Data Driven Dynamics: Tackling Nonlinear Systems

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Content

- Preliminary Math: Linear Algebra
- Ideas in Linear Dynamical systems
- Dynamic Mode Decomposition and Embedding - Theory and hands-on implementation

Linear Algebra - Recap

- matrices are Linear Transformations
- eigenvectors - subspaces invariant to the transformation
- eigenvalues - scaling factor
- find eigenvalues by solving characteristic equation: $\det(A - \lambda I) = 0$, solve for eigenvectors subsequently
- Rank of a matrix: number of independent cols/rows.

Matrix SVD

- A: an m$\times$n matrix
- can be decomposed as, $A = U\Sigma V^\dagger$
- U, $V \equiv$ Unitary with columns as eigenvectors of $AA^\dagger$ and $A^\dagger A$
- $\Sigma \equiv$ Rectangular/Diagonal matrix with entries as 'singular' values

eg: A = $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

Essence of SVD

- Data we typically work with has huge dimensionality.
- SVD gives us a ”Low-Rank” approximation of our high dimensional matrix
- Especially useful when patterns are known to occur in data, eg: image processing
- Foundation for Principal Component Analysis

Tackling DEs

- Try to find fixed points, characterize stability
- Qualitatively sketch the solutions based on eigenvalues of the matrix (Phase Potraits)

Tackling DEs

- **BUT**
- What if I don't know the equations governing a system?
- enter - Measurements

State space reconstruction: Delay Embedding

- Measurements: provide a single collapsed value arising from complex dynamics

  Is it possible to draw inferences about system's dimensional behaviour from repeated measurements?

State space reconstruction: Delay Embedding

- While embedding techniques can work fairly well, they pose some constraints
- dimension of reconstruction space must be higher than state-space of the object

  but we don't have a great estimate of the state space dimension typically

Linear System

- A system given by Linear DEs
- main feature: fully solvable
- it is thereby in our interest to try and map problems of Nonlinear nature to linear ones.

Dynamic Mode Decomposition

DMD

- Fails to work well with noisy data
- Better, more robust alternatives have been suggested
- However, the core idea remains the same