# Bayesian Modeling of Tide and Storm Surge Propagation in Mississippi Delta

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## 1. Introduction

Tidal rivers and estuaries are unique locations that experience flooding from both oceanographic and riverine origins. At the intersection of fluvial and coastal flooding are compound floods, which are caused by two or more flood drivers, such as storm surge and rainfall Wahl et al. (2015). One of the central questions in tidal-river hydraulics is how far upstream tides and storm surges can penetrate from the open coast. In reality, this propagation distance is not fixed but varies with river discharge, which can either bolster or attenuate the incoming tidal and surge waves. On top of that, surge, tide and freshwater flow interact nonlinearly high river flows can steepen and distort the tidal waveform, while a large surge can alter the effective river gradient. Finally, the rate at which those oscillations die out depends sensitively on channel geometry (width, depth, curvature) and on meteorological drivers (wind setup, atmospheric pressure), both of which are themselves uncertain. Together, these factors make it challenging to predict where and when compound flooding will occur without a flexible spatiotemporal model that jointly accounts for riverine forcing, coastal forcing, and their mutual uncertainty. Spatial-temporal Bayesian hierarchical models have become essential in hydrological analyses, capturing dependencies and heterogeneity across locations and time periods (Cressie, 2015).

## 2. Model

The goal is to model river water levels $Y_i$ observed at location $s(i)$ and time $t(i)$ using a spatiotemporal Bayesian hierarchical model:

$$Y_i = X_i^\top \beta + s_{s(i)} + u_{t(i)} + \varepsilon_i, \quad \varepsilon_i \sim \mathcal{N}(0, \tau_Y^{-1}) \tag{1}$$

where:

- $X_i = [1, X_{1i}, X_{2i}]$ includes intercept, storm surge, and discharge.
- $\beta$ is the vector of fixed-effect coefficients.
- $s_{s(i)}$ is a spatial random effect for site $s(i)$.

- $u_{t(i)}$ is a temporal random effect at time $t(i)$.
- $\varepsilon_i$ is the Gaussian residual noise.

# 3. Priors and Hyperparameters

## 3.1 Fixed Effects $\beta$: Horseshoe Prior

A global-local shrinkage prior is imposed on $\beta$ using the Horseshoe prior:

$$\beta_j \sim \mathcal{N}(0, \lambda_j^2 \tau^2), \tag{2}$$

$$\lambda_j \sim \text{Cauchy}^+(0, 1) \tag{3}$$

This prior effectively shrinks negligible predictors towards zero, particularly useful when many covariates might be irrelevant (Carvalho et al., 2010). However, it increases computational complexity. This encourages sparsity, shrinking irrelevant covariates while preserving signals.

## 3.2 Residual Precision $\tau_Y$

A weakly informative prior:

$$\tau_Y \sim \text{Gamma}(0.01, 0.01) \tag{4}$$

is used to model the inverse residual variance. It is updated via Gibbs sampling.

## 3.3 Spatial Effects: $s \sim \mathcal{N}(0, \sigma_s^2 R(\phi_s))$

The spatial random effect is modeled using an exponential covariance kernel:

$$R_{ij} = \exp\left(-\frac{D_{ij}}{\phi_s}\right) \tag{5}$$

where $D_{ij}$ is the great-circle distance in kilometers.

**Spatial Variance $\sigma_s^2$ Prior:**

$$p(\sigma_s^2) \propto \sigma_s^{-2} \exp\left(-\frac{\nu \cdot \sigma_0^2}{2\sigma_s^2}\right), \quad \nu = 0.1 \tag{6}$$

This prior is similar to an inverse-gamma, used with Metropolis-Hastings sampling.

**Spatial Range $\phi_s$ Prior:**

$$\phi_s \sim \text{Gamma}(2, \text{rate} = 1/50) \tag{7}$$

The model learns geographic structure using great-circle distances $D_{ij}$ between tide gauge coordinates. These are fed into the exponential kernel $R(\phi_s)$, allowing the model to learn how correlation decays over space.

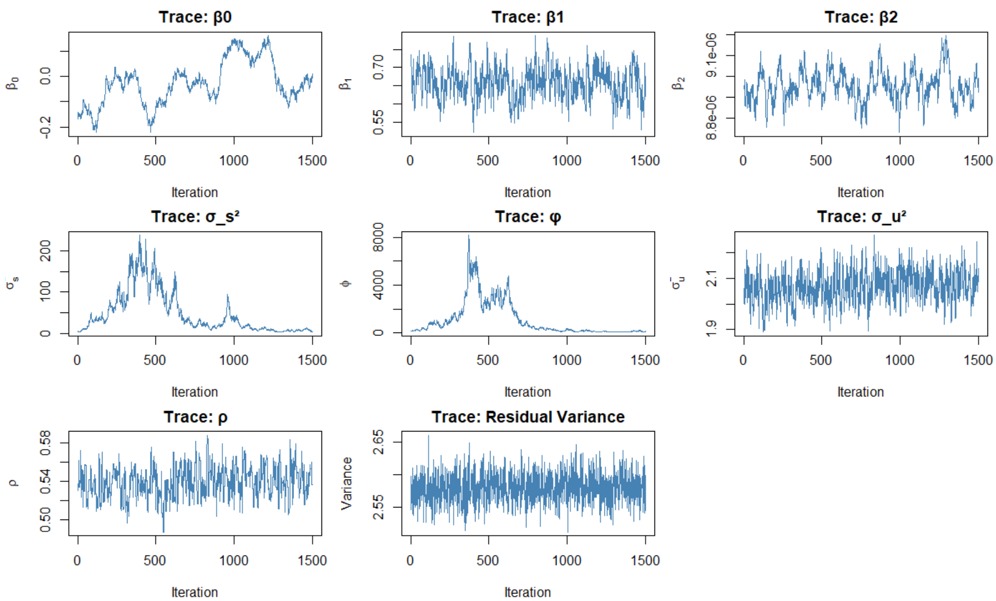


Figure 1: Trace Plots

### 3.4 Temporal Effects: $u_t$ as AR(1)

The temporal random effect follows a stationary AR(1) process:

$$u_t = \rho u_{t-1} + \eta_t, \quad \eta_t \sim \mathcal{N}(0, \sigma_u^2) \tag{8}$$

It captures smooth temporal fluctuations shared across all sites.

**Priors:**

$$\sigma_u^2 \sim \text{Inverse-Gamma}(2, 2) \tag{9}$$
$$\rho \sim \text{Uniform}(-1, 1) \tag{10}$$

The $u_t$ sequence is sampled via conditional Gibbs sampling, while $\rho$ is updated using Metropolis-Hastings.

## 4 Results

The trace plots in the figure 1 show that most parameters—especially $\beta_1$, $\beta_2$, $\rho$, $\sigma_u^2$, and the residual variance—mix tightly around their posterior modes, indicating good convergence. The spatial variance ($\sigma_s^2$) and range ($\phi$) chains initially explore very large values but then settle into a lower, stable regime, reflecting the Metropolis–Hastings sampler finding its high-probability region. Overall, after burn-in the chains appear well mixed with no obvious trends or stickiness.

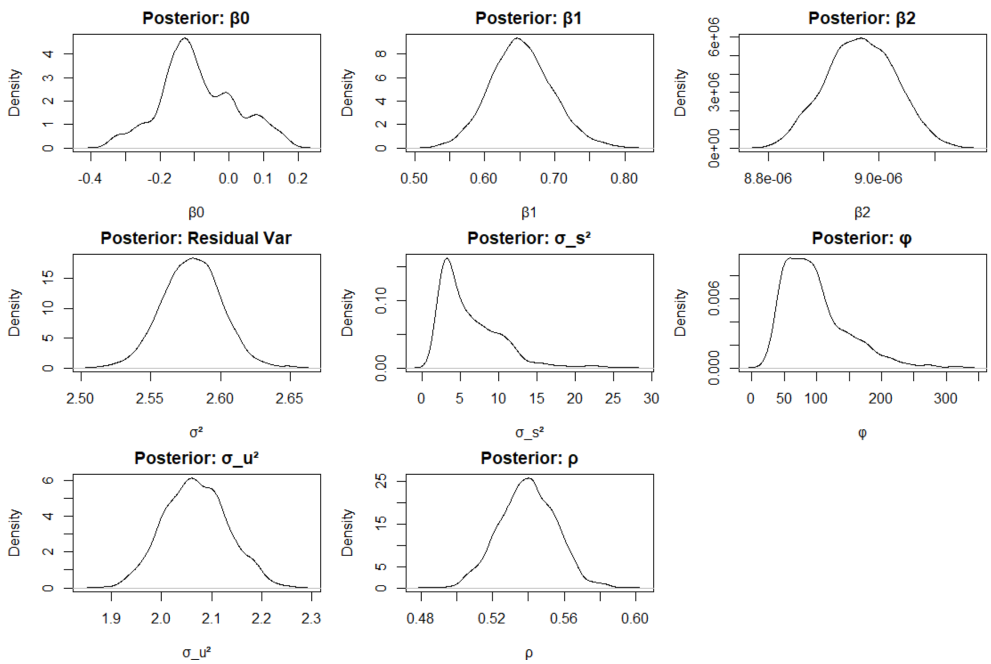


Figure 2: Posterior Density Plot

# 5 conclusion and Discussion

- **Storm surge contributes incrementally** Although its effect is smaller than discharge surge still exerts a consistent influence on water level underscoring the need to prioritize accurate surge forecasts in any predictive system.

- **River discharge as the primary driver** River discharge emerges as the strongest predictor of river stage, affirming the importance of including hydrologic flow data in coastal-river models.

- **Substantial site-to-site heterogeneity.** Even after accounting for surge and discharge, a considerable portion of variability is explained by location-specific effects. This suggests that local factors—such as channel geometry, bathymetry, or anthropogenic controls—play an important role.

- **Moderate spatial correlation range.** Residual spatial effects remain meaningfully correlated across distances comparable to the spacing of our gauges, implying that adding new sites within that range may yield diminishing returns, whereas more widely spaced stations capture distinct regimes.

- **Temporal persistence of anomalies.** Unmodeled daily fluctuations exhibit clear memory from one day to the next, reflecting lingering meteorological and hydrodynamic processes not explicitly included in our fixed covariates.

- **Residual uncertainty.** After modeling all fixed and random effects, some day-to-day noise remains, indicating opportunities to refine the model—particularly by incorporating additional drivers such as wind or atmospheric pressure.

## References


Carvalho, C. M., Polson, N. G., and Scott, J. G. (2010). The horseshoe estimator for sparse signals. *Biometrika*, 97:465–480.

Cressie, N. (2015). *Statistics for Spatial Data*. John Wiley & Sons.

Wahl, T., Jain, S., Bender, J., Meyers, S. D., and Luther, M. E. (2015). Increasing risk of compound flooding from storm surge and rainfall for major us cities. *Nature Climate Change*, 5(12):1093–1097.