

## Feedback — Week 4 - Problem Set

[Help](#)

You submitted this homework on **Wed 12 Feb 2014 12:04 PM PST**. You got a score of **8.50** out of **10.00**. You can [attempt again](#) in 10 minutes.

### Question 1

An attacker intercepts the following ciphertext (hex encoded):

20814804c1767293b99f1d9cab3bc3e7 ac1e37bfb15599e5f40eef805488281d

He knows that the plaintext is the ASCII encoding of the message "Pay Bob 100\$" (excluding the quotes). He also knows that the cipher used is CBC encryption with a random IV using AES as the underlying block cipher. Show that the attacker can change the ciphertext so that it will decrypt to "Pay Bob 500\$". What is the resulting ciphertext (hex encoded)? This shows that CBC provides no integrity.

**You entered:**

Your Answer	Score	Explanation
	✖ 0.00	
Total	0.00 / 1.00	

### Question 2

Let  $(E, D)$  be an encryption system with key space  $K$ , message space  $\{0, 1\}^n$  and ciphertext space  $\{0, 1\}^s$ . Suppose  $(E, D)$  provides authenticated encryption. Which of the following

systems provide authenticated encryption: (as usual, we use  $\parallel$  to denote string concatenation)

Your Answer	Score	Explanation
<input checked="" type="checkbox"/> $E'(k, m) = [c \leftarrow E(k, m), \text{ output } (c, c)]$ and $D'(k, (c_1, c_2)) = \begin{cases} D(k, c_1) & \text{if } c_1 = c_2 \\ \perp & \text{otherwise} \end{cases}$	<input checked="" type="checkbox"/> 0.25	$(E', D')$ provides authenticated encryption because an attack on $(E', D')$ directly gives an attack on $(E, D)$ .
<input checked="" type="checkbox"/> $E'(k, m) = (E(k, m), H(m))$ and $D'(k, (c, h)) = \begin{cases} D(k, c) & \text{if } H(D(k, c)) = h \\ \perp & \text{otherwise} \end{cases}$ (here $H$ is some collision resistant hash function)	<input checked="" type="checkbox"/> 0.00	This system is not CPA secure because $H(m)$ leaks information about the message in the ciphertext.
<input type="checkbox"/> $E'((k_1, k_2), m) = E(k_2, E(k_1, m))$ and $D'((k_1, k_2), c) = \begin{cases} D(k_1, D(k_2, c)) & \text{if } D(k_2, c) \neq \perp \\ \perp & \text{otherwise} \end{cases}$	<input checked="" type="checkbox"/> 0.00	$(E', D')$ provides authenticated encryption because an attack on $(E', D')$ gives an attack on $(E, D)$ . It's an interesting exercise to work out the ciphertext integrity attack on $(E, D)$ given a ciphertext integrity attacker on

$(E', D')$ .

- ☐  $E'(k, m) = (E(k, m), E(k, m))$  and  
 $D'(k, (c_1, c_2)) = D(k, c_1)$

✓ 0.25

This system does not provide ciphertext integrity. The attacker can query for  $E'(k, 0^n)$  to obtain  $(c_1, c_2)$ . It then outputs  $(c_1, 0^s)$  and wins the ciphertext integrity game.

Total

0.50 /  
1.00

### Question 3

If you need to build an application that needs to encrypt multiple messages using a single key, what encryption method should you use? (for now, we ignore the question of key generation and management)

Your Answer

Score

Explanation

☐ use a standard implementation of CBC encryption with a random IV.

☐ implement Encrypt-and-MAC yourself

☐ implement OCB by yourself

☒ use a standard implementation of one of the authenticated encryption modes GCM, CCM, EAX or OCB.

✓ 1.00

Total

1.00 /  
1.00

## Question 4

Let  $(E, D)$  be a symmetric encryption system with message space  $M$  (think of  $M$  as only consisting for short messages, say 32 bytes). Define the following MAC  $(S, V)$  for messages in  $M$ :

$$S(k, m) := E(k, m) \quad ; \quad V(k, m, t) := \begin{cases} 1 & \text{if } D(k, t) = m \\ 0 & \text{otherwise} \end{cases}$$

What is the property that the encryption system  $(E, D)$  needs to satisfy for this MAC system to be secure?

Your Answer	Score	Explanation
<input checked="" type="radio"/> ciphertext integrity	✓ 1.00	Indeed, ciphertext integrity prevents existential forgery under a chosen message attack.
<input type="radio"/> semantic security under a chosen plaintext attack		
<input type="radio"/> perfect secrecy		
<input type="radio"/> chosen ciphertext security		
Total	1.00 / 1.00	

## Question 5

In [lecture 8.1](#) we discussed how to derive session keys from a shared secret. The problem is what to do when the shared secret is non-uniform. In this question we show that using a PRF with a *non-uniform* key may result in non-uniform values. This shows that session keys cannot be derived by directly using a *non-uniform* secret as a key in a PRF. Instead, one has to use a key derivation function like HKDF.

Suppose  $k$  is a *non-uniform* secret key sampled from the key space  $\{0, 1\}^{256}$ . In particular,  $k$  is sampled uniformly from the set of all keys whose most significant 128 bits are all 0. In other words,

$k$  is chosen uniformly from a small subset of the key space. More precisely,

$$\text{for all } c \in \{0, 1\}^{256} : \quad \Pr[k = c] = \begin{cases} 1/2^{128} & \text{if } \text{MSB}_{128}(c) = 0^{128} \\ 0 & \text{otherwise} \end{cases}$$

Let  $F(k, x)$  be a secure PRF with input space  $\{0, 1\}^{256}$ . Which of the following is a secure PRF when the key  $k$  is uniform in the key space  $\{0, 1\}^{256}$ , but is insecure when the key is sampled from the *non-uniform* distribution described above?

Your Answer	Score	Explanation
<input checked="" type="radio"/> $F'(k, x) = \begin{cases} F(k, x) & \text{if } \text{MSB}_{128}(k) \neq 0^{128} \\ 0^{256} & \text{otherwise} \end{cases}$	<div>✓</div> 1.00	$F'(k, x)$ is a secure PRF because for a uniform key $k$ the probability that $\text{MSB}_{128}(k) = 0^{128}$ is negligible. However, for the *non-uniform* key $k$ this PRF always outputs 0 and is therefore completely insecure. This PRF cannot be used as a key derivation function for the distribution of keys described in the problem.
<input type="radio"/> $F'(k, x) = \begin{cases} F(k, x) & \text{if } \text{MSB}_{128}(k) = 0^{128} \\ 0^{256} & \text{otherwise} \end{cases}$		
<input type="radio"/> $F'(k, x) = \begin{cases} F(k, x) & \text{if } \text{MSB}_{128}(k) \neq 1^{128} \\ 1^{256} & \text{otherwise} \end{cases}$		
<input type="radio"/> $F'(k, x) = \begin{cases} F(k, x) & \text{if } \text{MSB}_{128}(k) \neq 1^{128} \\ 0^{256} & \text{otherwise} \end{cases}$		
Total	1.00 / 1.00	

## Question 6

In what settings is it acceptable to use *deterministic* authenticated encryption (DAE) like SIV?

Your Answer	Score	Explanation
<input type="radio"/> to encrypt many records in a database with a single key when the same record may repeat multiple times.		
<input type="radio"/> when a fixed message is repeatedly encrypted using a single key.		
<input checked="" type="radio"/> when the encryption key is used to encrypt only one message.	✓ 1.00	Deterministic encryption is safe to use when the message/key pair is never used more than once.
<input type="radio"/> to individually encrypt many packets in a voice conversation with a single key.		
Total	1.00 / 1.00	

## Question 7

Let  $E(k, x)$  be a secure block cipher. Consider the following tweakable block cipher:

$$E'((k_1, k_2), t, x) = E(k_1, x) \oplus E(k_2, t).$$

Is this tweakable block cipher secure?

Your Answer	Score	Explanation
<input type="radio"/> yes, it is secure assuming $E$ is a secure block cipher.		
<input checked="" type="radio"/> no because for $t \neq t'$ we have $E'((k_1, k_2), t, 0) \oplus E'((k_1, k_2), t, 1) = E'((k_1, k_2), t', 0) \oplus E'((k_1, k_2), t', 1)$	✓ 1.00	since this relation holds, an

attacker can make 4 queries to  $E'$  and distinguish  $E'$  from a random collection of one-to-one functions.

☐ no because for  $t \neq t'$  we have  
 $E'((k_1, k_2), t, 0) \oplus E'((k_1, k_2), t', 1) = E'((k_1, k_2), t', 1) \oplus E'((k_1, k_2), t, 0)$

☐ no because for  $x \neq x'$  and  $t \neq t'$  we have  
 $E'((k_1, k_2), t, x) \oplus E'((k_1, k_2), t', x) = E'((k_1, k_2), t, x') \oplus E'((k_1, k_2), t', x')$

☐ no because for  $x \neq x'$  we have  
 $E'((k_1, k_2), 0, x) \oplus E'((k_1, k_2), 0, x) = E'((k_1, k_2), 0, x') \oplus E'((k_1, k_2), 0, x')$

Total	1.00 /
	1.00

## Question 8

In [lecture 8.5](#) we discussed format preserving encryption which is a PRP on a domain  $\{0, \dots, s-1\}$  for some pre-specified value of  $s$ . Recall that the construction we presented worked in two steps, where the second step worked by iterating the PRP until the output fell into the set  $\{0, \dots, s-1\}$ .

Suppose we try to build a format preserving credit card encryption system from AES using \*only\* the second step. That is, we start with a PRP with domain  $\{0, 1\}^{128}$  from which we want to build a PRP with domain  $10^{16}$ . If we only used step (2), how many iterations of AES would be needed in expectation for each evaluation of the PRP with domain  $10^{16}$ ?

Your Answer

Score Explanation

☒  $2^{128}/10^{16} \approx 3.4 \times 10^{22}$  ✔ 1.00 On every iteration we have a probability of  $10^{16}/2^{128}$  of falling into the set  $\{0, \dots, 10^{16}\}$  and therefore in expectation we will need  $2^{128}/10^{16}$  iterations. This should explain why

step (1) is needed.

☐ 4

☐ 2

☐  $10^{16}$

Total	1.00 /
	1.00

## Question 9

Let  $(E, D)$  be a secure tweakable block cipher. Define the following MAC  $(S, V)$ :

$$S(k, m) := E(k, m, 0) \quad ; \quad V(k, m, \text{tag}) := \begin{cases} 1 & \text{if } E(k, m, 0) = \text{tag} \\ 0 & \text{otherwise} \end{cases}$$

In other words, the message  $m$  is used as the tweak and the plaintext given to  $E$  is always set to 0. Is this MAC secure?

Your Answer	Score	Explanation
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☐ no

☐ it depends on the tweakable block cipher.

<input checked="" type="radio"/> yes	✓ 1.00	A tweakable block cipher is indistinguishable from a collection of random permutations. The chosen message attack on the MAC gives the attacker the image of 0 under a number of the permutations in the family. But that tells the attacker nothing about the image of 0 under some other member of the family.
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Total	1.00 /
	1.00



## Question 10

In [Lecture 7.6](#) we discussed padding oracle attacks. These chosen-ciphertext attacks can break poor implementations of MAC-then-encrypt. Consider a system that implements MAC-then-encrypt where encryption is done using CBC with a random IV using AES as the block cipher. Suppose the system is vulnerable to a padding oracle attack. An attacker intercepts a 64-byte ciphertext  $c$  (the first 16 bytes of  $c$  are the IV and the remaining 48 bytes are the encrypted payload). How many chosen ciphertext queries would the attacker need *in the worst case* in order to decrypt the entire 48 byte payload? Recall that padding oracle attacks decrypt the payload one byte at a time.

Your Answer	Score	Explanation
<input checked="" type="radio"/> 12288	✓ 1.00	Correct. Padding oracle attacks decrypt the payload one byte at a time. For each byte the attacker needs no more than 256 guesses in the worst case. Since there are 48 bytes total, the number queries needed is $256 \times 48 = 12288$ .
<input type="radio"/> 48		
<input type="radio"/> 12240		
<input type="radio"/> 1024		
Total	1.00 / 1.00	