Bayes-Stein Estimation for Portfolio Analysis

Philippe Jorion, JFQA, Sep 1986

Krishnan Raman

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- Individual averages that exceed grand mean shrink, those that are smaller inflate.

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- **Data, Distribution, Parameter**
  $X_i \sim N(\theta_i, 1), i = 1..p, p > 2, \theta = (\theta_1, \theta_2, ..., \theta_p)'$

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- Roy's idea: Maximize $\frac{\mu_p - d}{\sigma_p^2}$, d = disaster return

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  Certainity Equivalence bad idea: Barry,Brown,Klein, Dickenson et al.
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- Step 2: Solve *Markowitz problem* using moments computed above.

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- *Central thesis of this paper is that the usual sample mean is not admissible for portfolio estimation*

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