Spatio Temporal Gaussian Process Real

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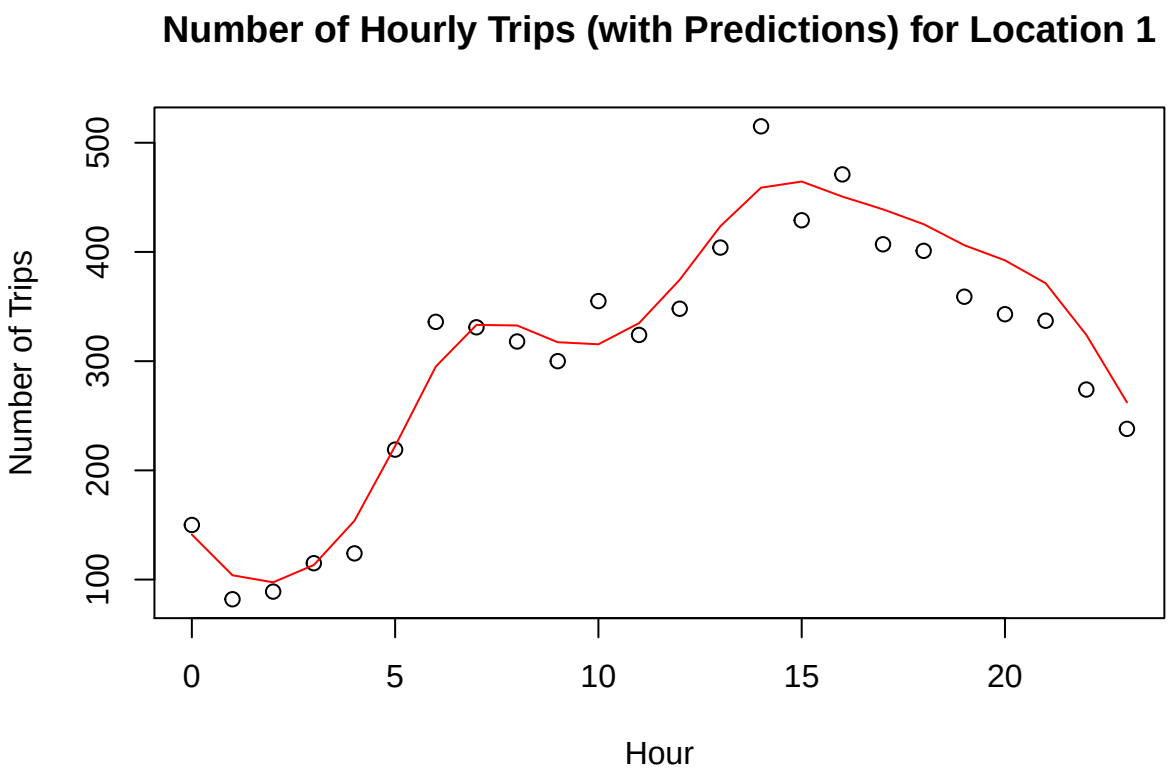
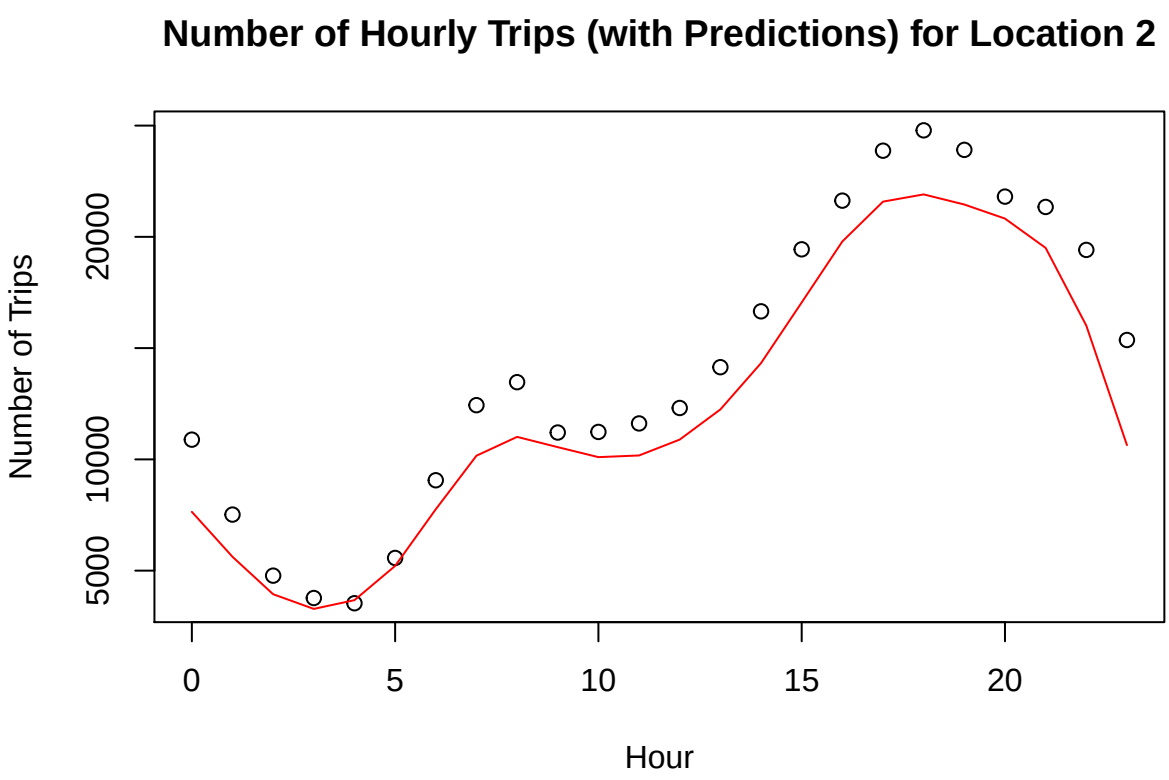
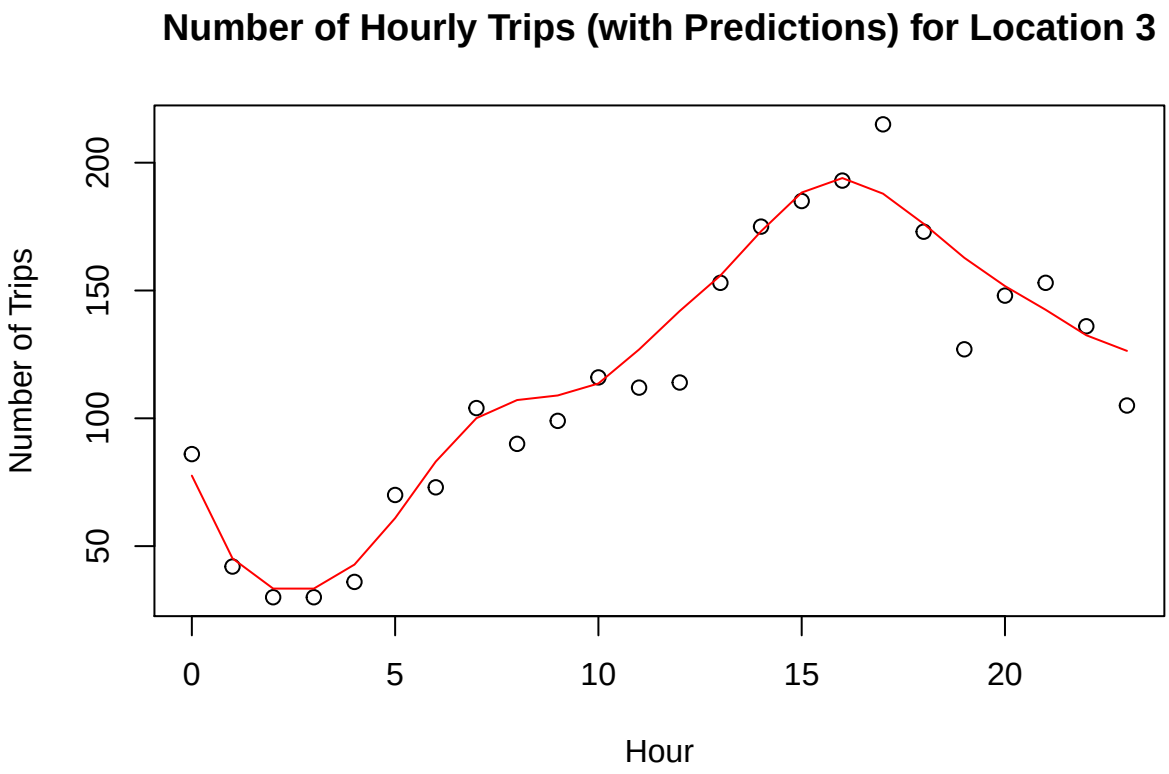
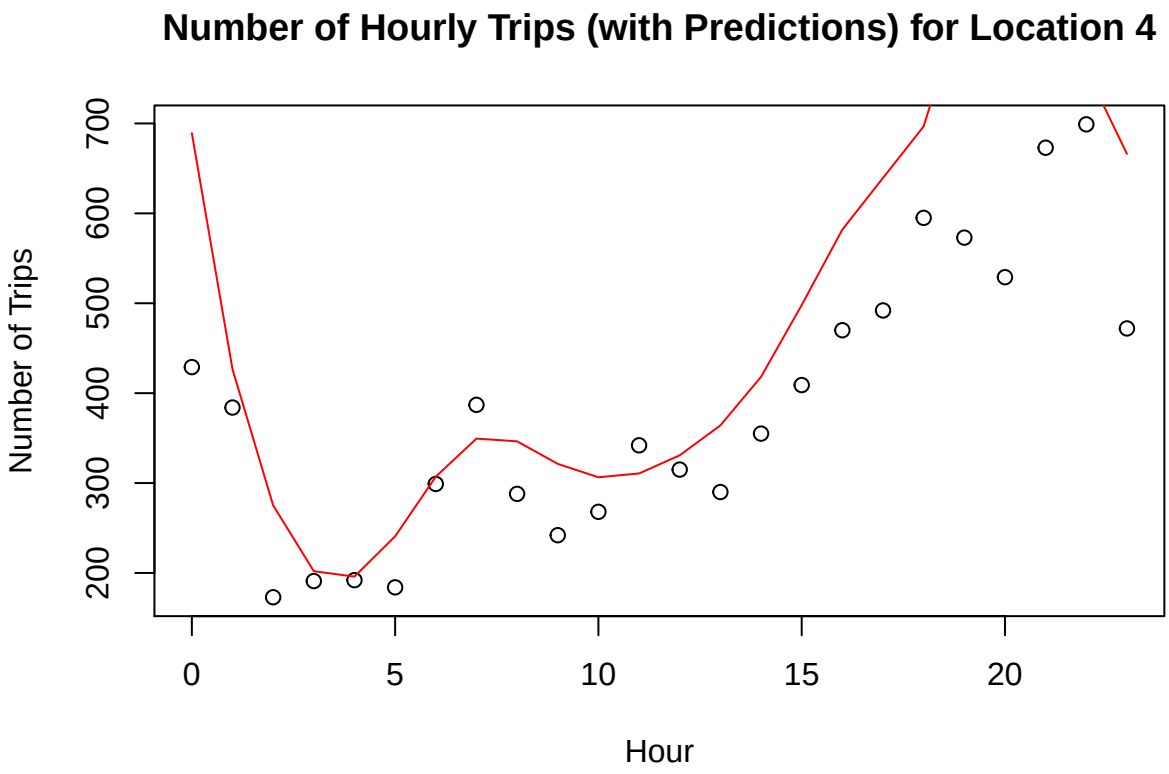
```
library(mvtnorm)

space.kern <- function(D, l2){
  exp(-1/(2*l2)*D)
}

space.D <- function(lat_long){
  D <- as.matrix(dist(lat_long, diag = TRUE, upper = TRUE))^2
}

time.kern <- function(D, l2){
  exp(-1/(2*l2)*D)
}

time.D <- function(tt){
  D <- as.matrix(dist(tt, diag = TRUE, upper = TRUE))^2
}

gau.pro.spat.temp <- function(data, niter, l2_space = 0.005, l2_time = 10,
                              prior_a = 2, prior_b = 2, var_init = 2){
  Y <- log(data[,3]+1)
  tt <- unique(data[,2])

  Ymean <- mean(Y)
  Ysd <- sd(Y)
  Y <- (Y-Ymean)/Ysd

  lat_long <- unique(data[,4:5])
  rownames(lat_long) <- NULL

  N <- length(tt)*nrow(lat_long)

  D_space <- space.D(lat_long)
  K_space <- space.kern(D_space, l2_space) + diag(10e-06, nrow(lat_long))
  D_time <- time.D(tt)
  K_time <- time.kern(D_time, l2_time) + diag(10e-06, length(tt))

  K_space_inv <- solve(K_space)
  K_time_inv <- solve(K_time)

  f_save <- matrix(0, niter, length(Y))
  sig2_save <- rep(0, niter)
  tau2_space_save <- rep(0, niter)
  tau2_time_save <- rep(0, niter)

  sig2_a <- prior_a
  sig2_b <- prior_b
```

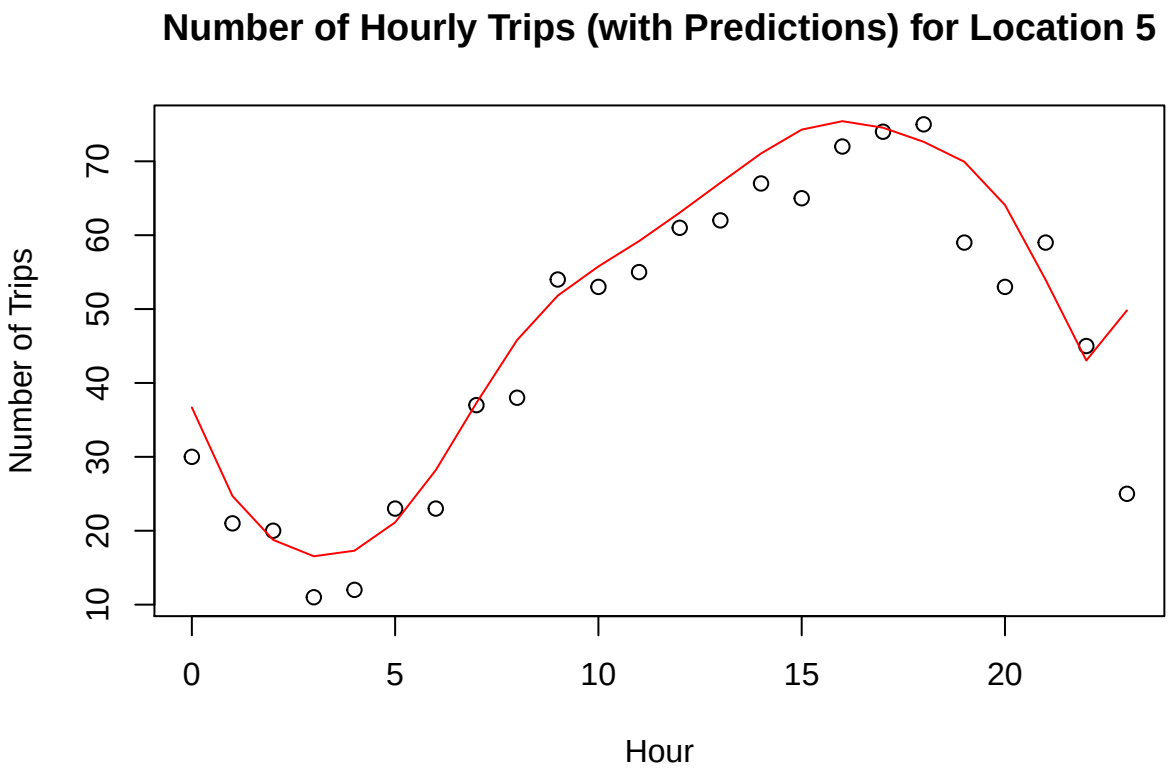
```
  tau2_time_a <- prior_a
  tau2_time_b <- prior_b
  tau2_space_a <- prior_a
  tau2_space_b <- prior_b

  sig2_save[1] <- var_init
  tau2_space_save[1] <- var_init
  tau2_time_save[1] <- var_init

  for (i in 2:niter){
    K_space_time <- kronecker(tau2_space_save[i-1]*K_space,
                          tau2_time_save[i-1]*K_time)
    sig2I <- diag(sig2_save[i-1], N)
    f_Sigma <- K_space_time + sig2I

    f_Mu <- K_space_time%*%solve(f_Sigma, Y)
    f_Sigma <- K_space_time - K_space_time%*%solve(f_Sigma, K_space_time)

    f_save[i,] <- rmvnorm(1, f_Mu, f_Sigma)

    sig2_save[i] <- 1/rgamma(1, sig2_a + N/2,
                             sig2_b + 0.5*t(Y-f_save[i,])%*%(Y-f_save[i,]))

    tau2_space_Sigma <- kronecker(K_space_inv, 1/tau2_time_save[i-1]*K_time_inv)

    tau2_space_save[i] <- 1/rgamma(1, tau2_space_a + N/2,
                                   tau2_space_b + 0.5*t(f_save[i,])%*%tau2_space_Sigma%*%f_save[i,])

    tau2_time_Sigma <- kronecker(1/tau2_space_save[i]*K_space_inv, K_time_inv)

    tau2_time_save[i] <- 1/rgamma(1, tau2_time_a + N/2,
                                  tau2_time_b + 0.5*t(f_save[i,])%*%tau2_time_Sigma%*%f_save[i,])
  }

  cbind(f_save, sig2_save, tau2_space_save, tau2_time_save)
}

load(file = "C://Users/david/Documents/R/Stat 695/Project/uber/trip_data.RData")

gaupro_out <- gau.pro.spat.temp(trip_data, niter = 100)

f_out <- gaupro_out[,1:600]
f_pred <- colMeans(exp(f_out*sd(log(trip_data$n_trips+1)) + mean(log(trip_data$n_trips+1))))

f_pred_mat <- matrix(f_pred, nrow = 24, ncol = 25, byrow = FALSE)
trip_mat <- matrix(trip_data$n_trips, nrow = 24, ncol = 25, byrow = FALSE)

for (i in 1:25){
  plot(0:23, trip_mat[,i], main = paste("Number of Hourly Trips (with Predictions) for Location", i),
          xlab = "Hour", ylab = "Number of Trips")
  lines(0:23, f_pred_mat[,i], col = "red")
}
```

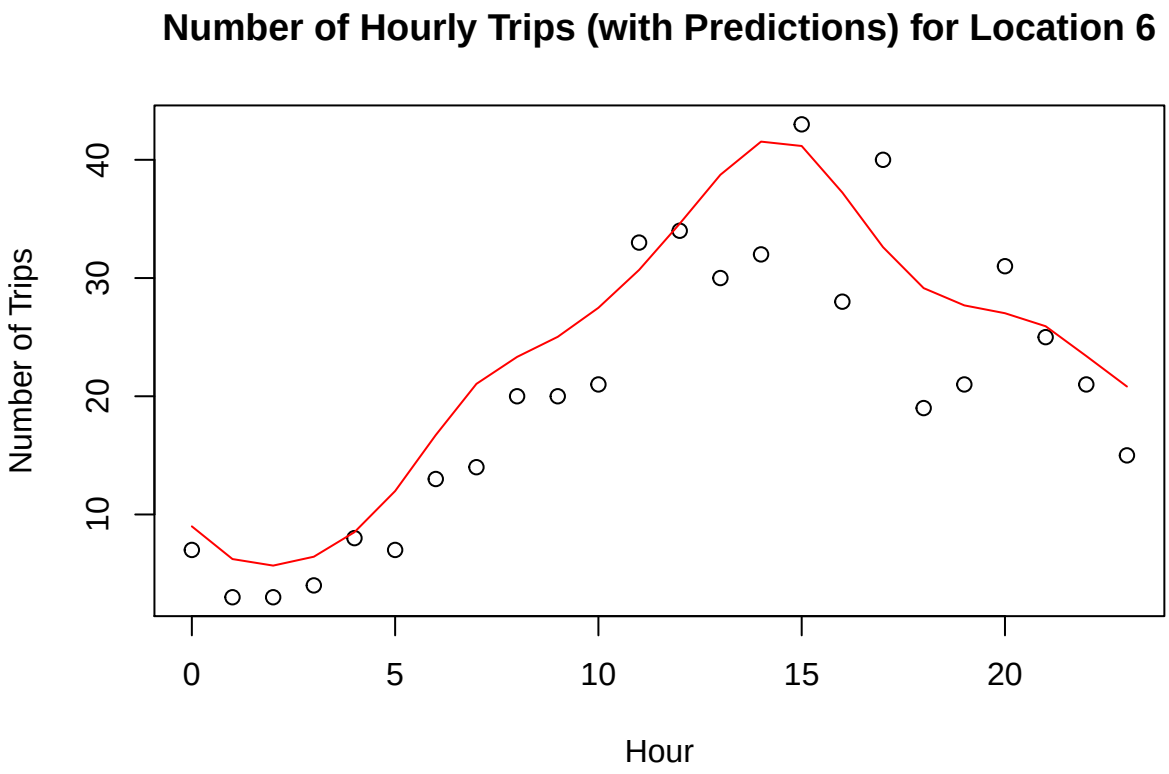

Number of Hourly Trips (with Predictions) for Location 1
Number of Trips
100
200
300
400
500
0
5
10
15
20
Hour

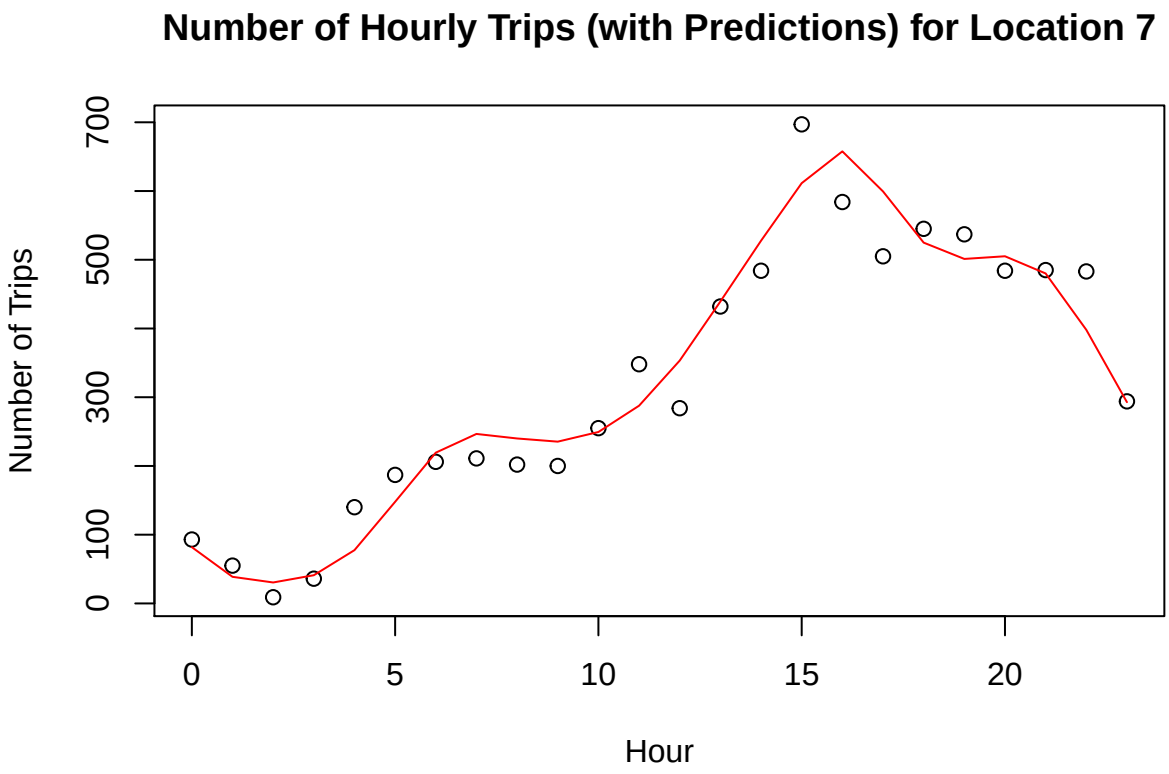

Number of Hourly Trips (with Predictions) for Location 2
Number of Trips
5000
10000
20000
0
5
10
15
20
Hour

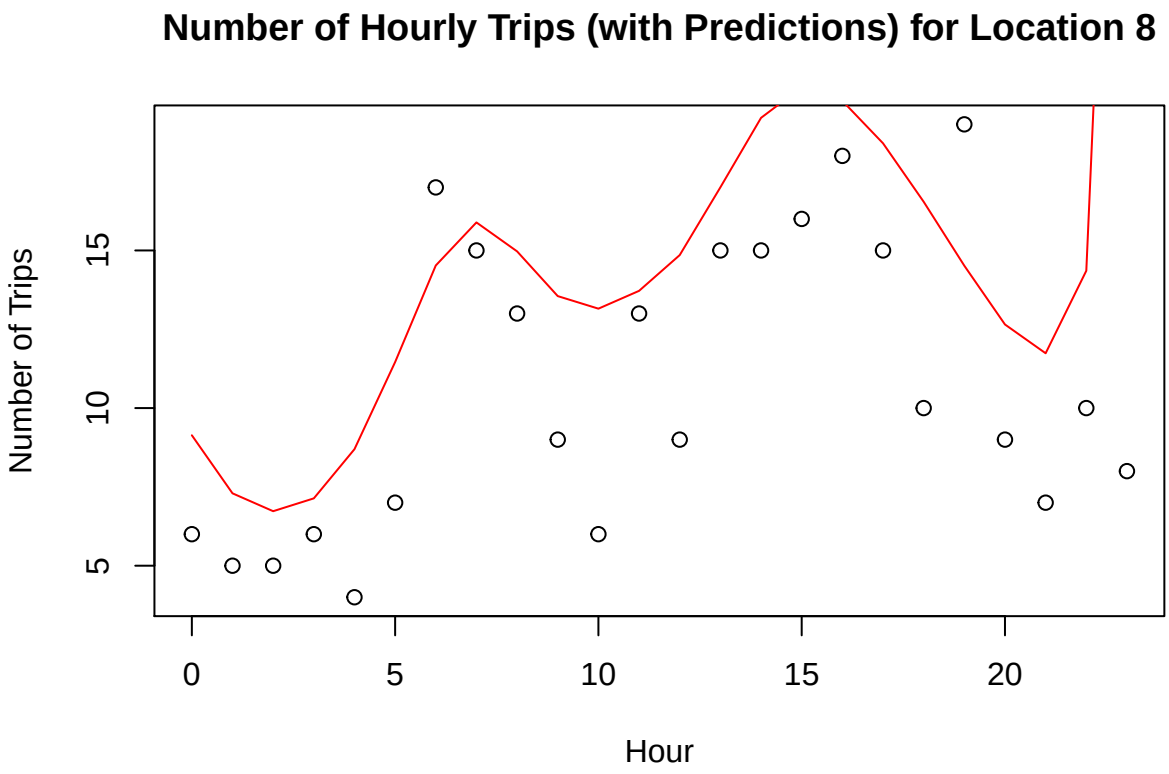

Number of Hourly Trips (with Predictions) for Location 3
Number of Trips
50
100
150
200
0
5
10
15
20
Hour

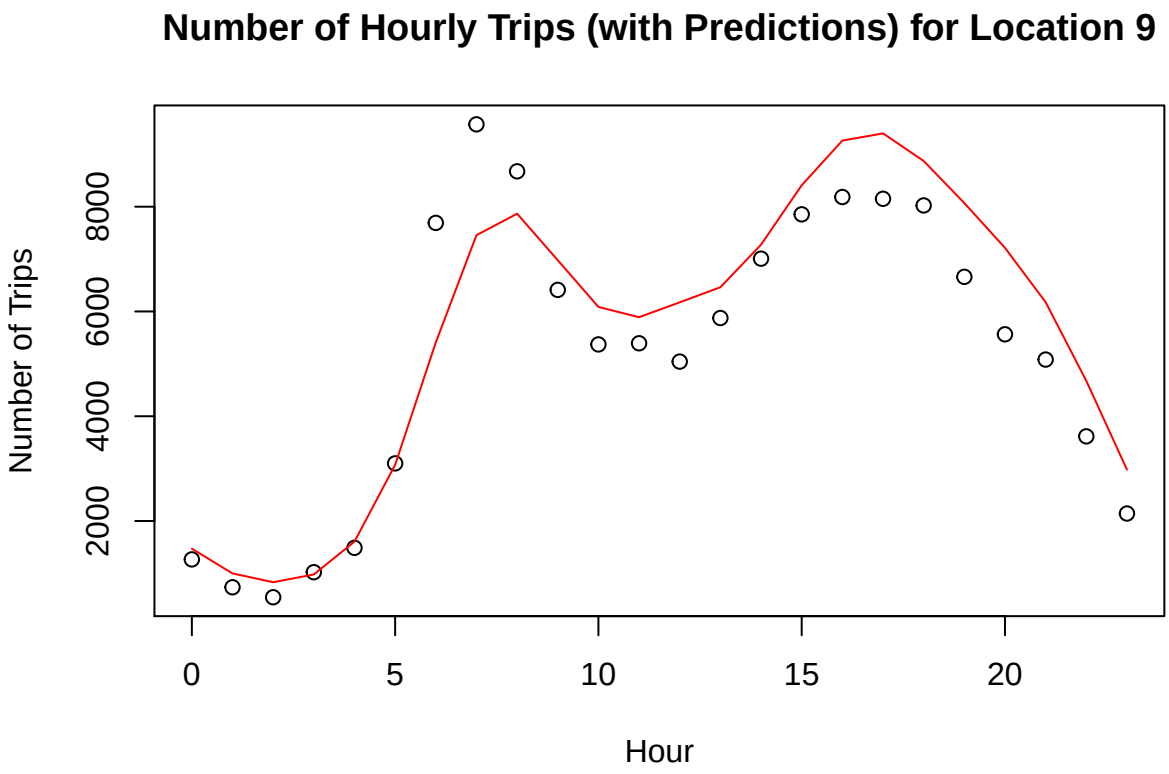

Number of Hourly Trips (with Predictions) for Location 4
Number of Trips
200
300
400
500
600
700
0
5
10
15
20
Hour

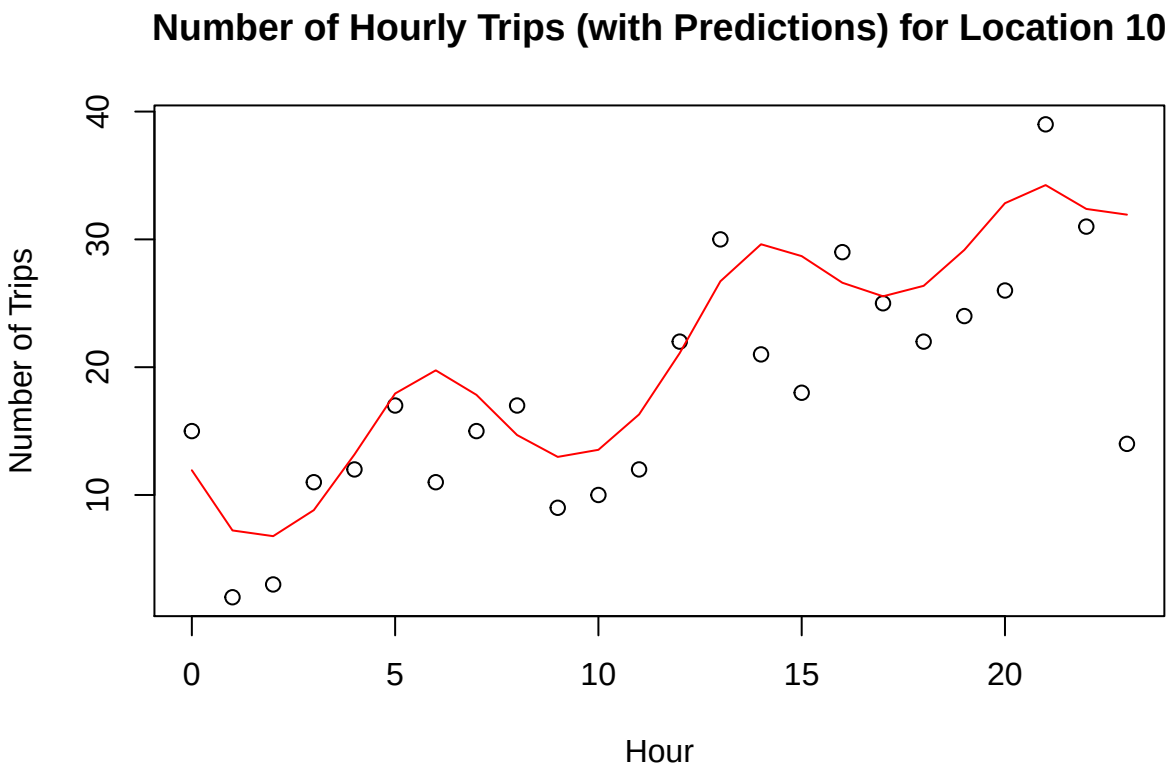

Number of Hourly Trips (with Predictions) for Location 5
Number of Trips
10
20
30
40
50
60
70
0
5
10
15
20
Hour

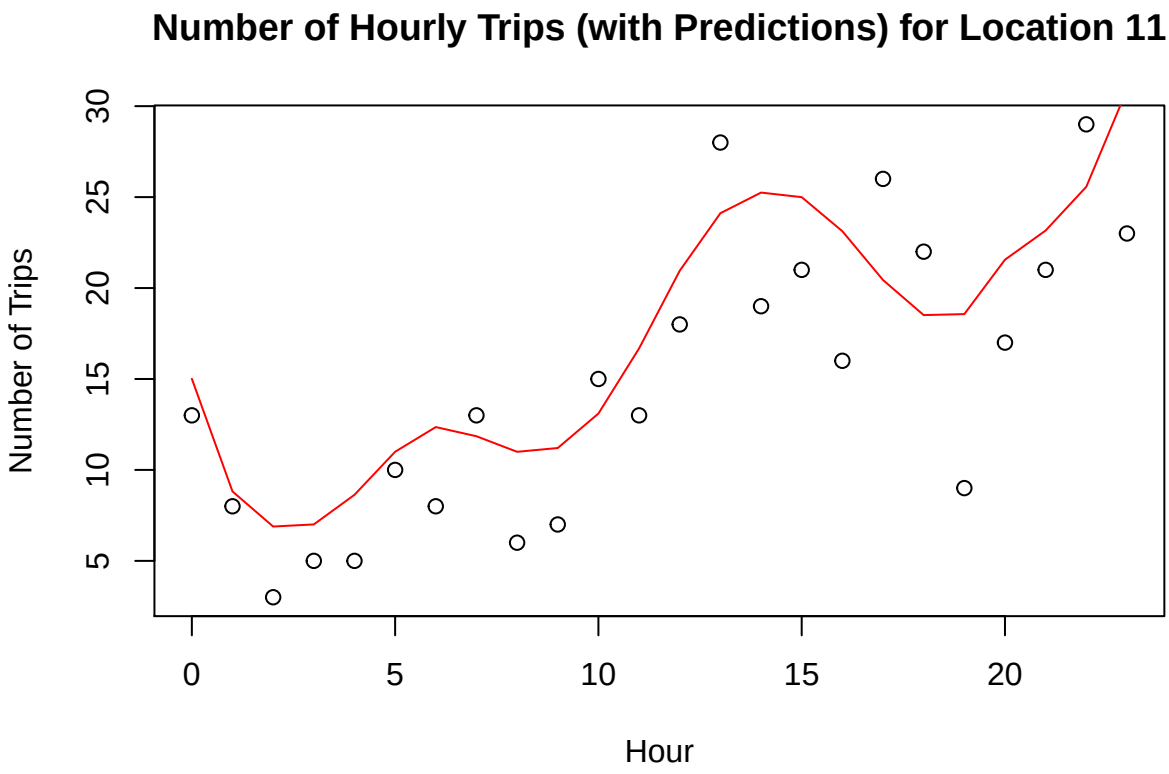

Number of Hourly Trips (with Predictions) for Location 6
Number of Trips
10
20
30
40
0
5
10
15
20
Hour

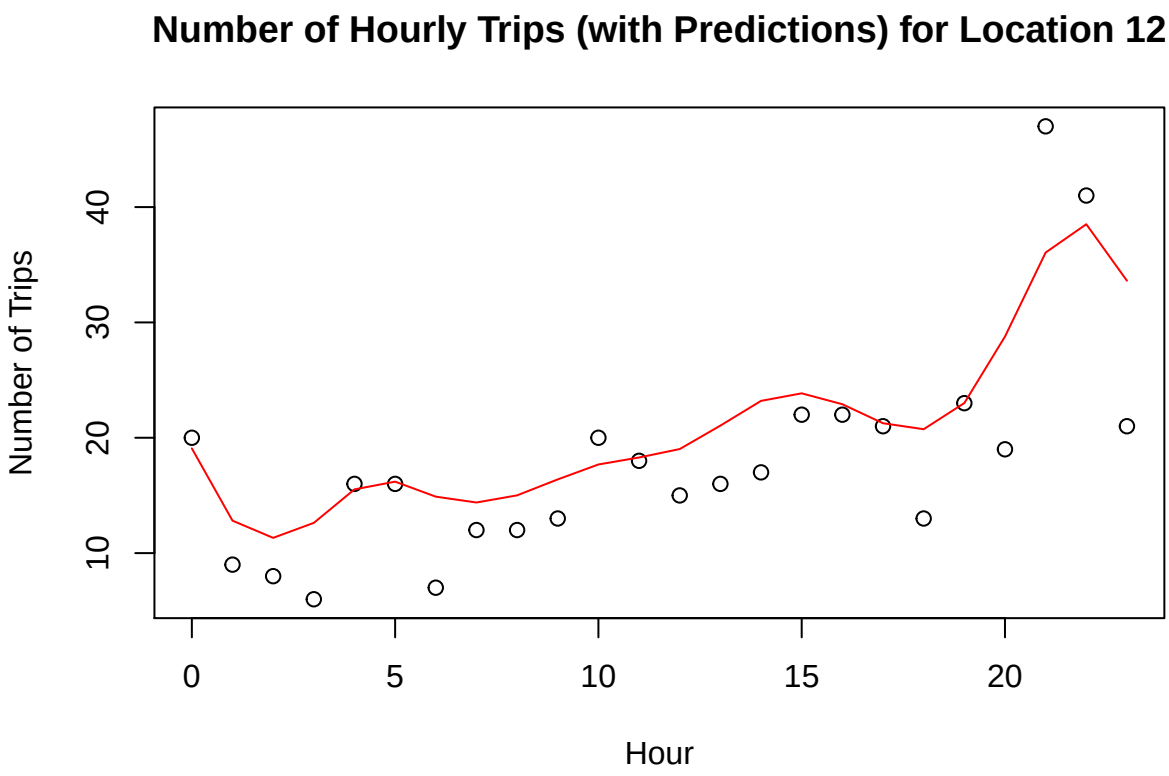

Number of Hourly Trips (with Predictions) for Location 7
Number of Trips
0
100
300
500
700
0
5
10
15
20
Hour

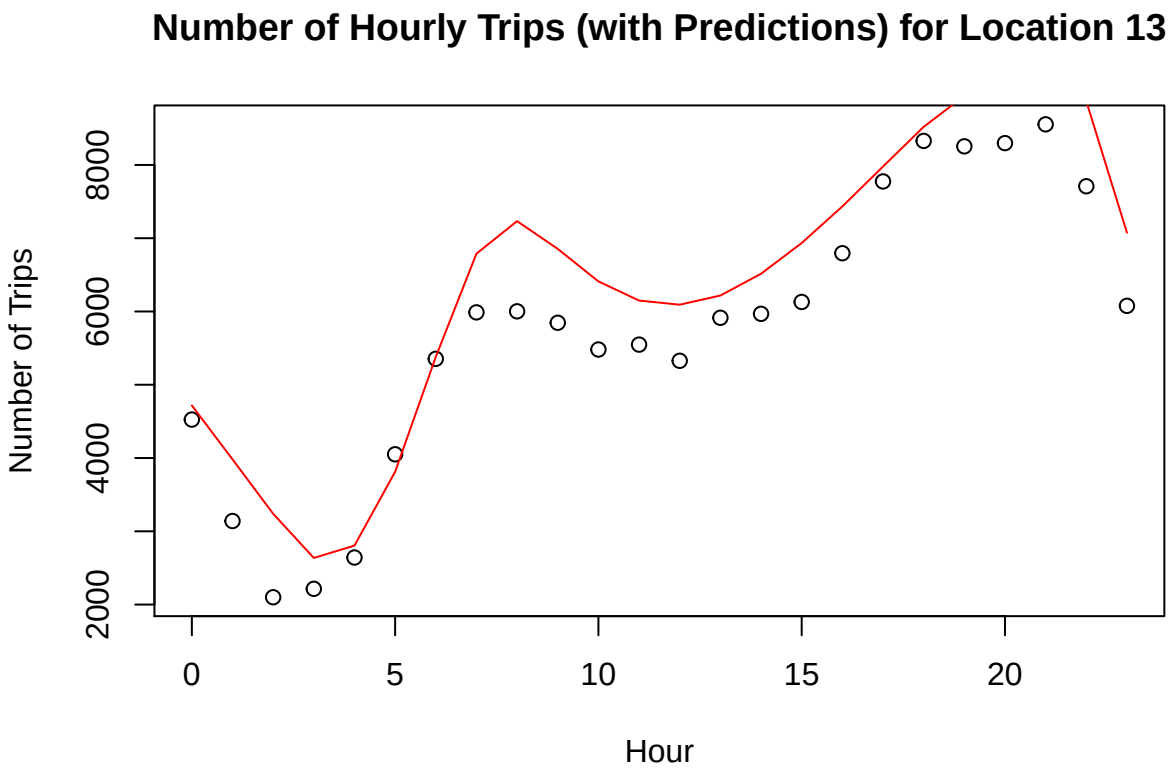

Number of Hourly Trips (with Predictions) for Location 8
Number of Trips
5
10
15
0
5
10
15
20
Hour

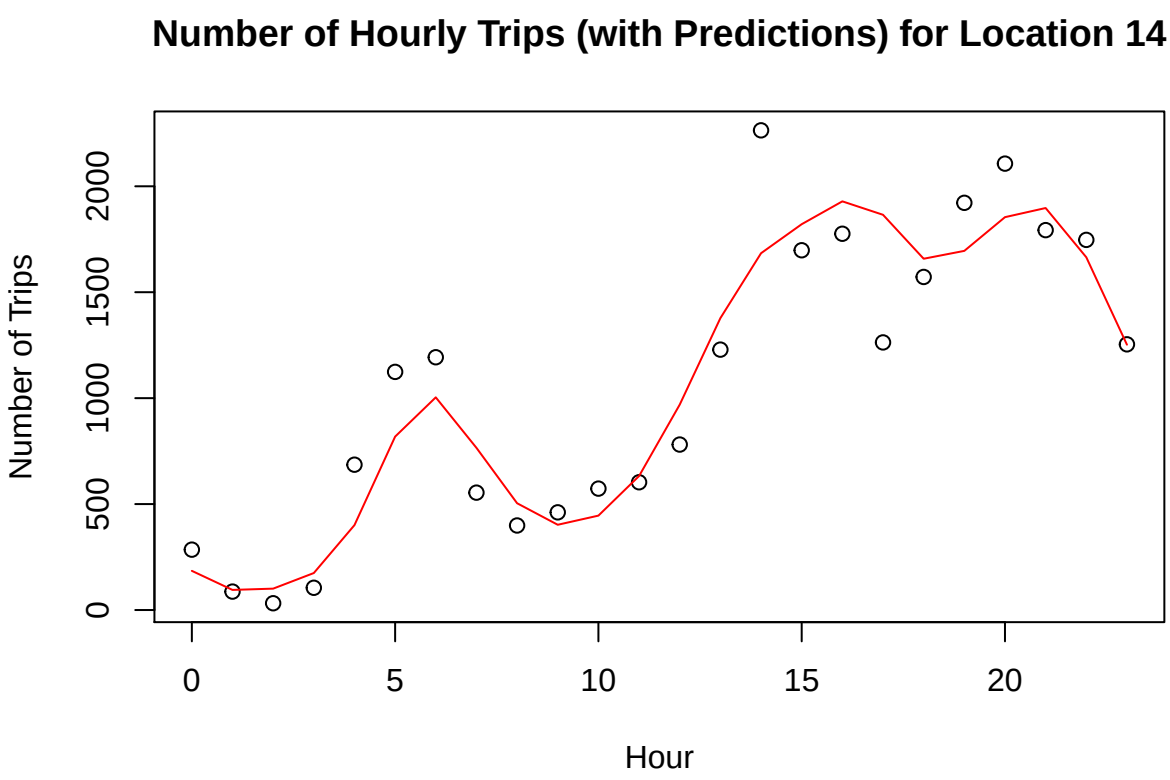

Number of Hourly Trips (with Predictions) for Location 9
Number of Trips
2000
4000
6000
8000
0
5
10
15
20
Hour

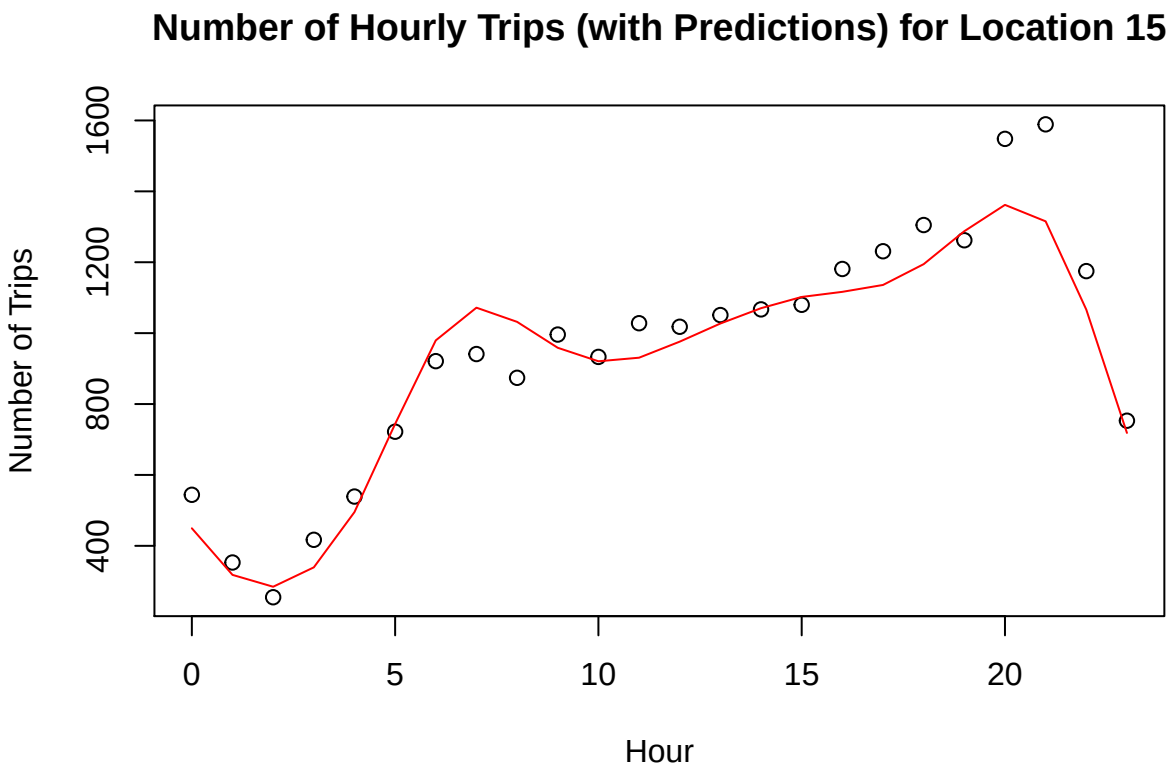

Number of Hourly Trips (with Predictions) for Location 10
Number of Trips
10
20
30
40
0
5
10
15
20
Hour

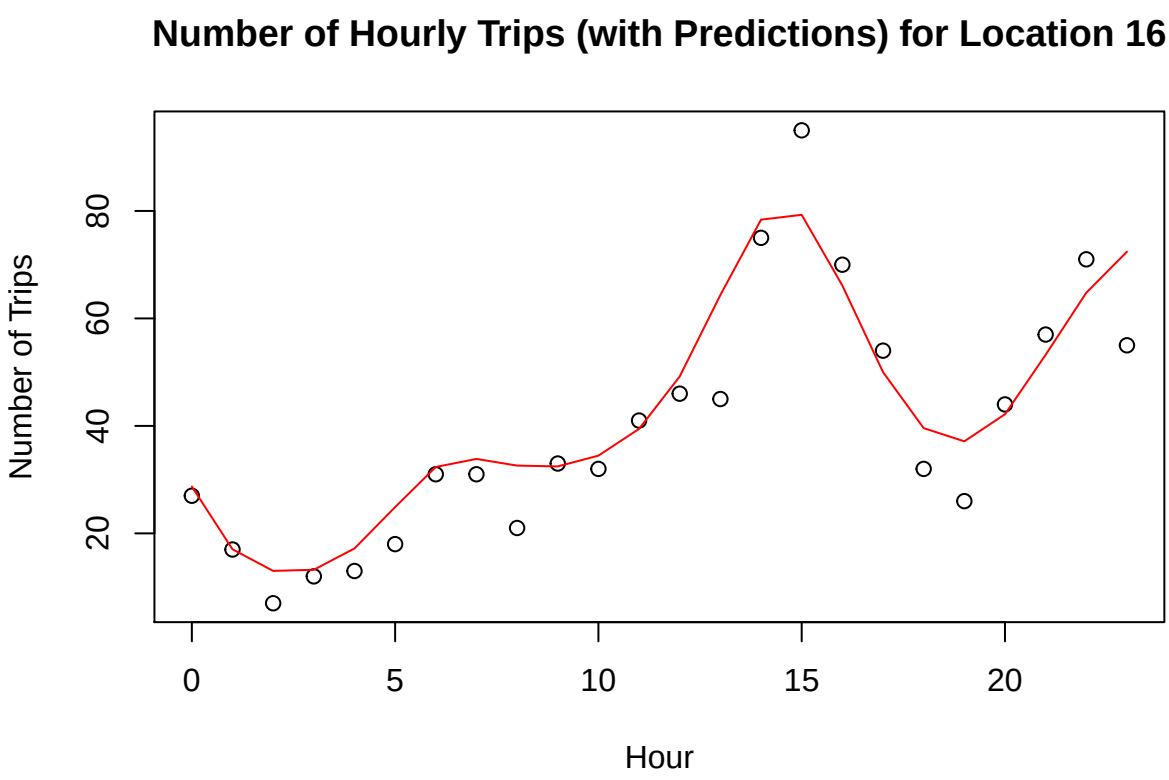

Number of Hourly Trips (with Predictions) for Location 11
Number of Trips
5
10
15
20
25
30
0
5
10
15
20
Hour

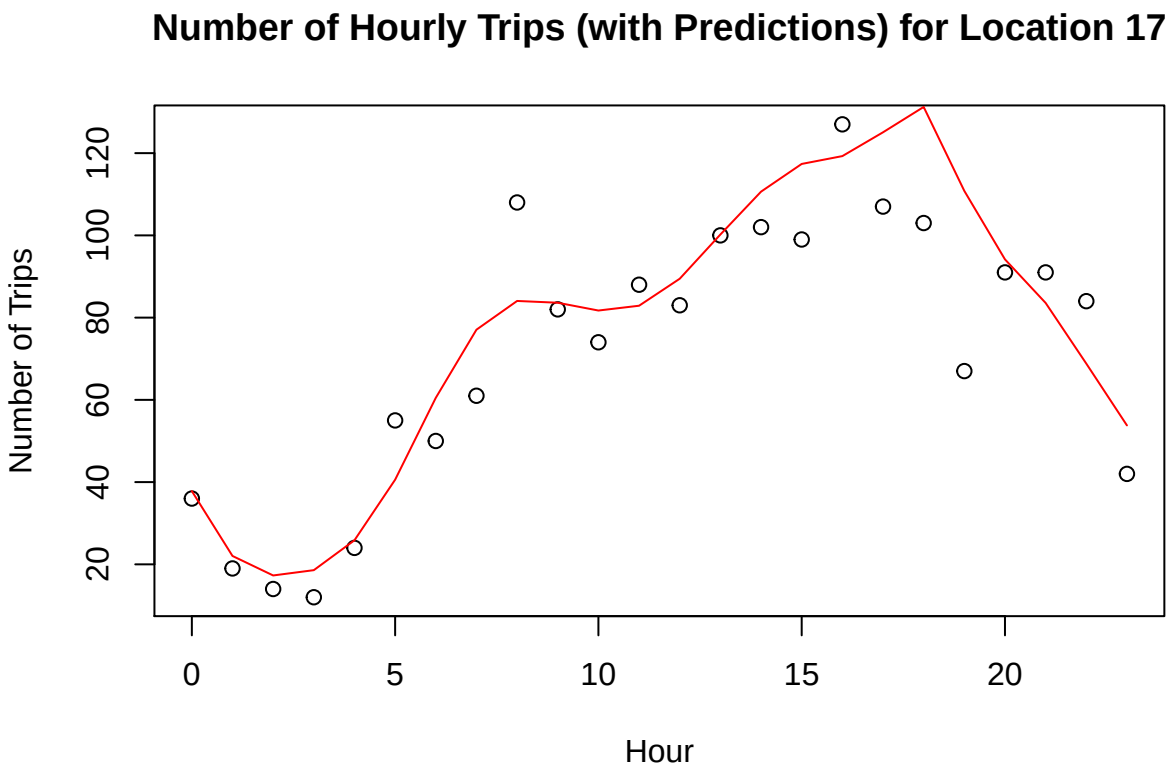

Number of Hourly Trips (with Predictions) for Location 12
Number of Trips
10
20
30
40
0
5
10
15
20
Hour

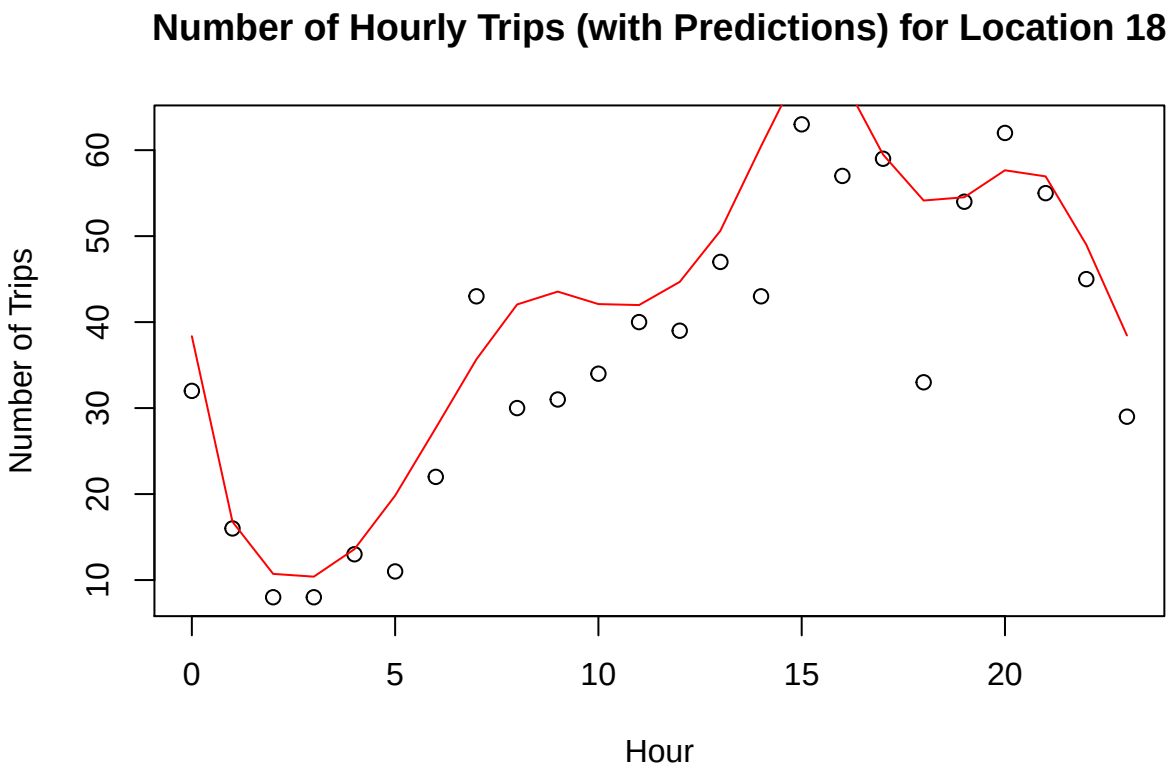

Number of Hourly Trips (with Predictions) for Location 13
Number of Trips
2000
4000
6000
8000
0
5
10
15
20
Hour

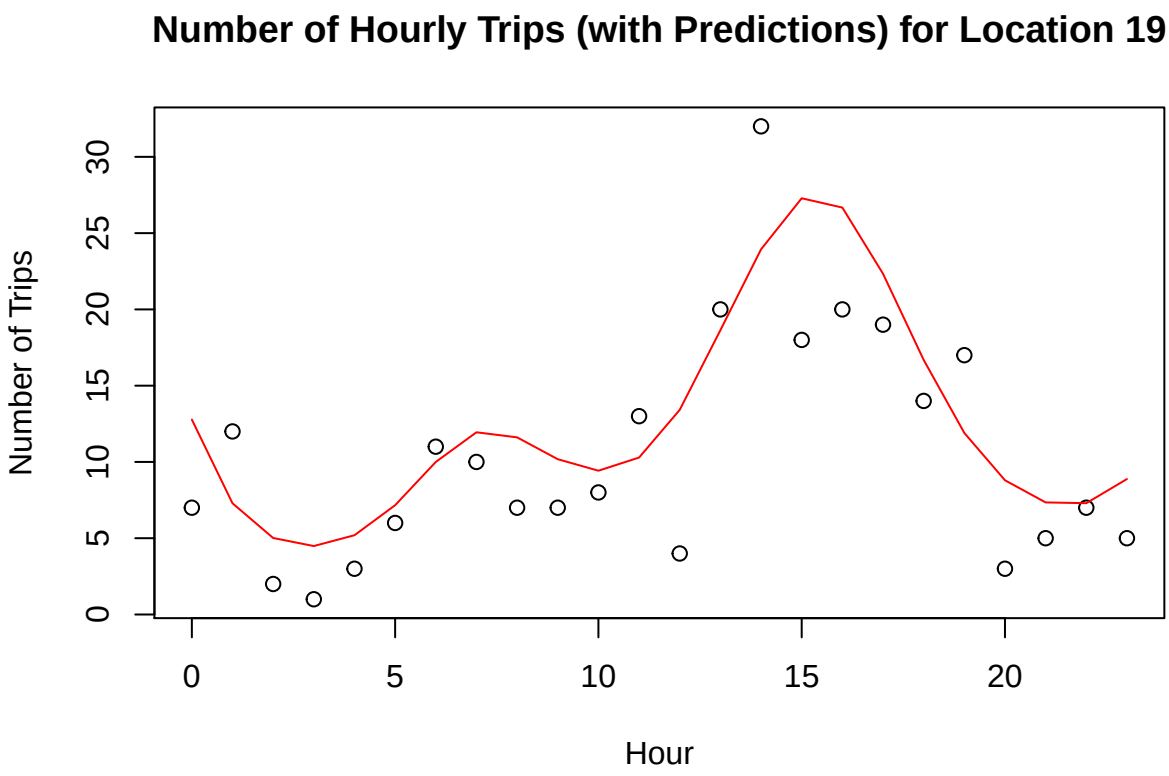

Number of Hourly Trips (with Predictions) for Location 14
Number of Trips
0
500
1000
1500
2000
0
5
10
15
20
Hour

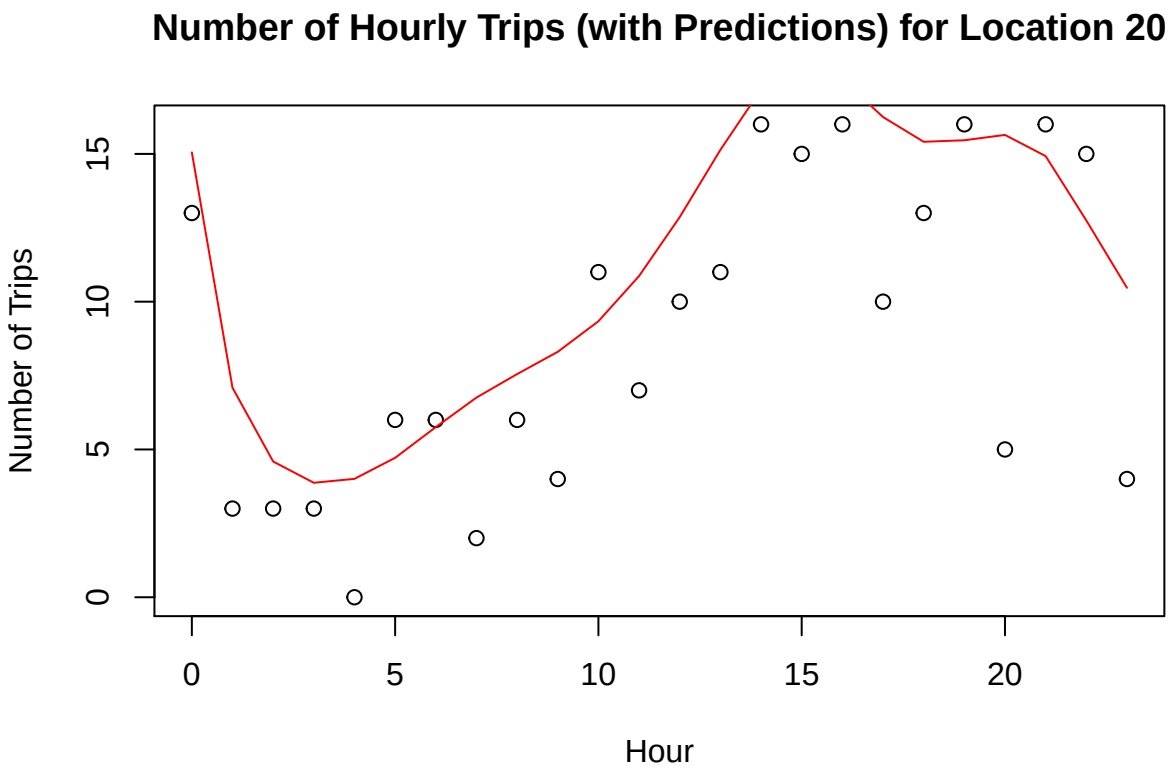

Number of Hourly Trips (with Predictions) for Location 15
Number of Trips
400
800
1200
1600
0
5
10
15
20
Hour

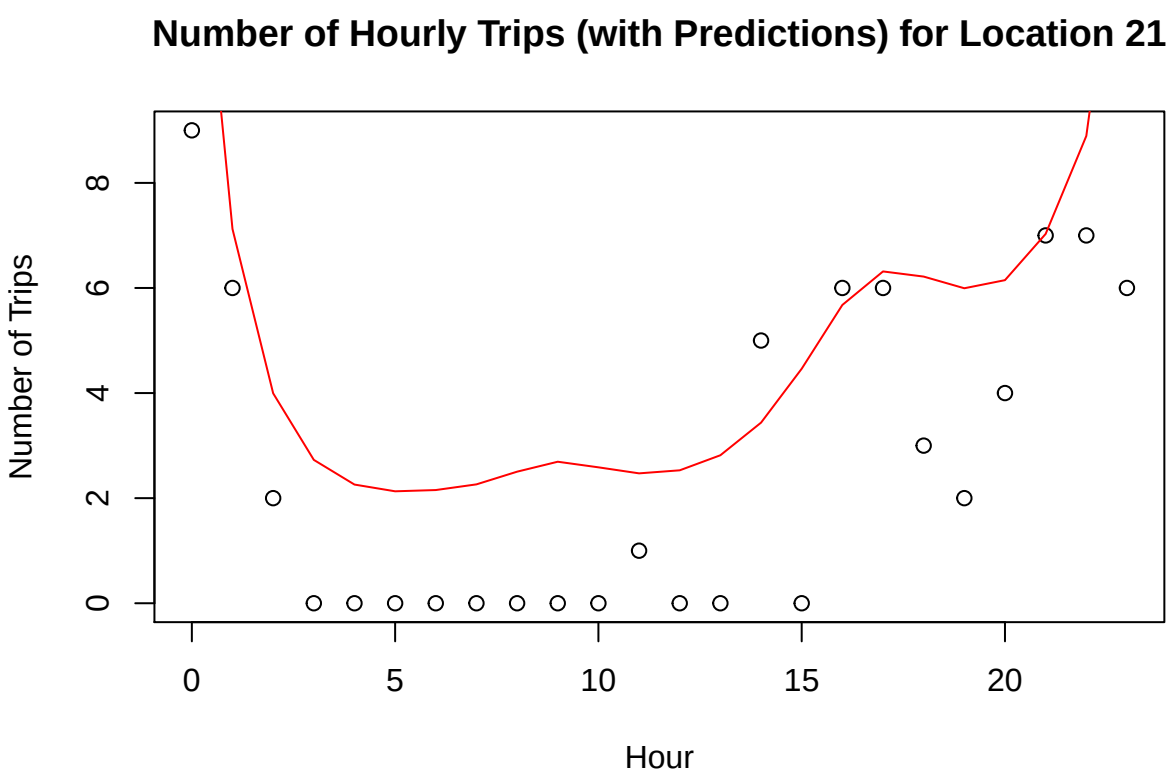

Number of Hourly Trips (with Predictions) for Location 16
Number of Trips
20
40
60
80
0
5
10
15
20
Hour

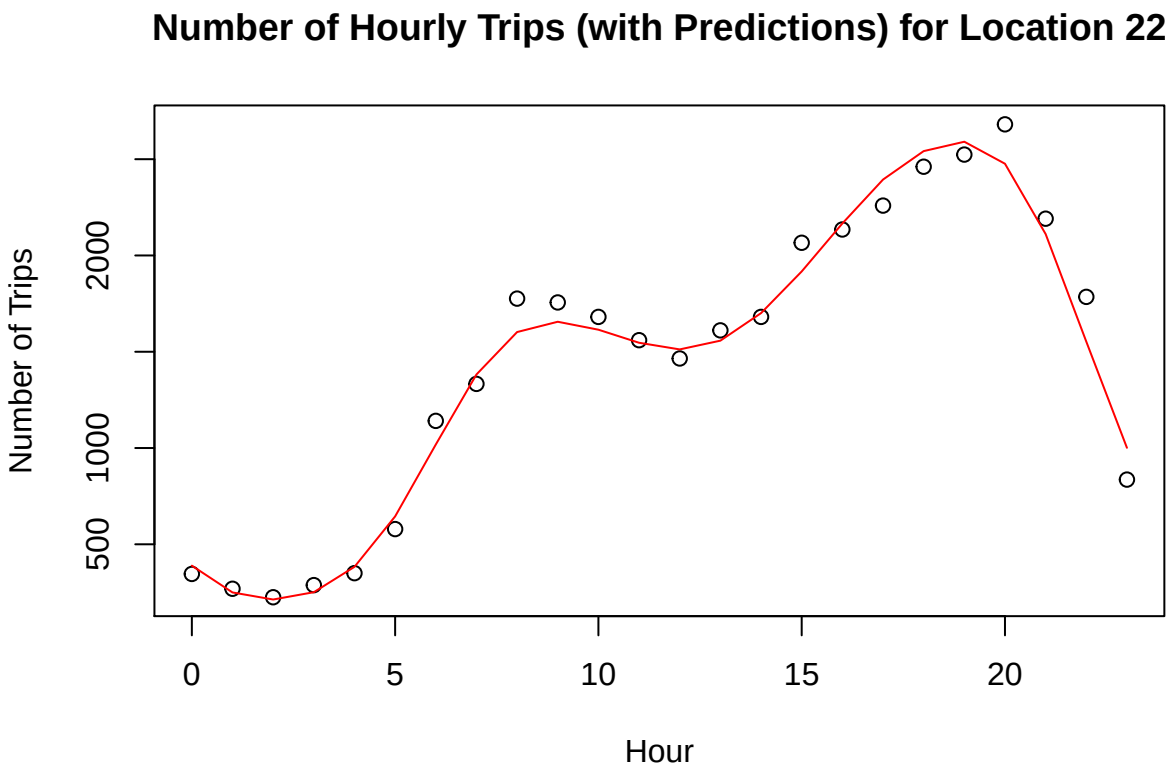

Number of Hourly Trips (with Predictions) for Location 17
Number of Trips
20
40
60
80
100
120
0
5
10
15
20
Hour

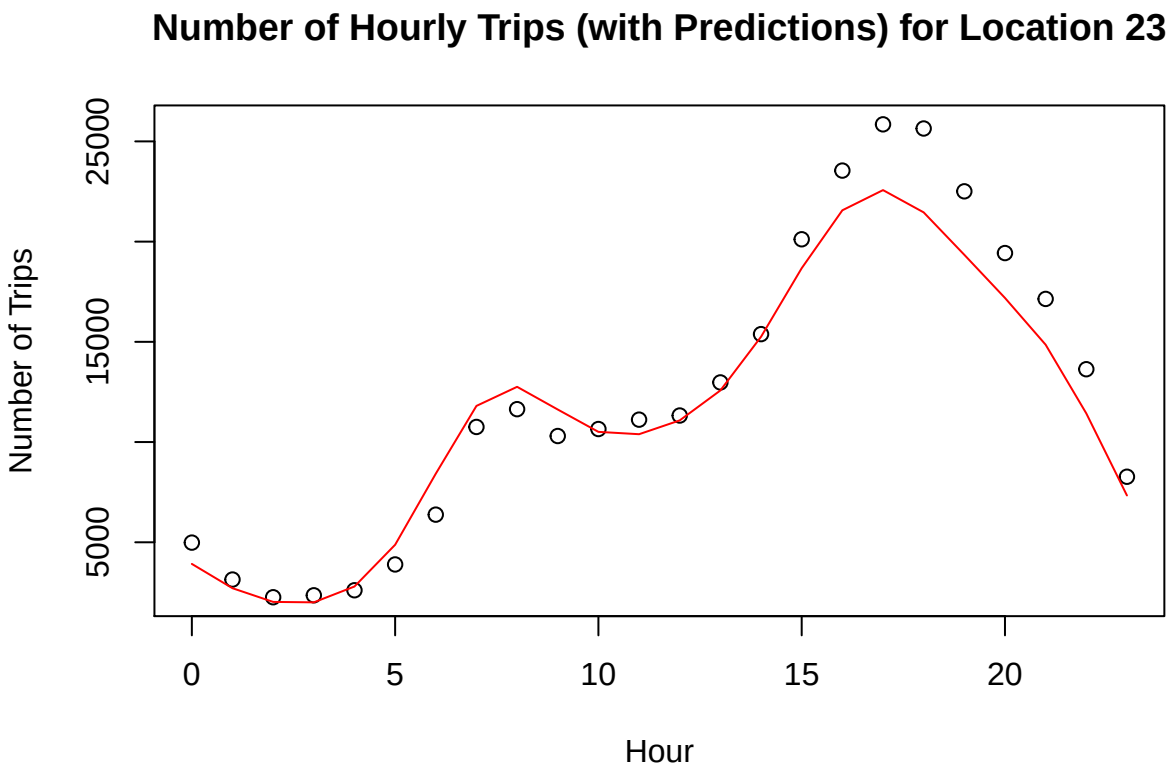

Number of Hourly Trips (with Predictions) for Location 18
Number of Trips
10
20
30
40
50
60
0
5
10
15
20
Hour

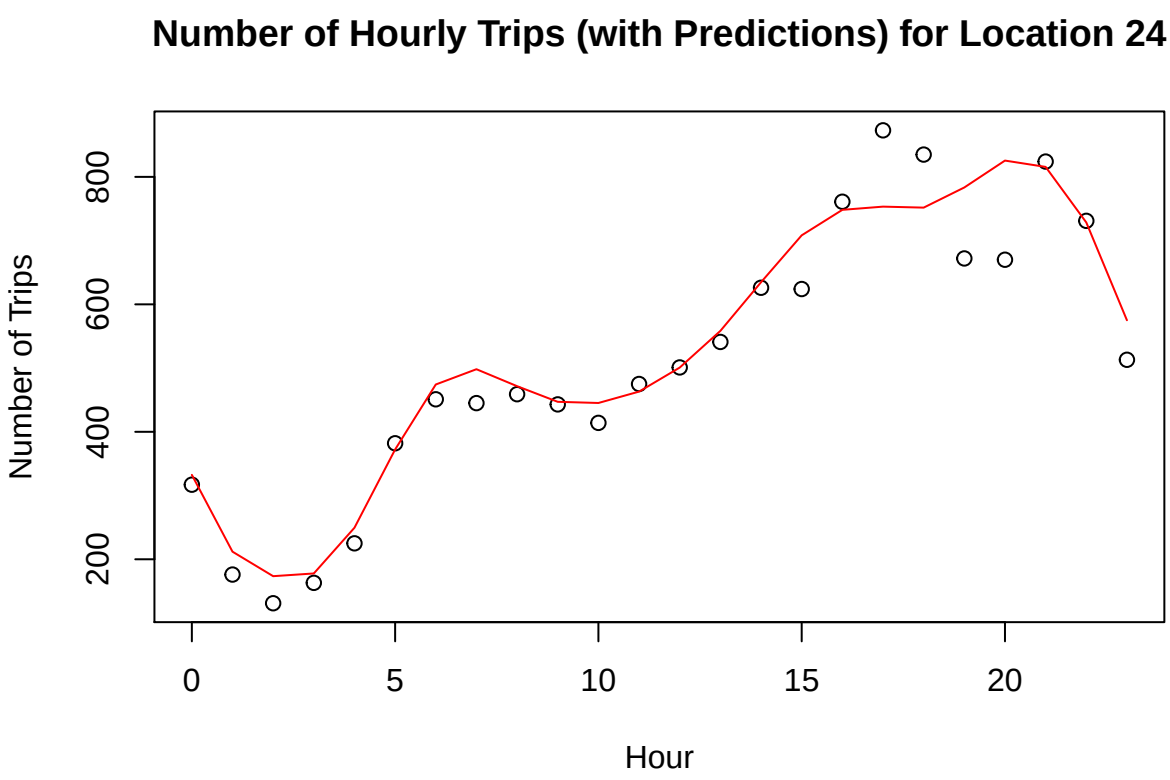

Number of Hourly Trips (with Predictions) for Location 19
Number of Trips
0
5
10
15
20
25
30
0
5
10
15
20
Hour

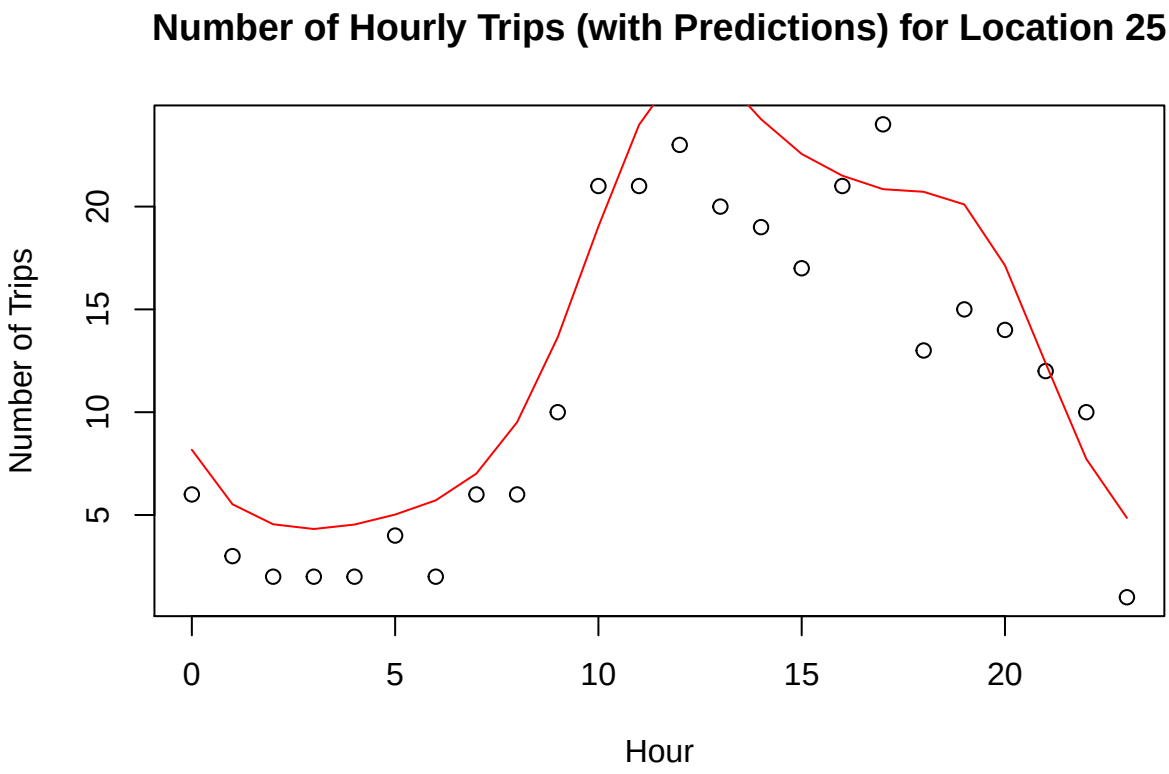

Number of Hourly Trips (with Predictions) for Location 20
Number of Trips
0
5
10
15
0
5
10
15
20
Hour


Number of Hourly Trips (with Predictions) for Location 21
Number of Trips
0
2
4
6
8
0
5
10
15
20
Hour


Number of Hourly Trips (with Predictions) for Location 22
Number of Trips
500
1000
2000
0
5
10
15
20
Hour


Number of Hourly Trips (with Predictions) for Location 23
Number of Trips
5000
15000
25000
0
5
10
15
20
Hour


Number of Hourly Trips (with Predictions) for Location 24
Number of Trips
200
400
600
800
0
5
10
15
20
Hour


Number of Hourly Trips (with Predictions) for Location 25
Number of Trips
5
10
15
20
0
5
10
15
20
Hour