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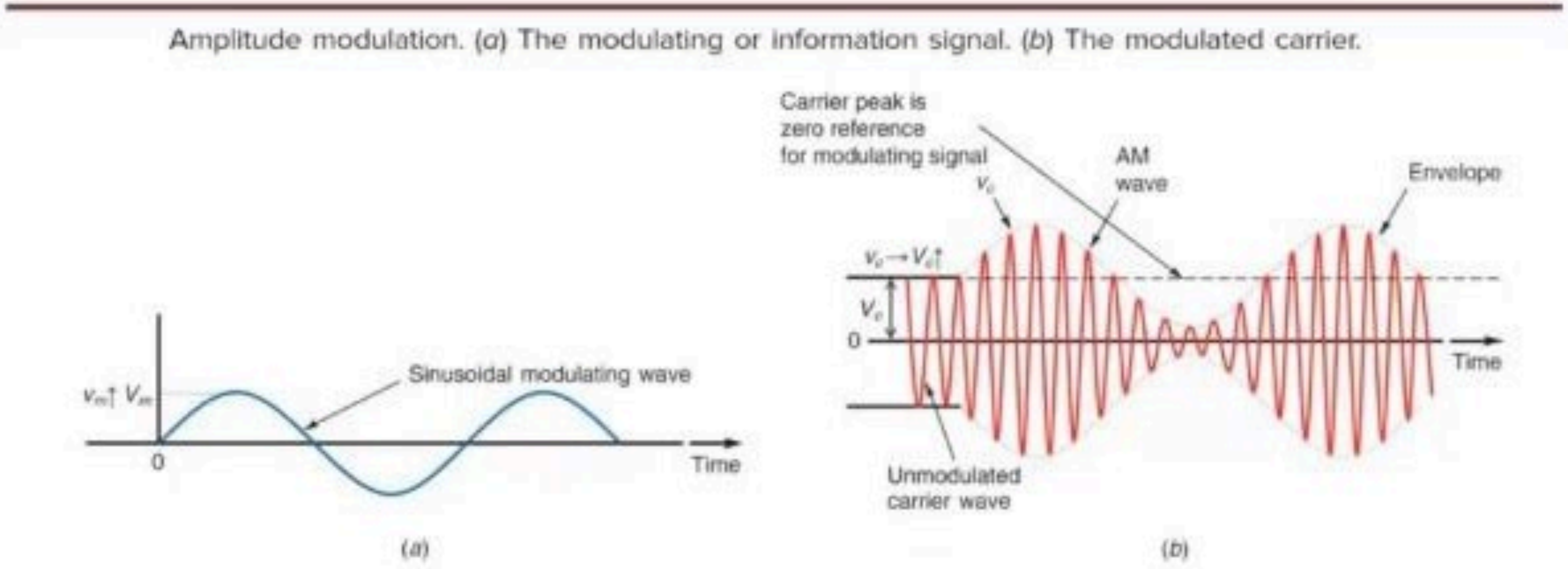
DIGITAL COMMUNICATION

- Prof. N. B. Kanirkar

- Amplitude Modulation
- AM Index
- Modulation Index for Sinusoidal AM
- Frequency Spectrum for Sinusoidal AM

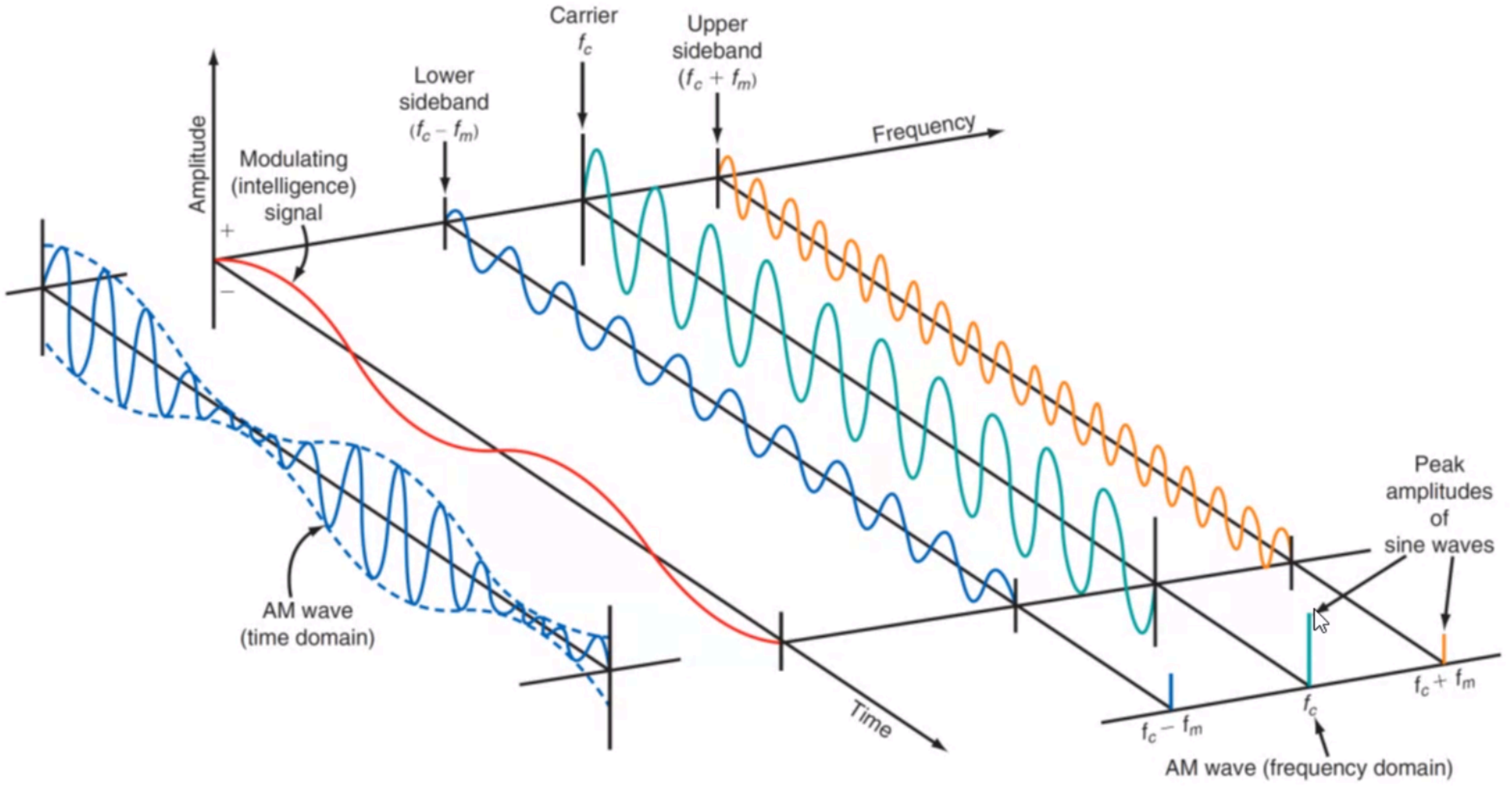
Good Morning

Amplitude Modulation



Amplitude Modulation Fundamentals

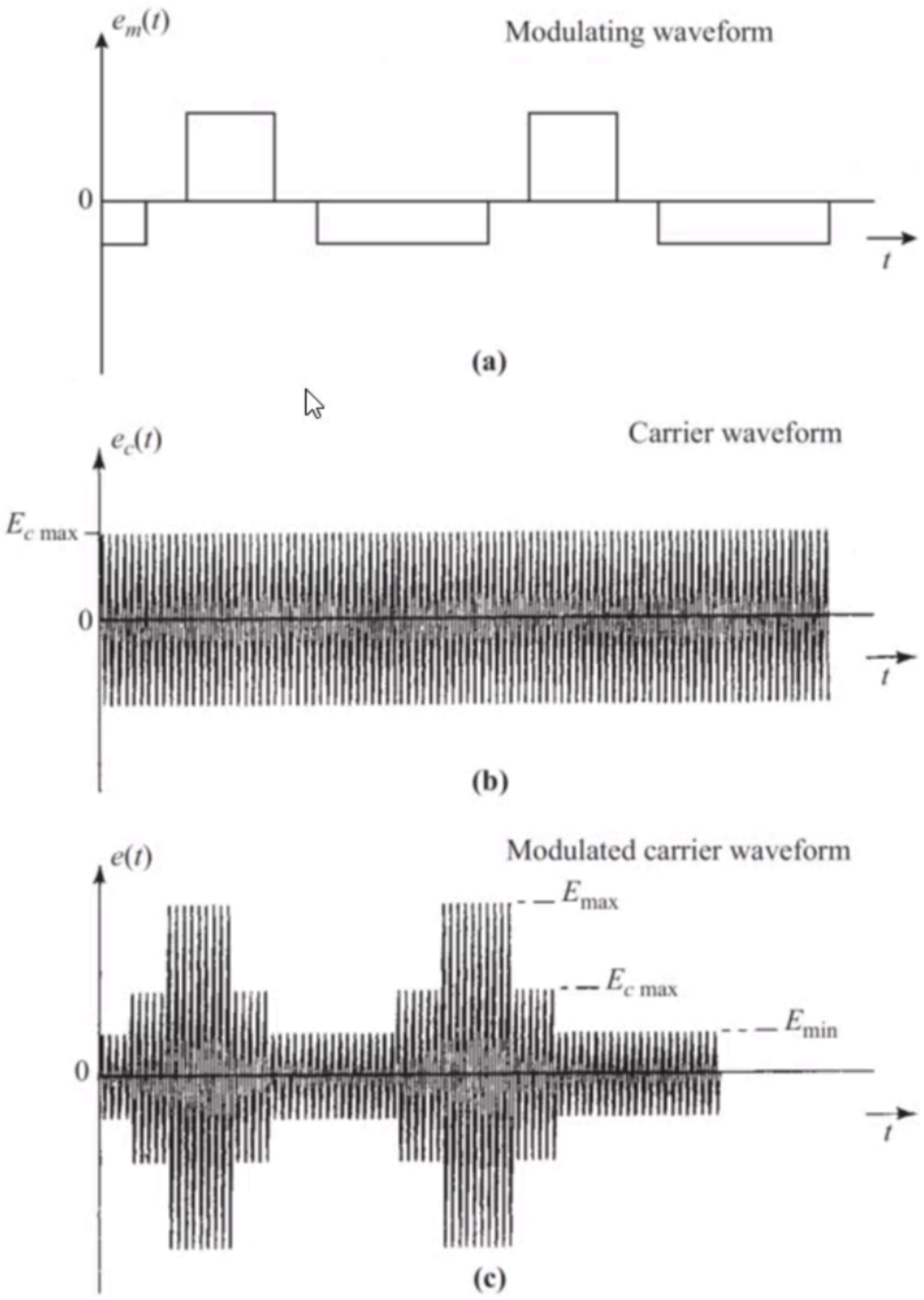
The relationship between the time and frequency domains.



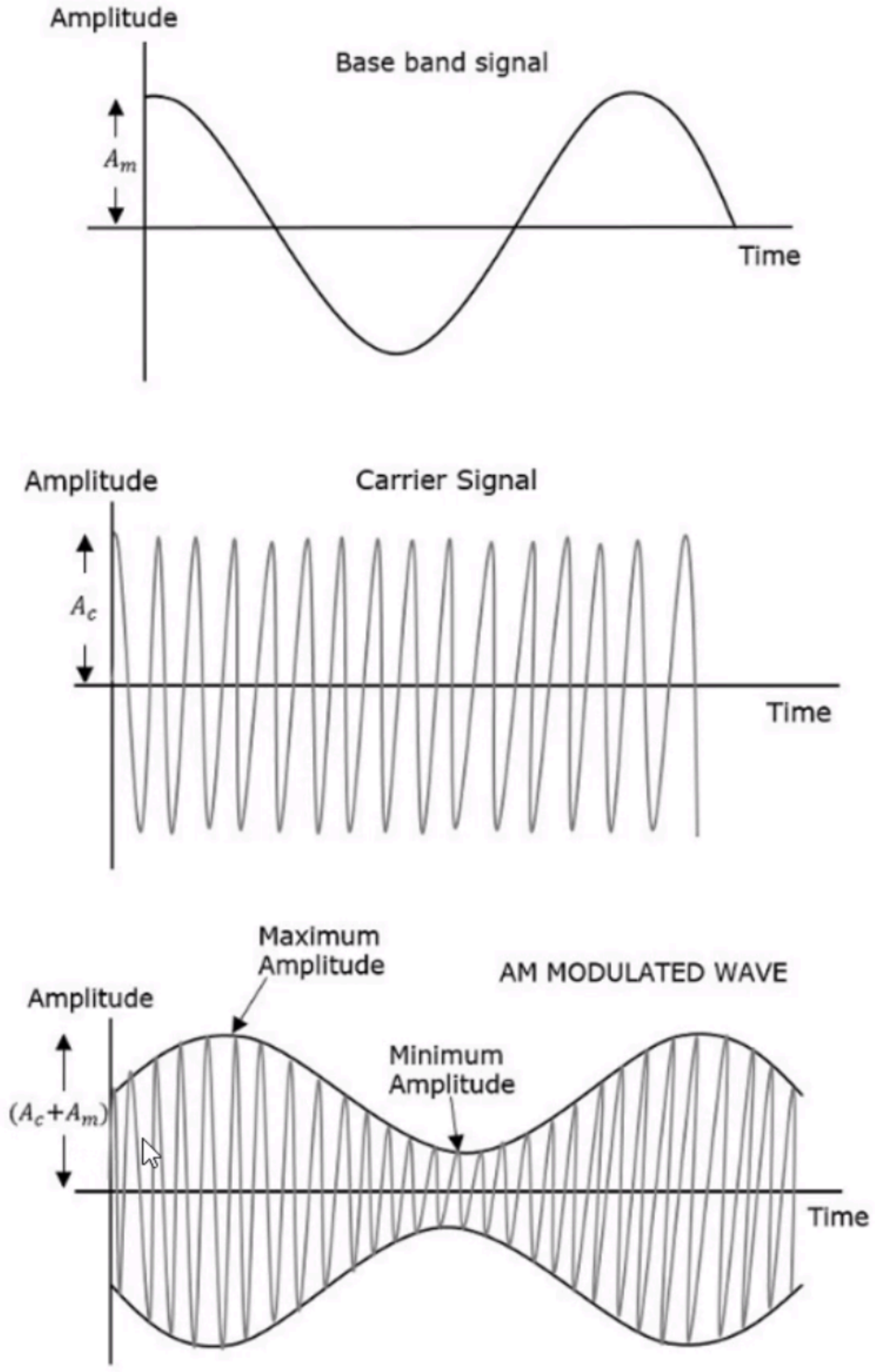
In amplitude modulation a voltage proportional to the modulating signal is added to the carrier amplitude. Let the added component of voltage be represented in functional notation as $e_m(t)$; then the modulated carrier wave is given by

$$e(t) = [E_{c\,\max} + e_m(t)]\cos(2\pi f_c t + \phi_c)$$

The term $[E_{c\,\max} + e_m(t)]$ describes the *envelope* of the modulated wave.



$e_m(t)$
Modulating waveform
0
t
(a)
$e_c(t)$
Carrier waveform
$E_{c\ max}$
0
t
(b)
$e(t)$
Modulated carrier waveform
E_{max}
$E_{c\ max}$
E_{min}
0
t
(c)


Amplitude
Base band signal
A_m
Time
Amplitude
Carrier Signal
A_c
Time
Maximum Amplitude
AM MODULATED WAVE
Amplitude
Minimum Amplitude
(A_c+A_m)
Time

Rearrange the Equation 1 as below.

$$s(t) = A_c \left[1 + \left(\frac{A_m}{A_c}\right)\cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$

$$\Rightarrow s\,(t) = A_c\,[1 + \mu\cos(2\pi f_m t)]\cos(2\pi f_c t)$$

$$\mu = \frac{A_m}{A_c}$$

$$A_{\max} + A_{\min} = A_c + A_m + A_c - A_m = 2A_c$$

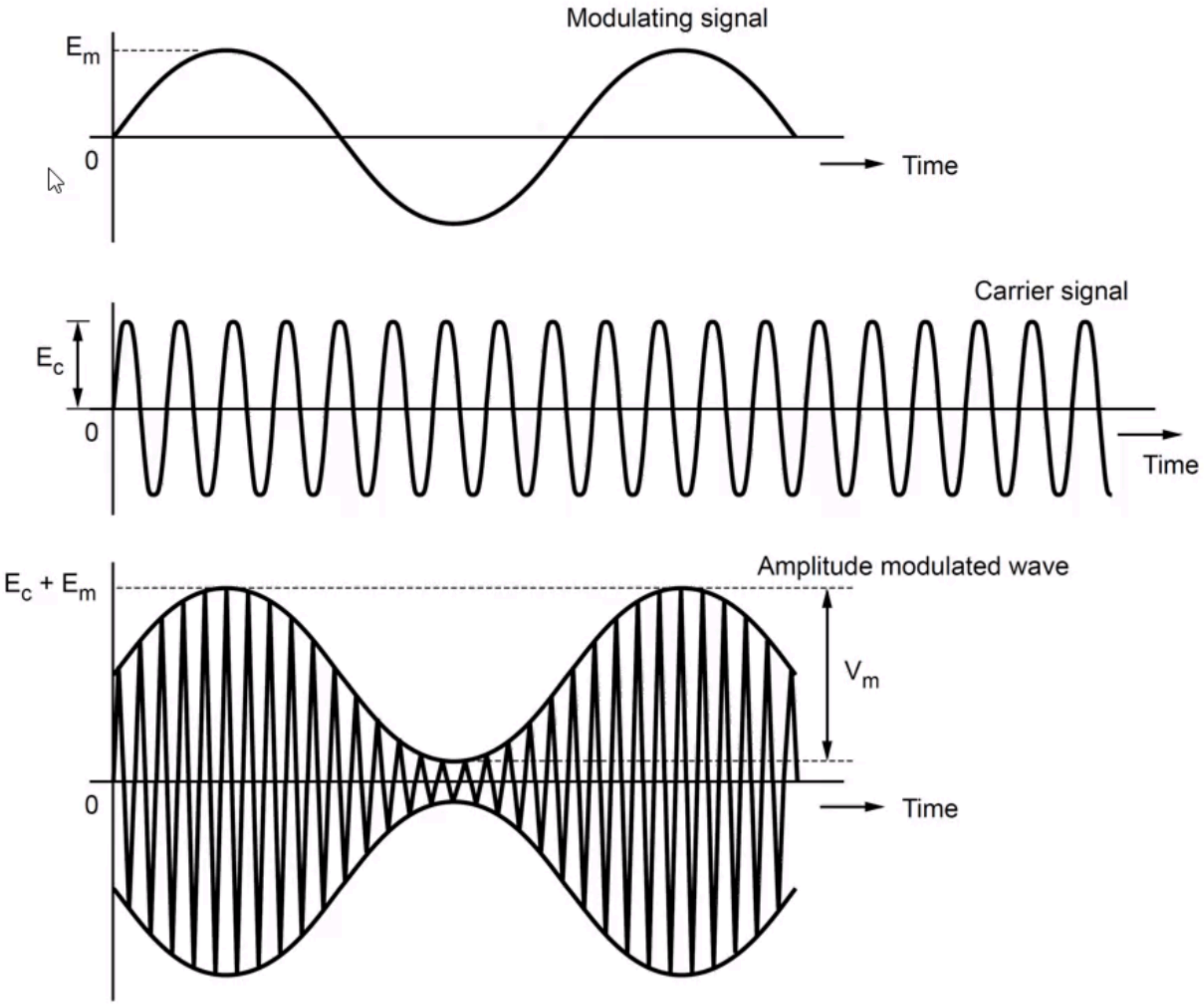
$$\Rightarrow A_c = \frac{A_{\max} + A_{\min}}{2}$$

$$A_{\max} - A_{\min} = A_c + A_m - (A_c - A_m) = 2A_m$$

$$\Rightarrow A_m = \frac{A_{\max} - A_{\min}}{2}$$

$$\frac{A_m}{A_c} = \frac{(A_{max} - A_{min})/2}{(A_{max} + A_{min})/2}$$

$$\Rightarrow \mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$$


Modulating signal
E_m
0
Time
Carrier signal
E_c
0
Time
Amplitude modulated wave
$E_c + E_m$
V_m
0
Time

Let us represent the modulating signal by e_m and it is given as,

$$e_m = E_m \sin \omega_m t$$

And carrier signal can be represented by e_c as,

$$e_c = E_c \sin \omega_c t$$

Here E_m is maximum amplitude of modulating signal.

E_c is maximum amplitude of carrier signal.

ω_m is frequency of modulating signal.

and ω_c is frequency of carrier signal.

Using the above mathematical expressions for modulating and carrier signals, we can create a new mathematical expression for the complete modulated wave. It is given as,

$$E_{AM} = E_c + e_m$$

$$= E_c + E_m \sin \omega_m t$$

$\therefore$ The instantaneous value of the amplitude modulated wave can be given as,

$$e_{AM} = E_{AM} \sin \theta$$

$$= E_{AM} \sin \omega_c t$$

$$\therefore \quad \boxed{e_{AM} = (E_c + E_m \sin \omega_m t) \sin \omega_c t}$$

This is an equation of AM wave.

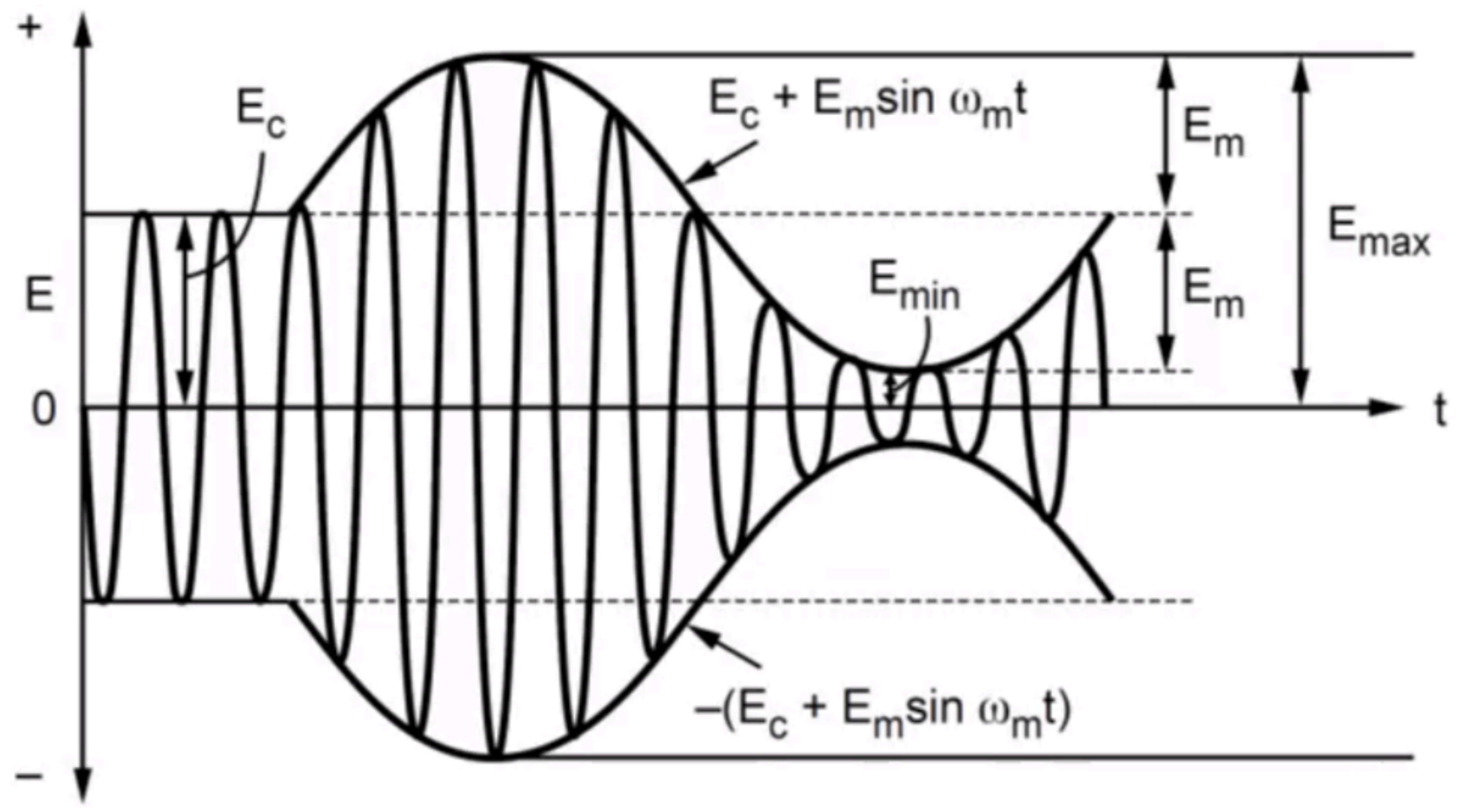
Modulation Index and Percent Modulation

The ratio of maximum amplitude of modulating signal to maximum amplitude of carrier signal is called modulation index. i.e.,

$$\boxed{\text{Modulation index, } m = \frac{E_m}{E_c}}$$

Value of E_m must be less than value of E_c to avoid any distortion in the modulated signal. Hence maximum value of modulation index will be equal to 1 when $E_m = E_c$. Minimum value will be zero. If modulation index is higher than 1, then it is called *over modulation*. Data is lost in such case. When modulation index is expressed in percentage, it is also called percentage modulation.

Calculation of modulation index from AM waveform



: AM wave

$$E_m = \frac{E_{max} - E_{min}}{2}$$

and $$E_c = E_{max} - E_m$$

$$= E_{max} - \frac{E_{max} - E_{min}}{2} \quad \text{... By putting for } E_m \text{ from equation}$$

$$= \frac{E_{max} + E_{min}}{2}$$

Taking the ratio of equation

$$m = \frac{E_m}{E_c} = \frac{\dfrac{E_{max} - E_{min}}{2}}{\dfrac{E_{max} + E_{min}}{2}}$$

$$\therefore \qquad \boxed{m = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}}$$

This equation gives the technique of calculating modulation index from AM wave.

Frequency Spectrum and Bandwidth

The modulated carrier has new signals at different frequencies, called side frequencies or sidebands. They occur above and below the carrier frequency.

i.e.

$$f_{USB} = f_c + f_m$$

$$f_{LSB} = f_c - f_m$$

Here f_c is carrier frequency.

f_m is modulating signal frequency.

Modulation Index for Sinusoidal AM

For sinusoidal AM, the modulating waveform is of the form

$$e_m(t) = E_{m\,\max}\cos(2\pi f_m t + \phi_m)$$

In general the fixed phase angle ϕ_m is unrelated to the fixed phase angle ϕ_c for the carrier, showing that these two signals are independent of each other in time. However, the amplitude modulation results are independent of these phase angles, which may therefore be set equal to zero to simplify the algebra and trigonometry used in the analysis. The equation for the sinusoidally modulated wave is therefore

$$e(t) = (E_{c\,\max} + E_{m\,\max}\cos 2\pi f_m t)\cos 2\pi f_c t$$

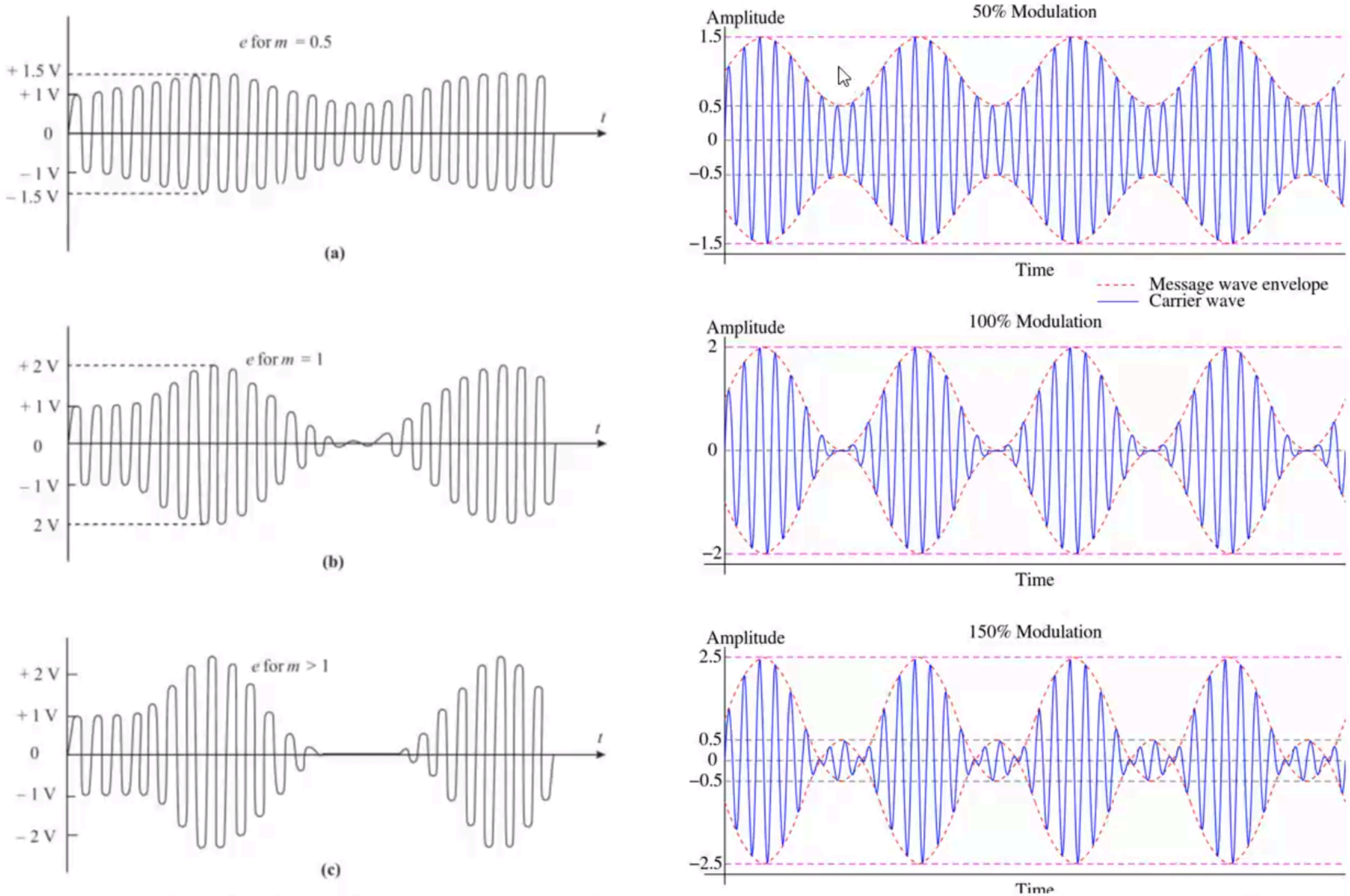
Since in this particular case $E_{\max} = E_{c\,\max} + E_{m\,\max}$ and $E_{\min} = E_{c\,\max} - E_{m\,\max}$ the modulation index is given by

$$m = \frac{E_{\max} - E_{\min}}{E_{\max} + E_{\min}}$$

$$= \frac{E_{m\,\max}}{E_{c\,\max}}$$

The equation for the sinusoidally amplitude modulated wave may therefore be written as

$$e(t) = E_{c\,\max}(1 + m\cos 2\pi f_m t)\cos 2\pi f_c t$$

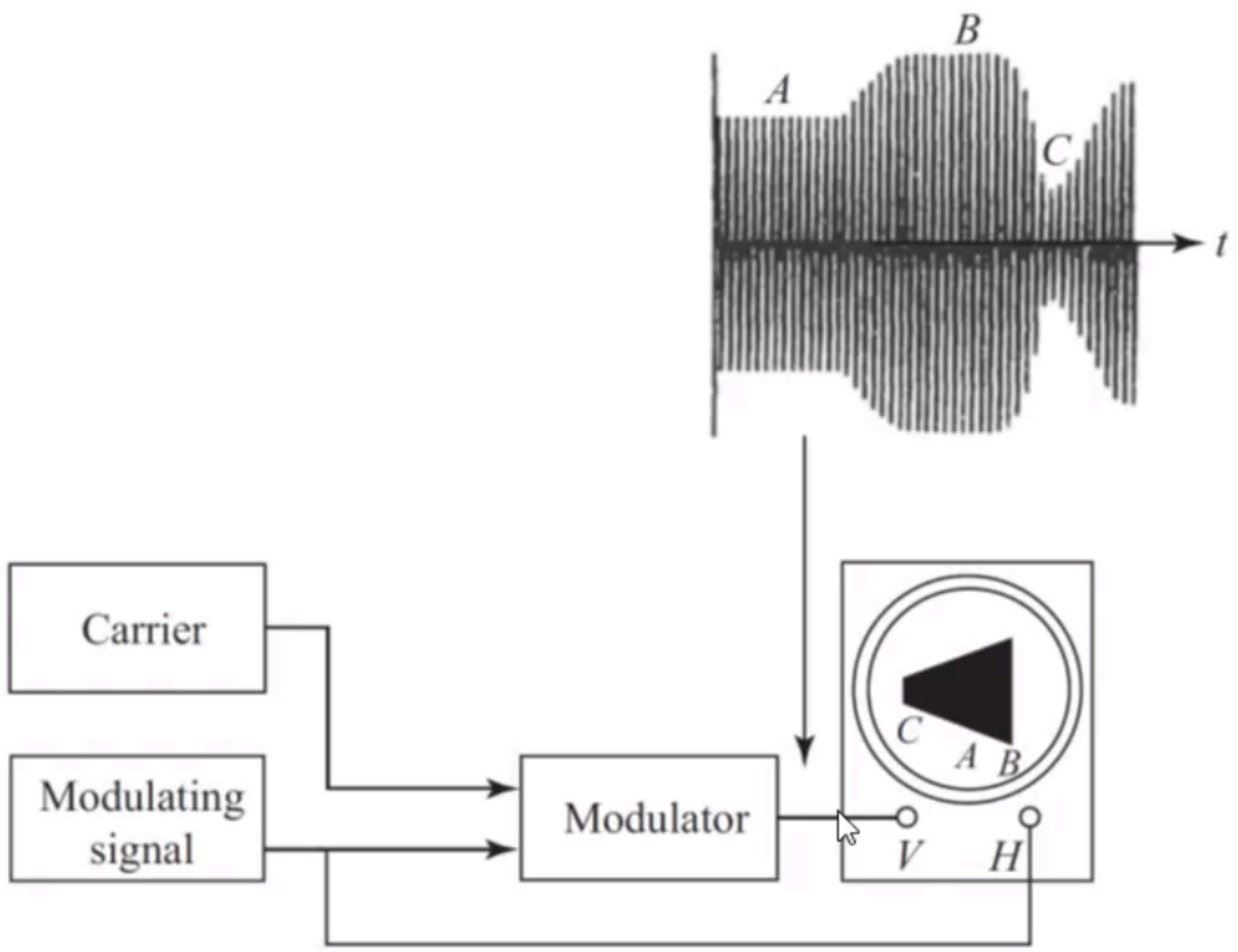


Sinusoidally amplitude modulated waveforms for (a) $m = 0.5$ (undermodulated), (b) $m = 1$ (fully modulated), and (c) $m > 1$ (overmodulated).

Amplitude Modulation Index

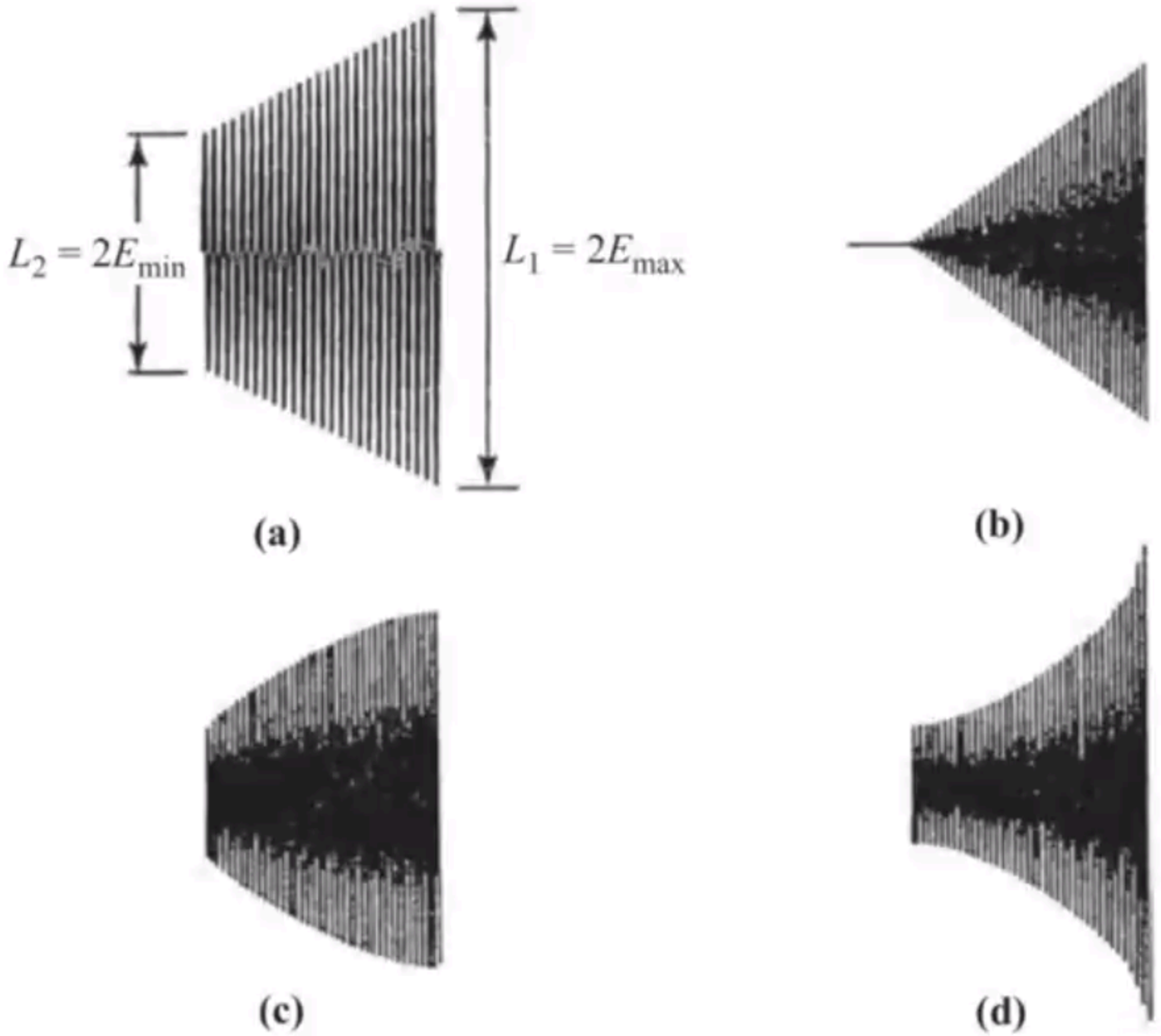
the *modulation index* is defined as

$$m = \frac{E_{\max} - E_{\min}}{E_{\max} + E_{\min}}$$



$$m = \frac{E_{\max} - E_{\min}}{E_{\max} + E_{\min}}$$

$$= \frac{L_1 - L_2}{L_1 + L_2}$$



(a) Normal trapezoidal pattern. (b) Trapezoidal pattern for $m > 1$. (c) Envelope distortion resulting from insufficient RF drive to the modulator. (d) Envelope distortion resulting from non-linearities in the modulator.

It will be seen that the modulation index is zero when $E_{\max} = E_{\min}\, E_{c\,\max}$, and it is unity when $E_{\min} = 0$. Thus, in practice, the modulation index should be in the range

$$0 \leq m \leq 1$$

A modulating signal consists of a symmetrical triangular wave having zero dc component and peak-to-peak voltage of 11 V. It is used to amplitude modulate a carrier of peak voltage 10 V. Calculate the modulation index and the ratio of the side lengths L_1/L_2 of the corresponding trapezoidal pattern.

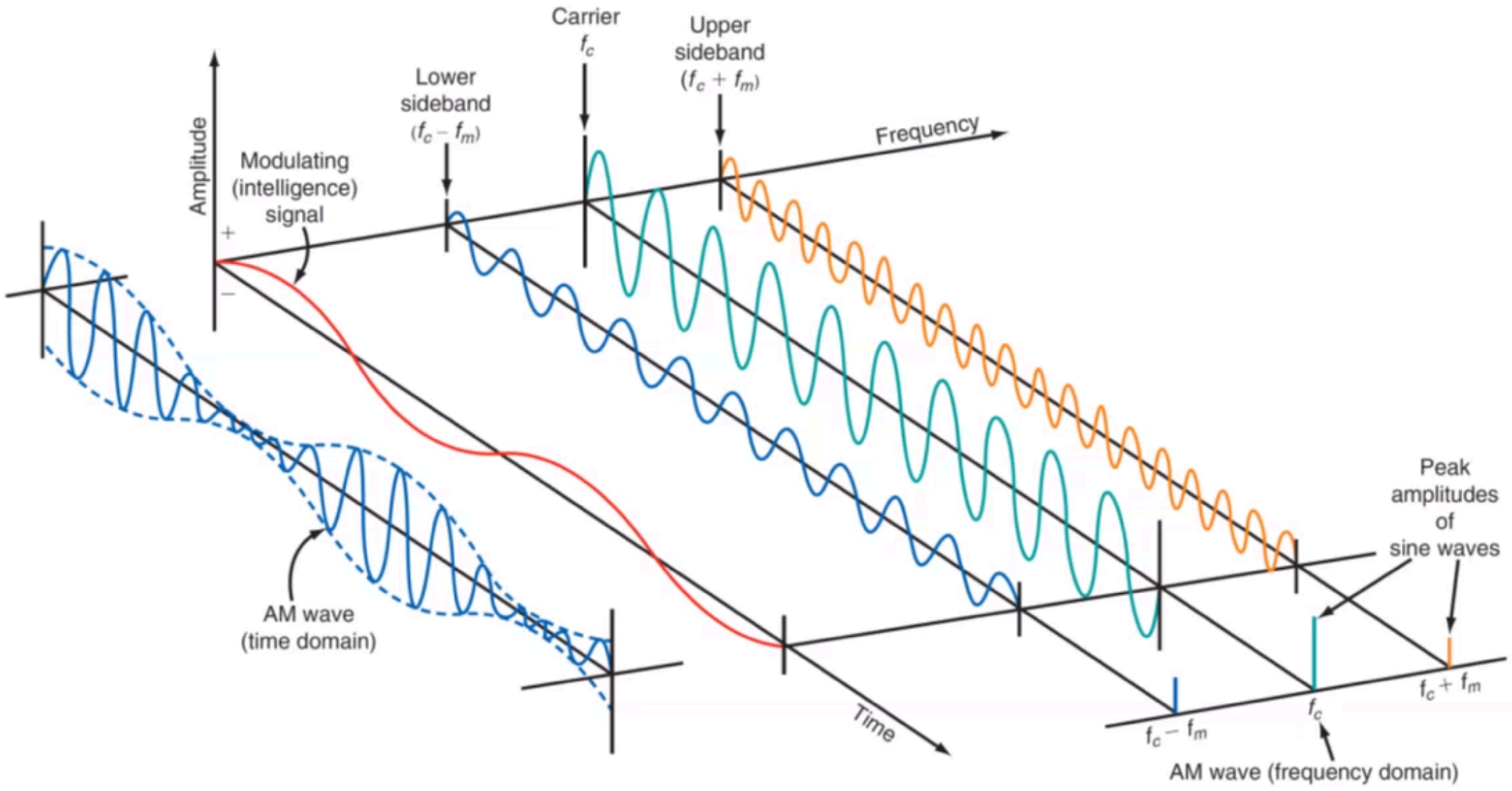
$$E_{\text{max}} = 10 + \frac{11}{2} = 15.5\,\text{V}$$

$$E_{\text{min}} = 10 - \frac{11}{2} = 4.5\,\text{V}$$

$$\therefore\ m = \frac{15.5 - 4.5}{15.5 + 4.5} = \mathbf{0.55}$$

L_1 is proportional to 15.5 V, L_2 to 4.5 V, and therefore $L_1/L_2 = 15.5/4.5 = \mathbf{3.44}$.

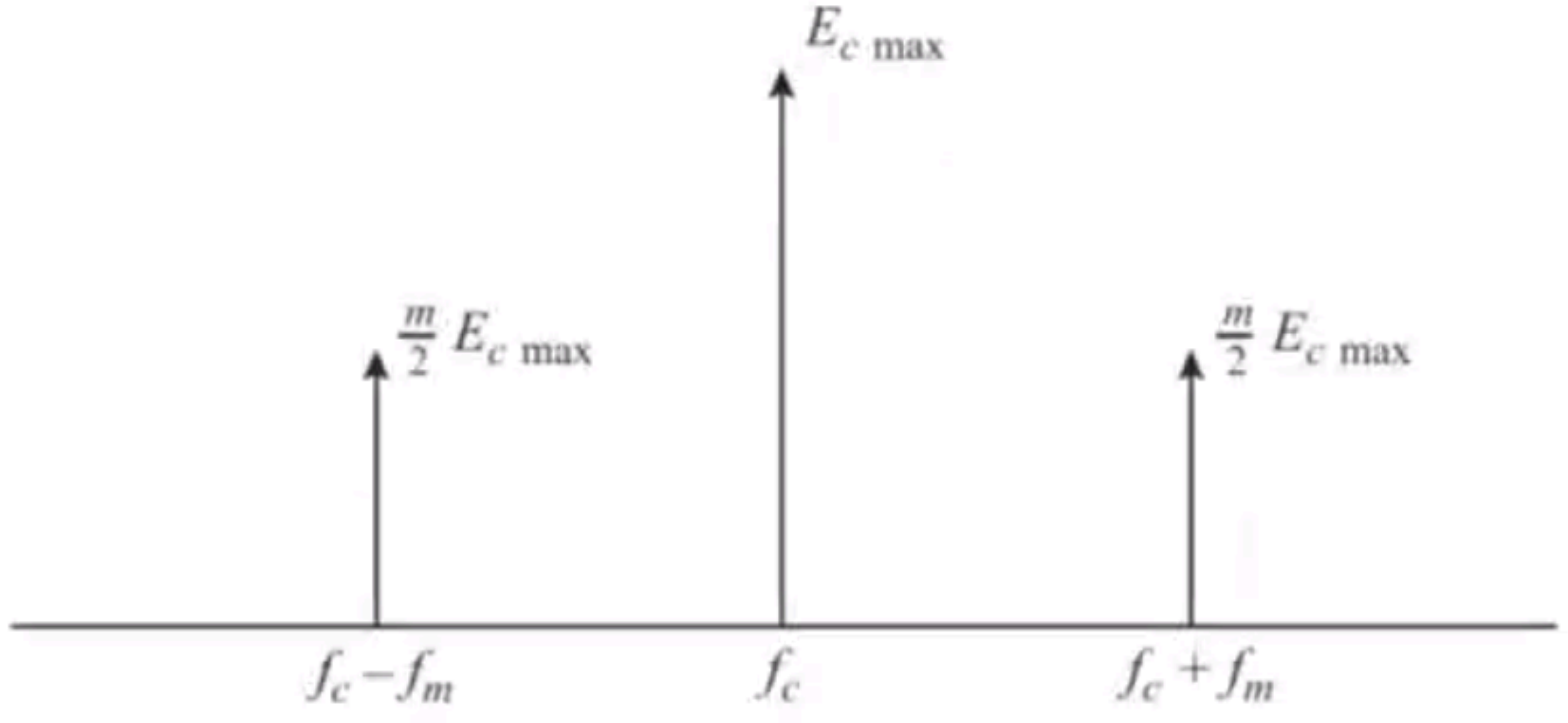
The relationship between the time and frequency domains.



Although the modulated waveform contains two frequencies f_c and f_m, the modulation process generates new frequencies that are the sum and difference of these. The spectrum is found by expanding the equation for the sinusoidally modulated AM as follows:

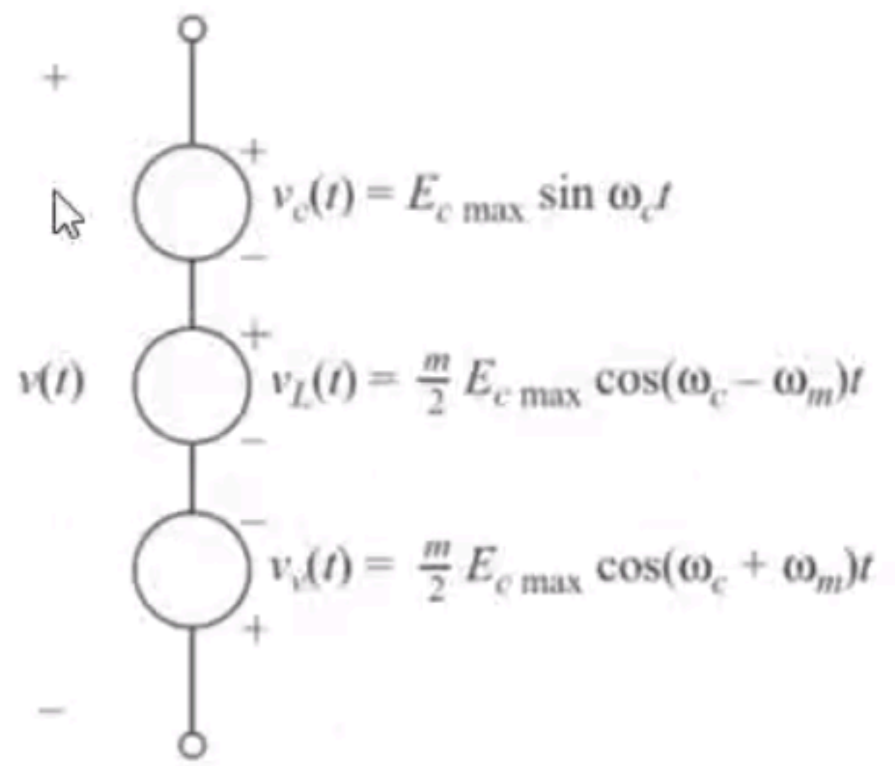
$$\begin{aligned} e(t) &= E_{c\,\max}(1 + m\cos 2\pi f_m t)\cos 2\pi f_c t \\ &= E_{c\,\max}\cos 2\pi f_c\, t + mE_{c\,\max}\cos 2\pi f_m t \times \cos 2\pi f_c t \\ &= E_{c\,\max}\cos 2\pi f_c t + \frac{m}{2}E_{c\,\max}\cos 2\pi(f_c - f_m)t + \frac{m}{2}E_{c\,\max}\cos 2\pi(f_c + f_m)t \end{aligned}$$

Equation shows that the sinusoidally modulated wave consists of three components: a carrier wave of amplitude $E_{c\,\max}$ and frequency f_c, a *lower side frequency* of amplitude $mE_{c\,\max}/2$ and frequency $f_c - f_m$, and an *upper side frequency* of amplitude $mE_{c\,\max}/2$ and frequency $f_c + f_m$. The amplitude spectrum is shown in Fig.



(a)

(a) Amplitude spectrum for a sinusoidally amplitude modulated wave.



The upper sideband f_{USB} and lower sideband f_{LSB} are computed as

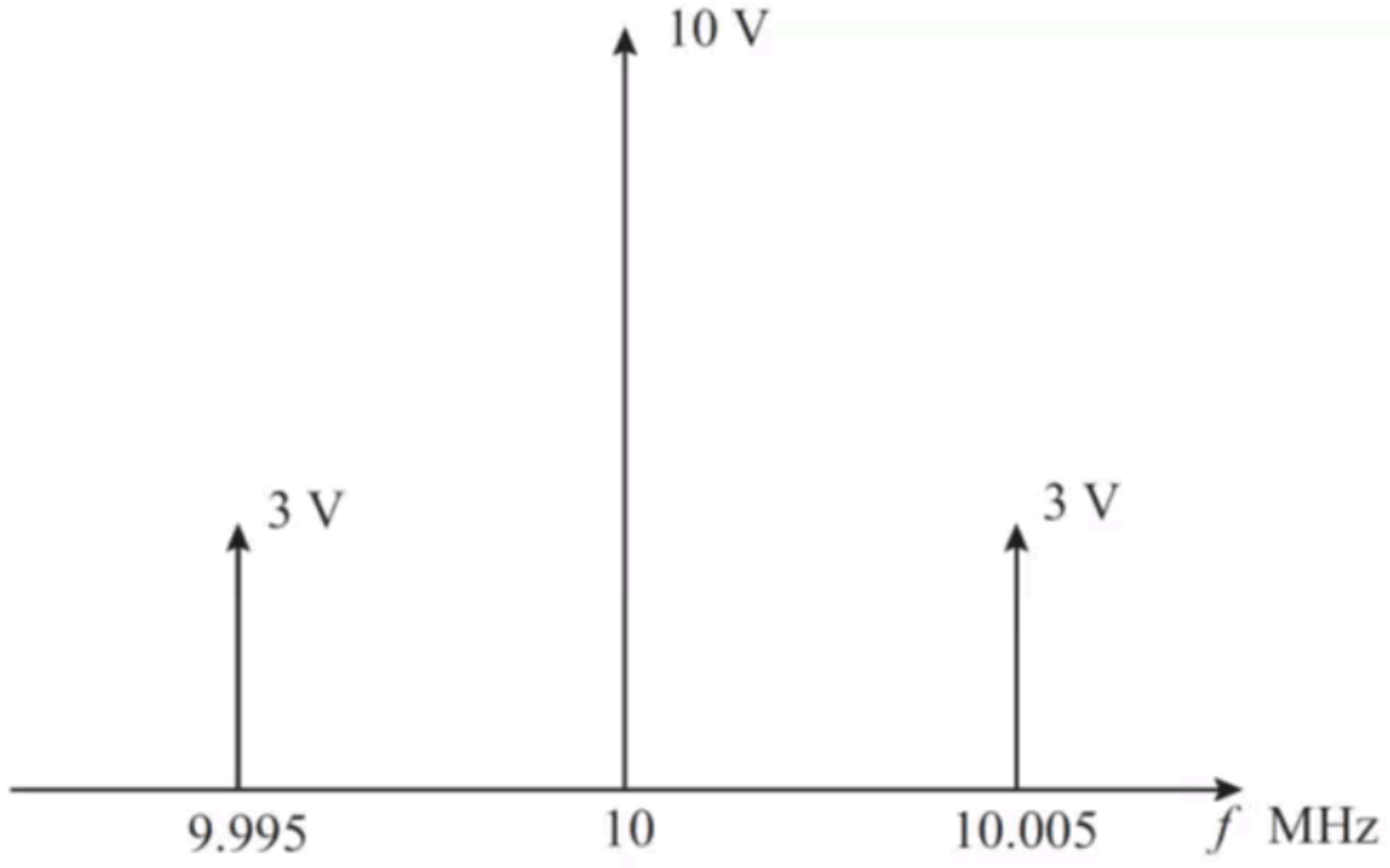
$$f_{USB} = f_c + f_m \quad \text{and} \quad f_{LSB} = f_c - f_m$$

EXAMPLE

A carrier wave of frequency 10 MHz and peak value 10 V is amplitude modulated by a 5-kHz sine wave of amplitude 6 V. Determine the modulation index and draw the amplitude spectrum.

SOLUTION $m = \dfrac{6}{10} = 0.6$

The side frequencies are $10 \pm 0.005 = 10.005$ and 9.995 MHz. The amplitude of each side frequency is $0.6 \times 10/2 = 3$ V. The spectrum is shown in Fig.



(b)

A standard AM broadcast station is allowed to transmit modulating frequencies up to 5 kHz. If the AM station is transmitting on a frequency of 980 kHz, compute the maximum and minimum upper and lower sidebands and the total bandwidth occupied by the AM station.

$$f_{\mathrm{USB}} = 980 + 5 = 985 \text{ kHz}$$

$$f_{\mathrm{LSB}} = 980 - 5 = 975 \text{ kHz}$$

$$\mathrm{BW} = f_{\mathrm{USB}} - f_{\mathrm{LSB}} = 985 - 975 = 10 \text{ kHz} \qquad \text{or}$$

$$\mathrm{BW} = 2(5 \text{ kHz}) = 10 \text{ kHz}$$


Bye


Thank you