

1) Reference String: 1, 2, 3, 4, 2, 1, 5, 6, 2, 1, 2, 3, 7, 6, 3, 2, 1, 2, 3, 6

a) LRU Replacement

→ 3 Frames

	1	2	3	4	2	1	5	6	2	1	2	3	7	6	3	2	1	2	3	6
1	1	1	1	4	4	4	5	5	5	1	1	1	7	7	7	2	2	2	2	2
2		2	2	2	2	2	2	6	6	6	6	3	3	3	3	3	3	3	3	3
3			3	3	3	1	1	1	2	2	2	2	2	6	6	6	1	1	1	6
					H					H				H			H		H	

Page Hit = 5

Hit ratio = $5/20 = 0.25$

Page Miss = 15

Miss ratio = $15/20 = 0.75$

→ 4 Frames

	1	2	3	4	2	1	5	6	2	1	2	3	7	6	3	2	1	2	3	6
1	1	1	1	1	1	1	1	1	1	1	1	1	1	6	6	6	6	6	6	6
2		2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3			3	3	3	3	5	5	5	5	5	3	3	3	3	3	3	3	3	3
4				4	4	4	4	6	6	6	6	6	7	7	7	7	1	1	1	1
					H	H			H	H	H			H	H		H	H	H	

Page Hit = 10

Hit ratio = 0.5

Page Miss = 10

Miss ratio = 0.5

→ 5 Frames

	1	2	3	4	2	1	5	6	2	1	2	3	7	6	3	2	1	2	3	6
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2		2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3			3	3	3	3	3	6	6	6	6	6	6	6	6	6	6	6	6	6
4				4	4	4	4	4	4	4	4	3	3	3	3	3	3	3	3	3
5							5	5	5	5	5	5	7	7	7	7	7	7	7	7
					H	H			H	H	H			H	H	H	H	H	H	H

Page Hit = 12

Hit ratio = $\frac{12}{20}$

= 0.6

Page Miss = 8

Miss ratio = $\frac{8}{20}$

= 0.4

→ 6 Frames

	1	2	3	4	2	1	5	6	2	1	2	3	7	6	3	2	1	2	3	6
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2		2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3			3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
4				4	4	4	4	4	4	4	4	4	7	7	7	7	7	7	7	7
5							5	5	5	5	5	5	5	5	5	5	5	5	5	5
6								6	6	6	6	6	6	6	6	6	6	6	6	6
					H	H			H	H	H	H		H	H	H	H	H	H	H

Page Hit = 13

Hit ratio = $13/20 = 0.65$

Page Miss = 7

Miss ratio = $7/20 = 0.35$

→ b) FIFO Replacement

→ 3 Frames

	1	2	3	4	2	1	5	6	2	1	2	3	7	6	3	2	1	2	3	6
1	1	1	1	4	4	4	4	6	6	6	6	3	3	3	3	2	2	2	2	6
2		2	2	2	2	1	1	1	2	2	2	2	7	7	7	7	1	1	1	1
3			3	3	3	3	5	5	5	1	1	1	1	6	6	6	6	6	3	3
					H					H				H			H			

Page Hit = 4

Hit ratio = $4/20 = 0.2$

Page Miss = 16

Miss ratio = 0.8

→ 4 Frames

	1	2	3	4	2	1	5	6	2	1	2	3	7	6	3	2	1	2	3	6
1	1	1	1	1	1	1	5	5	5	5	5	3	3	3	3	3	1	1	1	1
2		2	2	2	2	2	2	6	6	6	6	6	7	7	7	7	7	7	3	3
3			3	3	3	3	3	3	2	2	2	2	2	6	6	6	6	6	6	6
4				4	4	4	4	4	4	1	1	1	1	1	1	2	2	2	2	2
					H	H				H				H			H			H

Page Hit = 6

Hit ratio = $6/20 = 0.3$

Page Miss = 14

Miss ratio = 0.7

→ 5 Frames

	1	2	3	4	2	1	5	6	2	1	2	3	7	6	3	2	1	2	3	6
1	1	1	1	1	1	1	1	6	6	6	6	6	6	6	6	6	6	6	6	6
2		2	2	2	2	2	2	2	2	1	1	1	1	1	1	2	2	2	2	2
3			3	3	3	3	3	3	3	3	2	2	2	2	3	3	3	3	3	3
4				4	4	4	4	4	4	4	4	3	3	3	3	7	7	7	7	7
5							5	5	5	5	5	5	7	7	7	7	7	7	7	7
					H	H			H					H	H	H	H	H	H	H

Page Hit = 10

Hit ratio = 0.5

Page Miss = 10

Miss ratio = 0.5

→ 6 Frames

	1	2	3	4	2	1	5	6	2	1	2	3	7	6	3	2	1	2	3	6
1	1	1	1	1	1	1	1	1	1	1	1	1	7	7	7	7	7	7	7	7
2		2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1
3			3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	2	2
4				4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	3	3
5							5	5	5	5	5	5	5	5	5	5	5	5	5	5
6								6	6	6	6	6	6	6	6	6	6	6	6	6
					H	H			H	H	H	H		H	H	H				H

Page Hit = 10

Hit ratio = 0.5

Page Miss = 10

Miss ratio = 0.5

c) Optimal Replacement

→ 3 Frames

	1	2	3	4	2	1	5	6	2	1	2	3	7	6	3	2	1	2	3	6
1	1	1	1	1	1	1	1	1	1	1	1	3	3	3	3	3	3	3	3	3
2		2	2	2	2	2	2	2	2	2	2	2	7	7	7	2	1	2	2	2
3			3	4	4	4	5	6	6	6	6	6	6	6	6	6	6	6	6	6
					H	H			H	H	H			H	H				H	H

Page Hit = 9

Hit Ratio = 0.45

Page Miss = 11

Miss ratio = 0.55

→ 4 Frames

	1	2	3	4	2	1	5	6	2	1	2	3	7	6	3	2	1	2	3	6
1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	1	1	1	1
2		2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3			3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
4				4	4	4	5	6	6	6	6	6	6	6	6	6	6	6	6	6
					H	H			H	H	H	H		H	H	H		H	H	H

Page Hit = 12

Hit ratio = 0.6

Page Miss = 8

Miss ratio = 0.4

→ 5 Frames

	1	2	3	4	2	1	5	6	2	1	2	3	7	6	3	2	1	2	3	6
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2		2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3			3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
4				4	4	4	4	6	6	6	6	6	6	6	6	6	6	6	6	6
5							5	5	5	5	5	5	7	7	7	7	7	7	7	7
					H	H			H	H	H	H		H	H	H	H	H	H	H

Page Hit = 13

Hit ratio = 0.65

Page Miss = 7

Miss ratio = 0.35

7 faults is the least we can have with 7 pages.

2) Reference String: 7, 2, 3, 1, 2, 5, 3, 4, 6, 7, 7, 1, 0, 5, 4, 6, 2, 3, 0, 1

3 Frames

a) LRU Replacement

	7	2	3	1	2	5	3	4	6	7	7	1	0	5	4	6	2	3	0	1
1	7	7	7	1	1	1	3	3	3	7	7	7	7	5	5	5	2	2	2	1
2		2	2	2	2	2	2	4	4	4	4	1	1	1	4	4	4	3	3	1
3			3	3	3	5	5	5	6	6	6	6	0	0	0	6	6	6	0	0
	M	M	M	M	H	M	M	M	M	M	H	M	M	M	M	M	M	M	M	M

Page Hit = 2

Page Fault = 18

$$\text{Hit ratio} = \frac{2}{2+18} = \frac{2}{20} = \frac{1}{10} = 0.1$$

$$\text{Miss ratio} = \frac{18}{18+2} = \frac{18}{20} = 0.9$$

b) FIFO Replacement

	7	2	3	1	2	5	3	4	6	7	7	1	0	5	4	6	2	3	0	1
1	7	7	7	1	1	1	1	1	6	6	6	6	0	0	0	6	6	6	0	0
2		2	2	2	2	5	5	5	5	7	7	7	7	5	5	5	2	2	2	1
3			3	3	3	3	3	4	4	4	4	1	1	1	4	4	4	3	3	3
						H	H				H									

Page Hit = 3

Page Fault = 17

$$\text{Hit ratio} = \frac{3}{20} = 0.15$$

$$\text{Miss ratio} = \frac{17}{20} = 0.85$$

c) Optimal Replacement

	7	2	3	1	2	5	3	4	6	7	7	1	0	5	4	6	2	3	0	1
1	7	7	7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2		2	2	2	2	5	5	5	5	5	5	5	5	5	4	6	2	3	3	3
3			3	3	3	3	3	4	6	7	7	7	0	0	0	0	0	0	0	0
						H	H				H	H		H					H	H

Page Hit = 7

Page Miss = 13

$$\text{Hit Ratio} = \frac{7}{20} = 0.35$$

$$\text{Miss ratio} = \frac{13}{20} = 0.65$$

3) Reference string: 1, 2, 3, 4, 5, 3, 4, 1, 6, 7, 8, 7, 8, 9, 7, 8, 9, 5, 4, 5, 4, 2
3 Frames

a) LRU Replacement

	1	2	3	4	5	3	4	1	6	7	8	7	8	9	7	8	9	5	4	5	4	2
1	1	1	1	4	4	4	4	4	4	7	7	7	7	7	7	7	7	5	5	5	5	5
2		2	2	2	5	5	5	1	1	1	8	8	8	8	8	8	8	4	4	4	4	4
3			3	3	3	3	3	3	6	6	6	6	6	9	9	9	9	9	9	9	9	2
						H	H					H	H			H	H	H		H	H	

Page Hit = 9

Page Miss = 13

Hit ratio = $9/22 = 0.40$

Miss ratio = $13/22 = 0.60$

b) FIFO Replacement

	1	2	3	4	5	3	4	1	6	7	8	7	8	9	7	8	9	5	4	5	4	2
1	1	1	1	4	4	4	4	4	6	6	6	6	6	9	9	9	9	9	9	9	9	2
2		2	2	2	5	5	5	5	5	7	7	7	7	7	7	7	7	5	5	5	5	5
3			3	3	3	3	3	1	1	1	8	8	8	8	8	8	8	8	4	4	4	4
						H	H					H	H			H	H	H		H	H	

Page Hit = 9

Page Miss = 13

Hit ratio = 0.40

Miss ratio = 0.60

c) Optimal Replacement

	1	2	3	4	5	3	4	1	6	7	8	7	8	9	7	8	9	5	4	5	4	2
1	1	1	1	1	5	5	5	5	5	5	5	5	5	9	9	9	9	9	9	9	9	2
2		2	2	4	4	4	4	4	4	4	8	8	8	8	8	8	8	8	4	4	4	4
3			3	3	3	3	3	1	6	7	7	7	7	7	7	7	7	5	5	5	5	5
						H	H					H	H			H	H	H		H	H	

Page Hit = 9

Page Miss = 13

Hit ratio = $\frac{9}{22} = 0.40$

Miss ratio = 0.60

4) Reference String: 1, 2, 3, 4, 2, 1, 5, 6, 2, 1, 2, 3, 7, 6, 3, 2, 1, 2, 3, 6

4 Frames

a) LRU Replacement

	1	2	3	4	2	1	5	6	2	1	2	3	7	6	3	2	1	2	3	6
1	1	1	1	1	1	1	5	1	1	1	1	1	1	6	6	6	6	6	6	6
2		2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3			3	3	3	3	5	5	5	5	5	3	3	3	3	3	3	3	3	3
4				4	4	4	4	6	6	6	6	6	7	7	7	7	1	1	1	1
					H	H			H	H	H				H	H		H	H	H

Page Hit = 10

Hit ratio = $10/20 = 1/2 = 0.5$

Page Miss = 10

Miss ratio = $10/20 = 1/2 = 0.5$

b) FIFO Replacement

	1	2	3	4	2	1	5	6	2	1	2	3	7	6	3	2	1	2	3	6
1	1	1	1	1	1	1	5	5	5	5	5	3	3	3	3	3	1	1	1	1
2		2	2	2	2	2	2	6	6	6	6	6	7	7	7	7	7	7	3	3
3			3	3	3	3	3	3	2	2	2	2	2	6	6	6	6	6	6	6
4				4	4	4	4	4	4	1	1	1	1	1	1	2	2	2	2	2
					H	H				H				H			H		H	H

Page Hit = 6

Hit Ratio = $6/20 = 0.3$

Page Miss = 14

Miss ratio = $14/20 = 0.7$

c) Optimal Replacement

	1	2	3	4	2	1	5	6	2	1	2	3	7	6	3	2	1	2	3	6
1	1	1	1	1	1	1	1	1	1	1	1	1	7	7	7	7	1	1	1	1
2		2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3			3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
4				4	4	4	5	6	6	6	6	6	6	6	6	6	6	6	6	6
					H	H			H	H	H	H		H	H	H		H	H	H

Page Hit = 12

Hit ratio = $\frac{12}{20} = 0.6$

Page Miss = 8

Miss ratio = $\frac{8}{20} = 0.4$

5) Reference String: 7, 0, 1, 2, 0, 3, 0, 4, 2, 3, 0, 3, 2

4 Frames
a) LRU Replacement

	7	0	1	2	0	3	0	4	2	3	0	3	2
1	7	7	7	7	7	3	3	3	3	3	3	3	3
2		0	0	0	0	0	0	0	0	0	0	0	0
3			1	1	1	1	1	4	4	4	4	4	4
4				2	2	2	2	2	2	2	2	2	2
					H		H		H	H	H	H	H

Page Hit = 7

Hit ratio = $7/13 = 0.53$

Page Miss = 6

Miss ratio = $6/13 = 0.47$

b) FIFO Replacement

	7	0	1	2	0	3	0	4	2	3	0	3	2
1	7	7	7	7	7	3	3	3	3	3	3	3	3
2		0	0	0	0	0	0	4	4	4	4	4	4
3			1	1	1	1	1	1	1	1	0	0	0
4				2	2	2	2	2	2	2	2	2	2
					H		H		H	H	H	H	H

Page Hit = 6

Hit ratio = 0.47

Page Miss = 7

Miss ratio = 0.53

c) Optimal Replacement

	7	0	1	2	0	3	0	4	2	3	0	3	2
1	7	7	7	7	7	3	3	3	3	3	3	3	3
2		0	0	0	0	0	0	0	0	0	0	0	0
3			1	1	1	1	1	4	4	4	4	4	4
4				2	2	2	2	2	2	2	2	2	2
					H		H		H	H	H	H	H

Page Hit = 7

Hit ratio = $\frac{7}{13} = 0.53$

Page Miss = 6

Miss ratio = 0.47