Temporal Gaussian Process Real

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library(mvtnorm)
set.seed(1)

kern <- function(tt, l2){
  D <- as.matrix(dist(tt, diag = TRUE, upper = TRUE))^2
  exp(-1/(2*l2)*D)
}

gau.pro.temp <- function(data, niter, f1_l2 = 365, f2_l2 = 30, f3_l2 = 1,
                         prior_a = 2, prior_b = 2, var_init = 2){
  tt <- data[,1]
  Y <- data[,2]
  n <- length(Y)
  In <- diag(n)
  In_pert <- diag(10e-06, n)

  Ymean <- mean(Y)
  Ysd <- sd(Y)

  Y <- (Y-Ymean)/Ysd

  sig2a <- prior_a
  sig2b <- prior_b
  tau21a <- prior_a
  tau21b <- prior_b
  tau22a <- prior_a
  tau22b <- prior_b
  tau23a <- prior_a
  tau23b <- prior_b

  f1_save <- matrix(0, niter, n)
  f2_save <- matrix(0, niter, n)
  f3_save <- matrix(0, niter, n)
  sig2_save <- rep(0, niter)
  tau21_save <- rep(0, niter)
  tau22_save <- rep(0, niter)
  tau23_save <- rep(0, niter)

  sig2_save[1] <- var_init
  tau21_save[1] <- var_init
  tau22_save[1] <- var_init
  tau23_save[1] <- var_init

  for (i in 2:niter){
    K1 <- tau21_save[i-1]*kern(tt, f1_l2) + In_pert
    K2 <- tau22_save[i-1]*kern(tt, f2_l2) + In_pert
    K3 <- tau23_save[i-1]*kern(tt, f3_l2) + In_pert
```

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    YSig_inv <- solve(sig2_save[i-1]*In + K1 + K2 + K3, In)

    K1_Sigma <- K1 - K1%*%YSig_inv%*%t(K1)
    K1_Mu <- K1%*%YSig_inv%*%Y

    f1_save[i,] <- rmvnorm(1, K1_Mu, K1_Sigma)

    K2_Sigma <- K2 - K2%*%YSig_inv%*%t(K2)
    K2_Mu <- K2%*%YSig_inv%*%Y

    f2_save[i,] <- rmvnorm(1, K2_Mu, K2_Sigma)

    K3_Sigma <- K3 - K3%*%YSig_inv%*%t(K3)
    K3_Mu <- K3%*%YSig_inv%*%Y

    f3_save[i,] <- rmvnorm(1, K3_Mu, K3_Sigma)

    sig2_save[i] <- 1/rgamma(1, n/2 + sig2a,
                             sig2b + 0.5*t(Y-f1_save[i,]-
                                           f2_save[i,]-
                                           f3_save[i,])%*%(Y-f1_save[i,]-
                                                           f2_save[i,]-
                                                           f3_save[i,]))

    tau21_save[i] <- 1/rgamma(1, n/2 + tau21a,
                             tau21b + 0.5*t(f1_save[i,])%*%solve(K1, f1_save[i,]))
    tau22_save[i] <- 1/rgamma(1, n/2 + tau22a,
                             tau22b + 0.5*t(f2_save[i,])%*%solve(K2, f2_save[i,]))
    tau23_save[i] <- 1/rgamma(1, n/2 + tau23a,
                             tau23b + 0.5*t(f3_save[i,])%*%solve(K3, f3_save[i,]))
  }

  cbind(f1_save, f2_save, f3_save, sig2_save, tau21_save, tau22_save, tau23_save)
}

load(file = "C://Users/david/Documents/R/Stat 695/Project/uber/trip_counts.RData")

trip_data <- cbind(0:(nrow(trip_counts)-1), trip_counts$n_trips)
num_days <- nrow(trip_data)

gaupro_out <- gau.pro.temp(trip_data, niter = 50, f1_l2 = 365, f2_l2 = 30, f3_l2 = 1)

f1_out <- gaupro_out[,1:num_days]
f2_out <- gaupro_out[,(num_days+1):(num_days*2)]
f3_out <- gaupro_out[,(num_days*2+1):(num_days*3)]

f_pred <- colMeans(f1_out) + colMeans(f2_out) + colMeans(f3_out)

par(mfrow = c(1, 1))
plot(trip_counts, type = 'l', main = "Number of Daily Trips 9/2014 - 8/2015",
     xlab = "Day", ylab = "Number of Trips")
lines(trip_counts$date, f_pred*sd(trip_counts$n_trips) + mean(trip_counts$n_trips),
      col = 'red')
```
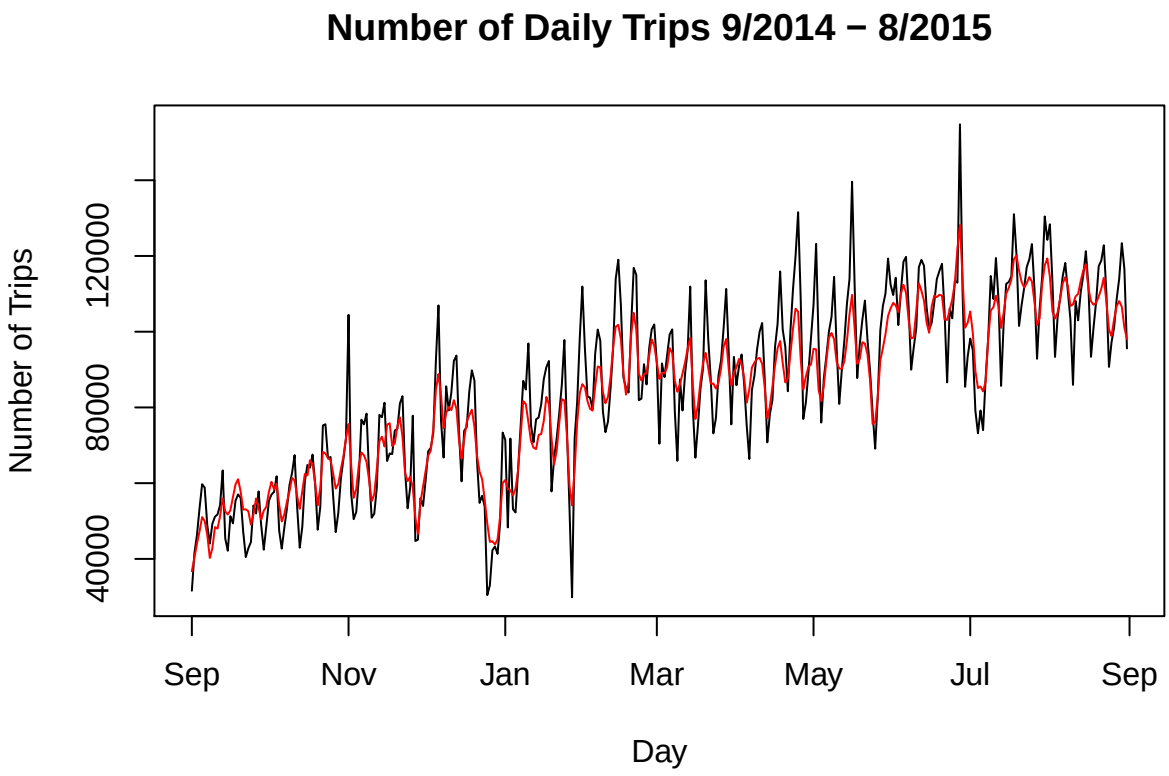
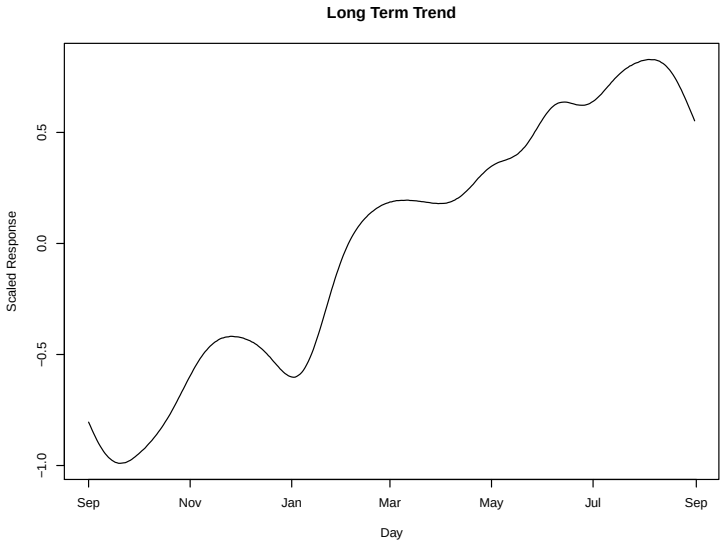
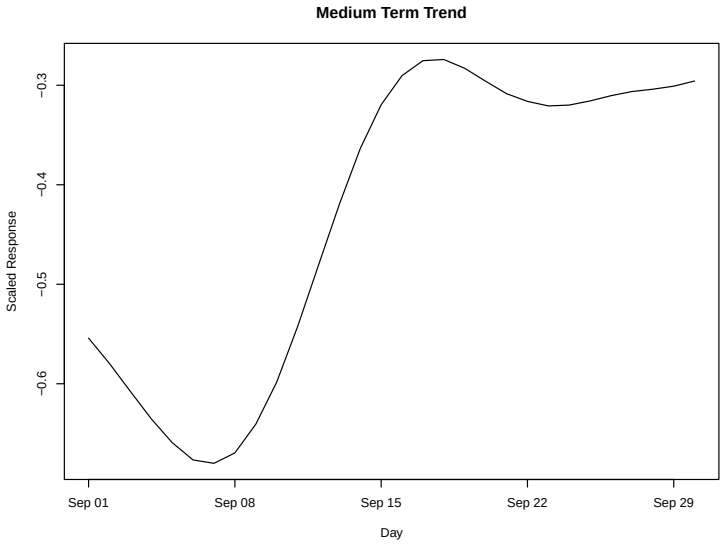
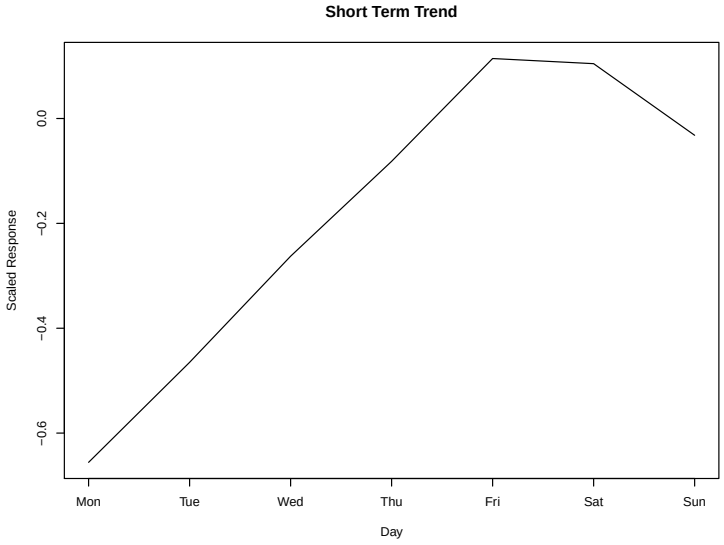




```
par(mfrow = c(3, 1))
plot(trip_counts$date, colMeans(f1_out), type = 'l',
     main = "Long Term Trend", xlab = "Day", ylab = "Scaled Response")
plot(trip_counts$date[1:30], colMeans(f2_out)[1:30], type = 'l',
     main = "Medium Term Trend", xlab = "Day", ylab = "Scaled Response")
plot(trip_counts$date[1:7], colMeans(f3_out)[1:7], type = 'l',
     main = "Short Term Trend", xlab = "Day", ylab = "Scaled Response")
```


Long Term Trend
Scaled Response
0.5
0.0
−0.5
−1.0
Sep
Nov
Jan
Mar
May
Jul
Sep
Day


Medium Term Trend
Scaled Response
−0.3
−0.4
−0.5
−0.6
Sep 01
Sep 08
Sep 15
Sep 22
Sep 29
Day


Short Term Trend
Scaled Response
0.0
−0.2
−0.4
−0.6
Mon
Tue
Wed
Thu
Fri
Sat
Sun
Day