# Unleashing the Power of Randomization in Auditing Differentially Private ML

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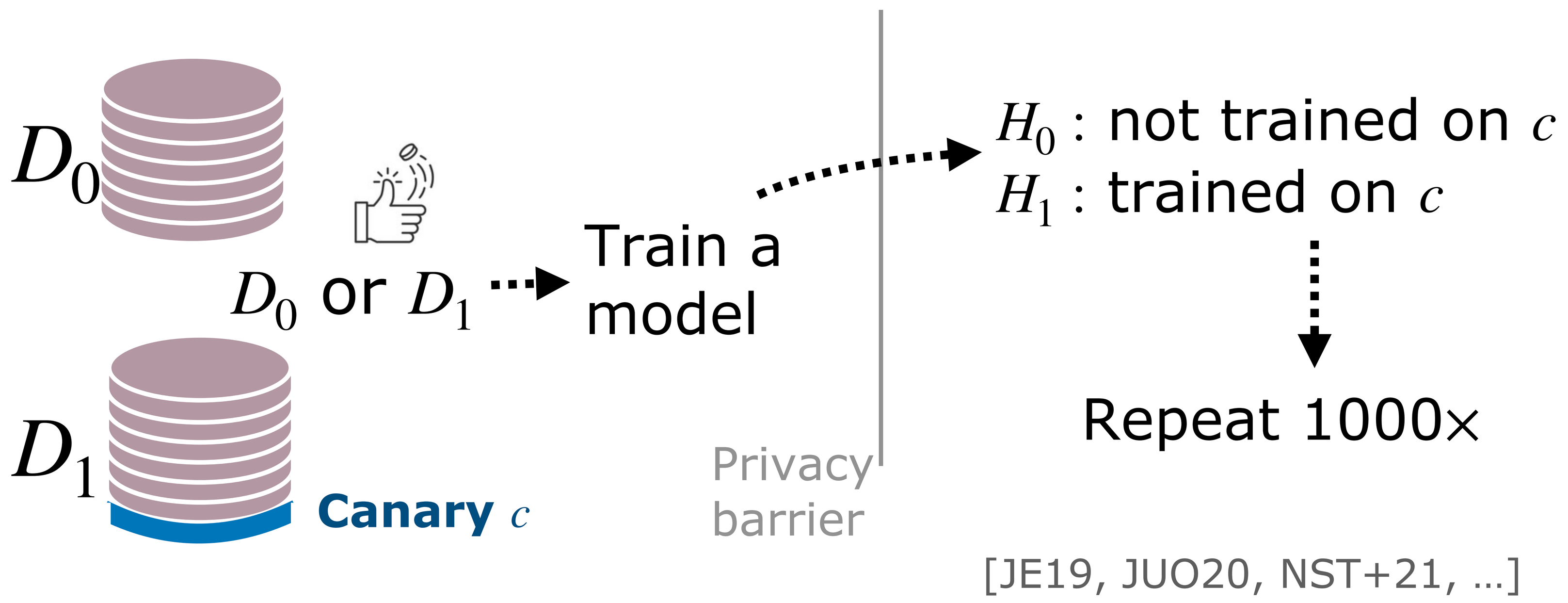
## Auditing DP guarantees in ML

### The standard approach: binary hypothesis testing




### The key components of this approach

**Step 1: DP definition**
For all neighboring $D_0, D_1$ and $R$, we have

$$\underbrace{\mathbb{P}(\mathscr{A}(D_1) \in R)}_{\text{True positive rate}} \le e^{\varepsilon}\, \underbrace{\mathbb{P}(\mathscr{A}(D_0) \in R)}_{\text{False positive rate}} + \delta \qquad \textbf{(1)}$$

**Step 2: Set up the hypothesis test**
Take $D_0$ = dataset, $D_1 = D_0 \cup \{\text{canary}\}$ and the test statistic as $R = \{\theta \,:\, \text{Loss}(\text{canary}\,;\theta) \le \tau\}$

**Step 3: Bernoulli Confidence intervals**
Run $n$ trials (each trial = one model training run)

$$\underbrace{\text{TPR}}_{\text{True rate}} \approx \underbrace{\frac{1}{n}\sum_{i=1}^{n} \text{Loss}_i(D_1) \le \tau)}_{\text{Empirical rate}} \pm \sqrt{\frac{\text{variance}}{n}} \qquad \textbf{(2)}$$

**Overall, (1) + (2) $\Longrightarrow$**

$$\varepsilon \ge \log\left(\frac{\text{TPR} - \delta}{\text{FPR}}\right) \ge \log\left(\frac{\widehat{\text{TPR}}_n - \frac{1}{\sqrt{n}} - \delta}{\widehat{\text{FPR}}_n + \frac{1}{\sqrt{n}}}\right)$$

**Problem**: the $1/\sqrt{n}$ term requires $n$ large
**How do we solve this?** Add multiple canaries
**Key**: Avoid group privacy with *randomization*

## Auditing **Lifted DP**

### Step 1: Lifted DP (LiDP) definition

**Def:** $\mathscr{A}$ is $(\varepsilon,\delta)$-LiDP if for all random $(\boldsymbol{D_0}, \boldsymbol{D_1}, \boldsymbol{R}) \sim \mathscr{P}$ independent of $\mathscr{A}$ s.t. $\boldsymbol{D_0}, \boldsymbol{D_1}$ are neighboring, we have

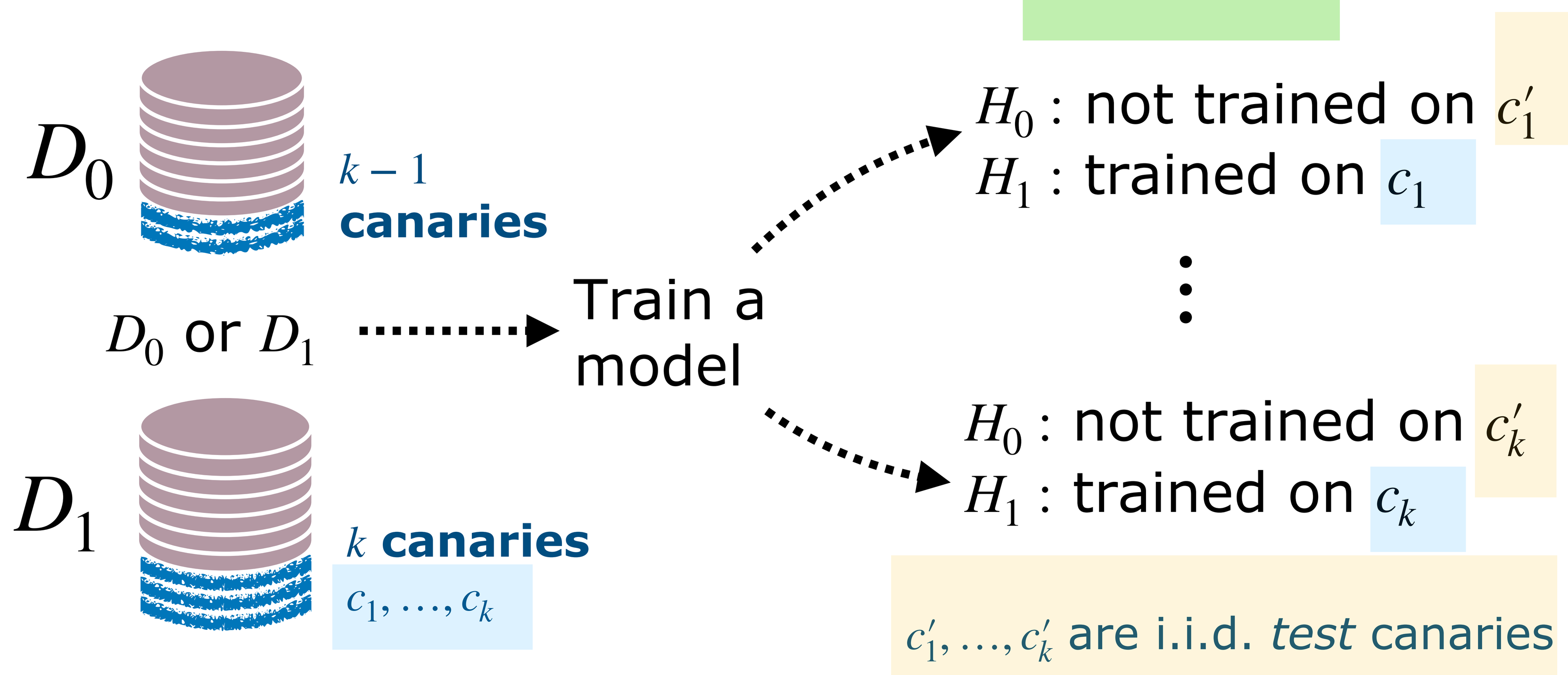
$$\mathbb{P}(\mathscr{A}(\boldsymbol{D_1}) \in \boldsymbol{R}) \le e^{\varepsilon}\, \mathbb{P}(\mathscr{A}(\boldsymbol{D_0}) \in \boldsymbol{R}) + \delta \qquad \textbf{(3)}$$

**Theorem**: $\mathscr{A}$ is $(\varepsilon,\delta)$-DP $\Longleftrightarrow$ $\mathscr{A}$ is $(\varepsilon,\delta)$-LiDP

**Consequence**: We can have *random* canaries!

### Step 2: Randomized hypothesis tests

Test for $k$ vs. $k-1$ canaries that are drawn i.i.d. from $P$




**Consequence**: Get $k$ statistics from each trial

### Step 3: Adaptive higher-order confidence

**Challenge**: the statistics are *correlated* (not i.i.d.)

We derive novel CIs using *empirical* correlations!

$$\left|\text{TPR} - \widehat{\text{TPR}}_{n,k}\right| \le \sqrt{\frac{1}{n}\left(\text{corr.} + \frac{1}{k} + \sqrt{\frac{\text{4th moment}}{n}}\right)} \qquad \textbf{(4)}$$

If $\text{corr.} = O(1/k)$, **improvement**: (3) + (4) $\Longrightarrow$

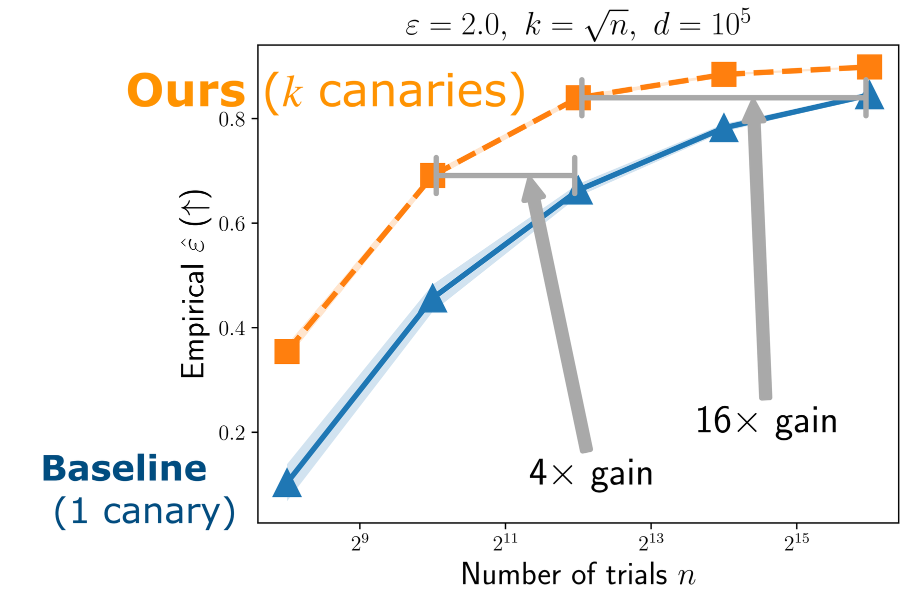
$$\varepsilon \ge \log\left(\frac{\text{TPR} - \delta}{\text{FPR}}\right) \ge \log\left(\frac{\widehat{\text{TPR}}_{n,k} - \frac{1}{\sqrt{nk}} - \frac{M_4}{n^{3/4}} - \delta}{\widehat{\text{FPR}}_{n,k} + \frac{1}{\sqrt{nk}} + \frac{M_4}{n^{3/4}}}\right)$$

## Experiments

### Auditing a Gaussian mechanism

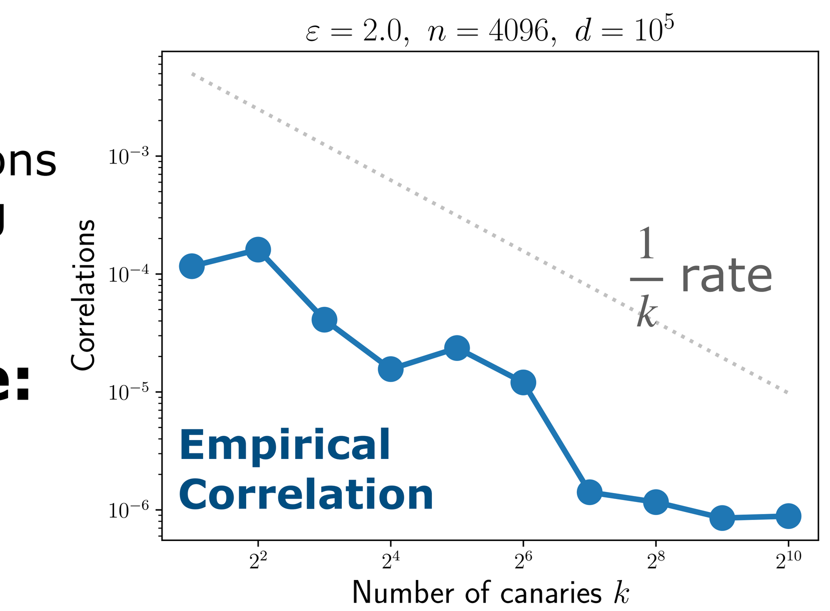
**Setup**:
- Sum query
- Canaries: uniform over unit sphere
- Test: inner product

**Result**:
4-16 × gain in sample complexity

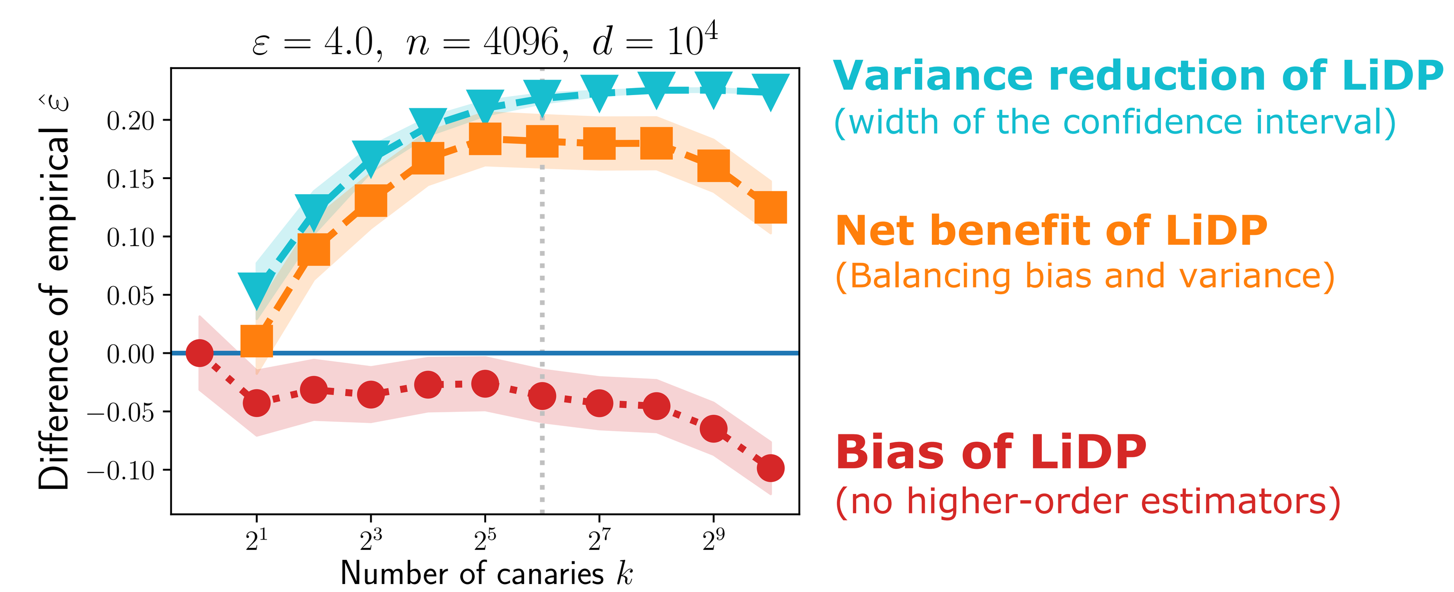



**Analysis:**
Empirical canary correlations are small, so LiDP auditing gives large wins.

**Practical Guidance:**
Multiple canaries should be "orthogonal"

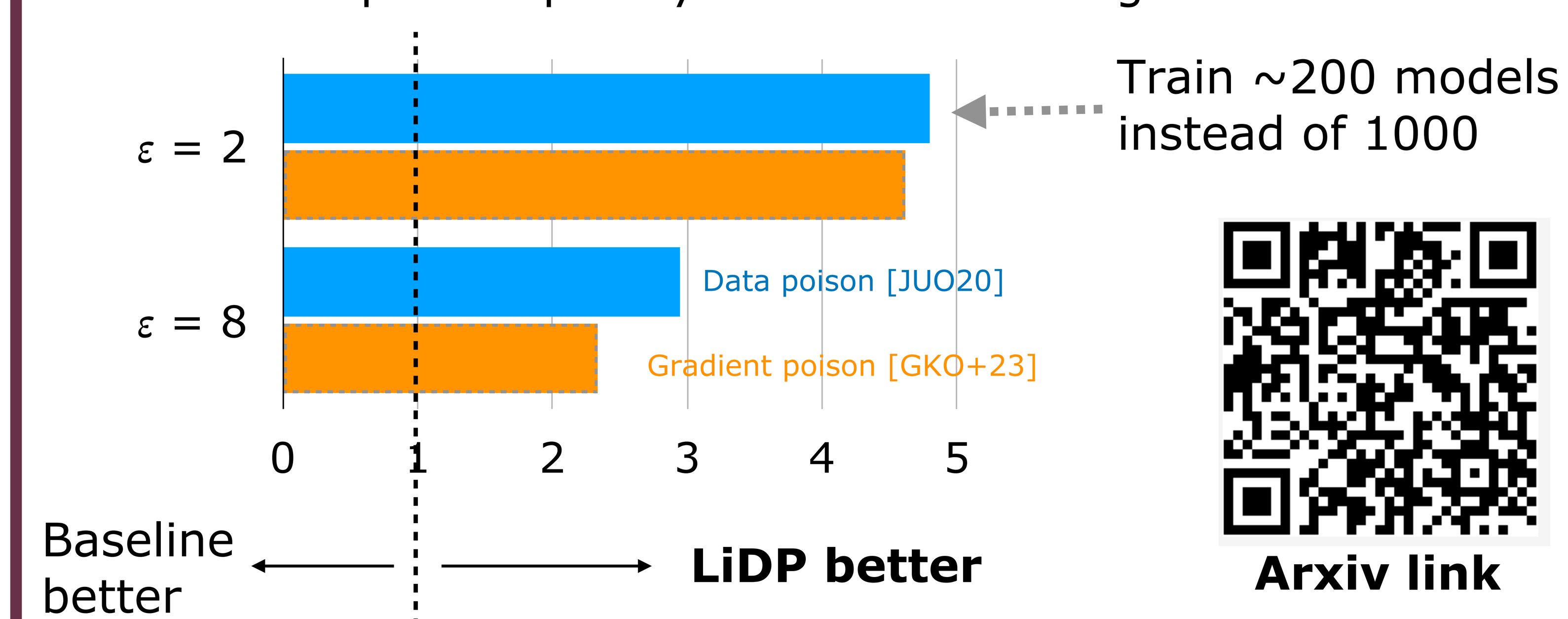



**Bias-Variance Tradeoff of LiDP:**




### **Experiments:** FashionMNIST + MLP model

Gain in sample complexity from LiDP auditing




Arxiv link