Bayesian Analysis of New York City's Uber Rides Data

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Introduction

Uber is a popular ridesharing platform that services millions of customers each day. Uber drivers are interested in making the most money. This involves answering the following questions:

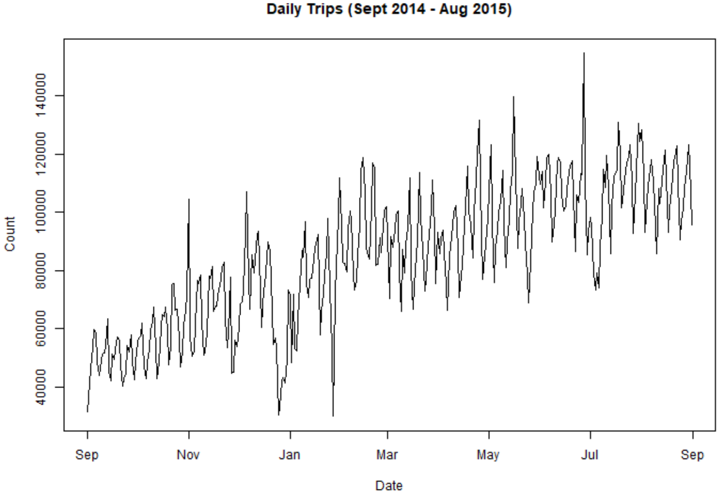
- What times of the year and week are drivers most needed?
- What times of day are the busiest and does this vary by location?
- For an optimal route, how much can a driver expect to make?
- Is the optimal route really that much better than a random route?

Two Datasets

	orig	dest	pickup_datetime	distance	duration	fare
1	7C	6A	2014-09-01 09:00:00	4.25	15.183	15.30
2	7B	15	2014-09-01 18:00:00	10.17	34.083	32.28
3	11	2A	2014-09-01 17:00:00	4.02	17.100	15.57
4	3B	4A	2014-09-01 13:00:00	1.46	6.533	8.00
5	2A	10	2014-09-01 14:00:00	8.31	26.283	26.29
6	5B	4C	2014-09-01 12:00:00	1.04	8.583	8.00

	Date.Time	Lat	Lon	Base
1	9/1/2014 0:01:00	40.2201	-74.0021	B02512
2	9/1/2014 0:01:00	40.7500	-74.0027	B02512
3	9/1/2014 0:03:00	40.7559	-73.9864	B02512
4	9/1/2014 0:06:00	40.7450	-73.9889	B02512
5	9/1/2014 0:11:00	40.8145	-73.9444	B02512
6	9/1/2014 0:12:00	40.6735	-73.9918	B02512

First Question: The Data



First Question: The Model

The model for the number of trips on day t is:

$$y(t) = \beta_0 + \beta_1 t + \beta_2 t^2 + f_1(t) + f_2(t) + \epsilon(t) \text{ where } \epsilon(t) \sim \mathcal{N}(0, \sigma^2)$$

This can be more compactly written in matrix/vector notation as:

$$Y = X\beta + f_1 + f_2 + \epsilon \text{ where } \epsilon \sim \mathcal{N}(0, \sigma^2 I)$$

Here, f_1 and f_2 are Gaussian Processes that represent a smooth, yearly trend and a shorter, periodic trend respectively. The $X\beta$ is a structural component that accounts for an increasing trend over time.

First Question: Gaussian Process

We assume that:

$$f_1 \sim \mathcal{N}(0, \tau_1^2\ K_1),\ \ f_2 \sim \mathcal{N}(0, \tau_2^2\ K_2)$$

$$K_1(t, t') = \exp\left(-\frac{|t-t'|^2}{(2)(7^2)}\right),\ K_2(t, t') = \exp\left(-\frac{-2\ \sin^2(\pi|t-t'|/7)}{7^2}\right)$$

$$\pi(\beta) \propto 1$$

$$\sigma^2 \sim \text{InverseGamma}(3, 5)$$

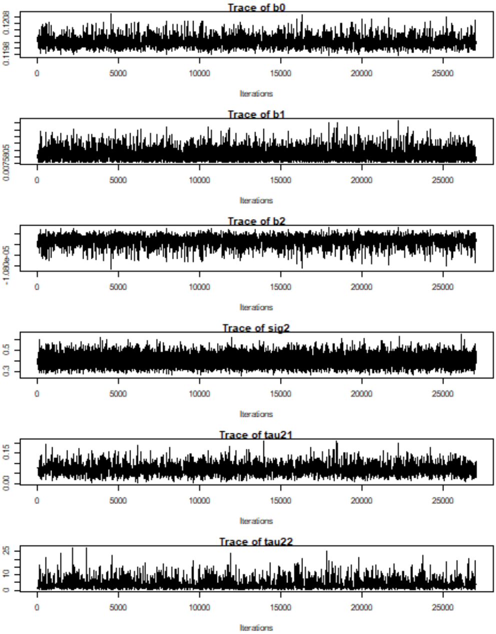
$$\tau_1^2, \tau_2^2 \sim \text{Gamma}(1.5, 0.2)$$

A Gibbs Sampler with MH-steps for τ_1^2 and τ_2^2 is implemented to sample from the marginal posterior $f(Y|\beta, \sigma^2, \tau_1^2, \tau_2^2)$ where $Y|\beta, \sigma^2, \tau_1^2, \tau_2^2 \sim \mathcal{N}(X\beta, K_1 + K_2 + \sigma^2 I)$ and then these can be used to estimate f_1 and f_2.

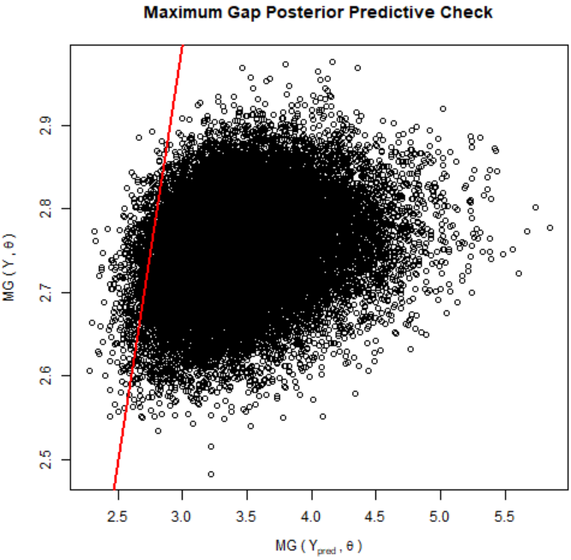
First Question: Model Checking

	GR Estimate	Upper C.I.	ESS
β_0	1.0015	1.0016	14032.3905
β_1	1.0006	1.0009	30000.0000
β_2	1.0015	1.0017	22429.7924
σ^2	1.0001	1.0004	21527.7282
τ_1^2	1.0008	1.0016	683.5401
τ_2^2	1.0011	1.0022	2434.2481

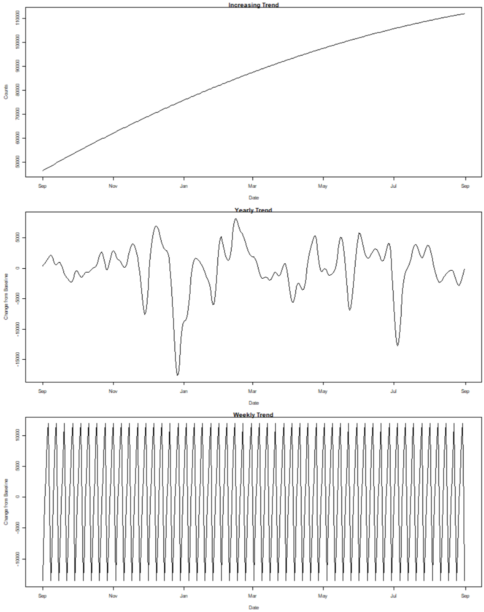
First Question: Model Checking



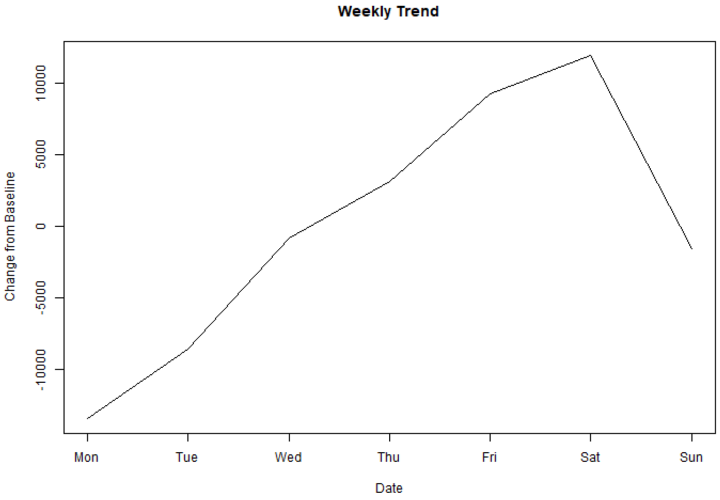
First Question: Posterior Predictive Check



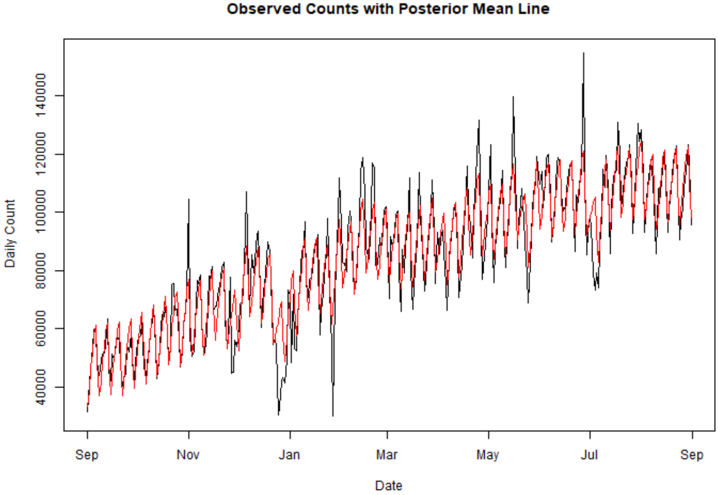
First Question: Posterior Estimates of Trends



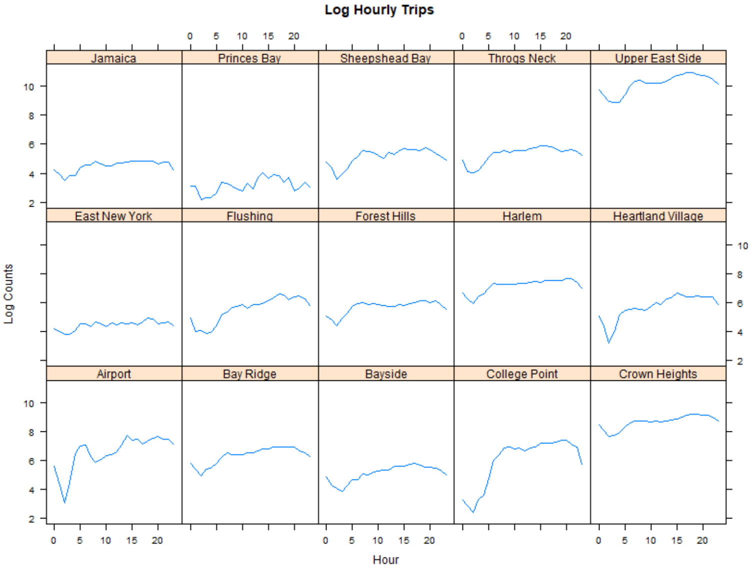
First Question: Weekly Trend



First Question: Predictions



Second Question: The Data



Second Question: The Model

The model for number of trips at location s at time t is:

$$y(s, t) = f(s, t) + \epsilon(s, t) \text{ where } \epsilon \sim \mathcal{N}(0, \sigma^2)$$

Or more compactly:

$$Y = f + \epsilon \text{ where } \epsilon \sim \mathcal{N}(0, \sigma^2 I)$$

Here, f just represents a spatio-temporal Gaussian Process that allows us to understand trends in trips across space and time simultaneously instead of considering them separately.

Question 2: Spatio-Temporal Gaussian Process

We assume that:

$$f \sim \mathcal{N}(0, K)$$

$$K(s, s', t, t') = \tau_s^2 \exp\left(- \frac{||s - s'||_2^2}{(2)(0.005^2)} \right) \times \tau_t^2 \exp\left(- \frac{|t - t'|^2}{(2)(12^2)} \right)$$

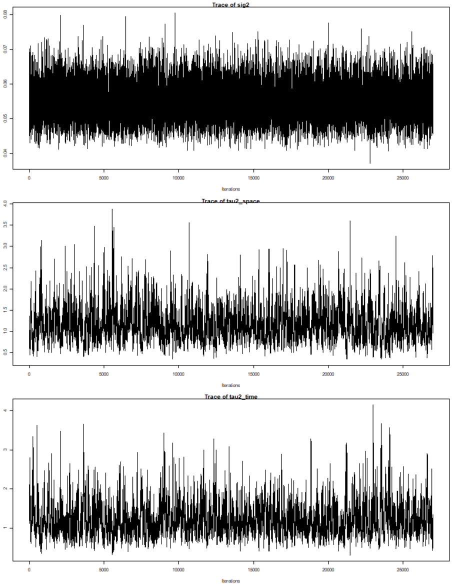
$$\sigma^2, \tau_s^2, \tau_t^2 \sim \text{InverseGamma}(3, 5)$$

A Gibbs Sampler is used to sample from the posterior distribution.

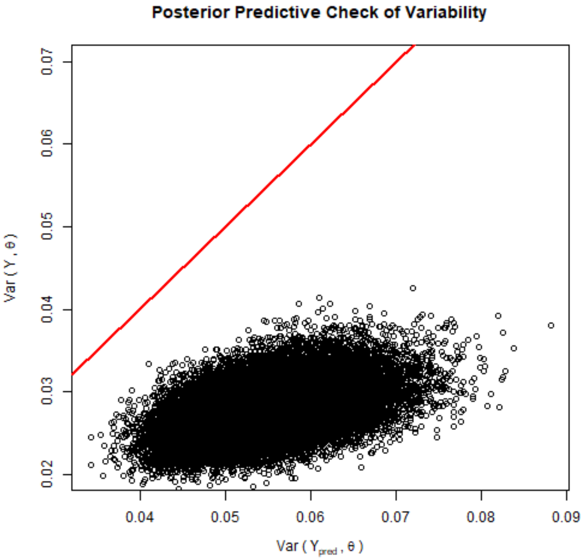
Second Question: Model Checking

	Point est.	Upper C.I.	ESS
σ^2	1.0000	1.0001	16561.97
τ_s^2	1.0200	1.0571	2727.01
τ_t^2	1.0193	1.0517	1115.583

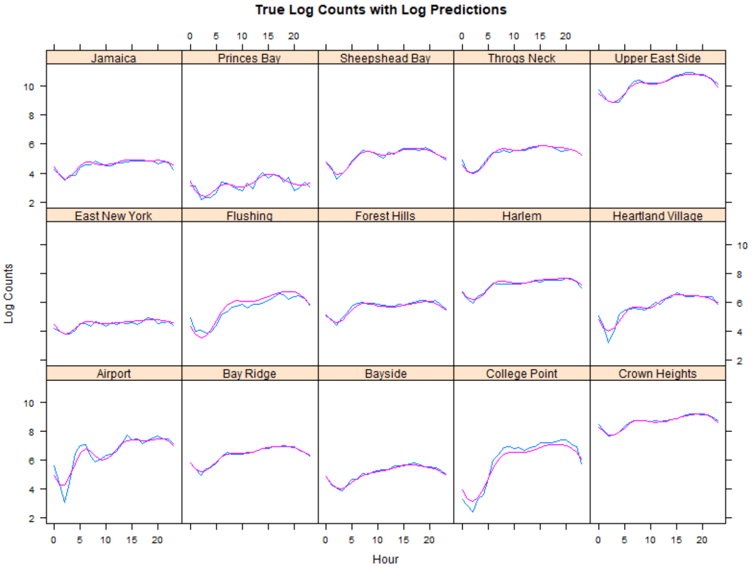
Second Question: Model Checking



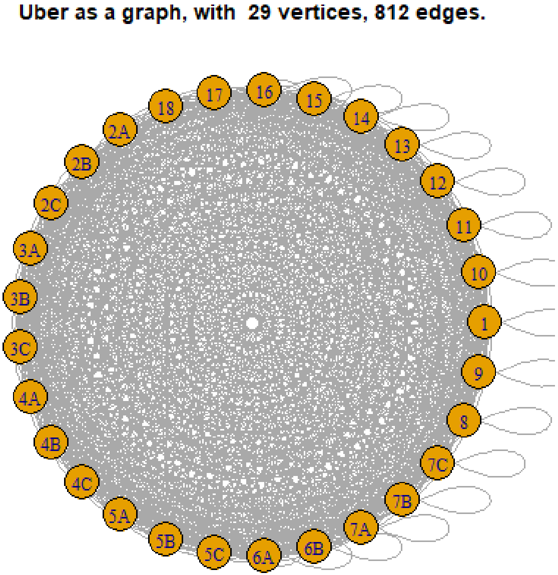
Second Question: Posterior Predictive Check



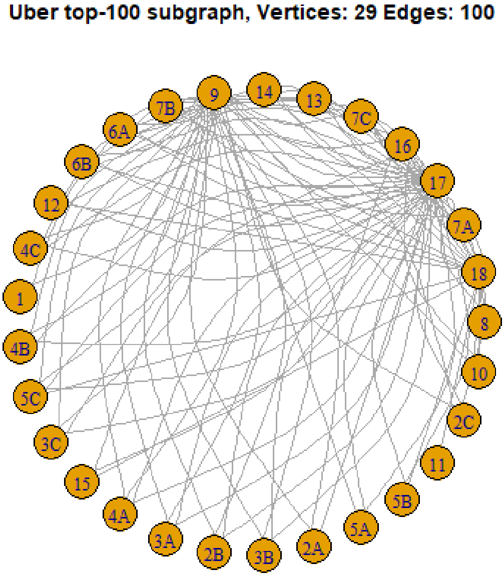
Second Question: Predictions



Third Question: Uber Graphs & Subgraphs



Third Question: Graph of Most Lucrative Edges



Third Question: What's an Optimal Circuit?

Some criterion for the optimal circuit:

- Circuit: Collection of edges
- Want a closed circuit aka Cycle or loop
- Want a "simple path" : each node visited only once

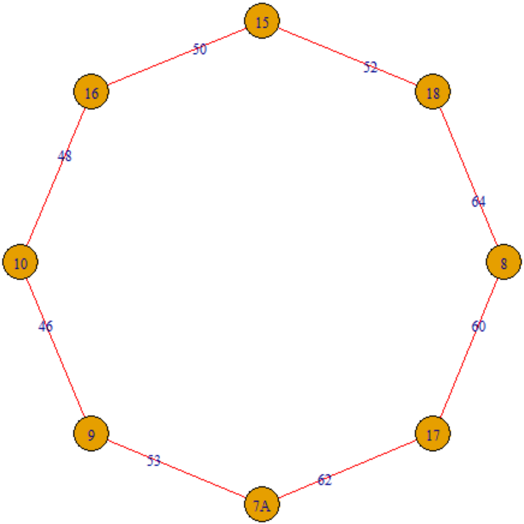
We want to find:

- Circuit that makes the most money for the Uber Driver

We are assuming:

- Driver works 9AM-5PM: 8 HOURS
- Make 1 Uber trip per hour
- Want an 8-cycle aka Simple Path with 8 Nodes aka 8-gon

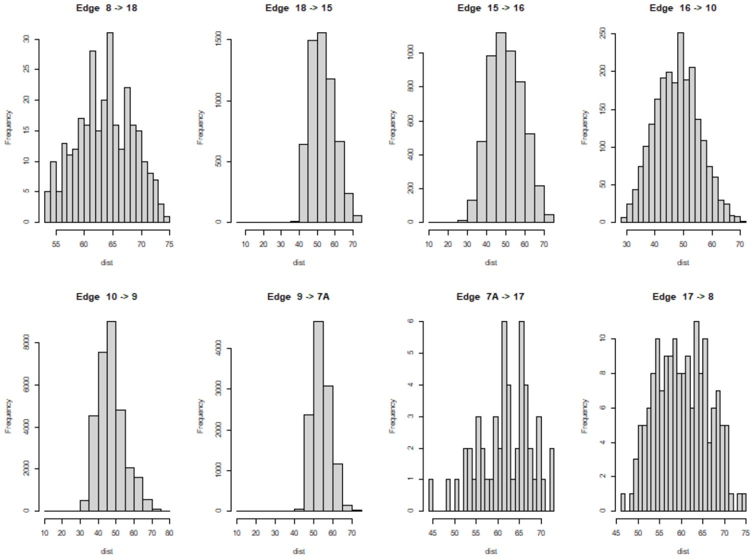
Third Question: The Optimal Circuit



Third Question: Summary Statistics for Optimal Circuit

Src	Dest	Mean	Median	Sigma	Var
8	18	63.54	63.69	4.90	23.97
18	15	53.06	52.44	6.71	45.06
15	16	50.18	49.72	8.65	74.84
16	10	47.70	47.86	7.69	59.07
10	9	47.20	46.39	7.40	54.79
9	7A	54.00	53.42	4.66	21.70
7A	17	61.56	61.96	6.22	38.72
17	8	60.19	59.89	6.04	36.47

Third Question: Fare Distributions for Each Edge



Third Question: Two Models

Edge Fare X_i = Observed fares for edge X_i over a year.
Circuit Fare $Y = \sum X_i$, i=1:8, Y is unobserved.

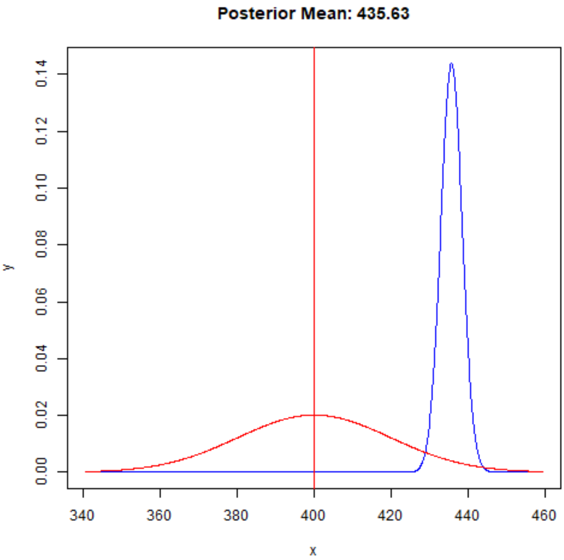
Model 1 : Gaussian Conjugate Prior

- $X_i \sim N(1/8\mu, 1/8\sigma^2) \rightarrow Y \sim N(\mu, \sigma^2)$
- $\mu \sim N(50 * 8, 8 * (7^2))$
- Posterior Dist of Y proportional to product of Prior & Likelihood
- Gaussian Conjugate prior yields Gaussian posterior

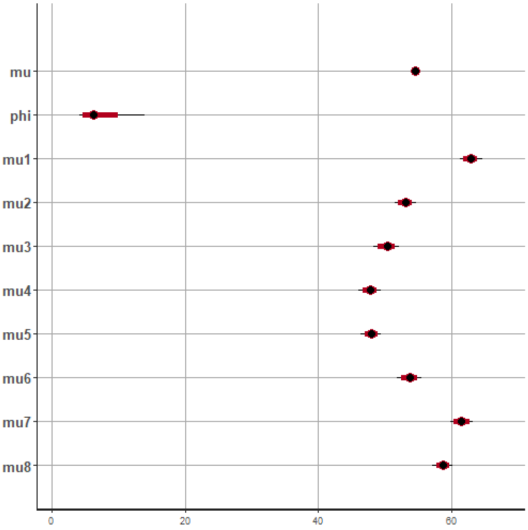
Model 2 : Hierarchical Model

- $X_i \sim N(\mu_i, \sigma)$
- $\mu_i \sim N(\mu, \phi)$
- $Y \sim N(8\mu, \sqrt{8}\sigma)$
- Estimate posterior mean of Y via MCMC

Third Question: Model 1 Posterior of Mean Fare

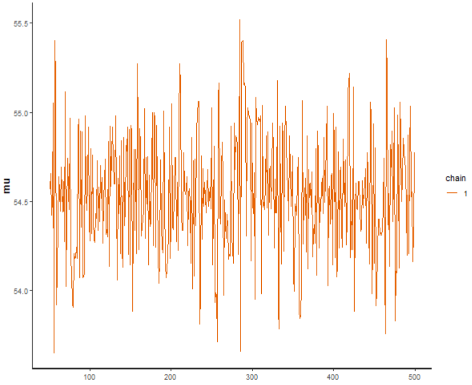


Third Question: Model 2 Posterior Distributions

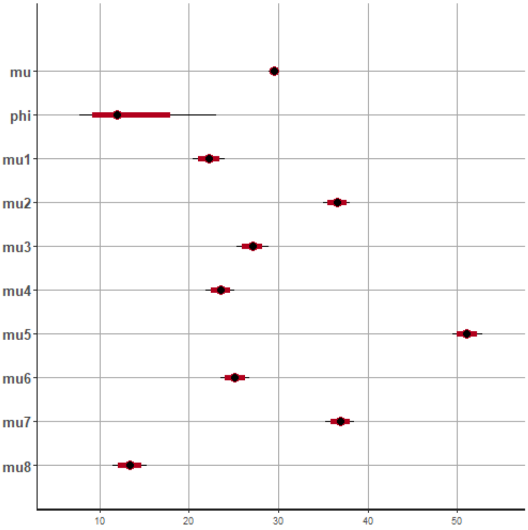


Third Question: Model 2 Posterior of Mean Fare

Mean Fare from Optimal Daily Circuit: $437.63



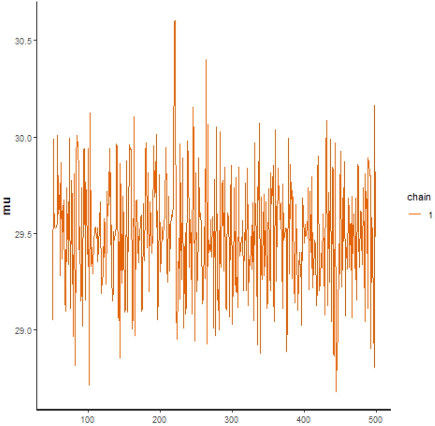
Fourth Question: Random Circuit Posterior Distributions



Fourth Question: Random Circuit Mean Fare Posterior

Mean Fare from Random Daily Circuit: $235.82

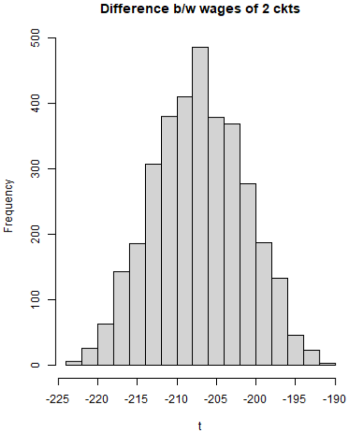
|Optimal-Random| Median Loss: $201.81

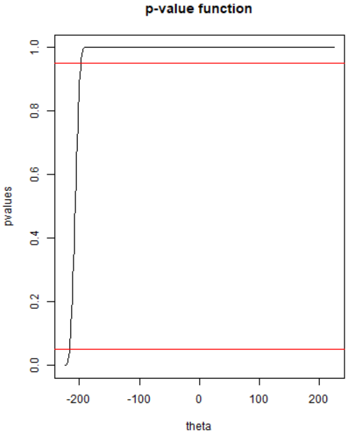


Fourth Questions: Confidence Distributions

- Professor Don Rubin: "Fisher Randomization Test is a Stochastic Proof-By-Contradiction"
- FRT = distribution-free test to compare two (multimodal) Uber circuits
- Fisher Sharp Null compares individual potential outcomes $Y_i(1)$ *vs* $Y_i(0)$ for every observation.
- An assignment is a Boolean vector over two weeks.
- An assignment simply means on the given day, the Uber driver drove Circuit c2.
- A non-assignment means the wages would come from Ckt c1 aka Null Hypothesis.
- Compute the biweekly average and compare with the null hypothesis biweekly average
- This comparison is a simple difference test statistic.

Fourth Question: Confidence Distributions





Conclusions

What we confirmed:

- Holidays are not times of high demand
- Weekends are when most people need an Uber
- Central New York is the busiest
- Most places see a spike in demand in the morning and evening
- You'll make a lot more money driving the optimal circuit

Room for Improvement:

- Fix estimation problems with length-scale parameters
- Address model inadequacies found with posterior predictive checks