X = underlying financial instrument
= usually called "underlying"

X_t = spot price of X on date t
= daily closing price of X on date t

Sometimes, we have daily open price as well.

$X.OPENPRC_t$ = open price of X on date t

$X.PRC_t$ = close price of X on date t

t : Denotes a date, <u>not time</u>.

For eg: $t = 3/1/2001$

Sample Space : $X \in \{SPY, DIA, QQQ\}$

a) SPY = An "exchange-traded fund" (ETF)
= proxy of the S&P 500
= $\sum_{i=1}^{500} w_i Y_i$
= weighted sum of 500 stocks
= represents the "whole USA market"

b) DIA = An ETF
= proxy of the Dow Jones
= $\sum_{i=1}^{30} w_i Y_i$
= weighted sum of 30 "large" stocks
= represents only "large industries"

c) QQQ = An ETF
= proxy of Nasdaq Index
$$= \sum_{i=1}^{100} w_i Y_i$$
= weighted sum of 100 stocks on Nasdaq
= represents mostly "tech stocks".

v = "volatility index" corresponding to underlying

d) vxd ⟺ DIA
e) vxn ⟺ QQQ
f) vix ⟺ SPY

} is the correspondence

Volatility index measures "implied volatility of the options on the underlying". For our purposes, volatility index = good estimator of $\hat{\sigma}$ = std deviation

So loosely speaking,
$$vxd \approx \hat{\sigma}_{DIA}$$
$$vxn \approx \hat{\sigma}_{QQQ}$$
$$vix \approx \hat{\sigma}_{SPY}$$

Since prices change continously, hence volatility indices are computed over short term.
"short term" = 30-day window.

Volatility index first invented by Prof. Whaley, in Fall 1993 (Journal of Derivatives)

Premise: "volatility index inversely correlated with underlying."
when "vol-index spikes, underlying crashes"

Goal: Explore this premise using contingency tables, glm's incl. Poisson Regression, multinomial logit models & other material in STAT 526.

Problems: Same as Exam 2, Problem 1+2.

vxd	dia < -10%	dia -5 - -10%	dia 0 - -5%	dia 0 - 5%	dia 5 - 10%	dia > 10%
< -10%						
-5 - -10%						
0 - -5%						
0 - 5%		//////				
5 - 10%						
> 10%						

We build a 6x6 contingency table with ordinal entries.

$\text{cell}_{(i,j)}$ = # of times when both i & j happened.

eg.

$\text{cell}_{(4,2)}$ = The number of times when vxd monthly return was between 0% and +5% while underlying DIA monthly return was between -5% and -10%

(shaded cell)

Cell entries are counts, not prices.
Cell entry is a nominal variable.
To find these cell entries:

Take any 30-day window.

e.g. X_t, X_{t+30}, v_t, v_{t+30}

where X = underlying, v = volatility index
say X = DIA, v = vxd, t = 3/1/2001

Then t_1 = 3/1/2001
t_2 = $t_1 + 30$ = 4/1/2001

Compute: $$\frac{\text{DIA.PRC}_{4/1/2001} - \text{DIA.PRC}_{3/1/2001}}{\text{DIA.PRC}_{3/1/2001}} = y$$

Compute: $$\frac{\text{vxd}_{4/1/2001} - \text{vxd}_{3/1/2001}}{\text{vxd}_{3/1/2001}} = x$$

Say $x = -4\%$, $y = 7\%$, $\Rightarrow$ cell(3,5) + 1
say $x = 2\%$, $y = -3\%$ $\Rightarrow$ cell(4,3) + 1
etc.
(All cell(i,j) start at 0.)

a) Since we have 3 underlying & 3 vol index, we have 3 contingency tables, 6x6 each.

b) Instead of 30-day window, we can also take 7-day window, to get dramatically different results.

c) Questions we can answer given this 6x6 table are same as Exam 2, Q1 & Q2.

1) pdr model, multinom models
2) glm model fits, comparisons of model,
3) cumulative logit model
4) compute conditional distribution of dia, if vxd goes up 5% in 30 day window.
5) Build a 2x2 contingency table by compressing 6x6 table like this -

		dia > 0%	dia < 0%
vxd	Return > 0%		
	Return < 0%		

Then compute odds ratio, statistical ⊥, predicted counts, best fitting log linear models.