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①

① Big omega Notation: prove that $g(n)=n^3+2n^2+4n$ is $\Omega(n^3)$

$$g(n) \geq c \cdot n^3$$

$$g(n) = n^3+2n^2+4n$$

for finding constants c and n_0

$$n^3+2n^2+4n \geq c \cdot n^3$$

Divide both sides with n^3

$$1+\frac{2n^2}{n^3}+\frac{4n}{n^3} \geq c$$

$$1+\frac{2}{n}+\frac{4}{n^2} \geq c$$

Here $\frac{2}{n}$ and $\frac{4}{n^2}$ approaches 0

$$1+\frac{2}{n}+\frac{4}{n^2} \geq 1$$

Example:- $c = 1/2$

$$1+\frac{2}{n}+\frac{4}{n^2} \geq \frac{1}{2}$$

$$1+\frac{2}{n}+\frac{4}{n^2} \geq 1 \quad (1 \geq 1/2;\ n \geq 1)$$

$$1+\frac{2}{n}+\frac{4}{n^2} \geq 1/2 \quad (n \geq 1;\ n_0 = 1)$$

$\therefore$ $g(n) = n^3+2n^2+4n$ is indeeded $\Omega(n^3)$.

② Big Theta Notation: Determine whether $h(n) = 4n^2 + 3n$ is $\Theta(n^2)$ or not.

$$c_1 n^2 \le h(n) \le c_2 n^2$$

In upper bound $h(n)$ is $O(n^2)$

In lower bound $h(n)$ is $\Omega(n^2)$

Upper Bound $(O(n^2))$:-

$$h(n) = 4n^2 + 3n$$

$$h(n) \le c_2 n^2$$

$$4n^2 + 3n \le c_2 n^2$$

$$\Rightarrow 4n^2 + 3n \le 5n^2$$

let's $c_2 = 5$

Divide both sides by n^2

$$4 + \frac{3}{n} \le 5$$

$h(n) = 4n^2 + 3n$ is $O(n^2)$ $(c_2 = 5; n_0 = 1)$.

Lower bound:-

$$h(n) = 4n^2 + 3n$$

$$h(n) \ge c_1 n^2$$

$$4n^2 + 3n \ge c_1 n^2$$

let's $c_1 = 4 \Rightarrow 4n^2 + 3n \ge 4n^2$

Divide both sides by n^2

$$4 + 3/n \ge 4$$

$h(n) = 4n^2 + 3n$ $(c_1 = 4; n_0 = 1)$

$h(n) = 4n^2 + 3n$ is $\Theta(n^2)$.

③ Let $f(n) = n^3 - 2n^2 + n$ and $g(n) = n^2$ show whether $f(n) = \Omega(g(n))$ is true or false and justify your answer.

$$f(n) \geq c \cdot g(n)$$

Substuting $f(n)$ and $g(n)$ into this inequality we

$$n^3 - 2n^2 + n \geq c \cdot (-n^2)$$

find c and n_0 holds $n \geq n_0$

$$n^3 - 2n^2 + n \geq -cn^2$$

$$n^3 - 2n^2 + n + cn^2 \geq 0$$

$$n^3 + ((c-2)n^2 + n \geq 0)$$

$$n^3 + ((c-2))\, n^2 + n \geq 0 \quad (n^3 \geq 0)$$

$$n^3 + (1-2)n^2 + n = n^3 - n^2 + n \geq 0 \quad (c=2)$$

$f(n) = n^3 - 2n^2 + n$ is $\Omega(g(n))$ is True.

$\therefore$ The statement $f(n) = \Omega(g(n))$ is True.

④ Determine whether $h(n) = n\log n + n$ is $\Theta(n\log n)$ prove a rigorous proof for your conclusion

$$c_1 n\log n \leq h(n) \leq c_2 n\log n$$

upper Bound:-

$$h(n) \leq c_2 n\log n$$

$$h(n) = n\log n + n$$

$$n\log n + n \leq c_2 n\log n$$

Divide both sides by $n\log n$

$$1 + \frac{n}{n\log n} \leq 2$$

$$1 + \frac{1}{\log n} \leq c_2 \quad \text{(simplify)}$$

$$1 + \frac{1}{\log n} \leq 2 \quad (c_2 = 2)$$

Then $h(n)$ is $O(n\log n)$

Lower bound:-

$$h(n) \geq c_1 n\log n$$

$$h(n) \geq n\log n + n$$

$$n\log n + n \geq c_1 n\log n$$

Divide both sides by $n\log n$

$$1 + \frac{n}{n\log n} > c_1$$

$$1 + \frac{1}{\log n} \geq c_1 \quad \text{(simplify)}$$

$$1 + \frac{1}{\log n} \geq 1 \quad (c_1 = 1)$$

$$\frac{1}{\log n} \geq 0$$

$h(n)$ is $\Omega(n\log n)$ $(c_1 = 1;\ n_0 = 1)$

$\therefore h(n) = n\log n + n$ is $\Theta(n\log n)$.

5 Solve the following recurrence relations and find the order of growth of solutions

$$T(n) = 4T(n/2) + n2, T(1) = 1.$$

Given,

$$T(n) = 4T(n/2) + n^2; \ T(1) = 1$$

$$T(n) = aT(n/b) + f(n)$$

$$a = 4; b = 2; \ f(n) = n^2$$

Applying master theorem

$$T(n) = aT(n/b) + f(n)$$

$$f(n) = O(n^{\log_b^a - E})$$

$$f(n) = \theta(n^{\log_b a}), \text{ then } T(n) = \theta(n^{\log_b^a} \log n)$$

$$f(n) = \Omega(n^{\log_b^a + E}), \text{ then } T(n) = \theta(n^{\log_b^a} \log n) \text{ (or) } f(n)$$

calculating $\log_b a$:

$$\log_b^a = \log_2^4 = 2$$

$$f(n) = n^2 = \theta(n^2)$$

$$f(n) = \theta(n^2) = \theta(n^{\log_b a})$$

$$T(n) = 4T(n/2) + n^2$$

$$T(n) = \theta(n^{\log_b^a} \log n) = \theta(n^2 \log n)$$

Order of growth:-

$T(n) = 4T(n/2) + n^2$ with $T(1) = 1$ is

$$\theta(n^2 \log n)$$