Lecture 1. Overview and Basic Principles

Montgomery: Chapter 1

- Experimentation is used to understand and/or improve a system (a product or a process).

$$\begin{array}{ccccc} \text{Inputs} & \longrightarrow & \boxed{\text{System: Product/Process}} & \longrightarrow & \text{Response(s)} \\ X & & f(X)+\epsilon & & Y \end{array}$$

- Have the system as a black box
- The system is quantified by the response(s)
- Uncertainty in system response to fixed inputs: nuisance factors/inherent noise
- Interested in studying the system:
  - Which inputs affect system response(s)?
  - What inputs lead to certain level of the response(s)?
  - What input combination results in low uncertainty?

- **Design of experiments** combines strategies of running an experiment with statistical tools for decision making
  - Develop empirical **linear model** to explain the system
  - Plan experiment to obtain objective conclusions from model
- **Features of experiment to consider**
  - Statement of problem
  - What response will be used? What change in response is important?
  - What inputs to study? Which inputs are most important?
  - How many observations to be taken? What are resources? Costs?
  - Are there uncontrollable nuisance factors?
  - Block on controllable nuisance factors?
  - What is experimental unit?

Design of Experiments

- Statement of the problem
  - What is the experiment intended to do?
    - Treatment Comparison
    - Variable Screening
    - Response Surface Exploration
    - System Optimization
    - System Robustness
  - Obvious question but often overlooked
  - Sound question goes long way towards solution
- Response(s) to be studied
  - Are variables measurable?
  - What sort of response is expected?
  - How accurately can response be measured?

- Inputs to be studied
  - What inputs may affect response?
  - What inputs are of interest?
  - Are factors to be held constant?
  - Varied at specific levels?
- Number of observations to be taken
  - How large a difference in response is important?
  - How much variation is present?
  - What costs and resources are available?
- Order of experiment
  - What is the timing of the experiment?
  - Is whole experiment to be randomized?
  - Are different factors randomized differently?

What is the experimental unit?

- **Experimental Unit:** Material to which a treatment is applied in a single trial of the experiment
  - Need to know experiment units in order to do proper analysis
  - May be different for different inputs
- Example: fertilizers (A, B, C) and 3 seed varieties (1, 2, 3) affect corn yield

C1	A1	A3
B1	B3	C2
A2	C3	B2

C1	C3	C2
B1	B3	B2
A1	A3	A2

- **Run:** an experimental condition or factor level combination at which responses are measured. Multiple executions of the same experimental conditions are considered separate runs and are called **replicates**.

Machine Tool Life Experiment

- An engineer is interested in the effects of **cutting speed** (A), **tool geometry** (B) and **cutting angle** (C) on the lifespan (in hours) of a machine tool.
- Two levels of each factor are chosen and three replicates of a 2^3 factorial design are run. The results from an experiment with 24 machine tools follow.

Factor			Replicate		
A	B	C	I	II	III
$-$	$-$	$-$	22	31	25
$+$	$-$	$-$	32	43	29
$-$	$+$	$-$	35	34	50
$+$	$+$	$-$	55	47	46
$-$	$-$	$+$	44	45	38
$+$	$-$	$+$	40	37	36
$-$	$+$	$+$	60	50	54
$+$	$+$	$+$	39	41	47

Fundamental Principles of Experimental Design

- **Replication** – multiple independent runs of the same factor combination.
  - To decrease uncertainty by averaging out experimental variability
  - If Y_i has mean μ and variance σ^2 then $E(\bar{Y}) = \mu$ and $Var(\bar{Y}) = \sigma^2/n$.
  - Different from **repetition** (e.g., multiple readings from the same experimental units with application of the same factor combination).
- **Randomization** - provides stronger basis for use of coincidence argument
  - Protection – averages out unknown factors
  - Independence of trials / Avoids biases
- **Blocking** - decrease uncertainty by adjusting for (controlling) specific nuisance factors

Randomized Design: Modified Fertilizer Mixtures for Tomato Plants

- An experiment was conducted by an amateur gardener whose object was to discover whether a change in the fertilizer mixture applied to his tomato plants would result in an improved yield.
- He had 11 plants set out in a single row; 5 were given the standard fertilizer mixture A, and the remaining 6 were fed a supposedly improved mixture B.
- The A's and B's were randomly applied to the positions in the row to give the design shown in next slide.
- The gardener arrived at this random arrangement by taking 11 playing cards, 5 red corresponding to fertilizer A and 6 black corresponding to fertilizer B.
- The cards were thoroughly shuffled and dealt to give the sequence shown in the design. The first card was red, the second was red, the third was black, and so forth.

Fertilizer Mixtures Experiment: Data

Pos	1	2	3	4	5	6	7	8	9	10	11
Trt	A	A	B	B	A	B	B	B	A	A	B
Yds	29.9	11.4	26.6	23.7	25.3	28.5	14.2	17.9	16.5	21.1	24.3

A	B
29.9	26.6
11.4	23.7
25.3	28.5
16.5	14.2
21.1	17.9
	24.3

$$n_A = 5 \qquad n_B = 6$$

$$\Sigma y_A = 104.2 \qquad \Sigma y_B = 135.2$$

$$\bar{y}_A = 20.84 \qquad \bar{y}_B = 22.53$$

Mean difference (modified minus standard)= $\bar{y}_B - \bar{y}_A = 1.69$

Testing Hypotheses

- H_0: $\mu_A = \mu_B$, i.e., the modified fertilizer does not improve the (mean) yield.
- H_a: $\mu_B > \mu_A$, i.e., the modified fertilizer improves the (mean) yield.
- Two sample t-test (refer to Page 38 of Montgomery)
  - $s_A^2 = 52.50, s_B^2 = 29.51$
  - $s_{pool}^2 = \frac{(n_A-1)s_A^2+(n_B-1)s_B^2}{n_A+n_B-2} = 39.73$
  - $t_0 = \frac{\bar{y}_B-\bar{y}_A}{s_{pool}\sqrt{1/n_A+1/n_B}} = .44$
  - P-value $= Pr(t > t_0 \mid t(n_A+n_B-2)) = Pr(t > .44 \mid t(9)) = .34$
  - Because P-value $\geq \alpha$, accept H_0.

Using SAS: Completely Randomized Design

```
TITLE 'Completely Randomized Design';
/* The unrandomized design */
DATA a;
    DO unit=1 to 11;
       IF (unit <= 5) then trt='A';
       ELSE trt='B';
       OUTPUT;
    END;
RUN;

/* Randomize the design in a */
PROC PLAN SEED=5140514;
    FACTORS unit=11;
    OUTPUT DATA=a OUT=b;
    RUN;

PROC SORT DATA=b; BY unit;
PROC PRINT; RUN; QUIT;
```

SAS Output

```
Completely Randomized Design

     Obs    unit    trt
       1       1      A
       2       2      B
       3       3      A
       4       4      B
       5       5      B
       6       6      A
       7       7      B
       8       8      A
       9       9      B
      10      10      B
      11      11      A
```

Blocking

- A **block** refers to a group of homogeneous units/runs.
  - Homogeneous according to certain factors, e.g., field patches in agriculture experiments
- Within-block variation and between-block variation
- Trade off between variation and the degrees of freedom
- **Block what you can and randomize what you cannot**
  - When doing an experiment, the run order should be randomized.

Randomization and Blocking: Typing Efficiency Experiment

- Compare the typing efficiency of two keyboards denoted by A and B. One typist uses the keyboards on six different manuscripts, denoted by 1-6.
- Design 1:

$$1.A - B, 2.A - B, 3.A - B, 4.A - B, 5.A - B, 6.A - B.$$

- Design 2:

$$1.A - B, 2.B - A, 3.A - B, 4.B - A, 5.A - B, 6.A - B.$$

- Design 3: Balanced Randomization (3 $A - B$s and 3 $B - A$s)

Using SAS: Typing Efficiency Experiment

```
TITLE 'Typing Efficiency Experiment';  /* Design 3 */
PROC PLAN SEED=20140505;
    FACTORS manuscript=6;
    TREATMENTS treatment=6 cyclic (1 1 1 2 2 2); /* 1: A-B; 2: B-A */
    OUTPUT OUT=typingdesign;

PROC PRINT; RUN;

PROC SORT DATA=typingdesign;
    BY manuscript;

DATA typingdesign; SET typingdesign;
    IF treatment=1 THEN keyboard='A-B';
    IF treatment=2 THEN keyboard='B-A';
    DROP treatment;

PROC PRINT; RUN; QUIT;
```

SAS Output

```
Typing Efficiency Experiment

Obs     manuscript     treatment
  1              4             1
  2              5             1
  3              1             1
  4              6             2
  5              2             2
  6              3             2

Obs     manuscript      keyboard
  1              1           A-B
  2              2           B-A
  3              3           B-A
  4              4           A-B
  5              5           A-B
  6              6           B-A
```

Randomization Test

$$H_0 : \mu_A = \mu_B \text{ vs. } H_a : \mu_B > \mu_A (\alpha = 5\%)$$

- Alternative to ANOVA F-test, a distribution free test.
- Under the null hypothesis, A and B are mere labels and should not affect the yield.
  - For example, the first plant would yield 29.9 pounds of tomatoes no matter it had been labeled as A or B (or fed A or B).
  - There are $\frac{11!}{5!6!} = 462$ ways of allocating 5 A's and 6 B's to the 11 plants, any one of which could equally be chosen.
  - The used design is just one of 462 equally likely possibilities. (why?)

For example:

Pos	1	2	3	4	5	6	7	8	9	10	11
Yds	29.9	11.4	26.6	23.7	25.3	28.5	14.2	17.9	16.5	21.1	24.3
LL1	A	A	A	A	A	B	B	B	B	B	B
LL2	A	A	A	A	B	A	B	B	B	B	B
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

LL1, LL2, etc are equally likely.

LL1: mean difference between B and A is -2.96

LL2: mean difference between B and A is -4.14

$\vdots$

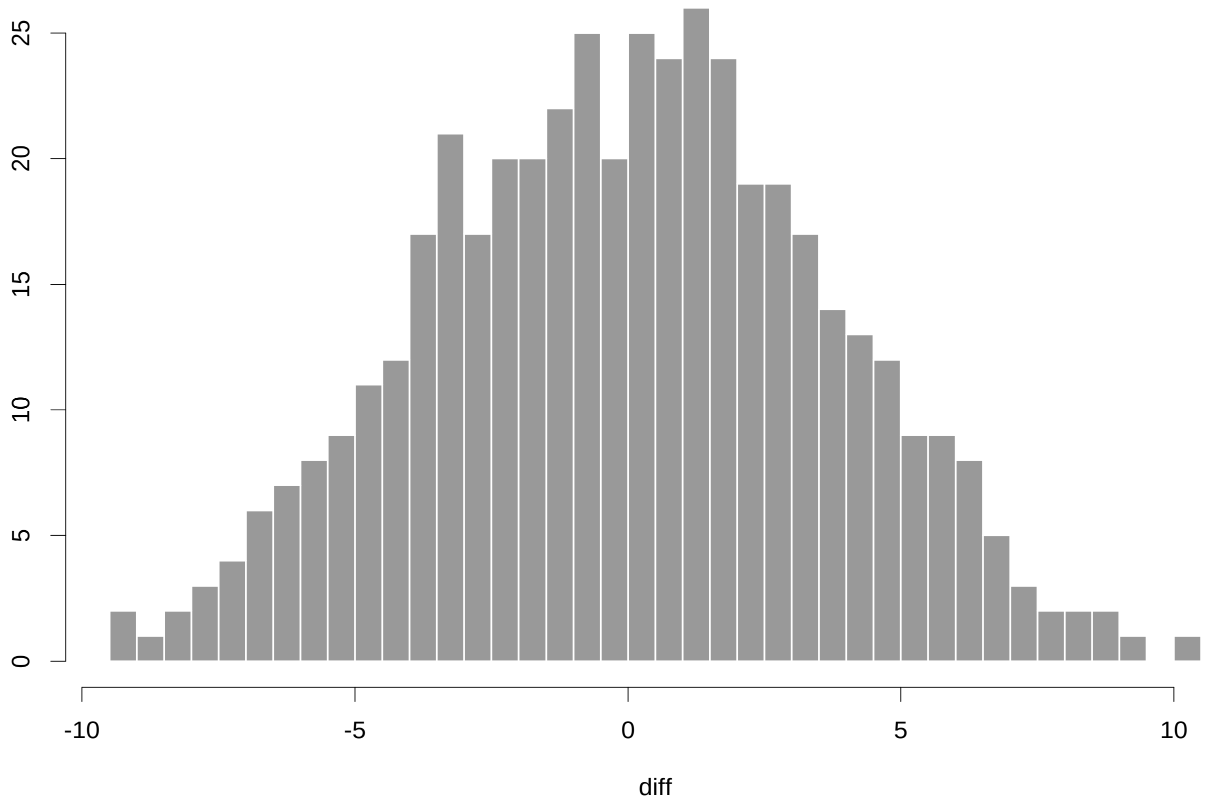
Under the null hypothesis, these differences are equally likely.

Significance of Observed Difference

A summary of possible allocations and their corresponding mean differences:

No	possible designs	$\bar{y}_A$	$\bar{y}_B$	mean difference
1	AAAAABBBBBB	23.38	20.42	-2.96
2	AAAABABBBBB	24.02	19.88	-4.14
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\cdot$	AABBABBBAAB	20.84	22.53	1.69
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
462	BBBBBBAAAAA	18.80	24.23	5.43

Randomization Distribution (Histogram) of the Mean Differences



- Observed Diff $= 1.69$
- P-value $= Pr\big($Diff $\geq 1.69 \mid$ randomization$\big) = \frac{155}{462} = .335$
- Because P-value $\geq \alpha$, accept H_0. (Same conclusion as ANOVA F-Test)