Professor Craig,

There seems to be some confusion about our project.

I will address 3 main concerns you've raised below -

1. There will automatically be an association between funds & that won't be interesting
2. The procedure to determine buckets in the contingency table
3. Lack of utility in real life.

Regards 1, 3:

ETF(Exchange Traded Funds) are the primary investment vehicle of the entire nation, since 401K, retirement & pension funds directly invest in the SPY ETF. The SPY is a weighted sum of 500 stocks. The purpose of weighting is to ensure diversification.

Our project <u>does NOT measure association among the ETFs, or among the ETF & its constituent stocks</u> – such association would be, as you noted, automatic & expected.

We are attempting to **<u>measure association between the ETF and an artificial metric</u>** that attempts to mimic the standard deviation of this ETF. For a long time, such a metric did not exist. It was unclear how to measure the standard deviation of a moving time series. It was clear that the ETF was volatile, because the individual stocks were volatile, so their weighted sum should be volatile as well. However, diversification theory postulates the weighted sum of X[i] will be less volatile than each of the individual X[I], i = 1..500.

However, there was no consensus on what time window to use in order to measure this volatility, nor which price (price at which stock opens, or closes, or average, max, min) would be most suitable. So there was no metric for an ETF's standard deviation.

In 1973, Professor Robert Whaley (Vanderbilt U) published a seminal article in the "Journal of Derivatives" that kicked off a trillion-dollar industry. He claimed he had invented a metric to measure the standard deviation (volatility) of an ETF.

Excerpts from this journal article are shown below -

DERIVATIVES ON MARKET VOLATILITY: HEDGING TOOLS LONG OVERDUE

Robert E. Whaley

The Chicago Board Options Exchange Market Volatility Index (ticker symbol VIX), which is based on the implied volatilities of eight different OEX option series, represents a market consensus forecast of stock market volatility over the next thirty calendar days. The Volatility Index can help the investment community in at least two important ways.

First, it provides a reliable estimate of expected short-term stock market volatility.[1] Expected market volatility is a critical piece of information to many investment decisions; the asset allocation decision is one. Second, it offers a market volatility "standard" upon which derivative contracts may be written. Such a standard must be based on a highly liquid underlying security market. In the case of VIX, the underlying security market is the OEX options market — by far the most active index option market in the U.S.

His artificial metric, so-called VIX, is computed using a tedious linear computation involving 8 derivatives. His original calculation is shown below -

INDEX CONSTRUCTION

The CBOE Market Volatility Index is constructed from the implied volatilities of the eight near-the-money, nearby, and second nearby OEX option series. The nearby OEX series are defined as the series with the shortest time to expiration but with at least eight calendar days to expiration.[25] The second nearby OEX series are the series of the adjacent contract month.

To explain the index construction, we denote the OEX option exercise price just below the current index level, S, as X_l and the exercise price just above the current index level as X_u. The implied volatilities of the nearby and second nearby OEX options are thus:

	Nearby Contract (1)		Second Nearby Contract (2)	
	Call	Put	Call	Put
$X_l\ (< S)$	$\sigma_{c,1}^{X_l}$	$\sigma_{p,1}^{X_l}$	$\sigma_{c,2}^{X_l}$	$\sigma_{p,2}^{X_l}$
$X_u\ (\geq S)$	$\sigma_{c,1}^{X_u}$	$\sigma_{p,1}^{X_u}$	$\sigma_{c,2}^{X_u}$	$\sigma_{p,2}^{X_u}$

The first step in computation of the index level is to average the call and put implied volatilities in each of the four categories of options, that is,

$$\sigma_1^{X_l} = \left(\sigma_{c,1}^{X_l} + \sigma_{p,1}^{X_l}\right)/2 \qquad \text{(A3A)}$$

$$\sigma_2^{X_l} = \left(\sigma_{c,2}^{X_l} + \sigma_{p,2}^{X_l}\right)/2$$

$$\sigma_1^{X_u} = \left(\sigma_{c,1}^{X_u} + \sigma_{p,1}^{X_u}\right)/2 \qquad \text{(A3C)}$$

$$\sigma_2^{X_u} = \left(\sigma_{c,2}^{X_u} + \sigma_{p,2}^{X_u}\right)/2. \qquad \text{(A3D)}$$

Recall that the averaging mitigates the effects that the staleness of the reported stock index level may have on computation of individual call and put option implied volatilities.

Next, interpolate between the nearby implied volatilities and the second nearby implied volatilities to create "at-the-money" implied volatilities. More specifically,

$$\sigma_1 = \sigma_1^{X_l}\left(\frac{X_u - S}{X_u - X_l}\right) + \sigma_1^{X_u}\left(\frac{S - X_l}{X_u - X_l}\right) \qquad \text{(A4A)}$$

$$\sigma_2 = \sigma_2^{X_l}\left(\frac{X_u - S}{X_u - X_l}\right) + \sigma_2^{X_u}\left(\frac{S - X_l}{X_u - X_l}\right) \qquad \text{(A4B)}$$

Finally, interpolate (or, occasionally, extrapolate) between the nearby and second nearby implied volatilities to create a thirty-calendar day (30 − 2 × int(30/7) ≡ twenty-two-trading day) implied volatility. If N_{t_1} is the number of trading days to expiration of the nearby contract, and N_{t_2} is the number of trading days of the second nearby contract, the CBOE Market Volatility Index is

$$VIX = \sigma_1\left(\frac{N_{t_2} - 22}{N_{t_2} - N_{t_1}}\right) + \sigma_2\left(\frac{22 - N_{t_1}}{N_{t_2} - N_{t_1}}\right). \qquad \text{(A5)}$$

Since 1973, this calculation has been revised & updated several times, and market practitioners now claim that for all practical purposes, the VIX as it exists today, is a robust metric for 30-day standard deviation of the SPY.

However, it is not a perfect metric by any means. In general, if the metric was sound, were the metric to go up, standard deviation I.e. volatility is rising, so returns must fall. If the metric goes down, returns must rise. We propose to test if this effect really happens, by counting the number of times this happens over an 18-year period (2001 to 2018). That dataset gives us roughly 5000 days, which we divide into the days when the events played out as claimed, & when they didn't.

To summarize, we are testing the association between the ETF & the artificial metric VIX constructed by Professor Whaley.

Here are some preliminary results:

	V1	V2	V3	V4	V5	V6	V7	V8
1	14	17	65	159	404	516	374	387
2	4	3	13	31	51	42	9	3
3	3	4	15	22	45	33	13	8
4	2	6	12	29	56	24	6	1
5	3	8	19	38	58	31	6	3
6	3	11	14	26	50	25	7	4
7	9	12	10	40	47	13	6	2
8	421	236	304	372	230	78	34	13

We address concern #2 (Bucket construction) using above table:

```
The columns mean
Col 1. < -3%
Col 2. -2 - -3%
Col 3. -1 to -2%
Col 4. 0 to -1%
Col 5. 0 to 1%
Col 6. 1 to 2%
Col 7. 2 to 3%
Col 8. > 3%

The rows mean the same as columns.

Along the column we have the ETF
Down the row we have the volatility index

So for example,
Cell (5,6) = 31 => there were 31 days when vix went up between 0-1%, and spy went up 1-2%
Cell (4,7) = 6 => there were 6 days when vix went up between 0 to -1%, and spy went up 2-3%
```

This 8x8 contingency table can be collapsed into a much simpler 2x2 table shown below:

	V1	V2
1	399	1972
2	1526	600

Of the ~4500 days, there were 1972 days when the vix went down & the SPY ETF went up.

There were 1526 days when the vix went up & the ETF went down. This table vindicates Dr. Whaley's attempt to provide an artificial metric that **concurrently** mimics the standard deviation of the ETF. The odds-ratio indicates statistical significance of the VIX-SPY contingency table & polr models can be fit etc.

However, the counts in the table pertain to the number of days when WEEKLY return of the vix & the WEEKLY return of the ETF simultaneously varied (**concurrently**).

But what if we seek to build a **predictive** model? We compute ETF returns over the next day (future) based on the vix return over the past 2 weeks (past).

Can past vix returns predict the future ETF?

This is a question of enormous real-life utility. The results (shown below) are not promising!!!

	V1	V2
1	1141	1287
2	914	1147

Unlike the strong signal in the concurrent model, the predictive table is no better than a fair coin toss.

At this point, we have built over a 100 tables by varying the time period (past 1 day, 1 week, 2 weeks, 1 month, vs future 1 day, 1 week etc) as well as varying the bucket construction (1% return, 5% , 10% etc). None of them provide anything predictive. They are all significant if the time periods are concurrent.

For now, we propose to restrict model building exercises to the concurrent time period, & continue exploration on the predictive front.

Further, we have 3 ETFs (SPY, DIA & QQQ). Each ETF has a standard deviation metric (vix, vxd, vxn). Thus, we get 3 separate tables & models for the S&P Dow Jones & Nasdaq. Each team member gets an ETF-volatility pair to build separate models.