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PAPER ID : 39906	Roll No.										

B. Tech. Examination 2021-22

(Even Semester)

DIFFERENTIAL EQUATIONS AND FOURIER ANALYSIS

Time : Three Hours] *[Maximum Marks : 60*

Note :– Attempt all questions.

SECTION–A

1. Attempt all parts of the following : $8\times1=8$

(a) Find the order and degree of the differential equation :

$$\left(\frac{d^2y}{dx^2}\right)^2-\left(\frac{dy}{dx}\right)^3-1=0$$

(b) Find the particular integral of the differential equation :

$$(D^2-1)\,y=1$$

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(c) Show that $x = 0$ is not an ordinary point of the differential equation :

$$3xy'' + 2y' + y = 0$$

(d) Find the value of :

$$\int_{-1}^{1} P_5^2(x)\,dx$$

(e) If $f(x) = 1$ is expanded in fourier sine series in $(0, x)$ then find the value of b_n.

(f) If the function $f(x)$ is expanded in fourier series in $(-c, c)$ then write the constant term.

(g) Form the partial differential equation from :

$$z = f(x^2 - y^2)$$

(h) Classify the partial differential equation :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, c > 0.$$

SECTION – B

2. Attempt any two parts of the following : $2 \times 6 = 12$

(a) Solve the simultaneous differential equations :

$$\frac{dx}{dt} + 5x - 2y = t$$

$$\frac{dy}{dt} + 2x + y = 0$$

(b) Find the power series solutionof the differential equation :

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0 \text{ about } x = 0$$

(c) Find the fourier series of the function $f(x) = \frac{1}{4}(\pi - x)^2$ in the interval $0 \leq x \leq 2\pi$.

Hence obtain the relation :

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots\dots\dots = \frac{\pi^2}{6}$$

(d) Solve completely the equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

representing the variations of a string of length 'ℓ', fixed at both ends, given that $y(0, t) = 0$, $y(\ell, t) = 0$, $y(x, 0) = f(x)$ and $\frac{\partial}{\partial t} y(x, 0) = 0$, $0 < x < \ell$.

SECTION – C

Note :- Attempt all questions. Attempt any two parts from each questions. 5×8=40

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3. (a) Solve the differential equation :

$$(D^2 + 4)\, y = \cos 2x$$

(b) Solve the differential equation :

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1 + e^x}$$

(c) Solve :

$$y'' - 4xy' + (4x^2 - 2)\, y = 0$$

given that $y = e^{x^2}$ is an integral included in the complementary fucntion.

4. (a) Prove that :

$$x J_n' = x J_{n-1} - n J_n$$

(b) Express $J_5(x)$ in terms of $J_1(x)$ and $J_2(x)$.

(c) Prove that :

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} \left(x^2 - 1\right)^n$$

5. (a) Find the fourier series of the function :

$$f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases}$$

(b) Expand $f(x) = x$ as a half range sine series in $0 < x < 2$.

(c) Obtain the half range cosine series for $f(x) = x^2$ in $0 < x < \pi$.

6. (a) Solve:

$$(D^2 - D'^2)\, z = x - y$$

(b) Solve:

$$(D + 1)(D + D' - 1)\, Z = \sin(x + 2y)$$

(c) Solve:

$$\frac{\partial u}{\partial x} = 3\frac{\partial u}{\partial t}$$

using method of separable of variables.

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