Date
Page

29/8/21

Tutorial-2

Q1. (i) Hamming distance :- the no. of positions at ~~which wi~~ which the digits are different ~~in~~ in two different elements of B^n is known a Hamming distance.

~~2. Hamming weight~~

* Example :- $x_1, x_2 \in B^5$

∴ Let $x_1 = 10101$ and $x_2 = 01110$

∴ $d(x_1, x_2) = \begin{matrix}1&0&1&0&1\\0&1&1&1&0\end{matrix} = 4.$

(ii) Hamming weight :- Let $x \in B^n$, then the no. of 1's in x is known as Hamming weight of 'x'.

* Example :- $x \in B^{10}$.

Let $x = 1001011101$

∴ $w(x) = 1001011101 = 6.$

(iii) ~~Minimum~~ distance of a set (or code) :- among all the pairs of the ~~B~~ group 'B^n', the distance ~~which is min^m~~ is ~~t~~ known as the minimum distance of a set

(iii) Minimum distance of a set (or code):-

⇒ Let $x_1, x_2 \in C$ then $\forall$ such x_1 and x_2. ~~the distance~~ pair having the least distance among the set (or code) is known as min^m^ distance of a set (or code).

(iv) Minimum weight of a set (or code):-

⇒ ~~Let $x \in C$ then $\forall$ such x which is having the least no. of 1's is equal to the min^m^ weight of the set 'C'.~~

⇒ Min^m^ weight of a set (or code) is equal to the weight of an element $x \in C$ which is having the least no. of 1's in the set 'C'.

(v) Generator matrix:-

⇒ Let $x \in B^m$ where B^m is our message grp

then $\exists$ 'G' such that $xG = y$

⇒ where $y \in C$ and $C \leq B^n$.

where 'B^n' is our word grp and 'C' is our code word grp.

⇒ then 'G' is Generator matrix.

(vi) Parity check matrix:-

⇒ ~~Let $x \in B^n$ where B^n is our word grp.~~

⇒ Let $x \in B^n$ where B^n is our word grp.

then $\exists$ 'H' such,

⇒ $xH^T = 0$ if $x \in C$ and $xH^T \neq 0$ if $x \notin C$,

where 'C' is our code word grp i.e, $C \leq B^n$.

⇒ then, 'H' is known as parity check matrix.

(vii) Codewords:-

⇒ images of a corresponding messege maped through an encoding fnc are known as code words.

(viii) Code:-

⇒ The set of all the code words is known as code.

(ix) Group code:-

⇒ If the set of all the code words is a group then it is called as group code.

(x) Encoding function:-

⇒ ~~An~~ 1-1 function mapping all the messages to their corresponding code words is known ~~an~~ as an Encoding function.

(XI) Message :-

⇒ all the elements of our message grp are known as message.

(XII) Undetectable error :-

⇒ errors which are beyond the error dectectn capacity are known as undetectable errors.

2. we have,

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}_{3\times 6}$$

⇒ also, $B^3 = \{000, 001, 010, 100, 011, 101, 110, 111\}$

⇒ $[000]G = 000000$

⇒ $[001]G = 001101$

⇒ $[010]G = 010011$

⇒ $[100]G = 100110$

⇒ $[011]G = 011110$

⇒ $[101]G = 101011$

⇒ $[110]G = 110101$

⇒ $[111]G = 111000$

∴ code $C = \{000000, 100110, 010011, 001101, 011110, 101011, 110101, 111000\}$

⇒ as we know that $H = [A^T / I_{(n-m)}]_{(n-m)\times n}$

and $G = [I_m / A]_{m\times n}$.

∴ $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

⇒

∴ $H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

∴ & $H^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

~~(i) ⇒ 110101~~

⇒ all the coset leaders :-

⇒ 100000, 010000, 001000, 000100, 000010, 000001, 100001,

(i) 110101

$\Rightarrow [110101] H^T = \cancel{[000000]}\ [000]$

$\therefore$ 110101 is a code word.

(ii) 001111

$\Rightarrow [001111] H^T = [010]$

$\therefore$ as we know that $w = v + c$

$\Rightarrow \boxed{c = w + v}$

$\therefore \quad c = 001111 + 000010$

$= 001101$ is our code word.

(iii) 110001

$\Rightarrow [110001] H^T = [100]$

$\therefore \quad c = 110001 + 000100$

$= 110101$ is our code word.

(iv) 111111

$\Rightarrow [111111] H^T = [111]$

$\Rightarrow \quad c = 111111 + 100001$

$= 011110$ is our code word.

classmate

Date

Page

⇒ the Last word is not decoded uniquely due to the possiblities of having more than one coset Leaders.

3.
$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}_{3\times 6}$$

~~∴ code word $\in B^6$~~

~~⇒ Let $[x_1 x_2 x_3 x_4 x_5 x_6] \in B^6$.~~

∴ ∴ code word $\in C \leq B^6$

⇒ Let $x_1 x_2 x_3 x_4 x_5 x_6 \in C$.

∴ $[x_1 x_2 x_3 x_4 x_5 x_6] H^T = [0]$

$$\Rightarrow [x_1\ x_2\ x_3\ x_4\ x_5\ x_6] \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [0]$$

$$\Rightarrow \quad x_1 + x_2 + x_3 + x_4 = 0$$
$$x_1 + x_3 + x_5 = 0.$$
$$x_2 + x_3 + x_6 = 0.$$

⇒ 3 - variables must be free variables.

$\Rightarrow$ Let x_2, x_3, and x_4 are the Free variables.

$\Rightarrow$ then, $\Rightarrow x_1 = x_2 + x_3 + x_4$. — (1)

$\Rightarrow x_5 + x_1 = x_3$.

$\Rightarrow x_5 = x_2 + x_4$ — (2)

$\Rightarrow x_6 = x_2 + x_3$ — (3).

$\Rightarrow$ ∴ we have, $[x_2+x_3+x_4, x_2, x_3, x_4, x_2+x_4, x_2+x_3]$

$\Rightarrow x_2 = 0, x_3 = 0, x_4 = 0$.

$\Rightarrow [0, 0, 0, 0, 0, 0] \Rightarrow$ 000000

$\Rightarrow x_2 = 0, x_3 = 0, x_4 = 1$.

$\Rightarrow [1, 0, 0, 1, 1, 0] \Rightarrow$ 100110

$\Rightarrow x_2 = 0, x_3 = 1, x_4 = 0$

$[1, 0, 1, 0, 0, 1] \Rightarrow$ 101001

$\Rightarrow x_2 = 0, x_3 = 1, x_4 = 1$.

$[0, 0, 1, 1, 1, 1] \Rightarrow$ 001111

$\Rightarrow x_2 = 1, x_3 = 0, x_4 = 0$

$[1, 1, 0, 0, 1, 1] \Rightarrow$ 110011

$\Rightarrow x_2 = 1, x_3 = 0, x_4 = 1$

$[0, 1, 0, 1, 0, 1] \Rightarrow$ 010101

$\Rightarrow x_2 = 1, x_3 = 1, x_4 = 0$

$[0, 1, 1, 0, 1, 0] \Rightarrow$ 011010

Date ______
Page ______

$\Rightarrow x_2 = 1, x_3 = 1, x_4 = 1$

$[1, 1, 1, 1, 0, 0] \Rightarrow \underline{\underline{111100}}$

$\therefore C = \{000000, 100110, 101001, 001111, 110011, 010101, 011010, 111100.\}$

also, ~~$w(000000) = 0$~~

$w(100110) = 3$

$w(101001) = 3$

$w(001111) = 4.$

$w(110011) = 4.$

$w(010101) = 3$

$w(011010) = 3$

$w(111100) = 4.$

$\Rightarrow \therefore$ min^m distance in the above code is $\boxed{3}$

$\therefore$ error dectecting capacity $= d_{min} - 1.$

$= \boxed{2}.$

and error correcting capacity $= \frac{d_{min} - 1}{2}$

$= \boxed{1}$

4. $e : B^2 \rightarrow B^5$

$\Rightarrow$

$00 \rightarrow 00000$

$01 \rightarrow 01110$

$10 \rightarrow 10101$

$11 \rightarrow 11011$

$\Rightarrow$

$\oplus$	00000	01110	10101	11011
00000	00000	01110	10101	11011
01110	01110	00000	11011	10101
10101	10101	11011	00000	01110
11011	11011	10101	01110	00000

$\Rightarrow$ from the above composition table, we see that our code follows closure property.

$\therefore$ $e : B^2 \to B^5$ is a group code.

5. $H = \{0, 3, 6, 9, 12\}$ $G = \{0, 1, 2, 3, \ldots 14\}$

$\Rightarrow$ $\therefore$ no. of distinct cosets $= \dfrac{O(G)}{O(H)} = \dfrac{15}{5} = 3$

$\Rightarrow$

$\therefore \Rightarrow 0 +_{15} H = \{0, 3, 6, 9, 12\}$

$\Rightarrow 1 +_{15} H = \{1, 4, 7, 10, 13\}$

$\Rightarrow 2 +_{15} H = \{2, 5, 8, 11, 14\}$

6. we have,

$$M = \left\{ \begin{bmatrix} a, & b \\ c & d \end{bmatrix} \middle| \forall a, b, c, d \in R \right\}$$

$\therefore$ we have a map $D : M \to \mathbb{R}$

$\therefore$ Let $M_1, M_2 \in M$.

$\Rightarrow M_1 = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$ and $M_2 = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$

$\Rightarrow D(M_1) = (a_1 d_1 - b_1 c_1)$ and $D(M_2) = (a_2 d_2 - b_2 c_2)$

also, we know that 'M' is grp under addition

$\therefore$ ~~M_2~~ $M_1 + M_2 = \begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{bmatrix} \in M$

$\therefore$ ~~&~~ $D(M_1+M_2) = \left[(a_1+a_2)(d_1+d_2) - (c_1+c_2)(b_1+b_2)\right]$

$= \left[(a_1 d_1 + a_1 d_2 + a_2 d_1 + a_2 d_2) - (c_1 b_1 + c_1 b_2 + c_2 b_1 + c_2 b_2)\right.$

$\Rightarrow D(M_1+M_2) = D(M_1) + D(M_2) + (a_1 d_2 + a_2 d_1 - c_1 b_2 - c_2 b_1)$

$\therefore$ clearly $D(M_1+M_2) \neq D(M_1) + D(M_2)$.

$\therefore$ $D : M \to \mathbb{R}$ is not a homomorphism.

7. ~~where~~ we have, $\phi : G \to G'$

where, $G \to$ set of all nonzero real nos
$G' = \{1, -1\}$

$\Rightarrow \phi = \begin{Bmatrix} R^+ \to 1 \\ R^- \to -1 \end{Bmatrix}$

$\Rightarrow$ Let take three cases, for $R_1, R_2 \in \mathbb{R}-\{0\}$.

(i) $R_1 \in R^+$ and $R_2 \in R^+$

(ii) $R_1 \in R^-$ and $R_2 \in R^-$

(iii) $R_1 \in R^+$ and $R_2 \in R^-$ or $R_1 \in R^-$ and $R_2 \in R^+$

(i) $\phi(R_1) = 1$ and $\phi(R_2) = 1$.

also, $\cancel{R_1 * R_2 \in}$ $R_1 \times R_2 \in R^+$

$\therefore$ $\phi(R_1 \times R_2) = 1 = \phi(R_1) \times \phi(R_2)$.

$\therefore$ for this case 'ϕ' is a homomorphism.

(ii) $\phi(R_1) = -1$ and $\phi(R_2) = -1$.

also, $R_1 \times R_2 \in R^+$

$\therefore$ $\phi(R_1 \times R_2) = +1 = \phi(R_1) \times \phi(R_2)$.

$\therefore$ for this case 'ϕ' is a homomorphism.

(iii) $\cancel{\phi(R_1)}$ Let $R_1 \in R^+$ and $R_2 \in R^-$

$\therefore$ $\phi(R_1) = 1$ and $\phi(R_2) = -1$.

also, $R_1 \times R_2 \in R^-$

$\therefore$ $\phi(R_1 \times R_2) = -1 = \phi(R_1) \times \phi(R_2)$

$\therefore$ for this case also, 'ϕ' is a homomorphism.

classmate

Date

Page

$\therefore$ For all cases, 'ϕ' turns out to be homomorphism.

8. we have, $\phi : G \rightarrow G'$

$\Rightarrow$ $Im(\phi) = \{\phi(x) \mid x \in G\} \subset G'$

$\Rightarrow$ Let $a, b \in G$.

$\therefore$ as 'ϕ' is homomorphism,

$$\boxed{\phi(a *_1 b) = \phi(a) *_2 \phi(b)} \quad \text{——— } (1).$$

where, $\phi(a)$, $\phi(b)$ and $\phi(a *_1 b) \in Im(\phi)$.

$\therefore$ due to ~~propert~~ Eqn-(1), $Im(\phi)$ is closed under '$*_2$'. $\Rightarrow$ It follows closure property.

$\Rightarrow$ also, $a, a^{-1} \in G$ such that $a *_1 a^{-1} = e$. where ~~e a~~ 'e' is identity of 'G'.

$$\boxed{[\phi(a)]^{-1} = \phi(a^{-1})} \quad \text{——— } (2).$$

$\therefore$ due to Eqn-(2), for all element of $Im(\phi)$, there exist ~~ar~~ inverse member for it.

∴ as ~~the~~ Im(ϕ) follows closure and inverse property under $*_2$, it is a subgroup of G'.

∴ $\boxed{Im(\phi) \leq G'}$

9. we have $H = \{(x, 3x) \mid x \in R\}$

and $G = R^2$

⇒ here G' represents our cartisian coordinate and H represents $y = 3x$ line on it.

∴ $H = \{(x, y) \mid x \in R, y = 3x\}$

∴ the coset $(3,7) + H = \{(x+3, y+7) / x \in R, y = 3x + 7\}$.

here $X = x+3$
$Y = y+7 \Rightarrow y = Y - 7$.

∴ $(3,7) + H = \{(X, Y) / X \in R, Y = 3X\}$

∴ ~~$Y = 3X + 7$~~
⇒ $Y = 3X$.
⇒ $y + 7 = 3(x+3)$
⇒ $\boxed{y = 3x + 2}$

∴ the coset $(3,7) + H$ represents $y = 3x + 2$ line.

Date
Page

∴ as we can see ~~from the~~ that the slope of the lines is equal.

$\Rightarrow$ $y = 3x$ is parallel to $y = 3x + 2$.

10. we have

$C = \{0000000, 1110100, 0111010, 0011101, 1001110, 0100111, 1010011, 1101001\}$.

$\Rightarrow$ as the above code follows the closure property, it is a group code.

also,
$w(1110100) = 4$
$w(0111010) = 4$
$w(0011101) = 4$
$w(1001110) = 4$
$w(0100111) = 4$.
$w(1010011) = 4$.
$w(1101001) = 4$.

∴ the min^m distance = 4.

∴ the error-detecting capacity of $c = 4 - 1 = \boxed{3}$

and the error-correcting capacity of $c = 4/2 = \boxed{2}$