Discrete Cosine Transform

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The Discrete Cosine Transform

- **Discrete Cosine Transform (DCT)** has emerged as the image transformation in most visual systems. DCT has been widely developed by modern video coding standards, for Example, MPEG, JVT etc.
- It is the **same family as the Fourier transform**
    - Converts data to frequency domain

The Discrete Cosine Transform

- Represents data via summation of variable frequency cosine waves.
- Captures only real components of the function.
    - Discrete Sine Transform (DST) captures odd (Imaginary) components → Not as useful.
    - Discrete Fourier Transform (DFT) captures both odd and even components → computationally intense.

The Discrete Cosine Transform

The discrete cosine transform (DCT) represents an image as a sum of sinusoids of varying magnitudes and frequencies. The dct2 function computes the two-dimensional discrete cosine transform (DCT) of an image.

The DCT has the property that, for a typical image, most of the visually significant information about the image is concentrated in just a few coefficients of the DCT.

For this reason, the DCT is often used in image compression applications. For example, the DCT is at the heart of the international standard lossy image compression algorithm known as JPEG. (The name comes from the working group that developed the standard: the Joint Photographic Experts Group.)

The Discrete Cosine Transform

The Discrete Cosine Transform (DCT) helps a separate the image into Parts of differing importance (With respect to the image’s visual quality.)

Mathematical Basis

- 1D DCT:

$$C(u) = \alpha(u) \sum_{x=0}^{N-1} f(x) \cos\left[\frac{\pi(2x+1)u}{2N}\right]$$

Where:

$$\alpha(u) = \begin{cases} \sqrt{\frac{1}{N}} & for \quad u = 0 \\ \sqrt{\frac{2}{N}} & for \quad u \neq 0. \end{cases}$$

- 1D DCT is ***O(n²)***

- 2D DCT:

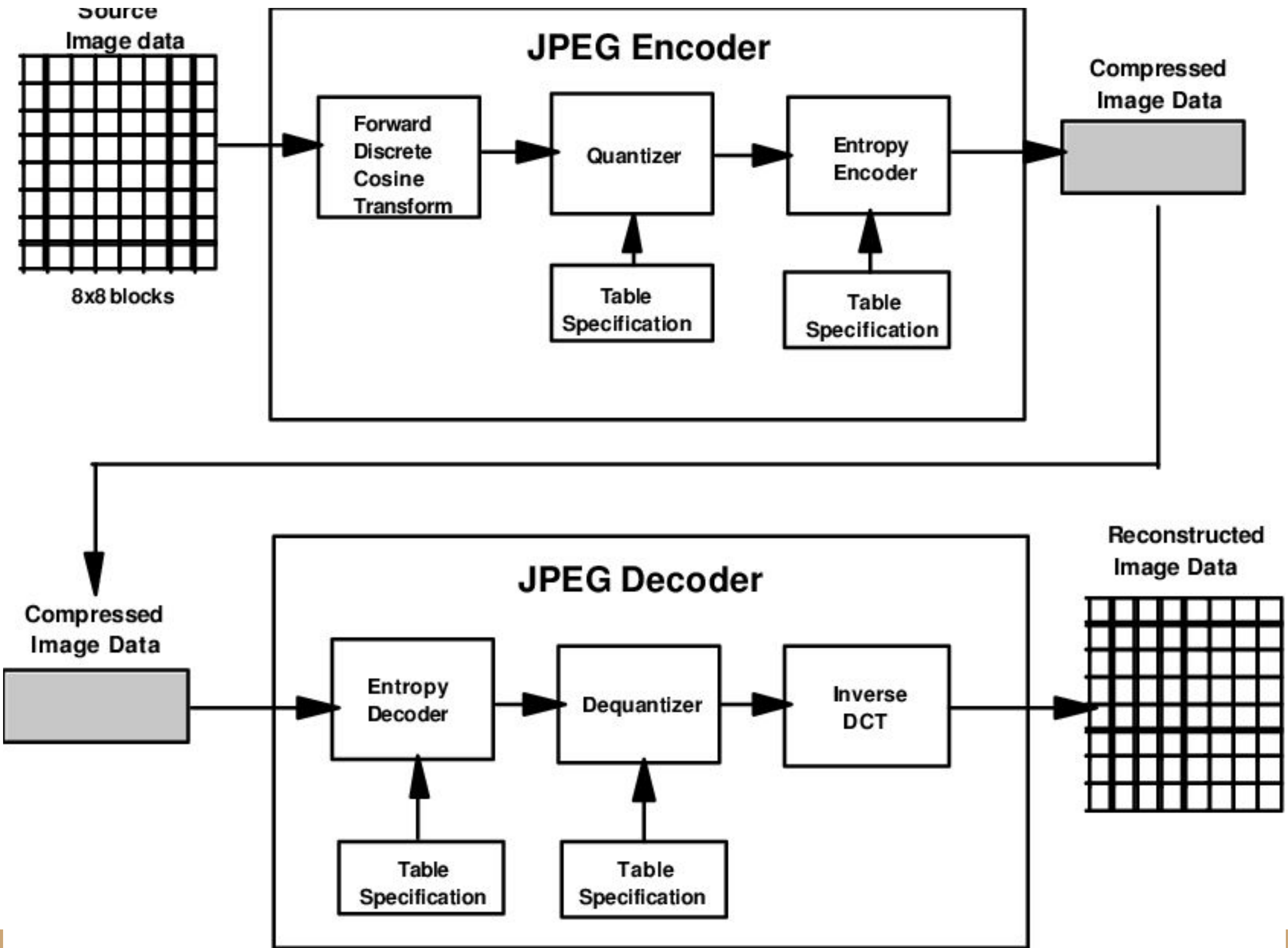
$$C(u,v) = \alpha(u)\alpha(v) \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y) \cos\left[\frac{\pi(2x+1)u}{2N}\right] \cos\left[\frac{\pi(2y+1)v}{2N}\right]$$

  - Where α(u) and α(v) are defined as shown in the 1D case.
  - 2D DCT is ***O(n³)***

The Process

The following is a general overview of the JPEG process. Later, we will take the reader through a detailed tour of JPEG's method so that a more comprehensive understanding of the process may be acquired.

1. The image is broken into 8x8 blocks of pixels.
2. Working from left to right, top to bottom, the DCT is applied to each block.
3. Each block is compressed through quantization.
4. The array of compressed blocks that constitute the image is stored in a drastically reduced amount of space.
5. When desired, the image is reconstructed through decompression, a process that uses the Inverse Discrete Cosine Transform (IDCT).



Source
Image data
8x8 blocks
JPEG Encoder
Forward Discrete Cosine Transform
Quantizer
Entropy Encoder
Table Specification
Table Specification
Compressed Image Data
Compressed Image Data
JPEG Decoder
Entropy Decoder
Dequantizer
Inverse DCT
Table Specification
Table Specification
Reconstructed Image Data

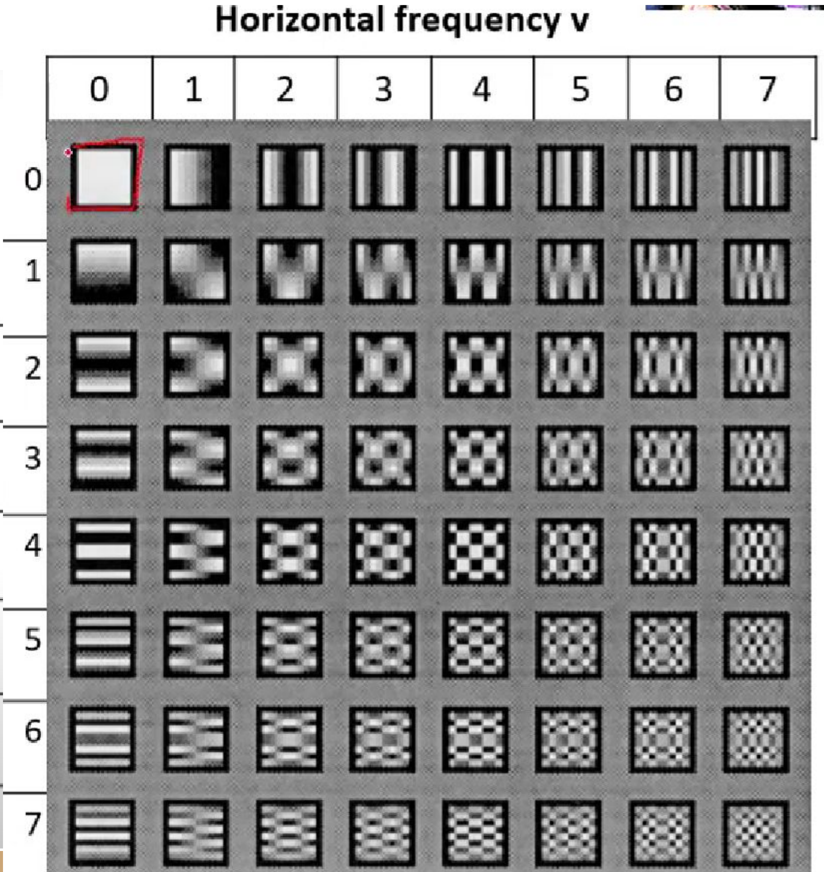
2D Basis DCT signal

Let N=8

Each image in figure is 8X8 image hence there are 64 possible images, that represents 64 basis functions of 8X 8 image.

$$C_{x,y}(u,v)=$$

vertical frequency u



That means we decomposed image into 64 basis functions. These basis functions are fixed.

continue..

- Horizontal frequencies increase from left to right, and Vertical frequencies increase from top to bottom.
- The constant valued basis function at upper left is called as DC basis function and corresponding coefficient is called DC coefficient. Rest of the coefficients are called AC coefficients.

Distribution of Coefficients in DCT

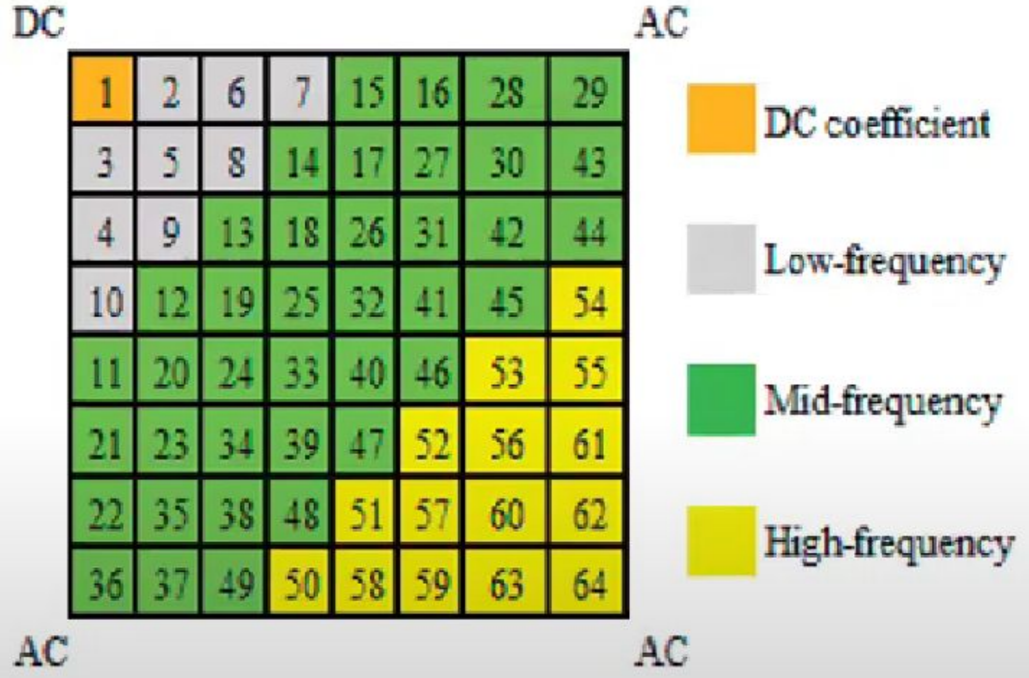
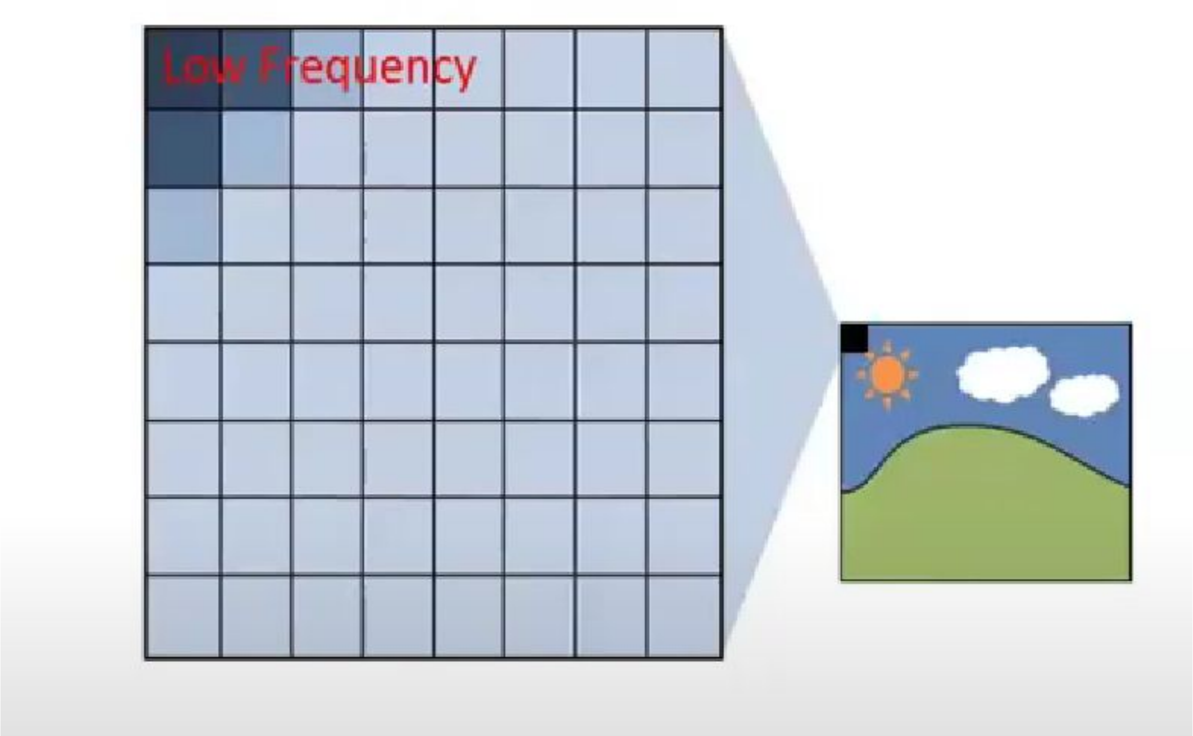
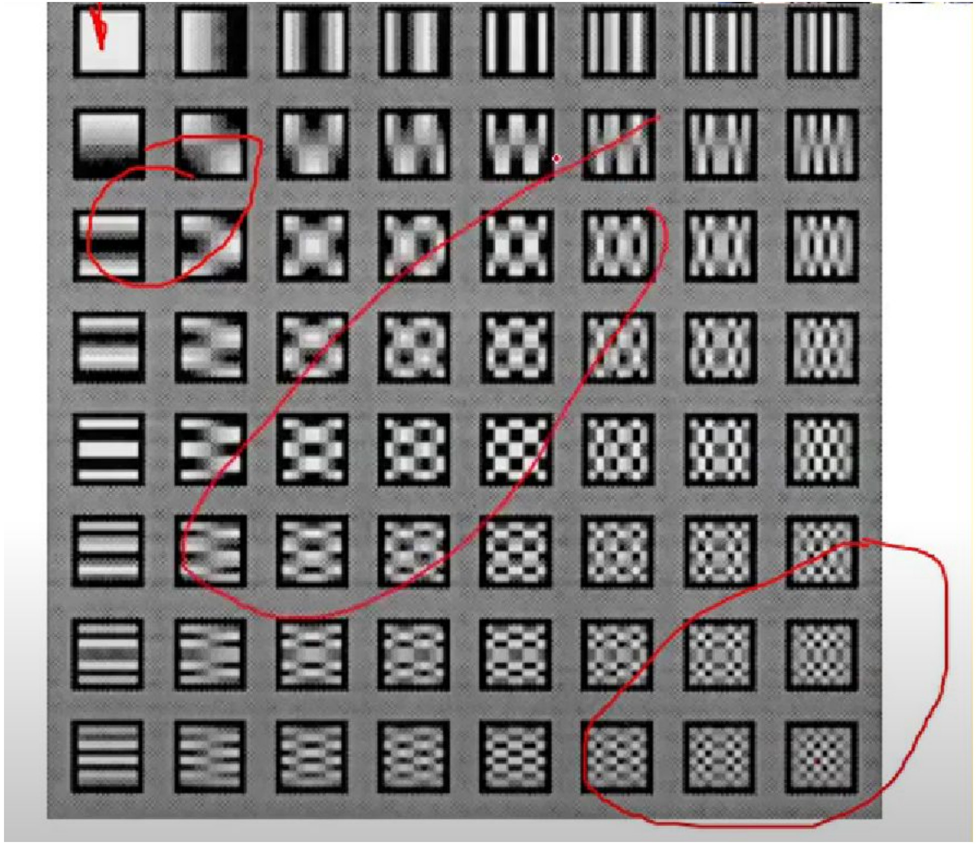


Figure 1. DCT coefficients.

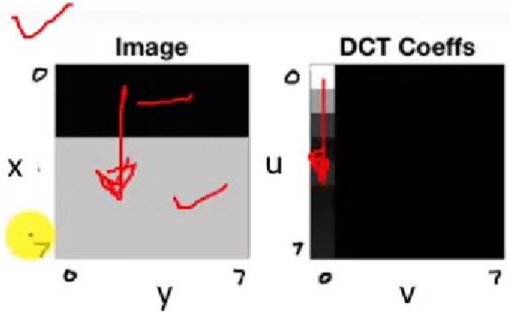
Distribution of Coefficients in DCT

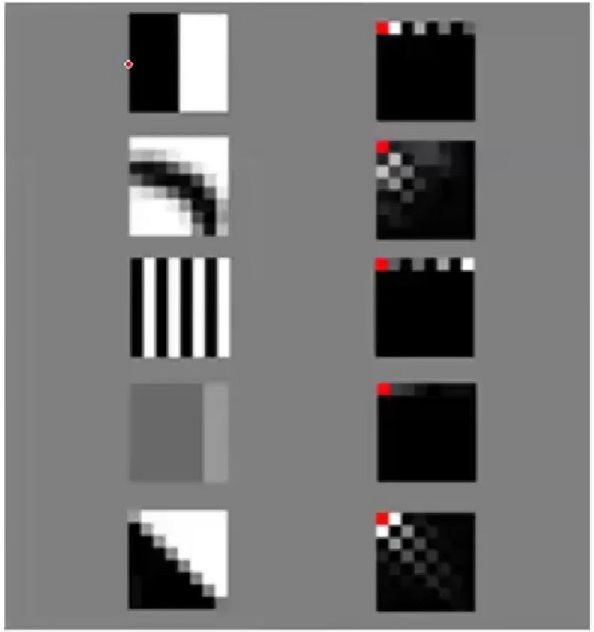


Distribution of Coefficients in DCT

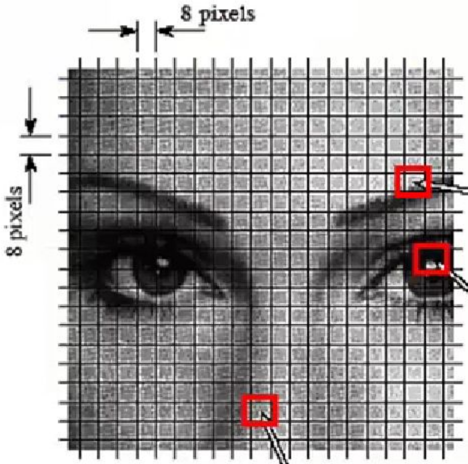


Example:
Horizontal
Edge





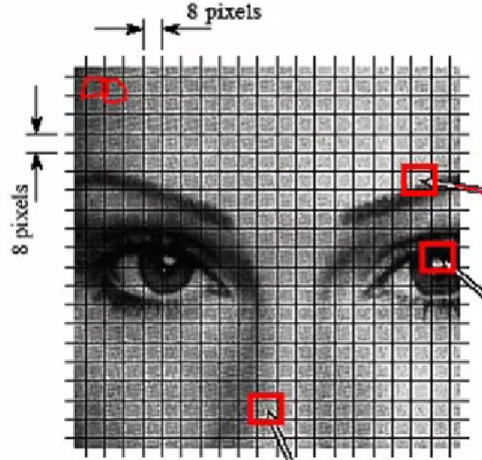
Discrete Cosine Transform



231	224	224	217	217	203	189	196
210	217	203	189	203	224	217	224
196	217	210	224	203	203	196	189
210	203	196	203	182	203	182	189
203	224	203	217	196	175	154	140
182	189	168	161	154	126	119	112
175	154	126	105	140	105	119	84
154	98	105	98	105	63	112	84

42	28	35	28	42	49	35	42
49	49	35	28	35	35	35	42
42	21	21	28	42	35	42	28
21	35	35	42	42	28	28	14
56	70	77	84	91	28	28	21
70	126	133	147	161	91	35	14
126	203	189	182	175	175	35	21
49	189	245	210	182	84	21	35

154	154	175	182	189	168	217	175
154	147	168	154	168	168	196	175
175	154	203	175	189	182	196	182
175	168	168	168	140	175	168	203
133	168	154	196	175	189	203	154
168	161	161	168	154	154	189	189
147	161	175	182	189	175	217	175
175	175	203	175	189	175	175	182



231	224	224	217	217	203	189	196
210	217	203	189	203	224	217	224
196	217	210	224	203	203	196	189
210	203	196	203	182	203	182	189
203	224	203	217	196	175	154	140
182	189	168	161	154	126	119	112
175	154	126	105	140	105	119	84
154	98	105	98	105	63	112	84

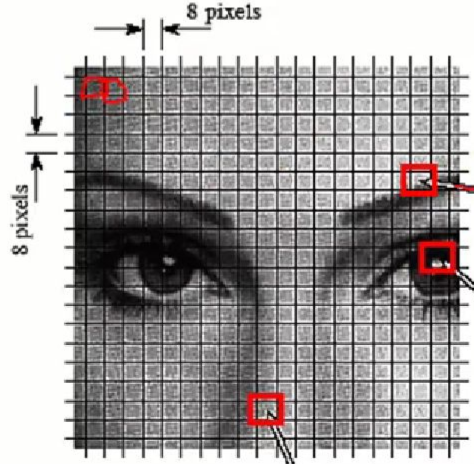
DCT Spectrum

d. Eyebrow spectrum

174	19	0	3	1	0	-3	1
52	-13	-3	-4	-4	-4	5	-8
-18	-4	8	3	3	2	0	9
5	12	-4	0	0	-5	-1	0
1	2	-2	-1	4	4	2	0
-1	2	1	3	0	0	1	1
-2	5	-5	-5	3	2	-1	-1
3	5	-7	0	0	0	-4	0

42	28	35	28	42	49	35	42
49	49	35	28	35	35	35	42
42	21	21	28	42	35	42	28
21	35	35	42	42	28	28	14
56	70	77	84	91	28	28	21
70	126	133	147	161	91	35	14
126	203	189	182	175	175	35	21
49	189	245	210	182	84	21	35

154	154	175	182	189	168	217	175
154	147	168	154	168	168	196	175
175	154	203	175	189	182	196	182
175	168	168	168	140	175	168	203
133	168	154	196	175	189	203	154
168	161	161	168	154	154	189	189
147	161	175	182	189	175	217	175
175	175	203	175	189	175	175	182



231	224	224	217	217	203	189	196
210	217	203	189	203	224	217	224
196	217	210	224	203	203	196	189
210	203	196	203	182	203	182	189
203	224	203	217	196	175	154	140
182	189	168	161	154	126	119	112
175	154	126	105	140	105	119	84
154	98	105	98	105	63	112	84

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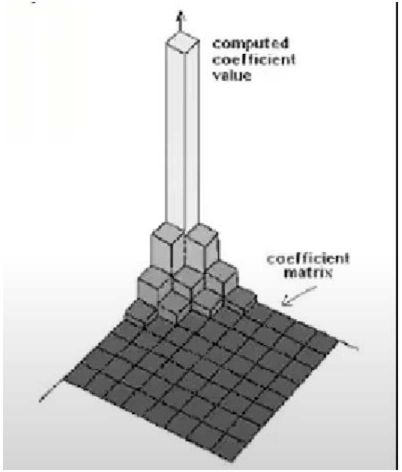
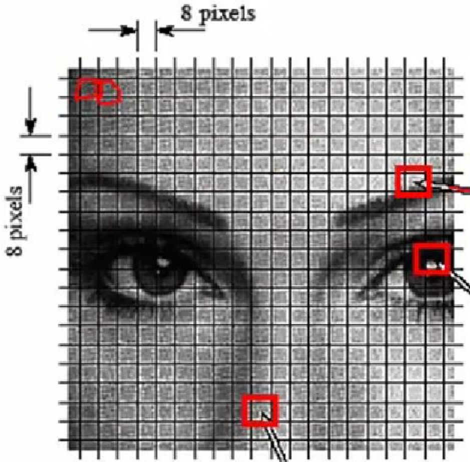
5	12	-4	0	0	-5	-1	0
1	2	-2	-1	4	4	2	0
-1	2	1	3	0	0	1	1
-2	5	-5	-5	3	2	-1	-1
3	5	-7	0	0	0	-4	0

42	28	35	28	42	49	35	42
49	49	35	28	35	35	35	42
42	21	21	28	42	35	42	28
21	35	35	42	42	28	28	14
56	70	77	84	91	28	28	21
70	126	133	147	161	91	35	14
126	203	189	182	175	175	35	21
49	189	245	210	182	84	21	35

154	154	175	182	189	168	217	175
154	147	168	154	168	168	196	175
175	154	203	175	189	182	196	182
175	168	168	168	140	175	168	203
133	168	154	196	175	189	203	154
168	161	161	168	154	154	189	189
147	161	175	182	189	175	217	175
175	175	203	175	189	175	175	182





231	224	224	217	217	203	189	196
210	217	203	189	203	224	217	224
196	217	210	224	203	203	196	189
210	203	196	203	182	203	182	189
203	224	203	217	196	175	154	140
182	189	168	161	154	126	119	112
175	154	126	105	140	105	119	84
154	98	105	98	105	63	112	84

DCT Spectrum

d. Eyebrow spectrum

174	19	0	3	1	0	-3	1
52	-13	-3	-4	-4	-4	5	-8
-18	-4	8	3	3	2	0	9
5	12	-4	0	0	-5	-1	0
1	2	-2	-1	4	4	2	0
-1	2	1	3	0	0	1	1
-2	5	-5	-5	3	2	-1	-1
3	5	-7	0	0	0	-4	0

42	28	35	28	42	49	35	42
49	49	35	28	35	35	35	42
42	21	21	28	42	35	42	28
21	35	35	42	42	28	28	14
56	70	77	84	91	28	28	21
70	126	133	147	161	91	35	14
126	203	189	182	175	175	35	21
49	189	245	210	182	84	21	35

154	154	175	182	189	168	217	175
154	147	168	154	168	168	196	175
175	154	203	175	189	182	196	182
175	168	168	168	140	175	168	203
133	168	154	196	175	189	203	154
168	161	161	168	154	154	189	189
147	161	175	182	189	175	217	175
175	175	203	175	189	175	175	182

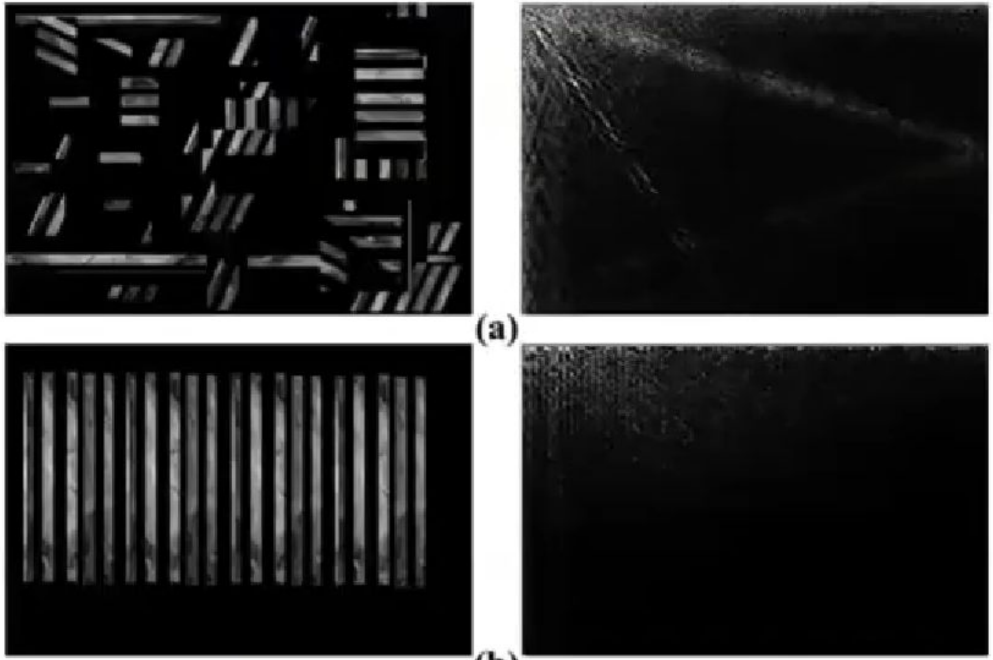
e. Eye spectrum

70	24	-28	-4	-2	-10	-1	0
-53	-35	43	13	7	13	1	3
23	9	-10	-8	-7	-6	5	-3
6	2	-2	8	2	-1	0	-1
-10	-2	-1	-12	2	1	-1	4
3	0	0	11	-4	-1	5	6
-3	-5	-5	-4	3	2	-3	5
3	0	4	5	1	2	1	0

f. Nose spectrum

174	-11	-2	-3	-3	6	-3	4
-2	-3	1	2	0	3	1	2
3	0	-4	0	0	0	-1	9
-4	-6	-2	1	-1	4	-10	-3
1	2	-2	0	0	-2	0	-5
3	-1	3	-2	2	1	1	0
3	5	2	-2	3	0	4	3
4	-3	-13	3	-4	3	-5	3

Discrete Cosine Transform



(a)

(b)

We can do the DCT and quantization process on the peppers image.

Figure 2 – Peppers

Each eight by eight block is hit with the DCT, resulting in the image shown in Figure 3.

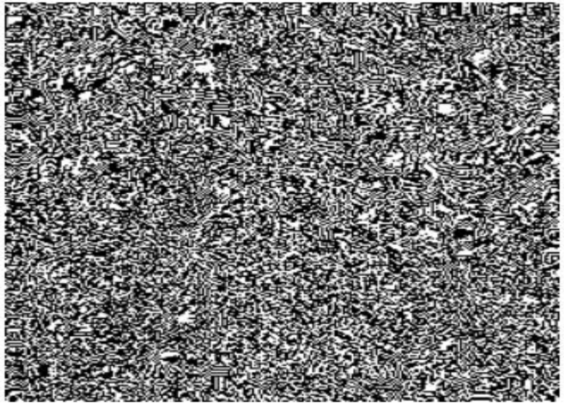


Figure 3 – DCT of Peppers

Figure 5 – Original Peppers

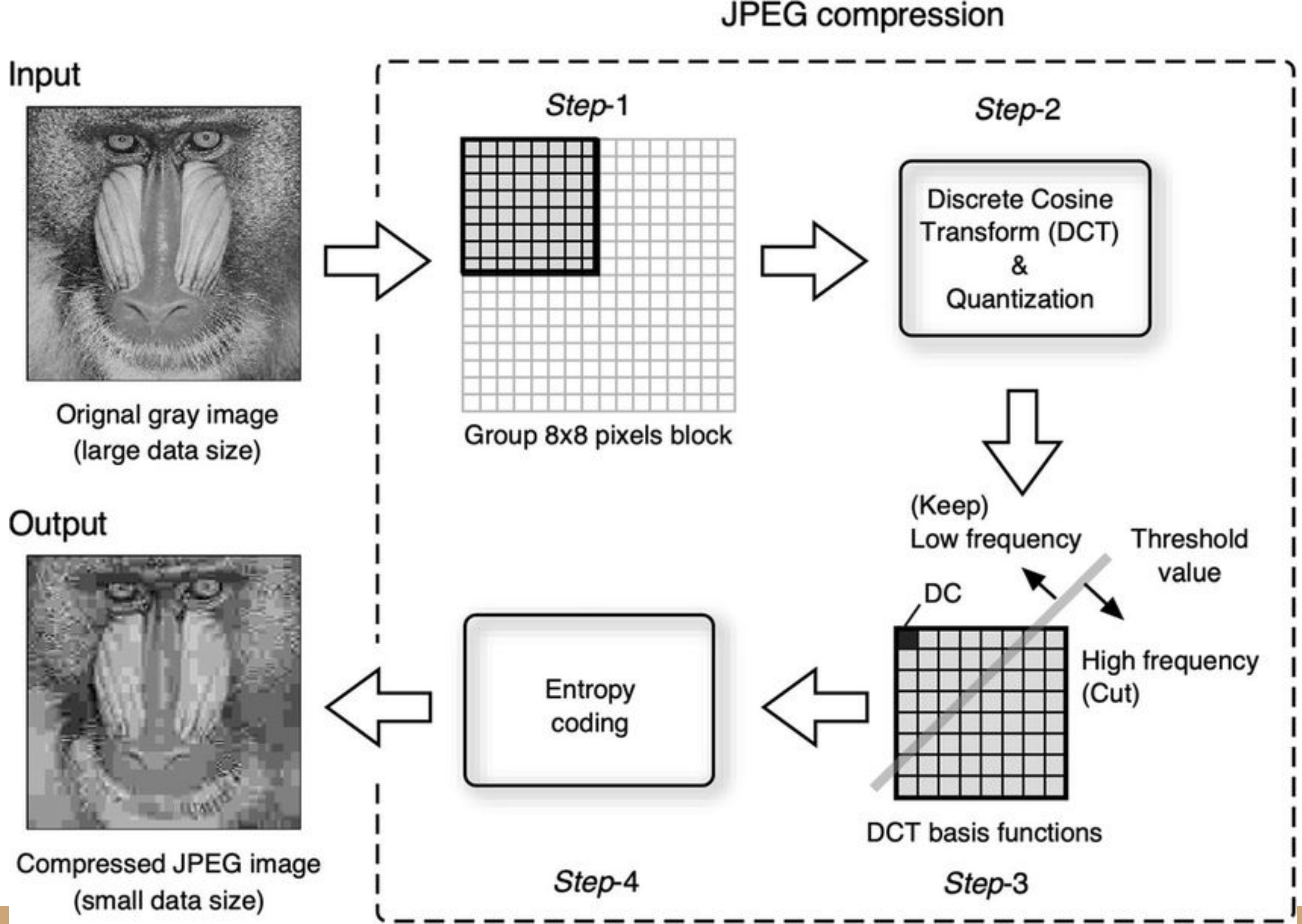
Figure 6 – Quality 50 – 84% Zeros

More Examples

We can see what the compression does to other images. High contrast images, or images with a lot of high frequencies do not compress as well as smooth, low frequency images.

Figure 9 – Original

Figure 10 – Quality 15 – 90% Zeros


JPEG compression
Input
Orignal gray image
(large data size)
Step-1
Group 8x8 pixels block
Step-2
Discrete Cosine
Transform (DCT)
&
Quantization
(Keep)
Low frequency
Threshold
value
DC
High frequency
(Cut)
DCT basis functions
Step-3
Entropy
coding
Step-4
Output
Compressed JPEG image
(small data size)

Significance/Where is this DCT used?

- Image Processing
  - Compression - Ex.) JPEG
  - Scientific Analysis - Ex.) Radio Telescope Data
- Audio Processing
  - Compression - Ex.) MPEG – Layer 3, aka. MP3
- Scientific Computing /
  High Performance Computing (HPC)
  - Partial Differential Equation Solvers

- Advantages
  - Computational efficient (easy to implement in hardware)
  - High compression performance (closely approximates performance of KLT for many images)
- Given these benefits, DCT has become an international standard for transform coding