Bayes-Stein Estimation for Portfolio Analysis

Philippe Jorion, JFQA, Sep 1986

Krishnan Raman

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Prelude: Stein's Paradox

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- Individual averages that exceed grand mean shrink, those that are smaller inflate.

Stein's Paradox, Dr. Richard Samworth , Statslab Cambridge

- **Data, Distribution, Parameter**
  $X_i \sim N(\theta_i, 1), i = 1..p, p > 2, \theta = (\theta_1, \theta_2, \ldots, \theta_p)'$

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  MLE & UMVUE: $\hat{\theta}^0(X) = (X_1, X_2, \ldots, X_p)'$
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- Roy's idea: Maximize $\frac{\mu_p - d}{\sigma_p^2}$, d = disaster return

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- Output of the Markowitz Problem: The x_j, optimal proportion of weights that sum to 1.

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  Certainity Equivalence : Treat estimates as population parameters !!!
  Certainity Equivalence bad idea: Barry,Brown,Klein, Dickenson et al.
  Markowitz procedure is inherently multivariate
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- Step 2: Solve *Markowitz problem* using moments computed above.

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  But we don't know θ

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- *Central thesis of this paper is that the usual sample mean is not admissible for portfolio estimation*

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Empirical Bayes

Jorion: Section IV

- Goal: Find the predictive distribution of future returns, conditioned on the prior, data, covariance matrix & scale factor
- Data $y = (y_1, y_2, \ldots y_t)$
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- Why doesn't Sample mean work ?
  Special Case $\lambda = 0$, corresponds to Diffuse Prior
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  In this special case Bayes rule won't be admissible (Berger: Statistical Decision Theory)

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- Another interpretation of above prior: Means μ vary around the common grand mean η

Empirical Bayes

Jorion: Section IV: Derivation

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- After integration over η, μ, we find the predictive density is Normal with mean vector & covariance matrix
  Mean Vector $E(r) = (1 - w)Y + w1Y_0$
  w = shrinkage factor = $\frac{\lambda}{T+\lambda}$
  Y = vector of sample means = $\frac{\sum_{t=1}^{T} y_t}{T}$
  Y_0 = grand mean = x'Y
  x' = weights of the minimum variance portfolio = $\frac{1'\Sigma^{-1}}{1'\Sigma^{-1}1}$
  Covariance matrix $V = \Sigma(1 + \frac{1}{T+\lambda}) + \frac{\lambda}{T(T+1+\lambda)} \frac{11'}{1'\Sigma^{-1}1}$

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  Third term is the uncertainity in the common factor

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- Large T: $w \rightarrow 0$, $r \rightarrow Y$, $V \rightarrow \Sigma$

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- Large T: $w \to 0, r \to Y, V \to \Sigma$
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Empirical Bayes

Jorion: Section IV: Simplified Working Formulas

- Y = vector of sample means of returns

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- $\hat{\Sigma} = \frac{T-1}{T-N-2} S$
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My Project

Krishnan: Markowitz for FAANG stocks

- Consider Daily returns for Facebook, Amazon, Apple, Netflix & Google for the year 2020

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- Compare the allocations for the remaining 30 Days ***