ME 351 Machine Design Project

Bicycle Transmission System

Reverse Engineering Analysis

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Introduction

This report presents a comprehensive reverse engineering analysis of a bicycle transmission system as shown in the attached 2D drawing. The analysis focuses on estimating loads on various components, calculating safety factors against potential failure modes, and providing detailed design specifications as required by Professor Pankaj Wahi for the Machine Design course project. The system represents an alternative to traditional chain-driven bicycle transmissions, utilizing a shaft-driven mechanism for power transfer from pedals to wheel.

Key Terms and Definitions

Gear Terminology:

- **Pitch Circle:** An imaginary circle drawn on a gear profile that represents the theoretical contact surface during pure rolling motion between meshing gears.
- **Pitch Point:** The point where two pitch circles of mating gears touch each other during operation.
- **Module (m):** The ratio of pitch circle diameter in millimeters to the number of teeth ($m = D/T$). This is a fundamental parameter in gear design that determines tooth size.
- **Pressure Angle (α):** The angle between the common normal at the point of contact and the tangent to the pitch circle at the pitch point. Standard values are 14.5°, 20°, and 25°. This affects load distribution and tooth strength.
- **Addendum:** The radial distance between the pitch circle and the top of the tooth (typically equal to 1 module).
- **Dedendum:** The radial distance between the pitch circle and the bottom of the tooth.
- **Circular Pitch:** The distance along the pitch circle from one point on a tooth to the corresponding point on the next tooth ($C = \pi D/T$).
- **Gear Ratio (G):** The ratio of the number of teeth on the driven gear to the number of teeth on the driving gear ($G = T/t$).
- **Contact Ratio (CR):** Represents the average number of teeth in contact during gear operation. A higher contact ratio typically results in smoother operation and better load distribution.
- **Lewis Form Factor (Y):** A dimensionless parameter used in the Lewis equation for calculating bending stress in gear teeth. It depends on tooth shape, number of teeth, and pressure angle.

Stress Terminology:

- **Bending Stress (σb):** The stress created at the root of a gear tooth due to the tangential force applied, calculated using the Lewis equation.
- **Shear Stress (τ):** Stress that acts parallel to the cross-section of a material, commonly experienced in shafts under torsion.
- **Safety Factor (SF):** The ratio of ultimate stress (or yield stress) to the allowable or working stress, providing a margin of safety in design calculations. SF = Ultimate Stress/Allowable Stress.

- **Von Mises Stress:** A scalar stress value that combines the effects of all stress components, used to predict yielding of materials under complex loading conditions.

System Description and Components

The bicycle transmission system consists of a shaft-driven mechanism that replaces the traditional chain drive. Based on the 2D drawing, the system has:

- Overall dimensions: 524.25 mm length × 122.56 mm height
- Main gear diameter: 200 mm
- Secondary gears: 70 mm and 80 mm
- Main shaft diameter: 63.94 mm

Key Components:

- Bevel gears at front (connected to pedals)
- Drive shaft (transmission shaft)
- Bevel gears at rear (connected to wheel hub)
- Bearings for supporting the shaft
- Mounting framework

The design utilizes a crossed-axis configuration with bevel gears to transmit power from the pedals to a perpendicular shaft, and then to the rear wheel. This arrangement eliminates the need for a chain and chainrings, reducing maintenance requirements while providing a clean, enclosed power transmission system.

Design Assumptions

1. Rider mass: 80 kg (typical adult)
2. Maximum pedal force: 400 N
3. Pedal crank length: 170 mm (standard bicycle)
4. The shaft rotates at a constant speed about its axis
5. The material for gears is AISI 4140 chromium-molybdenum steel
6. The material for shaft is AISI 4340 alloy steel
7. The shaft is supported by two bearings

8. The bevel gears have a pressure angle of 20° (standard)
9. Module of gears: m = 8 mm
10. Face width of gears: b = 20 mm
11. The design will operate in normal cycling conditions, including occasional peak loads

Load Estimation

Pedal Force and Input Torque

Calculating the maximum torque at the pedal:
$T = F \times r = 400 \text{ N} \times 0.17 \text{ m} = 68 \text{ N·m}$

Where:

- F = pedal force (N)
- r = crank length (m)

This torque represents the peak input to the transmission system during strong acceleration or uphill climbing.

Transmission Forces

For the 200 mm main gear, the tangential force is:
$$F_t = \frac{2T}{d} = \frac{2 \times 68 \text{ N·m}}{0.2 \text{ m}} = 680 \text{ N}$$

The radial component is:
$F_r = F_t \times \tan(\alpha) = 680 \text{ N} \times \tan(20°) = 247.6 \text{ N}$

Where:

- α = pressure angle (20°)

Estimation of Dynamic Forces

When considering actual operating conditions, we must account for dynamic loading effects. Using Buckingham's equation for dynamic load:

$$F_d = F_t + \frac{9.84V(Cb + F_t)}{9.84V + \sqrt{Cb + F_t}}$$

Where:

- V = Pitch line velocity (m/s) = $\frac{\pi \times d \times n}{60} = \frac{\pi \times 0.2 \times 60}{60} = 0.628$ m/s (assuming 60 rpm pedaling)
- C = Manufacturing error factor (11000 N/mm for precision cut gears)
- b = Face width (20 mm)

$$F_d = 680 + \frac{9.84 \times 0.628 \times (11000 \times 20 + 680)}{9.84 \times 0.628 + \sqrt{11000 \times 20 + 680}} = 1287 \text{ N}$$

This dynamic load is approximately 1.9 times the static load, highlighting the importance of accounting for dynamic effects in gear design.

Gear Analysis

Lewis Equation for Bending Stress

The bending stress in gear teeth is calculated using the Lewis equation:

$$\sigma_b = \frac{F_t \times K_v}{b \times m \times Y}$$

Where:

- F_t = Tangential force (680 N)
- K_v = Dynamic factor (1.5 for moderate shock)
- b = Face width (20 mm)
- m = Module (8 mm)
- Y = Lewis form factor (0.322 for 20° pressure angle and assuming 25 teeth)

$$\sigma_b = \frac{680 \times 1.5}{20 \times 8 \times 0.322} = 79.1 \text{ MPa}$$

Hertzian Contact Stress

For surface durability, the Hertzian contact stress is:

$$\sigma_H = Z_E \sqrt{\frac{F_t}{d \times b} \times \frac{Z_R}{Z_I}}$$

Where:

- Z_E = Elastic coefficient (√190 MPa for steel)
- Z_R = Roughness factor (0.9 for ground teeth)
- Z_I = Geometry factor (0.08 for 20° pressure angle)
- d = Pitch diameter (200 mm)

$$\sigma_H = \sqrt{190}\sqrt{\frac{680}{200 \times 20} \times \frac{0.9}{0.08}} = 521 \text{ MPa}$$

Wear Tooth Load

The wear tooth load represents the maximum load that can be transmitted without causing excessive wear:

$$F_w = d \times b \times K \times Q$$

Where:

- K = Load-stress factor (dependent on material properties, approximately 0.9 for hardened steel)
- Q = Ratio factor (1.0 for equal size gears)

$$F_w = 200 \times 20 \times 0.9 \times 1.0 = 3600 \text{ N}$$

This wear tooth load is significantly higher than our calculated dynamic load of 1287 N, providing a safety factor of approximately 2.8 against wear failure.

Shaft Analysis

Torsional Stress Analysis

The torsional stress in the shaft is:

$$\tau = \frac{16T}{\pi d^3} = \frac{16 \times 68 \times 10^3}{\pi \times (63.94)^3} = 13.25 \text{ MPa}$$

Where:

- T = Torque (68 N·m)
- d = Shaft diameter (63.94 mm)

Bending Stress Analysis

The bending moment is estimated as:

$M = F_r \times L = 247.6 \text{ N} \times 0.2 \text{ m} = 49.52 \text{ N·m}$

Where:

- L = Estimated moment arm (0.2 m)

Bending stress:

$$\sigma_b = \frac{32M}{\pi d^3} = \frac{32 \times 49.52 \times 10^3}{\pi \times (63.94)^3} = 19.27 \text{ MPa}$$

Combined Stress Analysis

Using the von Mises theory for combined stress:

$$\sigma_{eq} = \sqrt{\sigma_b^2 + 3\tau^2} = \sqrt{(19.27)^2 + 3(13.25)^2} = 30.44 \text{ MPa}$$

Alternatively, using maximum shear stress theory:

$$\tau_{max} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{19.27}{2}\right)^2 + (13.25)^2} = 15.22 \text{ MPa}$$

Bearing Selection and Analysis

Bearing Load Calculation

The equivalent dynamic load on the bearing:

$P = XF_r + YF_a$

Where:

- X = Radial load factor (1.0 for purely radial loading)
- Y = Axial load factor (0 for no axial load)
- F_r = Radial force (247.6 N)
- F_a = Axial force (0 N, assuming no axial load)

$P = 1.0 \times 247.6 + 0 \times 0 = 247.6 \text{ N}$

Bearing Life Calculation

The L10 life of the bearing in hours:

$$L_{10} = \frac{(C/P)^3 \times 10^6}{60 \times n}$$

Where:

- C = Dynamic load rating (assume 10,000 N for a typical bicycle bearing)
- P = Equivalent dynamic load (247.6 N)
- n = Rotational speed (60 rpm)

$$L_{10} = \frac{(10000/247.6)^3 \times 10^6}{60 \times 60} = 67{,}911 \text{ hours}$$

This is equivalent to approximately 7.75 years of continuous operation, which is more than sufficient for bicycle application.

Bearing Selection Criteria

For this application, we would select a deep groove ball bearing with the following specifications:

- Inner diameter: 65 mm (to match shaft diameter with appropriate fit)
- Outer diameter: ~120 mm (typical for this shaft size)
- Width: ~23 mm
- Dynamic load rating: ≥10,000 N
- Static load rating: ≥5,000 N
- Sealed design to prevent contamination

Safety Factor Analysis

Gear Safety Factor

For AISI 4140 steel with yield strength of 600 MPa:

$$n_{gear} = \frac{\sigma_{yield}}{\sigma_b} = \frac{600}{79.1} = 7.58$$

This exceeds the recommended safety factor of 1.5-2.0 for gears under normal loading conditions[5].

Shaft Safety Factor

For AISI 4340 alloy steel with yield strength of 800 MPa:

$$n_{shaft} = \frac{\tau_{yield}}{\tau_{max}} = \frac{0.5 \times 800}{15.22} = 26.3$$

Where 0.5 is the factor for converting yield strength to shear yield strength.

This safety factor is significantly higher than the recommended 2.0-3.0 for normal operation[5], suggesting that the shaft diameter could potentially be reduced to optimize the design.

Bearing Safety Factor

$$n_{bearing} = \frac{C}{P} = \frac{10000}{247.6} = 40.39$$

This exceeds the recommended safety factor of 1.0-1.5 for bearings[5].

Fatigue Analysis

Since the bicycle transmission system undergoes cyclic loading, it's important to analyze potential fatigue failure:

Endurance Limit

For AISI 4340 steel:

$$S_e' = 0.5 \times S_{ut} = 0.5 \times 1200 = 600 \text{ MPa}$$

Where:

- S_e' = Theoretical endurance limit
- S_{ut} = Ultimate tensile strength (1200 MPa for AISI 4340)

Modified Endurance Limit

Applying modification factors:

$$S_e = k_a \times k_b \times k_c \times k_d \times k_e \times k_f \times S_e'$$

Where:

- k_a = Surface finish factor (0.7 for machined surface)
- k_b = Size factor (0.85 for 65 mm diameter)

- k_c = Loading factor (1.0 for bending)
- k_d = Temperature factor (1.0 for room temperature)
- k_e = Reliability factor (0.868 for 95% reliability)
- k_f = Miscellaneous factor (0.9 for typical applications)

$$S_e = 0.7 \times 0.85 \times 1.0 \times 1.0 \times 0.868 \times 0.9 \times 600 = 297 \text{ MPa}$$

Fatigue Safety Factor

Using the Goodman criterion:

$$n_f = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}}$$

Where:

- σ_a = Stress amplitude (approximately 15 MPa for our application)
- σ_m = Mean stress (approximately 15 MPa for our application)

$$n_f = \frac{1}{\frac{15}{297} + \frac{15}{1200}} = \frac{1}{0.051 + 0.013} = 15.6$$

This indicates excellent fatigue resistance for the design.

Failure Mode Analysis

Potential Failure Modes and Safety Factors

Component	Failure Mode	Calculated Stress	Material Strength	Safety Factor
Gear teeth	Bending	79.1 MPa	600 MPa	7.58
Gear teeth	Surface fatigue	521 MPa	1500 MPa*	2.88
Shaft	Torsional shear	13.25 MPa	400 MPa	30.2
Shaft	Combined stress	30.44 MPa	800 MPa	26.3
Bearings	Fatigue	-	-	40.39
Overall system	Fatigue	-	-	15.6

*Assumed hardened surface strength for AISI 4140 after heat treatment

Critical Failure Modes

Based on the analysis, the most critical potential failure modes are:

1. **Gear tooth surface fatigue (pitting):** With the lowest safety factor of 2.88, this would likely be the first failure mode to manifest after extended use. This could be mitigated through proper material selection, heat treatment, and lubrication.

2. **Gear tooth bending failure:** Despite having a safety factor of 7.58, this could occur under extreme shock loading or if material defects are present.

3. **Bearing wear and contamination:** While the calculated safety factor is very high, real-world performance would depend on proper sealing and lubrication to prevent contamination.

Material Selection Justification

For Gears:
AISI 4140 chromium-molybdenum steel is selected for the following reasons:

- Yield strength: 600 MPa
- Good hardenability allowing for case hardening
- Excellent wear resistance
- Common in precision gear applications
- Good machinability prior to heat treatment
- Cost-effective for medium production volumes

The material should be heat treated to 55-60 HRC surface hardness through carburizing or case hardening, while maintaining a tough core to resist shock loading.

For Shaft:
AISI 4340 alloy steel is selected for the following reasons:

- Yield strength: 800 MPa
- High toughness and fatigue resistance
- Good machinability
- Ideal for heavily loaded shafts
- Excellent resistance to shock and impact

- Good through-hardening capabilities

The shaft should be heat treated to 30-35 HRC to provide a good balance between strength and toughness.

For Bearings:
Standard chrome steel bearings (AISI 52100) are recommended with:

- Hardness: 60-65 HRC
- Sealed design with rubber shields
- Grease lubrication
- Cage material: Polyamide for reduced weight and noise

Geometric Dimensioning and Tolerancing Considerations

For proper functioning of the gear system, the following tolerances should be applied:

1. **Gear Tooth Profile:** ISO tolerance grade 7 (IT7) to ensure proper mesh and minimize noise.
2. **Shaft Diameter:** h6 tolerance for bearing seats to ensure proper fit.
3. **Bearing Seats:** N6 tolerance for proper interference fit.
4. **Gear Runout:** Total runout tolerance of 0.05 mm to prevent vibration and ensure smooth rotation.
5. **Perpendicularity:** 0.02 mm between shaft axis and gear mounting faces to ensure proper gear alignment.
6. **Concentricity:** 0.03 mm for gear pitch circle relative to mounting hole to ensure proper mesh.