Course Information

CSE 107 — Introduction to Modern Cryptography

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Cryptography usage

Did you use any cryptography today?

Cryptography usage

- https invokes the TLS protocol
- TLS uses cryptography
- TLS is in ubiquitous use for secure communication: shopping, banking, Netflix, gmail, Facebook, ...

Secure messaging apps

WhatsApp, Signal, iMessage/FaceTime, Viber, Telegram, LINE, Threema, ChatSecure, KakaoTalk, ...

Use them!

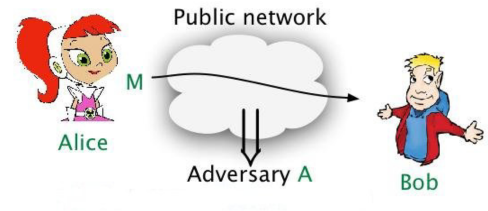
Cryptography usage

Other uses of cryptography

- ATM machines
- Bitcoin
- Tor: Anonymous web browsing
- Google authenticator
- ...

11,748 android apps use cryptography (encryption), and 10,327 get it wrong [EBFK13]

What is cryptography about?



Adversary: clever person with powerful computer

Security goals:

- **Data privacy:** Ensure adversary does not see or obtain the data (message) M.
- **Data integrity and authenticity:** Ensure M really originates with Alice and has not been modified in transit.

Example: Medical databases

Doctor — Database

Get Alice $\longrightarrow$

Alice	F_A
Bob	F_B

$\longleftarrow$ F_A

Reads F_A
Modifies F_A to F'_A

Put: Alice, F'_A $\longrightarrow$

Alice	F'_A
Bob	F_B

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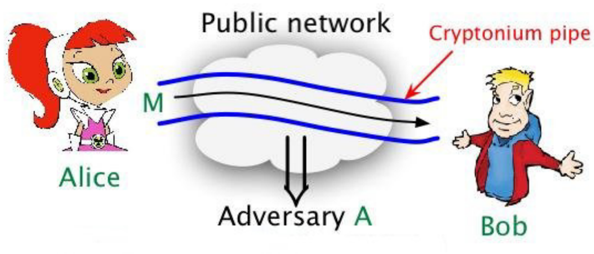
- Privacy: F_A, F'_A contain confidential information and we want to ensure the adversary does not obtain them
- Integrity and authenticity: Need to ensure
  - doctor is authorized to get Alice's file
  - F_A, F'_A are not modified in transit
  - F_A is really sent by database
  - F'_A is really sent by (authorized) doctor

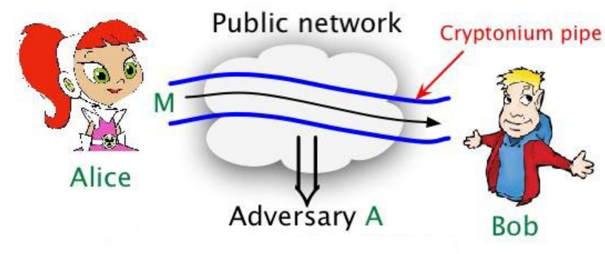
Ideal World



Cryptonium pipe: Cannot see inside or alter content.

All our goals would be achieved!

Ideal World

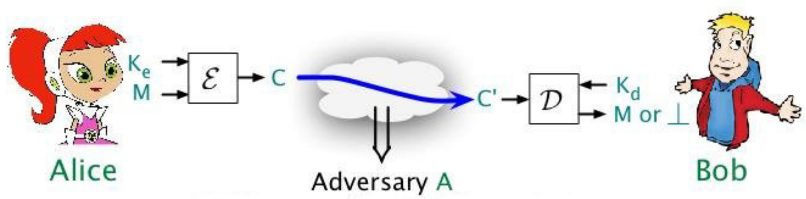


Cryptonium pipe: Cannot see inside or alter content.

All our goals would be achieved!

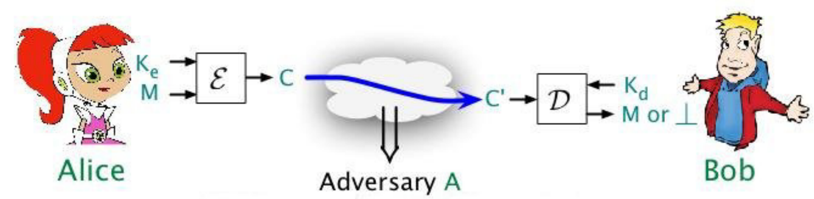
But cryptonium is only available on planet Crypton and is in short supply. ☹

Cryptographic schemes



$\mathcal{E}$: encryption algorithm $\quad K_e$: encryption key
$\mathcal{D}$: decryption algorithm $\quad K_d$: decryption key

Cryptographic schemes



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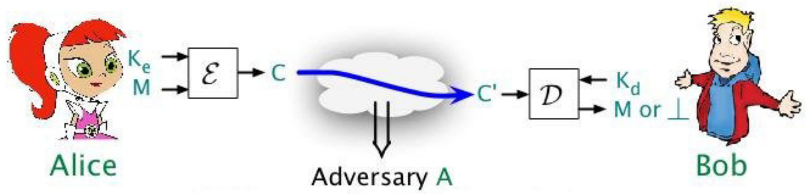
Algorithms: standardized, implemented, public!

Cryptographic schemes



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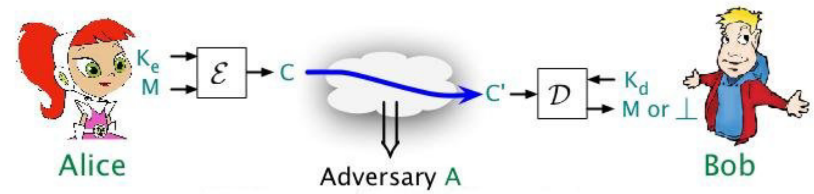
Settings:

- public-key (assymmetric): K_e public, K_d secret
- private-key (symmetric): $K_e = K_d$ secret

Cryptographic schemes



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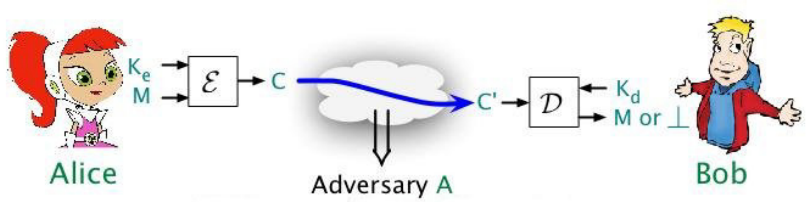
How do keys get distributed? Magic, for now!

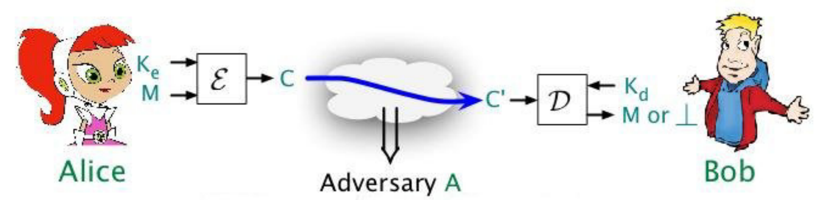
Cryptographic schemes



Our concerns:

- How to define security goals?
- How to design $\mathcal{E}$, $\mathcal{D}$?
- How to gain confidence that $\mathcal{E}$, $\mathcal{D}$ achieve our goals?

Cryptographic schemes



Computer Security: How does the computer/system protect K_e/K_d from break-in (viruses, worms, OS holes, ...)? (CSE 127,227)

Cryptography: How do we use K_e, K_d to ensure security of communication over an insecure network? (CSE 107,207)

Why is cryptography hard?

- One cannot anticipate an adversary strategy in advance; number of possibilities is infinite.
- "Testing" is not possible in this setting.

Early history

Substitution ciphers/Caesar ciphers:

$$K_e = K_d = \pi : \Sigma \to \Sigma, \text{a secret permutation}$$

e.g., $\Sigma = \{A, B, C, \ldots\}$ and π is as follows:

σ	A	B	C	D	$\cdots$
$\pi(\sigma)$	E	A	Z	U	$\cdots$

$$\begin{aligned} \mathcal{E}_\pi(CAB) &= \pi(C)\pi(A)\pi(B) \\ &= Z\ E\ A \\ \mathcal{D}_\pi(ZEA) &= \pi^{-1}(Z)\pi^{-1}(E)\pi^{-1}(A) \\ &= C\ A\ B \end{aligned}$$

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Not very secure! (Common newspaper puzzle)

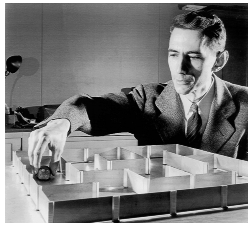
The age of machines

Enigma: German World War II machine

Broken by British in an effort led by Turing

Shannon and One-Time-Pad (OTP) Encryption

$K_e = K_d = \underbrace{K \xleftarrow{\$} \{0,1\}^k}_{\substack{K \textit{ chosen at random} \\ \textit{from } \{0,1\}^k}}$



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- $\mathcal{E}_K(M) = K \oplus M$
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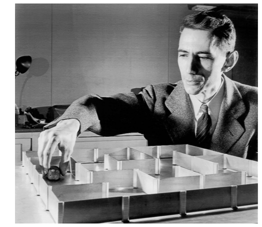

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Theorem (Shannon): OTP is perfectly secure as long as only one message encrypted.

"Perfect" secrecy, a notion Shannon defines, captures mathematical impossibility of breaking an encryption scheme.

Fact: if $|M| > |K|$, then no scheme is perfectly secure.

Modern Cryptography: A Computational Science

Security of a "practical" system must rely not on the impossibility but on the computational difficulty of breaking the system.

("Practical" = more message bits than key bits)

Rather than:

"It is impossible to break the scheme"

We might be able to say:

"No attack using $\leq 2^{160}$ time succeeds with probability $\geq 2^{-20}$"

I.e., Attacks can exist as long as cost to mount them is prohibitive, where Cost = computing time/memory, \$\$\$

Modern Cryptography: A Computational Science

Security of a "practical" system must rely not on the impossibility but on the computational difficulty of breaking the system.

Cryptography is now not just mathematics; it needs to draw on computer science

- Computational complexity theory (CSE 105,200)
- Algorithm design (CSE 101,202)

The factoring problem

Input: Composite integer N
Desired output: prime factors of N

Example:
Input: 85
Output:

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Can we write a factoring program? Easy!

Alg Factor(N) // N a product of 2 primes
For $i = 2, 3, \ldots, \lceil\sqrt{N}\rceil$ do
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But this is very slow ...
Prohibitive if N is large (e.g., 400 digits)

Can we factor fast?

- Gauss couldn't figure out how
- Today there is no known algorithm to factor a 400 digit number in a practical amount of time.

Factoring is an example of a problem believed to be computationally hard.

Note 1: A fast algorithm MAY exist.

Note 2: A quantum computer can factor fast! One has not yet been built but efforts are underway ...

Atomic Primitives or Problems

Examples:

- Factoring: Given large $N = pq$, find p, q
- Block cipher primitives: DES, AES, ...
- Hash functions: MD5, SHA1, SHA3, ...

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Drawback: Don't directly solve any security problem.

Higher Level Primitives

Goal: Solve security problem of direct interest.

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Features:

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Lego Approach

We typically design high-level primitives from atomic ones

Atomic primitive

↓

Transformer

↓

High-level primitive

Defining security

A great deal of design tries to produces schemes without first asking:

"What exactly is the security goal?"

This leads to schemes that are complex, unclear, and wrong.

Being able to precisely state what is the security goal of a design is challenging but important.

We will spend a lot of time developing and justifying strong, precise notions of security.

Thinking in terms of these precise goals and understanding the need for them may be the most important thing you get from this course!

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Does it mean that given $C = \mathcal{E}_{K_e}(M)$, adversary cannot

- recover M?
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- ...

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We will provide a formal definition for privacy, justify it, and show it implies the above (and more).

Cryptography in practice

Schemes designed via the principles we will study are in use (TLS, SSH, IPSec, . . .): HMAC, RSA-OAEP, ECIES, Ed25519, CMAC, GCM, . . .

New uses for old mathematics

Cryptography uses

- Number theory
- Combinatorics
- Modern algebra
- Probability theory

Modern Cryptography: Esoteric mathematics?

Hardy, in his essay A Mathematician's Apology writes:

> *"Both Gauss and lesser mathematicians may be justified in rejoicing that there is one such science [number theory] at any rate, and that their own, whose very remoteness from ordinary human activities should keep it gentle and clean"*

No longer: Number theory is the basis of modern public-key systems such as RSA.

Security today

- Server breaches, malware
- Compromise of people's private information leading to identity theft, credit-card fraud, ...
- Lack of privacy: Information about us is collected and harvested
- Mass surveillance: Snowden Revelations

2017 Equifax breach exposed 143 million social security numbers.

Cryptography is a central tool in getting more security and privacy.

Cryptography on the horizon

Computing on encrypted data

- Searchable encryption
- Homomorphic encryption
- multi-party computation
- garbled circuits
- ...

What you can get from this course

Be able to

- Identify threats
- Evaluate security solutions and technologies
- Design high-quality solutions
- Develop next-generation privacy tools
- ...

If nothing else, develop a healthy sense of paranoia!

How to do well in CSE 107

Characteristics of the successful 107 student:

- More interested in learning than grades
- Likes challenges, does not give up easily
- Tries to understand *all* the materiel, not just some of it
- Questions are more often about the materiel (slides) than about how to do the homework.
- Understands theory behind examples.

If you take the course with the view that you only want to pass, you increase the risk of not passing. If you take it aiming to get an A and are willing to work for it, you may very well get one.

How to do well in CSE 107

Doesn't work too well: Random access mode, in which you look at homework or quiz problem, then try to find something in slides that "matches" it.

Works well: Sequential mode, where you first go through all the slides, sequentially, and make sure you understand the materiel, and THEN attempt homework and quizzes.

Some students expect a recipe for success: "I am willing to work hard. Just tell me what to do!"

We are not aware of any such recipe. Different people understand things in different ways and have different paths to success. You will find your own!