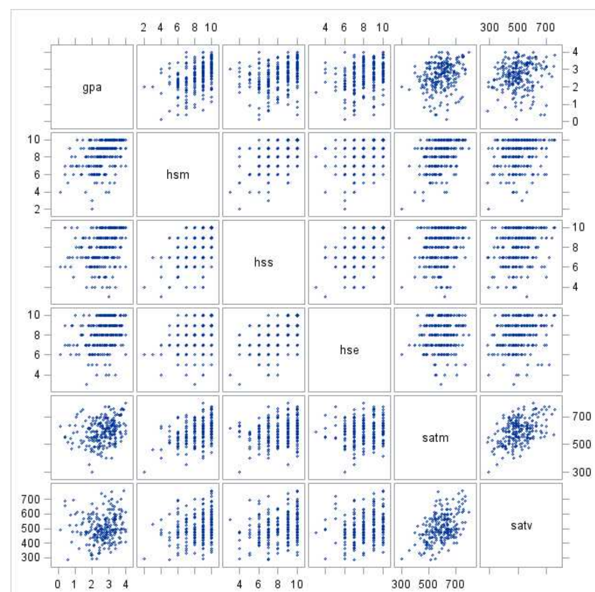





Distribution of hse
Percent
hse
2.8 3.6 4.4 5.2 6.0 6.8 7.6 8.4 9.2 10.0
0 5 10 15 20 25
Curve
Normal(Mu=8.0938 Sigma=1.5079)


Distribution of satm
Percent
satm
300 360 420 480 540 600 660 720 780
0 5 10 15 20 25 30
Curve
Normal(Mu=595.29 Sigma=86.401)


Distribution of satv
Percent
satv
300 350 400 450 500 550 600 650 700 750
0 5 10 15 20 25 30
Curve
Normal(Mu=504.55 Sigma=92.61)


gpa
hsm
hss
hse
satm
satv

Correlations

```
[1] proc corr data=a1;
    var hsm hss hse;

          hsm    hss    hse
    hsm  1.00   0.57    0.44
               <.0001  <.0001
    hss  0.57   1.00    0.57
       <.0001          <.0001
    hse  0.44   0.57   1.00
       <.0001 <.0001

[2] proc corr data=a1;
    var satm satv;

          satm     satv
    satm  1.00     0.46
                  <.0001
    satv  0.46     1.00
        <.0001
```


Correlations

```
[3] proc corr data=a1;
    var hsm hss hse satm satv;
    with gpa;

           hsm    hss     hse
     gpa  0.43   0.32    0.28
        <.0001 <.0001  <.0001


          satm    satv
    gpa   0.25    0.11
        0.0001  0.0873
```


Regression Models

- Will now investigate:

  Model 1: GPA = HSM HSS HSE

  Model 2: GPA = HSM HSE

  Model 3: GPA = HSM

  Model 4: GPA = SATM SATV

  Model 5: GPA = HSM HSS HSE SATM SATV

- Should check residuals prior to any inference

Model 1

```
                         Analysis of Variance
                                Sum of       Mean
Source               DF        Squares     Square  F Value  Pr > F
Model                 3       27.71233    9.23744    18.86  <.0001
Error               220      107.75046    0.48977
Corrected Total     223      135.46279

Root MSE              0.69984    R-Square     0.2046
Dependent Mean        2.63522    Adj R-Sq     0.1937
Coeff Var            26.55711

                         Parameter Estimates

                    Parameter    Standard
Variable      DF     Estimate       Error    t Value     Pr > |t|
Intercept      1      0.58988     0.29424       2.00       0.0462
hsm            1      0.16857     0.03549       4.75       <.0001
hss            1      0.03432     0.03756       0.91       0.3619
hse            1      0.04510     0.03870       1.17       0.2451
```

Model 2

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	27.30349	13.65175	27.89	<.0001
Error	221	108.15930	0.48941		
Corrected Total	223	135.46279			

Root MSE	0.69958	R-Square	0.2016
Dependent Mean	2.63522	Adj R-Sq	0.1943
Coeff Var	26.54718		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	0.62423	0.29172	2.14	0.0335
hsm	1	0.18265	0.03196	5.72	<.0001
hse	1	0.06067	0.03473	1.75	0.0820

Model 3

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	25.80989	25.80989	52.25	<.0001
Error	222	109.65290	0.49393		
Corrected Total	223	135.46279			

Root MSE	0.70280	R-Square	0.1905
Dependent Mean	2.63522	Adj R-Sq	0.1869
Coeff Var	26.66958		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	0.90768	0.24355	3.73	0.0002
hsm	1	0.20760	0.02872	7.23	<.0001

Model 4

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	8.58384	4.29192	7.48	0.0007
Error	221	126.87895	0.57411		
Corrected Total	223	135.46279			

Root MSE	0.75770	R-Square	0.0634
Dependent Mean	2.63522	Adj R-Sq	0.0549
Coeff Var	28.75287		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	1.28868	0.37604	3.43	0.0007
satm	1	0.00228	0.00066291	3.44	0.0007
satv	1	-0.00002456	0.00061847	-0.04	0.9684

Model 5

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	28.64364	5.72873	11.69	<.0001
Error	218	106.81914	0.49000		
Corrected Total	223	135.46279			

Root MSE	0.70000	R-Square	0.2115
Dependent Mean	2.63522	Adj R-Sq	0.1934
Coeff Var	26.56311		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	0.32672	0.40000	0.82	0.4149
satm	1	0.00094359	0.00068566	1.38	0.1702
satv	1	-0.00040785	0.00059189	-0.69	0.4915
hsm	1	0.14596	0.03926	3.72	0.0003
hss	1	0.03591	0.03780	0.95	0.3432
hse	1	0.05529	0.03957	1.40	0.1637

General Linear Test

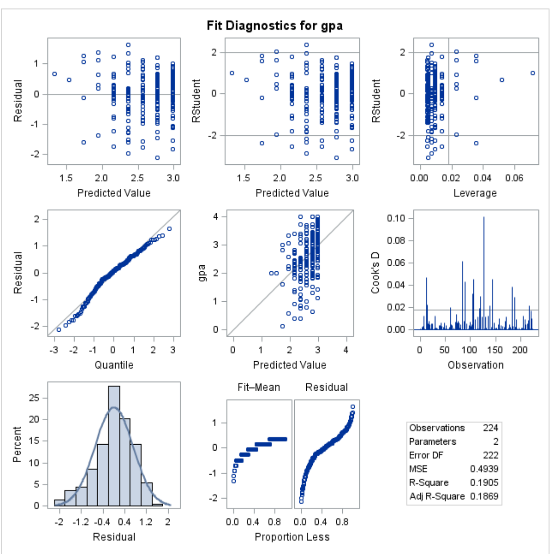
- Can use TEST statement in SAS

```
proc reg data=a1;
  model gpa=satm satv hsm hss hse;
  sat: test satm, satv;
  hs: test hsm, hss, hse;

Test sat
  Results for Dep Var gpa
             Mean
Source   DF  Square    F Pr > F
Num       2 0.46566 0.95 0.3882
Den     218 0.49000

Test hs
  Results for Dep Var gpa
            Mean
Source  DF  Square     F      P
Num      3 6.68660 13.65 <.0001
Den    218 0.49000
```


What's the Best Model?

- Will discuss selection approaches in Chpts 8,9, and 10
- Appears HSM only is best model
- Should also be looking at diagnostics
- Important:
  - Look at variables one at a time
  - Look at all pairwise relationships
  - PLOT! PLOT! PLOT!

Model Fit Diagnostics



Key Results

- The relationship between Y and X_j depends on the other predictors in the model
- A predictor may be significant alone but not significant when other variables are in the model
- Similarly, coefficients and standard errors depend on the variables that are in the model

Background Reading

- KNNL Sections 6.8-6.9
- KNNL Sections 7.1-7.3