Name: Krishna Chaitanya Sripada

Ans 1

The generated samples are:

```
1 [[-0.97977299]
 [ 0.00842229]
 [-0.81230254]
 ...,
 [-0.02064795]
 [-0.514107 ]
 [ 0.5813556 ]]
2 [[ 0.125146  -0.40207117]
 [-0.82553458 0.08322131]
 [-0.93928138 0.08880301]
 ...,
 [ 0.26225264 -0.85643974]
 [-0.0297685 -0.66172214]
 [ 0.2369696  0.73281328]]
3 [[ 0.1545676 -0.88476312 0.37538976]
 [-0.076204   0.77983112 -0.24302431]
 [-0.79693234 -0.19104474 0.08678971]
 ...,
 [ 0.46406434 -0.70798911 -0.42797605]
 [-0.0982297 -0.17806004 0.92219372]
 [-0.12246412 0.52546831 0.12582817]]
4 [[-0.10995823 -0.09060495 0.18391883 -0.25517966]
 [-0.44176387 0.00365563 -0.00300895 -0.66249913]
 [ 0.31395478 -0.28075808 0.07659535 0.57308922]
 ...,
 [-0.40635851 -0.69734769 0.05903802 -0.39497366]
 [-0.61739903 -0.44575521 0.10770923 0.06502449]
 [ 0.35055945 0.09720481 0.75759123 -0.47176295]]
```

I've just displayed the first 4 samples to show the generation of 10000 samples using the algorithm 1. Note that this is not the complete set of samples generated (as seen by the ellipsis in the output).

Ans 2

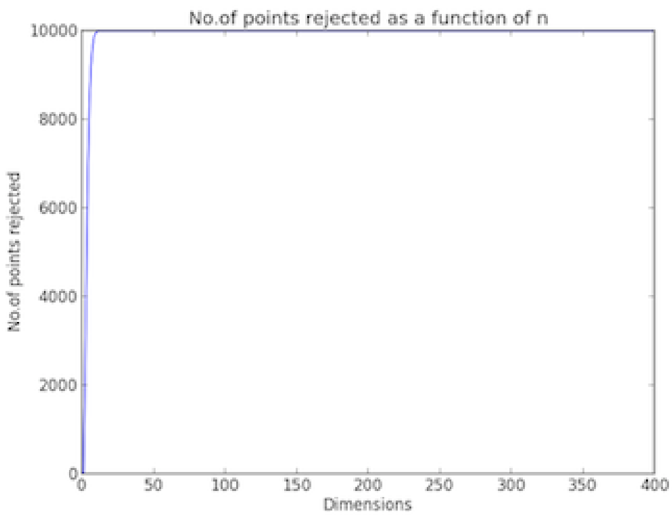

Figure 1 :No.of points rejected as a function of 'n'

As we can see from the above figure, for dimensions 1 to 13, the points rejected are 0, 2106, 4734, 6924, 8386, 9202, 9615, 9836, 9933, 9975, 9993, 9999, 9998 and from dimensions 14 to 400, we see that all the points are rejected i.e., all points are inside the cube $[-1, 1]^n$ but not inside the ball $B^n(1)$.

Ans 3

The generated samples are:

```
2 [[[-0.65692011 1.16783918]]

 [[ 0.82797408 -1.39857805]]

 [[ 0.1461279  0.13553733]]

 ...,
 [[-0.04946436 1.12446543]]

 [[ 0.91540538 -0.21908985]]

 [[ 0.81293634 -2.28027548]]]
5 [[[-1.2456768 1.80039919 1.17679543 -0.69789122 -0.76721011]]

 [[-2.08692172 0.51378863 2.156899  0.4165659 -0.4891643 ]]

 [[ 0.00469149 1.79521765 0.23825432 -0.1498503 -0.46991127]]

 ...,
 [[-0.72383131 -1.01382281 -0.30672538 0.68511039 0.17310003]]

 [[-0.27179642 -0.82979279 0.16814383 0.76172546 -0.53268273]]

 [[-1.19568478 -1.71578359 -0.35880254 0.32901967 0.41580998]]]
10 [[[-2.11815605 -0.35731693 1.6130568 ..., 1.25442419 1.56232948
    0.35447931]]

 [[-0.13652401 1.37226018 -0.76691262 ..., -0.95764159 0.09348221
   -1.41324988]]

 [[-1.41626515 0.42959637 -0.72194693 ..., 0.16525344 -1.43592195
   -0.41799285]]

 ...,
 [[-0.97485073 0.44934148 -0.38081946 ..., -0.07637263 1.10600349
    0.61022057]]

 [[ 0.24000133 -0.71633007 0.41135705 ..., -1.20362708 -0.6297904
    0.05049542]]

 [[-2.56336523 1.33890549 0.52369033 ..., -0.73657761 1.36940211
    1.38825666]]]
15 [[[-0.80481799 1.81004443 -0.02678096 ..., -0.3047872 0.57138172
   -0.23113882]]

 [[-0.13910619 1.08208882 -0.72344492 ..., 0.56388572 -1.4720767
    0.2459732 ]]

 [[-1.40338075 -0.48069908 0.01378676 ..., -0.44783218 -2.6447068
   -0.01177376]]

 ...,
 [[-1.62982492 -0.22021754 0.60822971 ..., -0.0808899 1.64250104
    0.78917796]]

 [[-0.25659009 -0.1753043 -1.20571977 ..., -1.35860634 1.46238483
   -0.31844546]]

 [[-1.0764558  0.27053731 0.16825497 ..., -0.78778908 0.64355262
   -1.47627777]]]
20 [[[ 0.29053213 -0.98661741 -1.12983281 ..., -1.02831458 0.31240548
   -0.21604852]]
```

```
[[-0.03431936 -0.2834155 -0.19163956 ..., -0.91225312 -0.19950807
  -0.58912561]]

[[-1.4425197  0.13178094 0.95207496 ..., -0.64722797 1.27522824
   1.29144633]]

...,
[[-1.42532757 -0.20938423 -0.67902917 ..., -0.48856252 -1.38059591
  -0.00405272]]

[[-0.91422073 -0.33843193 1.19571491 ..., -0.48680666 -0.64461441
   0.87500838]]

[[ 0.49749863 0.82310547 -0.95117333 ..., 0.09735673 0.79174279
  -1.3268248 ]]]
```

I've used a numpy array and because of that, not all the samples are displaced in the output. (as seen by the ellipsis in the output).

Ans 4

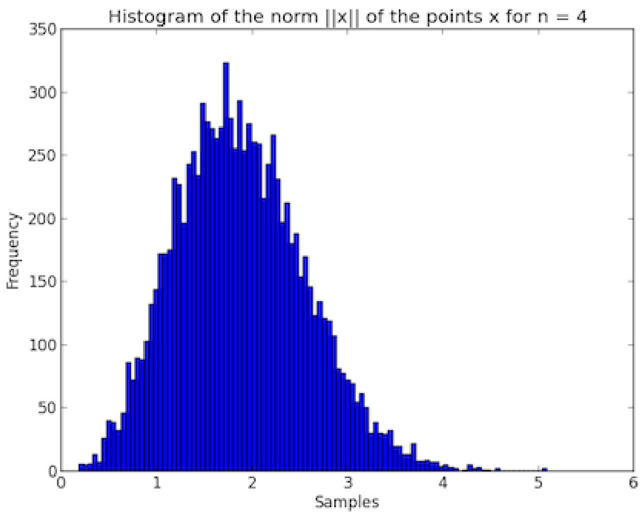

Figure 2 :Histogram for n =4

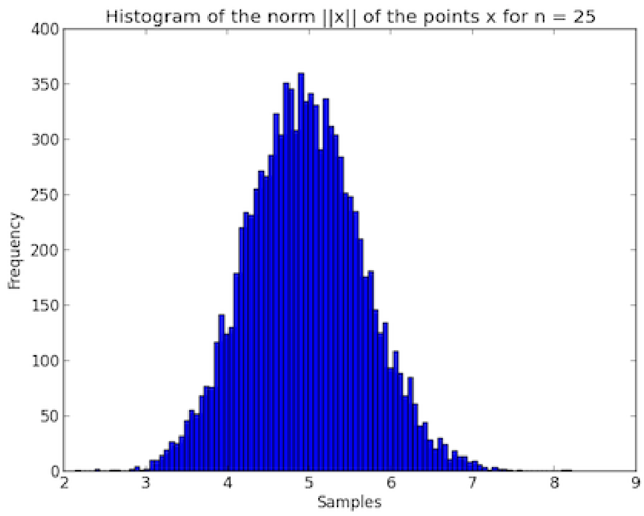

Figure 3 :Histogram for n =25

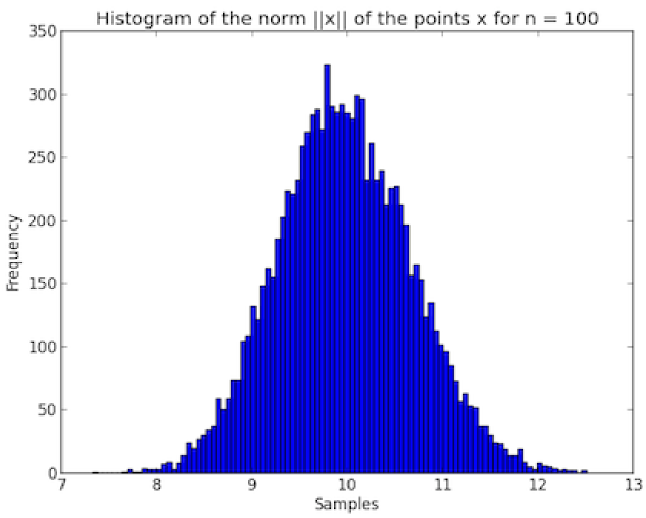

Figure 4 :Histogram for n =100

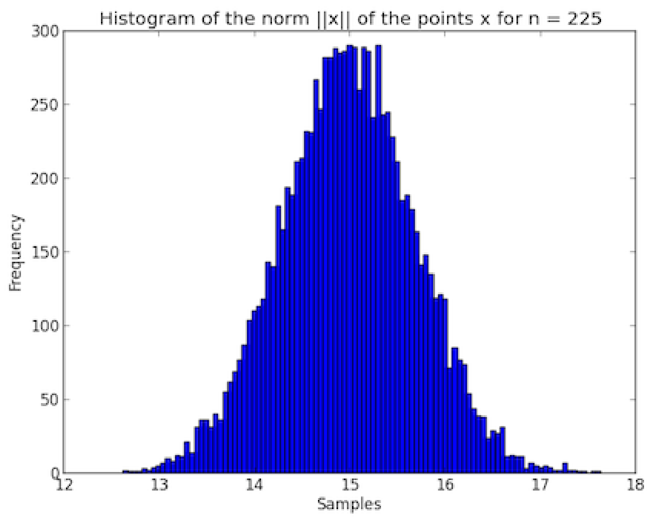

Figure 5 :Histogram for n =225

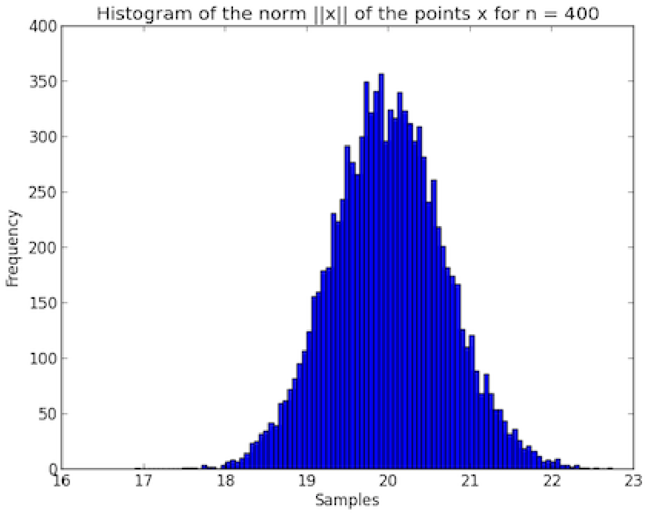

Figure 6 :Histogram for n =400

Ans 5

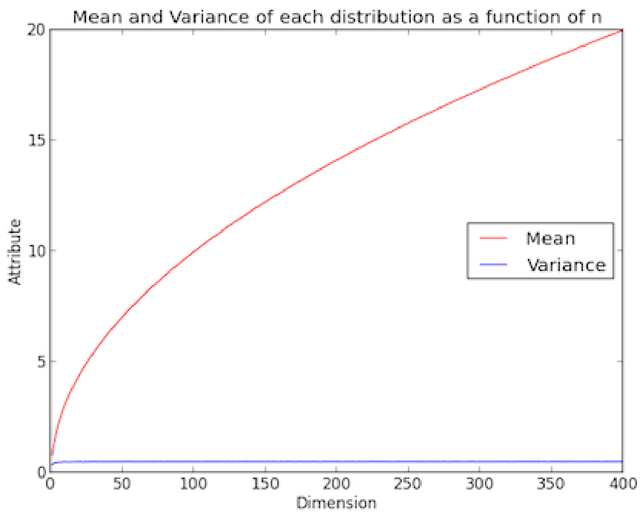

Figure 7 :Mean and Variance as a function of 'n'

We notice that the variance values don't change as the dimension increases. The mean varies as $\sqrt{n}$ as the dimension increases.

Ans 6

Given that,
$\gamma(A) = \frac{1}{(2\pi)^{\frac{n}{2}}} \int_A e^{-\frac{\|x\|^2}{2}} dx$
Therefore,
$\gamma(x \in \mathbb{R}^n : f(x) \leq a) = \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{x:f(x)\leq a} e^{-\frac{\|x\|^2}{2}} dx$
Expanding this, we get,
$\gamma(x \in \mathbb{R}^n : f(x) \leq a) \leq \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{x:f(x)\leq a} e^{-\frac{\sum_{i=1}^n x_i^2}{2}} dx_1 \dots dx_n$
Also given that,
$d\gamma(x) = \frac{1}{(2\pi)^{\frac{n}{2}}} e^{-\frac{\sum_{i=1}^n x_i^2}{2}} dx_1 \dots dx_n$
Using the above equation, we get,
$\gamma(x \in \mathbb{R}^n : f(x) \leq a) \leq \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{x:f(x)\leq a} (2\pi)^{\frac{n}{2}} d\gamma(x)$

$\gamma(x \in \mathbb{R}^n : f(x) \leq a) \leq \int_{x:f(x)\leq a} d\gamma(x)$
Multiplying $e^{-\lambda a}$, on both sides, we get,

$e^{-\lambda a}\gamma(x \in \mathbb{R}^n : f(x) \leq a) \leq e^{-\lambda a} \int_{x:f(x)\leq a} d\gamma(x)$

$e^{-\lambda a}\gamma(x \in \mathbb{R}^n : f(x) \leq a) \leq \int_{x:f(x)\leq a} e^{-\lambda f(x)} d\gamma(x)$

$\gamma(x \in \mathbb{R}^n : f(x) \leq a) \leq e^{\lambda a} \int_{x:f(x)\leq a} e^{-\lambda f(x)} d\gamma(x)$

Since $f : \mathbb{R}^n \to \mathbb{R}$,

$\gamma\{x \in \mathbb{R}^n : f(x) \leq a\} \leq e^{\lambda a} \int_{\mathbb{R}^n}^{\mathbb{R}} e^{-\lambda f(x)} d\gamma(x)$

Ans 7

Replacing $f(x) = \frac{\|x\|^2}{2}$ and using $a = \frac{(n-\delta)}{2}$, we get,

$\gamma(x \in \mathbb{R}^n : \frac{\|x\|^2}{2} \leq \frac{(n-\delta)}{2}) \leq e^{\frac{\lambda(n-\delta)}{2}} \int_{\mathbb{R}^n}^{\mathbb{R}} e^{\frac{-\lambda\|x\|^2}{2}} d\gamma(x)$

From the equations given in the question, we get,

$d\gamma(x) = \frac{1}{(2\pi)^{\frac{n}{2}}} e^{-\frac{\|x\|^2}{2}} dx$

Upon replacement, we get,

$\gamma(x \in \mathbb{R}^n : \frac{\|x\|^2}{2} \leq \frac{(n-\delta)}{2}) \leq e^{\frac{\lambda(n-\delta)}{2}} \int_{\mathbb{R}^n}^{\mathbb{R}} e^{\frac{-\lambda\|x\|^2}{2}} \frac{1}{(2\pi)^{\frac{n}{2}}} e^{-\frac{\|x\|^2}{2}} dx$

This simplifies to,

$\gamma\{x \in \mathbb{R}^n : \| x \|^2 \leq (n-\delta)\} \leq e^{\frac{\lambda(n-\delta)}{2}} \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{\mathbb{R}^n}^{\mathbb{R}} e^{\frac{-(\lambda+1)\|x\|^2}{2}} dx$

Ans 8

Given that $y = x\sqrt{1+\lambda}$ and

$\frac{1}{(2\pi)^{\frac{n}{2}}} \int_{\mathbb{R}^n} e^{\frac{-(\lambda+1)\|x\|^2}{2}} dx = \frac{1}{(1+\lambda)^{\frac{n}{2}}}$

If $\lambda = \frac{\delta}{(n-\delta)}$, we get,

$\gamma\{x \in \mathbb{R}^n : \| x \|^2 \leq (n-\delta)\} \leq e^{\frac{\frac{\delta}{(n-\delta)}(n-\delta)}{2}} \frac{1}{(1+\lambda)^{\frac{n}{2}}}$

$\gamma\{x \in \mathbb{R}^n : \| x \|^2 \leq (n-\delta)\} \leq e^{\frac{\delta}{2}} \frac{1}{(1+\frac{\delta}{(n-\delta)})^{\frac{n}{2}}}$

Upon simplification, we get,

$\gamma\{x \in \mathbb{R}^n : \| x \|^2 \leq (n-\delta)\} \leq e^{\frac{\delta}{2}} (\frac{n-\delta}{n})^{\frac{n}{2}}$

Ans 9

If $\epsilon = \frac{\delta}{n}$ and since $\ln(1-x) + x \leq \frac{-x^2}{2}$, we get,

$\gamma\{x \in \mathbb{R}^n : \| x \|^2 \leq (n-n\epsilon)\} \leq e^{\frac{n\epsilon}{2}} (\frac{n-n\epsilon}{n})^{\frac{n}{2}}$

$\gamma\{x \in \mathbb{R}^n : \| x \|^2 \leq n(1-\epsilon)\} \leq e^{\frac{n\epsilon}{2}} (1-\epsilon)^{\frac{n}{2}}$

Taking ln on both sides, we get,

$\ln(\gamma\{x \in \mathbb{R}^n : \| x \|^2 \leq n(1-\epsilon)\}) \leq \frac{n\epsilon}{2} + \frac{n}{2}\ln(1-\epsilon)$

$\ln(\gamma\{x \in \mathbb{R}^n : \| x \|^2 \leq n(1-\epsilon)\}) \leq \frac{n}{2}(\ln(1-\epsilon) + \epsilon)$

Upon simplification, we get,

$\ln(\gamma\{x \in \mathbb{R}^n : \| x \|^2 \leq n(1-\epsilon)\}) \leq -\frac{n}{2}(\frac{\epsilon^2}{2})$

$\gamma\{x \in \mathbb{R}^n : \| x \|^2 \leq n(1-\epsilon)\} \leq e^{-\frac{n\epsilon^2}{4}}$

Ans 10

Given,

$\gamma\{x \in \mathbb{R}^n : \parallel x \parallel^2 \leq n(1-\epsilon)\} \leq e^{-\frac{n\epsilon^2}{4}}$

and,

$\gamma\{x \in \mathbb{R}^n : \parallel x \parallel^2 \geq n(1+\epsilon)\} \leq e^{-\frac{n\epsilon^2}{8}}$

Combining the above two equations and taking ln on both sides, we get,

$\ln(\gamma\{x \in \mathbb{R}^n : \parallel x \parallel^2 \leq n(1-\epsilon)\}) \leq -\frac{n\epsilon^2}{4}$

$\ln(\gamma\{x \in \mathbb{R}^n : \parallel x \parallel^2 \geq n(1+\epsilon)\}) \leq -\frac{n\epsilon^2}{8}$

Subtracting above two equations, we get,

$\ln(\gamma\{x \in \mathbb{R}^n : \parallel x \parallel^2 \leq n(1-\epsilon)\}) - \ln(\gamma\{x \in \mathbb{R}^n : \parallel x \parallel^2 \geq n(1+\epsilon)\}) \leq -\frac{n\epsilon^2}{8}$

$\ln(\frac{\gamma\{x \in \mathbb{R}^n : \|x\|^2 \leq n(1-\epsilon)\}}{\gamma\{x \in \mathbb{R}^n : \|x\|^2 \geq n(1+\epsilon)\}}) \leq -\frac{n\epsilon^2}{8}$

$\frac{\gamma\{x \in \mathbb{R}^n : \|x\|^2 \leq n(1-\epsilon)\}}{\gamma\{x \in \mathbb{R}^n : \|x\|^2 \geq n(1+\epsilon)\}} \leq e^{-\frac{n\epsilon^2}{8}}$

$\frac{\gamma\{x \in \mathbb{R}^n : \|x\|^2 - n + n\epsilon \leq 0\}}{\gamma\{x \in \mathbb{R}^n : n + n\epsilon - \|x\|^2 \leq 0\}} \leq e^{-\frac{n\epsilon^2}{8}}$

Applying dividendo, we get,

$\gamma\{x \in \mathbb{R}^n : \frac{2(\|x\|^2 - n)}{2n\epsilon} \leq 0\} \leq \frac{e^{-\frac{n\epsilon^2}{8}} - 1}{e^{-\frac{n\epsilon^2}{8}} + 1} \leq e^{-\frac{n\epsilon^2}{8}} \leq 2e^{-\frac{n\epsilon^2}{8}}$

$\gamma\{x \in \mathbb{R}^n : \left| \frac{\|x\|^2}{n} - 1 \right| \geq \epsilon\} \leq 2e^{-\frac{n\epsilon^2}{8}}$

Ans 11

We can be that from Eq(11) and Eq(12), we get,

$\gamma\{x \in \mathbb{R}^n : \parallel x \parallel \leq \sqrt{n(1-\epsilon)}\} \leq e^{-\frac{n\epsilon^2}{4}}$

$\gamma\{x \in \mathbb{R}^n : \parallel x \parallel \geq \sqrt{n(1+\epsilon)}\} \leq e^{-\frac{n\epsilon^2}{8}}$

Since the concentration of measure is a little lesser than $\sqrt{n}$ as $\parallel x \parallel \leq \sqrt{n(1-\epsilon)}$ and little higher than $\sqrt{n}$ as $\parallel x \parallel \geq \sqrt{n(1+\epsilon)}$, we can say that the Gaussian measure is concentrated on the sphere of radius $\sqrt{n}$ with a decay of $e^{\frac{-\epsilon^2}{8}}$.

Ans 12

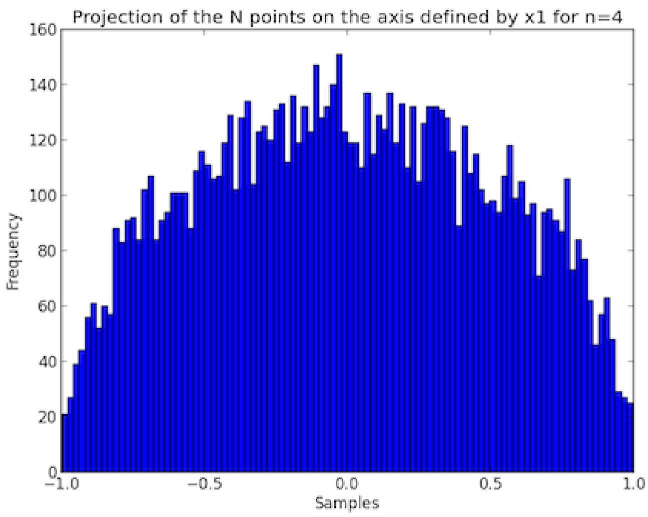

Figure 8 :Histogram for n = 4

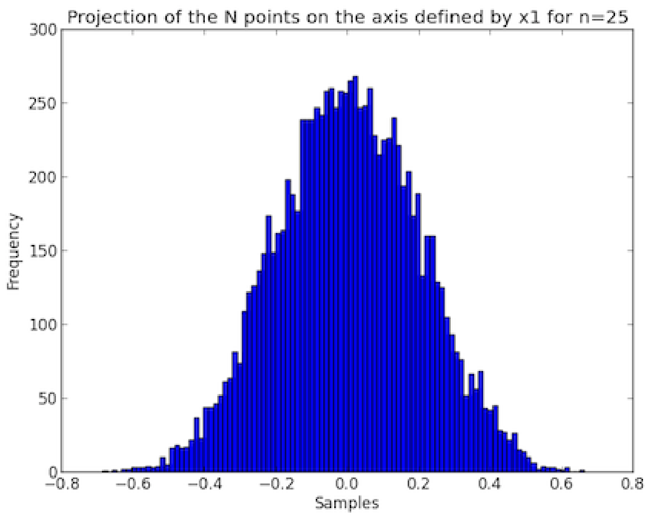

Figure 9 :Histogram for n =25

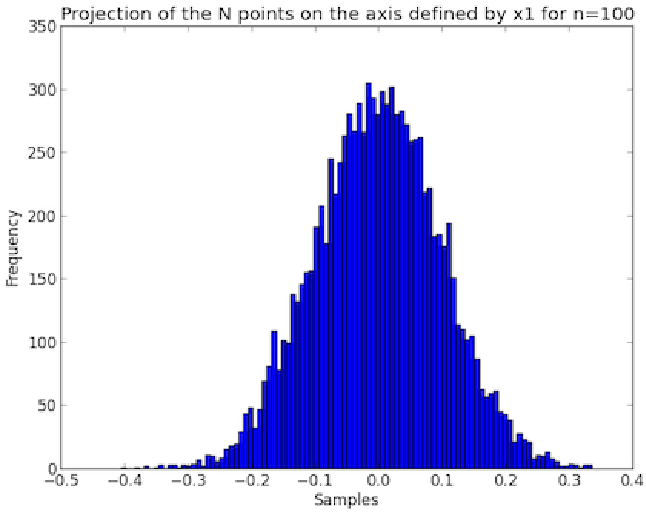

Figure 10 :Histogram for n = 100

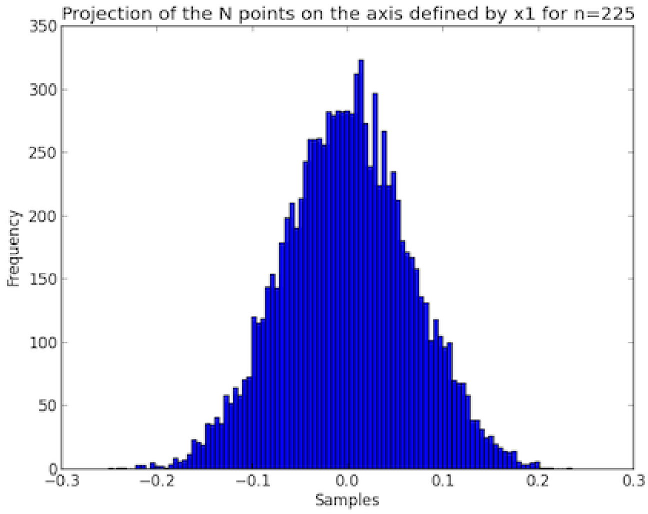

Figure 11 :Histogram for n =225

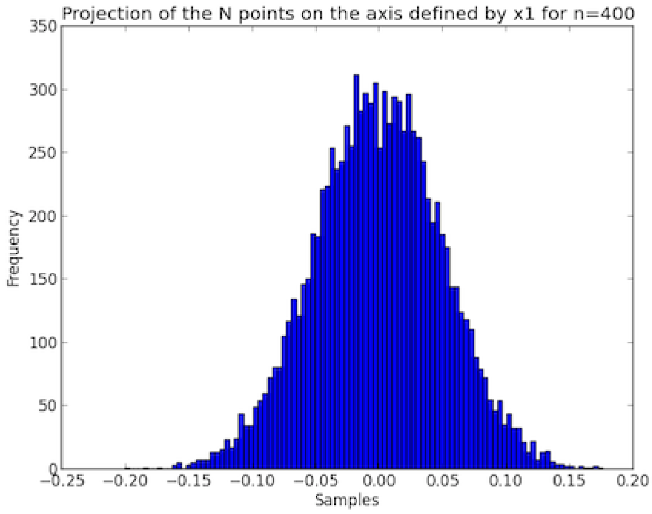

Figure 12 :Histogram for n = 400

Ans 13

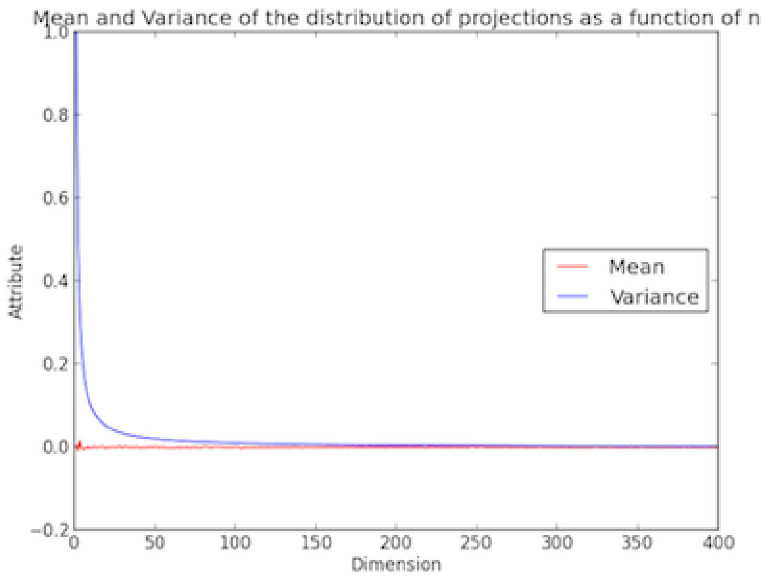

Figure 13 :Mean and Variance of the distribution of the projections

Ans 14

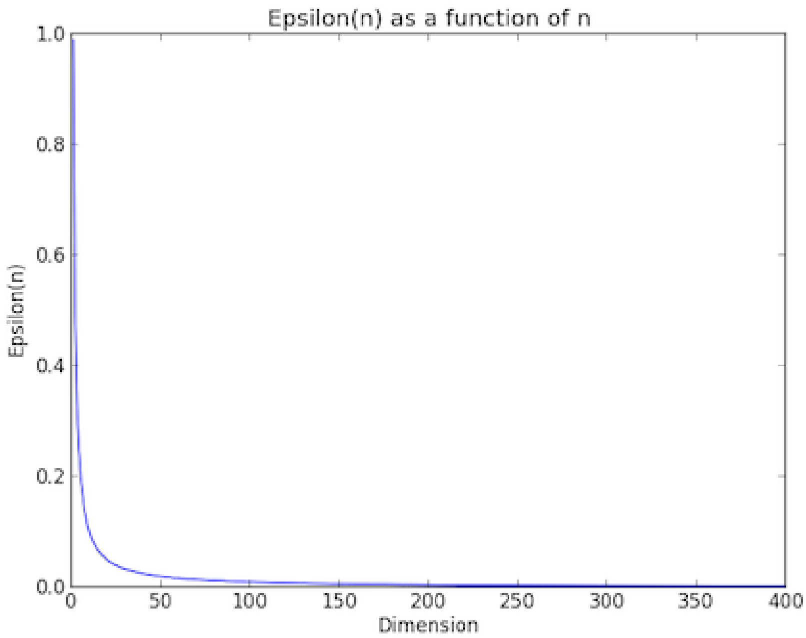

Figure 14 :Plot for $\epsilon(n)$ as a function of n

This curve is similar to the exponential decay curve where we can see that as the dimension increases, the value of $\epsilon(n)$ reaches close to 0.

Ans 15

The choice of axis (x_1 versus any other axis) is not important as the results of projecting the points onto the axis remains the same regardless of the axes chosen. Since the points are randomly chosen, they follow the same distribution and thus we notice the symmetry in the points.

Ans 16

```
4 [[ 0.63028936 -1.097736 0.93835393 1.2317479 ]
 [-1.20713363 1.27085457 -0.43553443 -0.85910816]
 [ 1.68404095 -0.30129244 0.37521483 -0.96563077]
 ...,
 [-0.18374822 -0.61523148 1.8802388 -0.22897355]
 [-0.9860119 -0.75993342 1.2928199 0.88255223]
 [-0.04117349 -0.44859436 -1.35485332 -1.40051431]]
25 [[ 0.68799096 -0.26786441 -0.68556769 ..., -0.07891129 2.22662855
  -0.66184881]
 [ 0.36996842 -1.05560762 -2.35133188 ..., 0.62801174 0.57403549
   1.16396758]
 [-1.0262364  1.72229894 -2.09451358 ..., -0.7085281 -0.6868293
  -0.28082034]
 ...,
 [-0.61785262 0.76006305 0.48852513 ..., -1.10566109 -0.72852901
  -1.1394798 ]
 [-1.21689115 1.69291832 1.49669937 ..., -0.58510604 -1.11935484
  -1.0218551 ]
 [ 0.46226122 1.32115449 1.36879548 ..., 1.67600523 0.62818761
   0.66409739]]
100 [[-1.41846347 -0.11382888 3.141689 ..., -0.55453294 -0.82674094
  -0.05748162]
```

```
 [-0.30089584 0.15039451 -1.38820549 ..., 0.898891 0.15666097
   2.06694431]
 [-0.85606152 -1.24574924 0.85788595 ..., 2.06564677 -2.22904304
   2.13899737]
 ...,
 [ 1.16194303 0.30843398 -0.06935013 ..., -1.06148569 -0.32450694
  -0.19570746]
 [ 0.47265695 -1.5127091 -0.48501618 ..., -1.12642233 0.11579328
   1.50321864]
 [-0.43713988 -0.20868236 0.36851884 ..., -0.05434654 0.05375461
  -0.15957454]]
225 [[ 0.07051839 -1.05894358 -1.61087903 ..., -0.9888515 1.87658628
   1.5854141 ]
 [ 1.27572116 0.7610539 -1.74757944 ..., 0.01515552 -0.73937378
  -0.03770229]
 [-2.21061224 0.45518723 -0.70741455 ..., 1.66380623 0.78883653
  -0.08602367]
 ...,
 [ 1.82828175 1.59886912 0.12101186 ..., 0.67458581 0.20624187
   0.75808681]
 [ 0.90236545 -0.39523239 -2.28663612 ..., 0.54438564 2.0427492
  -0.59887064]
 [ 0.33834693 -0.47356384 0.41578749 ..., -0.33740048 -0.93787427
   0.24766759]]
400 [[-1.56539675 -0.16155796 1.5748229 ..., 0.15596077 1.85347936
  -0.02557695]
 [ 0.40660968 2.04050084 -0.90966779 ..., 1.55023846 -0.98069676
   0.21059077]
 [-0.54338601 2.2325388 -2.04107933 ..., 0.39092473 -0.15738214
  -0.05136597]
 ...,
 [ 1.45327995 0.02588449 0.70836157 ..., 0.63640216 0.06611108
   0.18656551]
 [ 0.01239606 -1.71669257 0.60640558 ..., 0.58437235 0.04035378
  -0.62907416]
 [-0.50045222 0.40013465 -0.64167211 ..., 0.31144153 -2.17281477
  -1.0930534 ]]
```

I've used a numpy array and because of that, not all the samples are displaced in the output. (as seen by the ellipsis in the output).

Ans 17

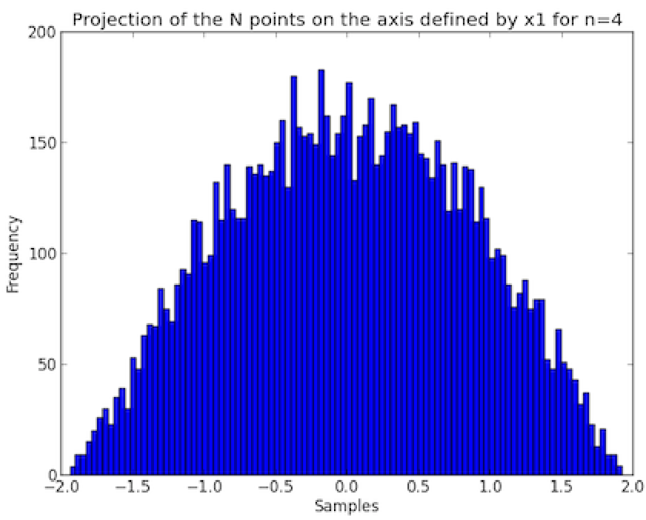

Figure 15 :Histogram for n = 4

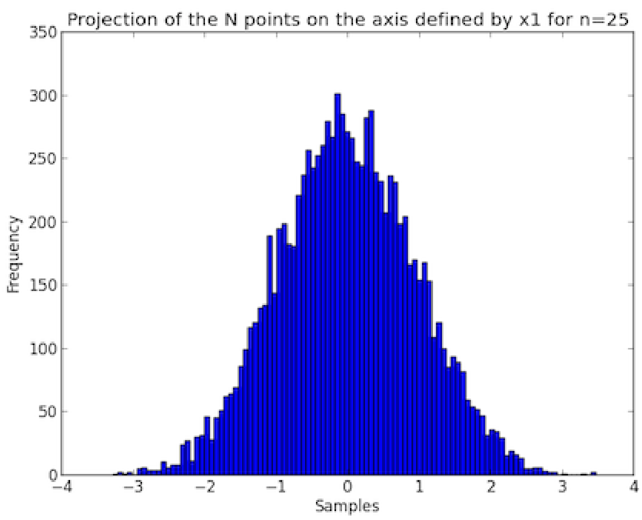

Figure 16 :Histogram for n =25

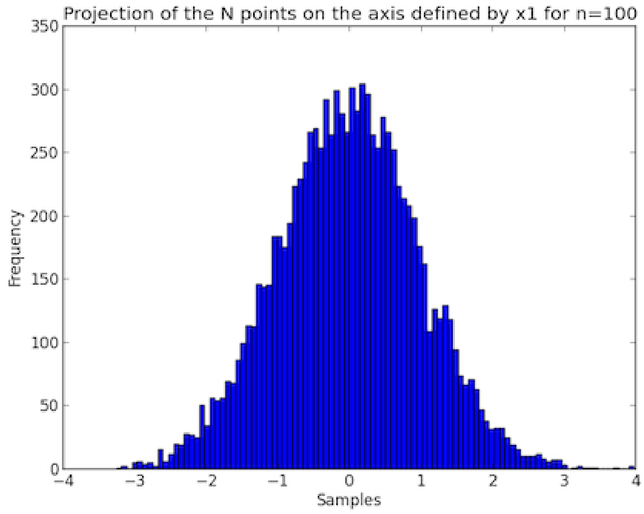

Figure 17 :Histogram for n = 100

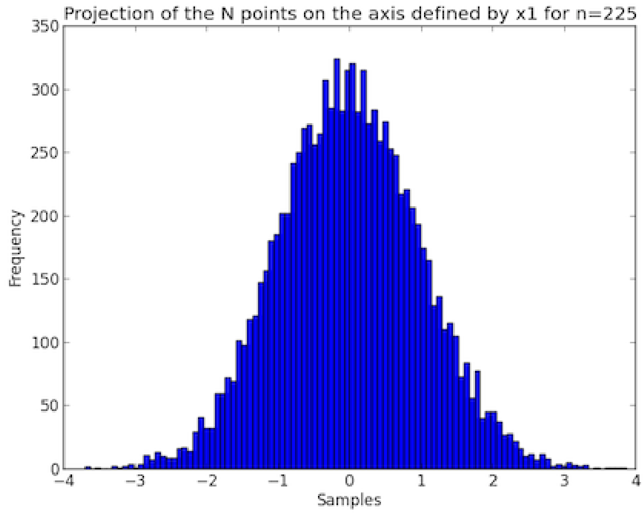

Figure 18 :Histogram for n =225

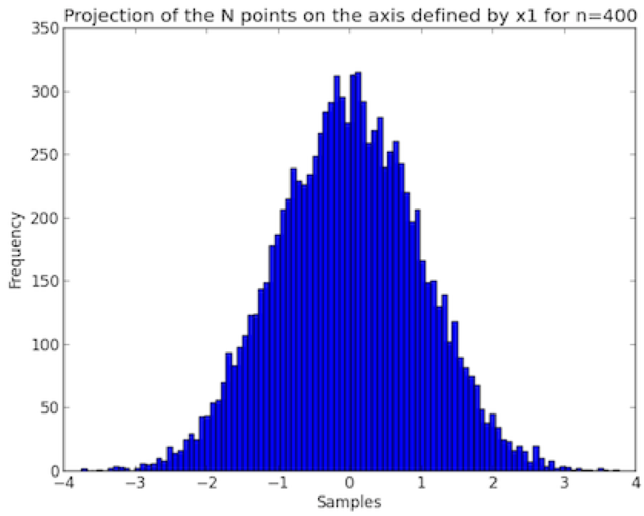

Figure 19 :Histogram for n = 400

Ans 18

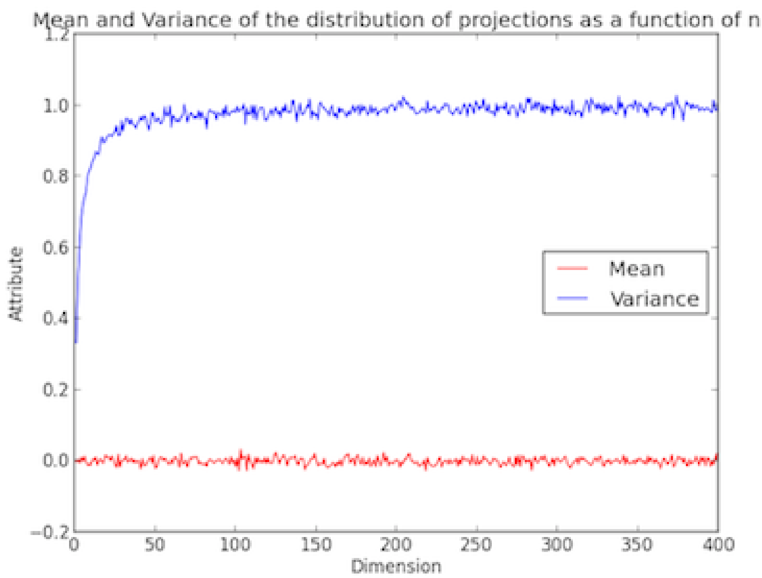

Figure 20 :Mean and Variance of the distribution of the projections

We observe that the mean is always close to 0 regardless of the dimension and the variance reaches 1 after a certain dimension and then remains close to 1 as the dimension keeps increasing.

Ans 19

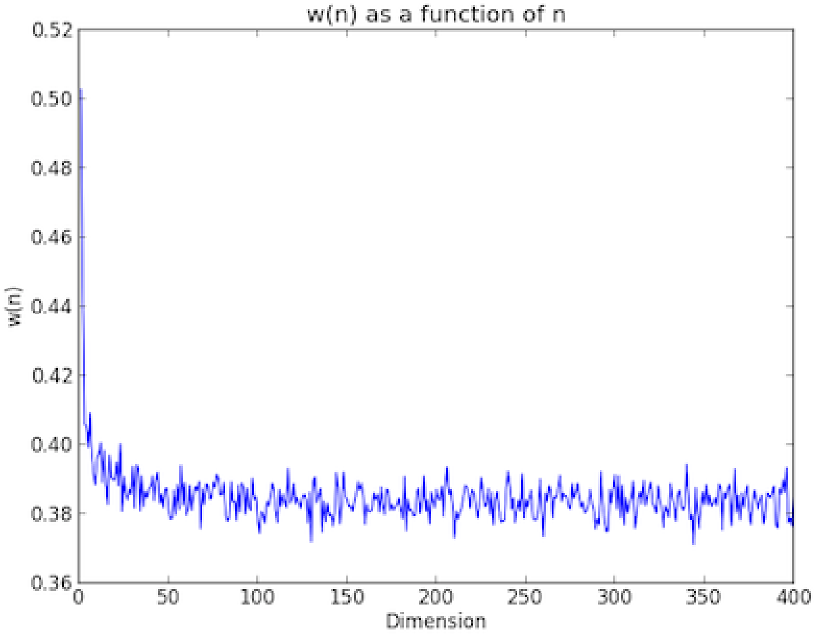

Figure 21 :Plot w(n) as a function of n

We observe an exponential decay curve where the relative volume is high as the dimension is low and once the dimension increases, we see that the volume falls down to a certain value and remains more or less around the same value for higher dimensions.

Ans 20

No, the choice of axis (x_1 versus any other axis) is not important for the results in the previous answer. Since the points are randomly chosen, they follow the same distribution and thus we notice the symmetry in the points.

Ans 21

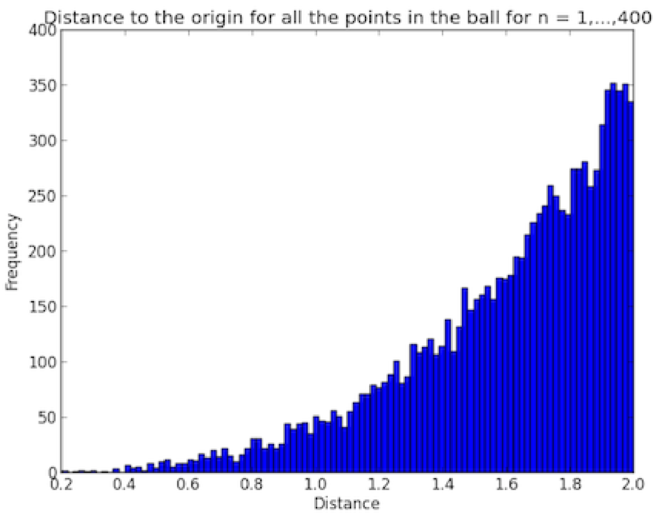

Figure 22 :Histogram for n = 4

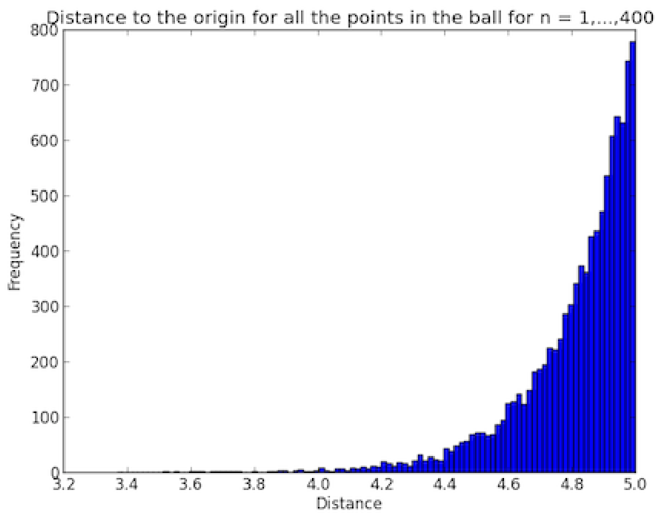

Figure 23 :Histogram for n =25

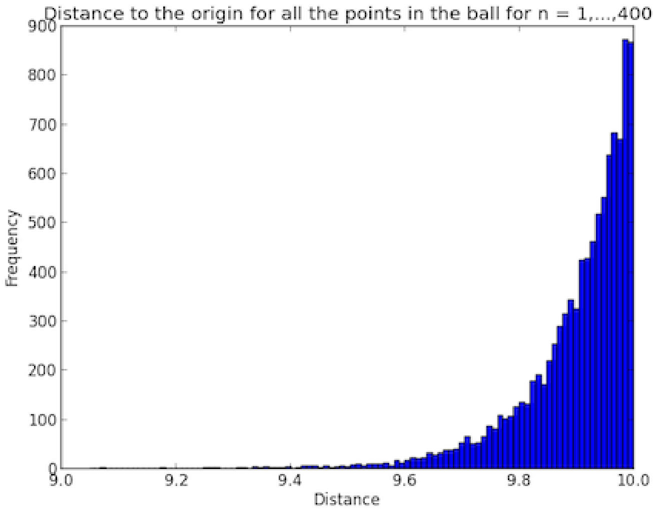

Figure 24 :Histogram for n = 100

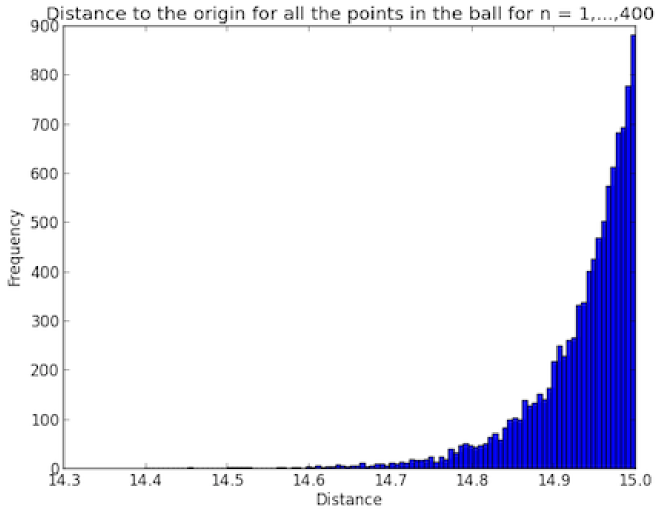

Figure 25 :Histogram for n =225

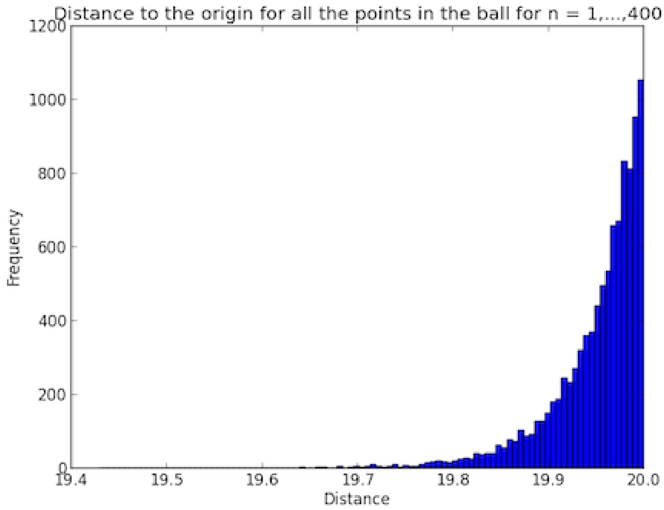

Figure 26 :Histogram for n = 400

The apparent paradox is that we see that the concentration points reaches its maximum at the radius value = $\sqrt{n}$ unlike the concentration measure which also has the highest concentration at the radius and follows the Gaussian distribution, this doesn't.

Ans 22

```
def generateWignerMatrix(matrix_size):
    matrix = np.zeros((matrix_size, matrix_size)) #Form a symmetric matrix
    newSize = (matrix_size*(matrix_size+1))/2
    bern = bernoulli.rvs(0.5, size=newSize) #Get the random bernoulli variates
    for i in range(0, len(bern)):
        if bern[i]==0:
            bern[i]=-1

    i = 0
    upperIndex = np.triu_indices(matrix_size)
    for x,y in zip(upperIndex[0], upperIndex[1]):
        matrix[x][y] = bern[i]
        i+=1

    upperIndex = np.triu_indices_from(matrix, k=1)
    lowerIndex = np.tril_indices_from(matrix, k=-1)
    matrix[lowerIndex] = matrix[upperIndex] #To make it symmetric
```

```
    return matrix

def gaussianOrthogonalEnsemble(matrix_size):
    matrix = np.zeros((matrix_size, matrix_size)) #Form a symmetric matrix
    newSize = (matrix_size* (matrix_size-1))/2
    upperValues = np.random.normal(0,1, newSize)
    diagonalValues = np.random.normal(0,2, matrix_size)

    i=0
    upperIndex = np.triu_indices_from(matrix, k=1)
    for x,y in zip(upperIndex[0], upperIndex[1]):
        matrix[x][y] = upperValues[i]
        i+=1

    i=0
    diagIndex = np.diag_indices(matrix_size)
    for x,y in zip(diagIndex[0], diagIndex[1]):
        matrix[x][y] = diagonalValues[i]
        i+=1

    upperIndex = np.triu_indices_from(matrix, k=1)
    lowerIndex = np.tril_indices_from(matrix, k=-1)
    matrix[lowerIndex] = matrix[upperIndex] #To make it symmetric
    return matrix
```

Result:

```
Wigner Matrix for Bernoulli
[[-1. -1. -1.]
 [-1. -1. -1.]
 [-1. -1. 1.]]
[[-1. -1. 1.  1.]
 [-1. -1. -1. -1.]
 [ 1.  1.  1.  1.]
 [-1. -1. 1.  1.]]
[[ 1.  1.  1. -1.  1.]
 [ 1.  1. -1. 1.  1.]
 [ 1. -1. -1. 1.  1.]
 [ 1. -1. 1. -1. -1.]
 [ 1.  1.  1. -1. -1.]]
[[ 1. -1. 1.  1. -1. -1.]
 [-1. -1. -1. 1. -1. 1.]
 [ 1.  1.  1. -1. -1. 1.]
 [-1. -1. -1. 1.  1.  1.]
 [ 1. -1. 1. -1. 1.  1.]
 [-1. 1.  1.  1.  1. -1.]]
[[ 1.  1. -1. 1. -1. 1.  1.]
 [ 1.  1. -1. 1.  1.  1. -1.]
 [-1.  1. -1. 1.  1. -1. 1.]
 [-1.  1.  1.  1. -1. 1. -1.]
 [-1.  1.  1.  1. -1. 1. -1.]
 [-1.  1.  1. -1. 1. -1. -1.]
 [-1.  1. -1. 1. -1. -1. 1.]]
Gaussian Orthogonal Ensemble
[[-0.58795889 -0.26558973 0.36388504]
 [-0.26558973 -3.62889251 0.87365577]
 [ 0.36388504 0.87365577 0.14558246]]
[[-3.19216509 1.23904826 -0.21510458 -0.61022329]
 [ 1.23904826 -0.84057953 -0.54634932 -0.44721636]
 [-0.21510458 -0.61022329 0.26963036 -1.63942638]
 [-0.54634932 -0.44721636 -1.63942638 4.41093031]]
[[  3.44946759e-01 -6.23968540e-01 -6.09764649e-01 -1.28609529e+00
   -3.49366306e-02]
 [ -6.23968540e-01 5.04447054e-01 -1.32138308e+00 -6.82824448e-01
    2.62230403e-01]
```

```
 [ -6.09764649e-01 -1.28609529e+00 5.25325302e-01 -7.12667576e-01
   9.12295017e-04]
 [ -3.49366306e-02 -1.32138308e+00 -6.82824448e-01 5.27916391e-01
   9.41917990e-01]
 [  2.62230403e-01 -7.12667576e-01 9.12295017e-04 9.41917990e-01
  -1.06396417e+00]]
[[ 2.87321601 0.62798658 -0.61421847 1.54056368 1.97696979 -0.07339508]
 [ 0.62798658 -3.14871772 -1.8496576 -0.85691044 -0.48947324 -0.57227096]
 [-0.61421847 1.54056368 2.30067746 -1.33874822 0.19141084 0.65019644]
 [ 1.97696979 -0.07339508 -1.8496576 1.85426215 2.4312348 0.48717158]
 [-0.85691044 -0.48947324 -0.57227096 -1.33874822 -1.62786253 -0.35539932]
 [ 0.19141084 0.65019644 2.4312348  0.48717158 -0.35539932 -1.09962205]]
[[ 0.39032708 -0.69772674 -0.48730364 0.97186259 -0.2714061 1.59037837
   1.09501161]
 [-0.69772674 -0.29306715 0.50910021 1.26603055 -1.58719921 0.11616505
  -1.23474255]
 [-0.48730364 0.97186259 0.10985546 1.85841202 0.8493826 1.76617943
  -1.43739731]
 [-0.2714061  1.59037837 1.09501161 1.10506865 0.86772239 -1.67945601
  -0.88857469]
 [ 0.50910021 1.26603055 -1.58719921 0.11616505 -0.89039518 1.23827375
  -1.64444492]
 [-1.23474255 1.85841202 0.8493826  1.76617943 -1.43739731 2.16447929
  -0.26594146]
 [ 0.86772239 -1.67945601 -0.88857469 1.23827375 -1.64444492 -0.26594146
  -3.74461914]]
```

Ans 23

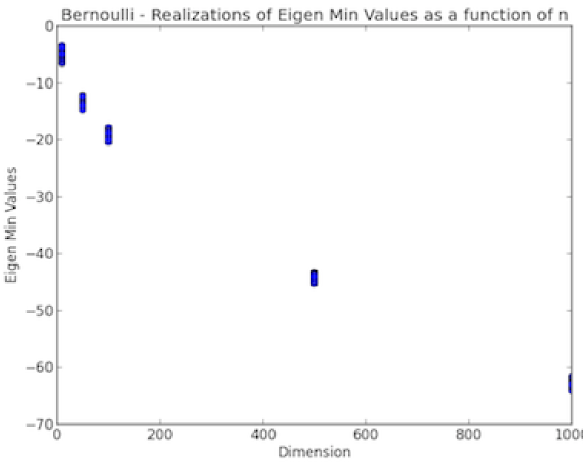

Figure 27 :Realization of eigen min values - Symmetric Bernoulli ensemble

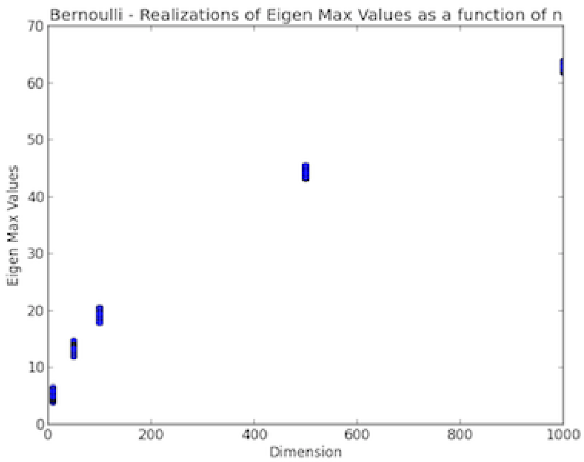

Figure 28 :Realization of eigen max values - Symmetric Bernoulli ensemble

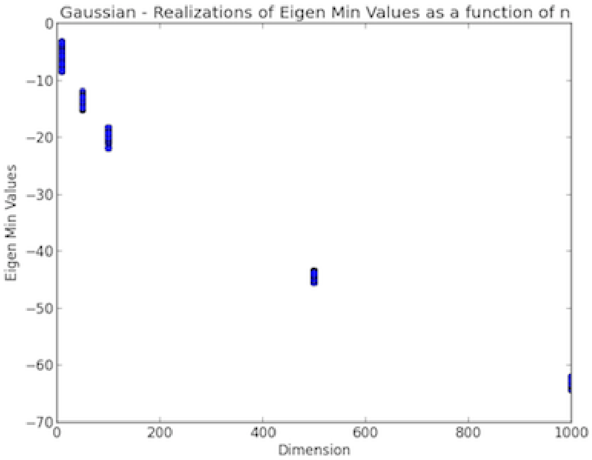

Figure 29 :Realization of eigen min values - Gaussian Orthogonal Ensemble

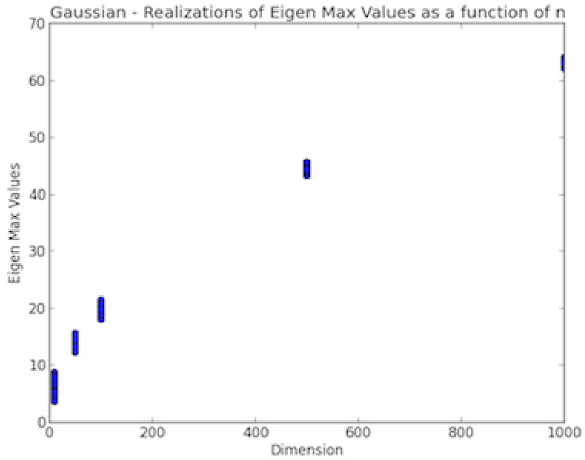

Figure 30 :Realization of eigen max values - Gaussian Orthogonal Ensemble

Ans 24

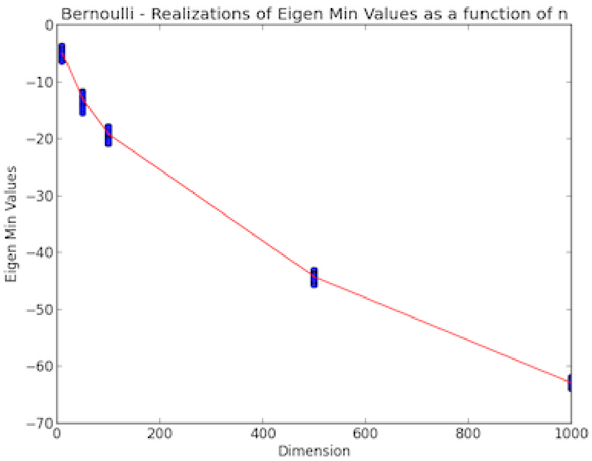

Figure 31 :Realization of eigen min values with curve fitting - Symmetric Bernoulli ensemble

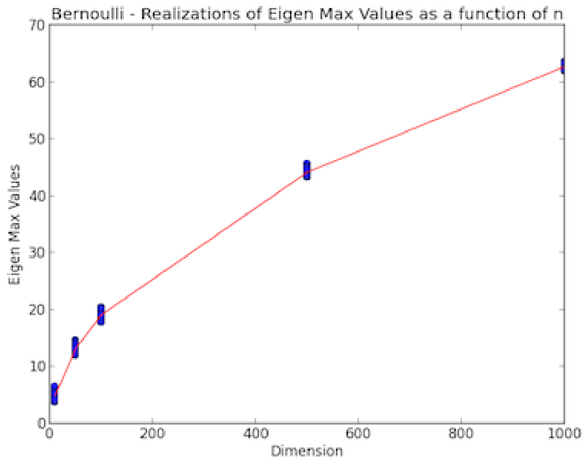

Figure 32 :Realization of eigen max values with curve fitting - Symmetric Bernoulli ensemble

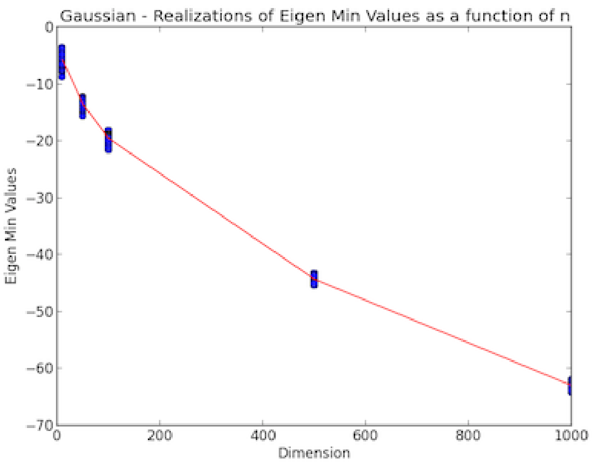

Figure 33 :Realization of eigen min values with curve fitting - Gaussian Orthogonal Ensemble

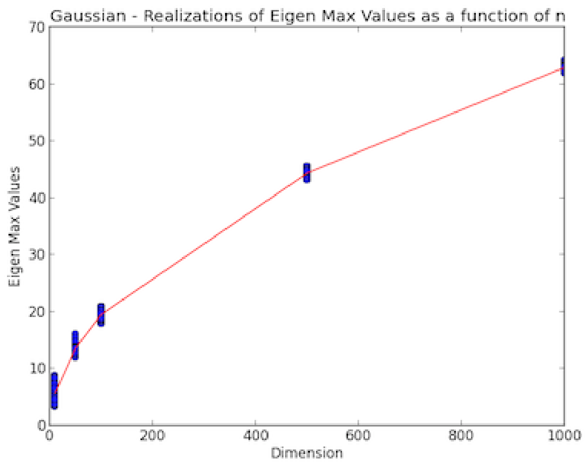

Figure 34 :Realization of eigen max values with curve fitting - Gaussian Orthogonal Ensemble

Ans 25

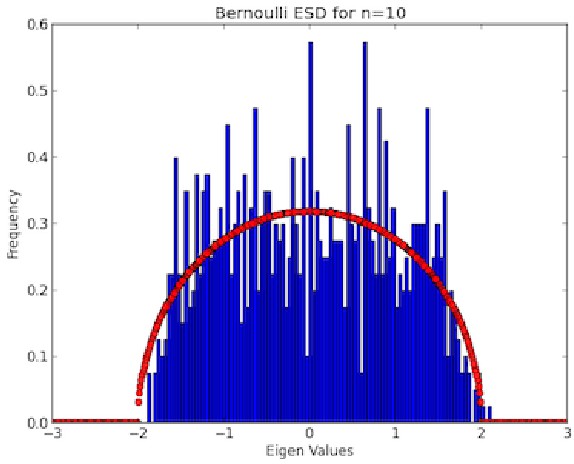

Figure 35 : Bernoulli - Empirical Spectral Distribution for n = 10

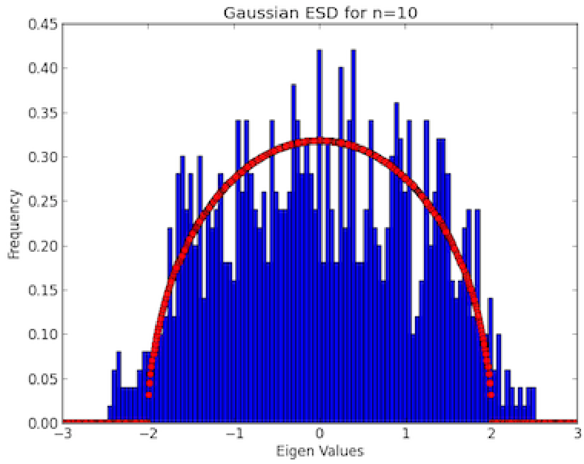

Figure 36 :Gaussian - Empirical Spectral Distribution for n = 10

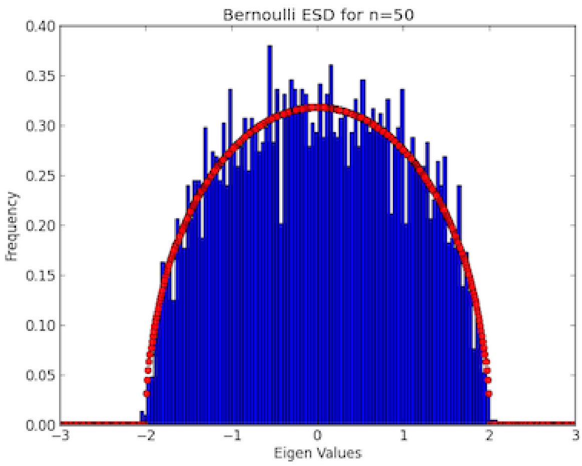

Figure 37 : Bernoulli - Empirical Spectral Distribution for n = 50

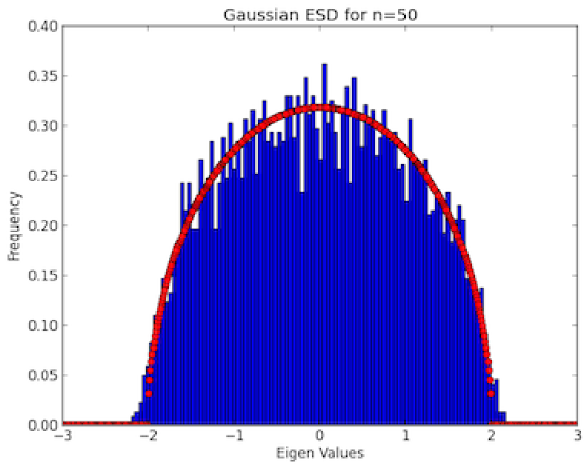

Figure 38 :Gaussian - Empirical Spectral Distribution for n = 50

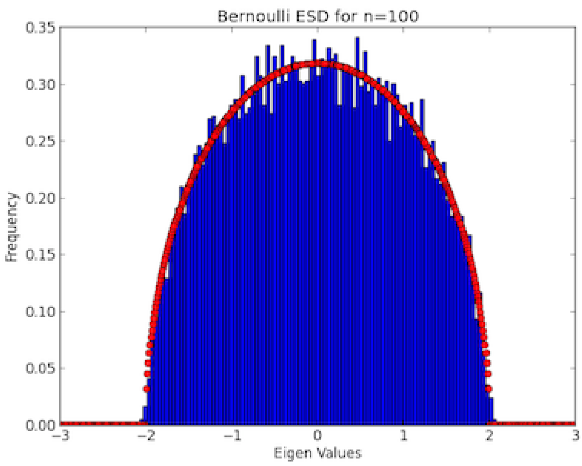

Figure 39 : Bernoulli - Empirical Spectral Distribution for n = 100

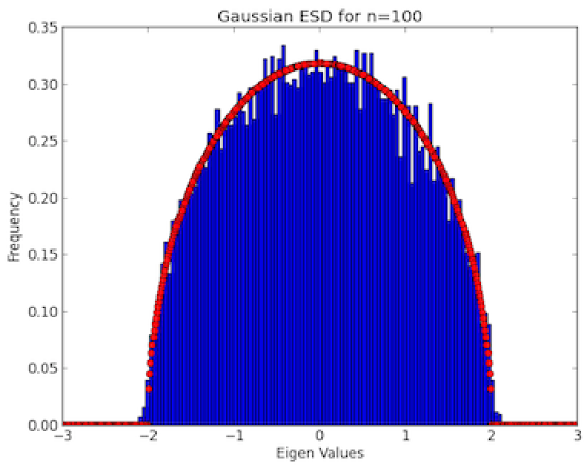

Figure 40 :Gaussian - Empirical Spectral Distribution for n = 100

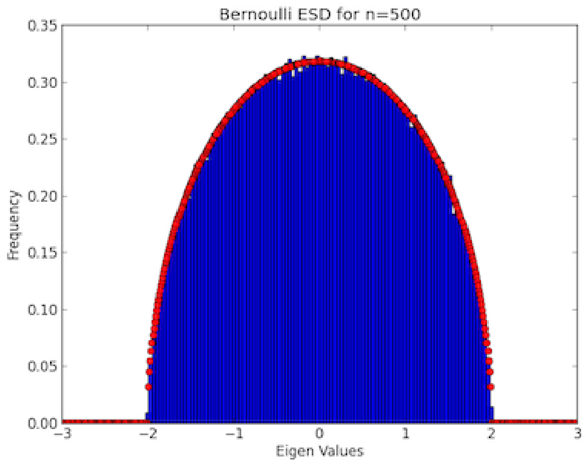

Figure 41 : Bernoulli - Empirical Spectral Distribution for n = 500

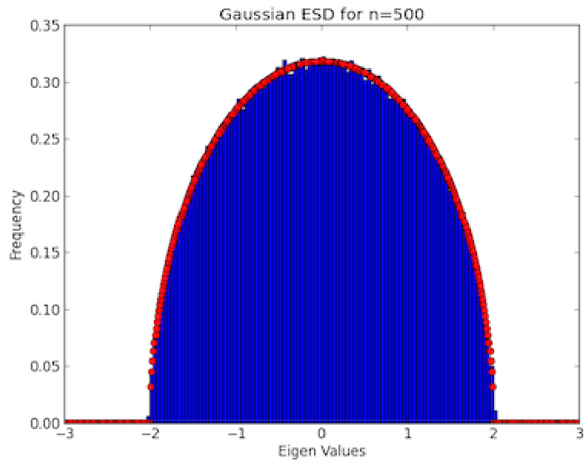

Figure 42 :Gaussian - Empirical Spectral Distribution for n = 500

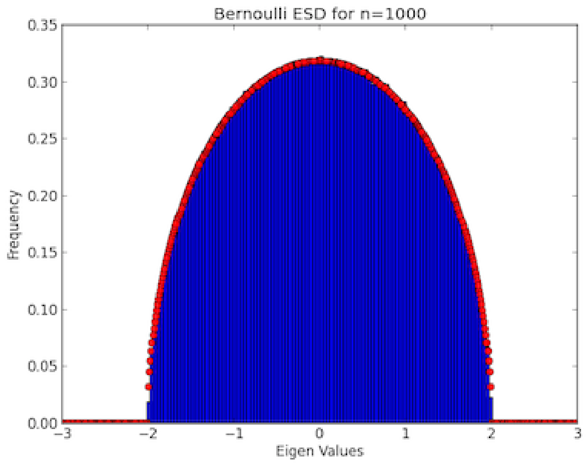

Figure 43 : Bernoulli - Empirical Spectral Distribution for n = 1000

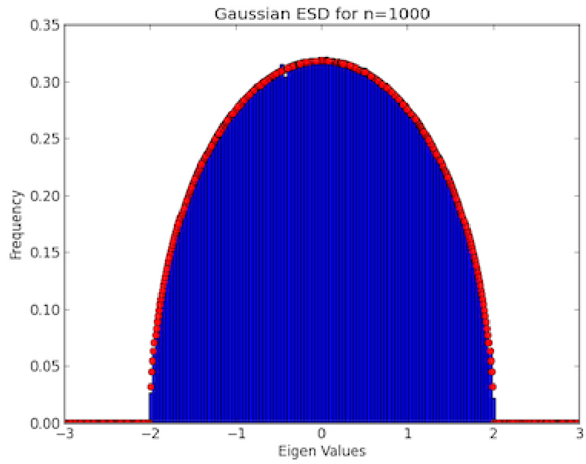

Figure 44 :Gaussian - Empirical Spectral Distribution for n = 1000

We can see that for symmetric matrices with random entries (Bernoulli and Gaussian) the Empirical Spectral Distribution follows the Wigner's Semicircular Law with radius 2 along the x-axis.

Ans 26

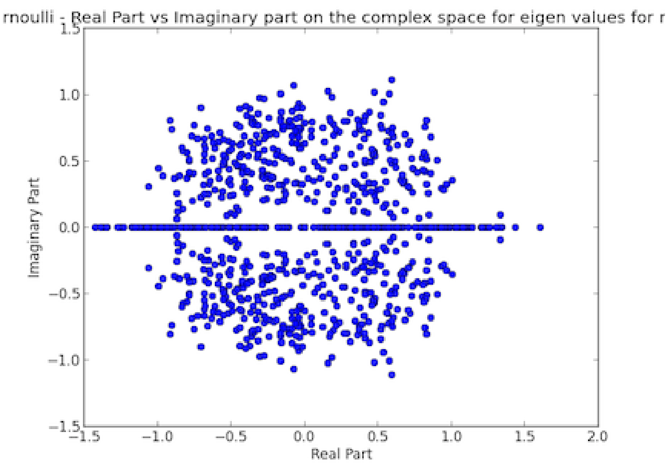

Figure 45 : Bernoulli - Normalized eigenvalues for n = 10

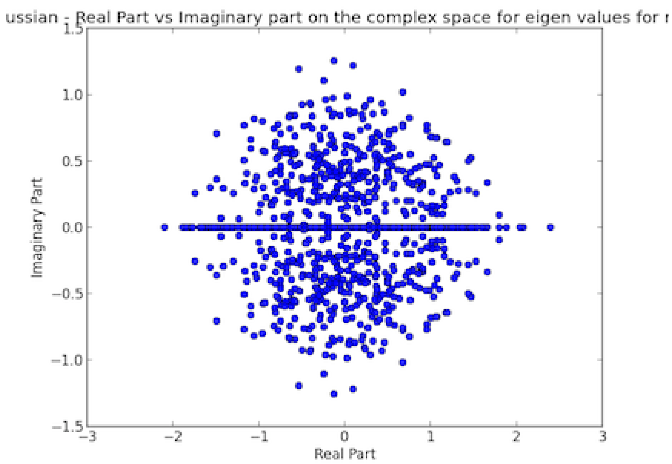

Figure 46 :Gaussian - Normalized eigenvalues for n = 10

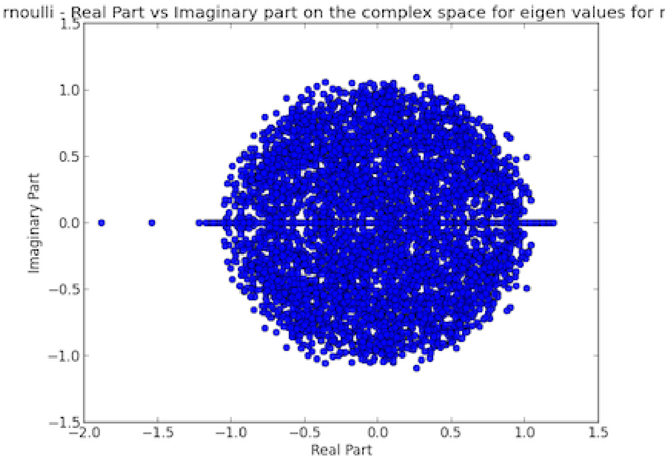

Figure 47 : Bernoulli - Normalized eigenvalues for n = 50

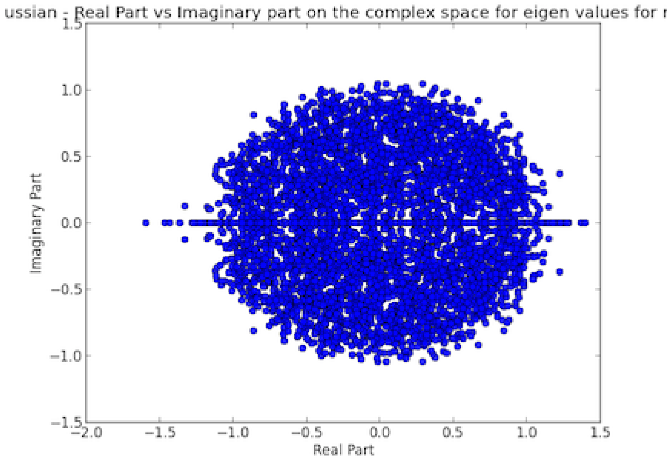

Figure 48 :Gaussian - Normalized eigenvalues for n = 50

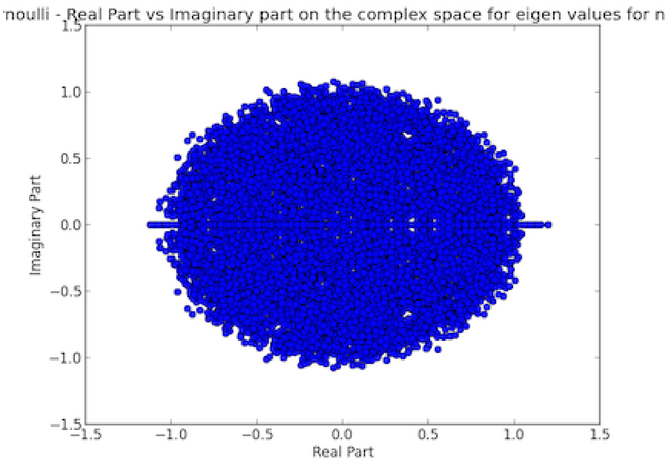

Figure 49 : Bernoulli - Normalized eigenvalues for n = 100

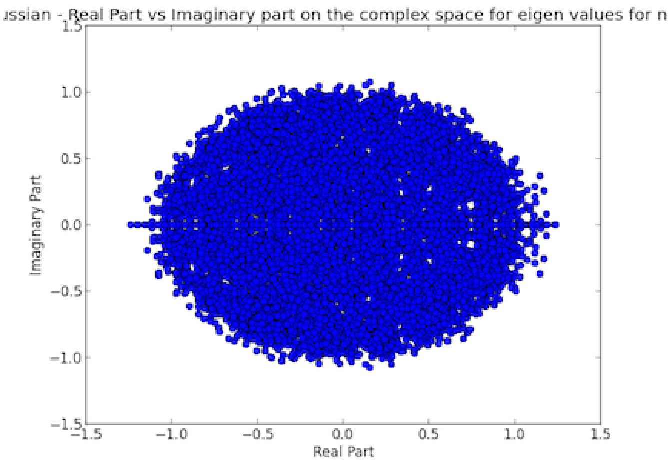

Figure 50 :Gaussian - Normalized eigenvalues for n = 100

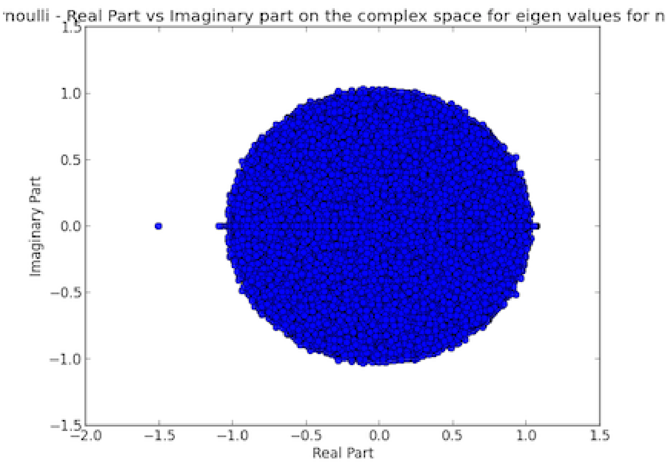

Figure 51 : Bernoulli - Normalized eigenvalues for n = 500

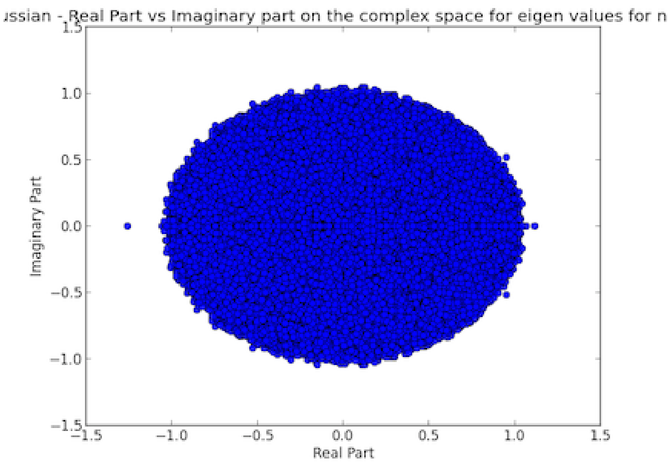

Figure 52 :Gaussian - Normalized eigenvalues for n = 500

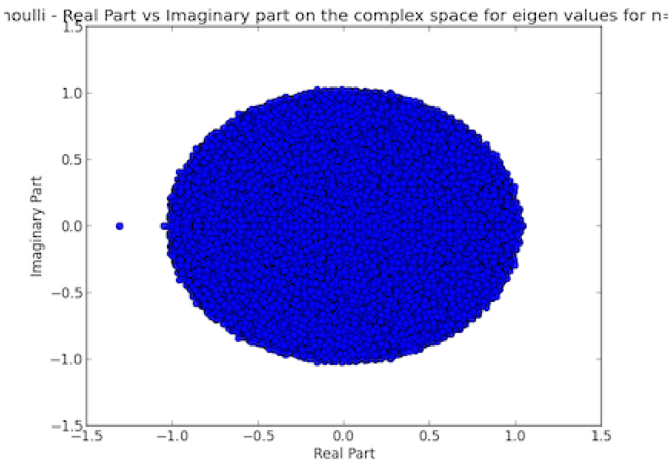

Figure 53 : Bernoulli - Normalized eigenvalues for n = 1000

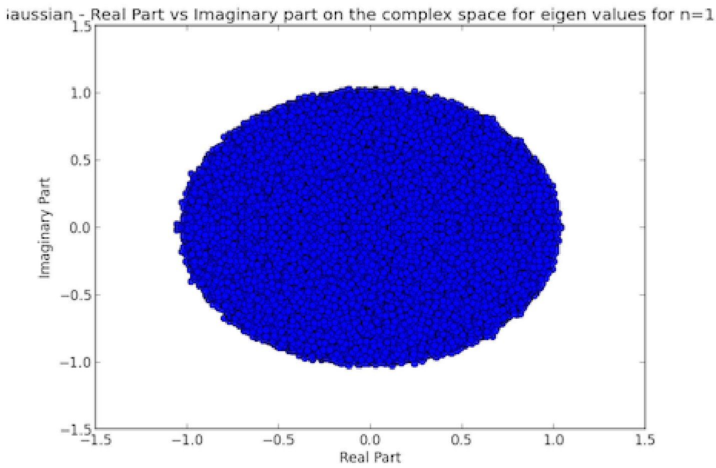

Figure 54 :Gaussian - Normalized eigenvalues for n = 1000

Ans 27

The minimum value for the eigenvalues seems to be increasing with dimension and the maximum value of the eigenvalues seems to be decreasing with dimension unlike the symmetric case.