5.1.Experiment No. 6

Aim: Implement Greedy search algorithm for any ofthe following application:
Prim's Algorithm | Minimum Spanning Tree (Python Code)

Objective:

Theory: **What is a Minimum Spanning Tree?**

As we all know, the graph which does not have edges pointing to any direction in a graph is called an undirected graph and the graph always has a path from a vertex to any other vertex. A spanning tree is a subgraph of the undirected connected graph where it includes all the nodes of the graph with the minimum possible number of edges. Remember, the subgraph should contain each and every node of the original graph. If any node is missed out then it is not a spanning tree and also, the spanning tree doesn't contain cycles. If the graph has n number of nodes, then the total number of spanning trees created from a complete graph is equal to $n^{(n-2)}$. In a spanning tree, the edges may or may not have weights associated with them. Therefore, the spanning tree in which the sum of edges is minimum as possible then that spanning tree is called the minimum spanning tree. One graph can have multiple spanning-tree but it can have only one unique minimum spanning tree. There are two different ways to find out the minimum spanning tree from the complete graph i.e Kruskal's algorithm and Prim's algorithm. Let us study prim's algorithm in detail below:

What is Prim's Algorithm?

Prim's algorithm is a minimum spanning tree algorithm which helps to find out the edges of the graph to form the tree including every node with the minimum sum of weights to form the minimum spanning tree. Prim's algorithm starts with the single source node and later explore all the adjacent nodes of the source node with all the connecting edges. While we are exploringthe graphs, we will choose the edges with the minimum weight and those which cannot cause the cycles in the graph.

Prim's Algorithm for Minimum Spanning Tree

Prim's algorithm basically follows the greedy algorithm approach to find the optimal solution. To find the minimum spanning tree using prim's algorithm, we will choose a source node and keep adding the edges with the lowest weight.

The algorithm is as given below:

- Initialize the algorithm by choosing the source vertex
- Find the minimum weight edge connected to the source node and another node and add it to the tree
- Keep repeating this process until we find the minimum spanning tree

Step 1: Determine an arbitrary vertex as the starting vertex of the MST.
Step 2: Follow steps 3 to 5 till there are vertices that are not included in the MST (known as fringe vertex).
Step 3: Find edges connecting any tree vertex with the fringe vertices.
Step 4: Find the minimum among these edges.
Step 5: Add the chosen edge to the MST if it does not form any cycle.
Step 6: Return the MST and exit

Pseudocode

T = ∅;

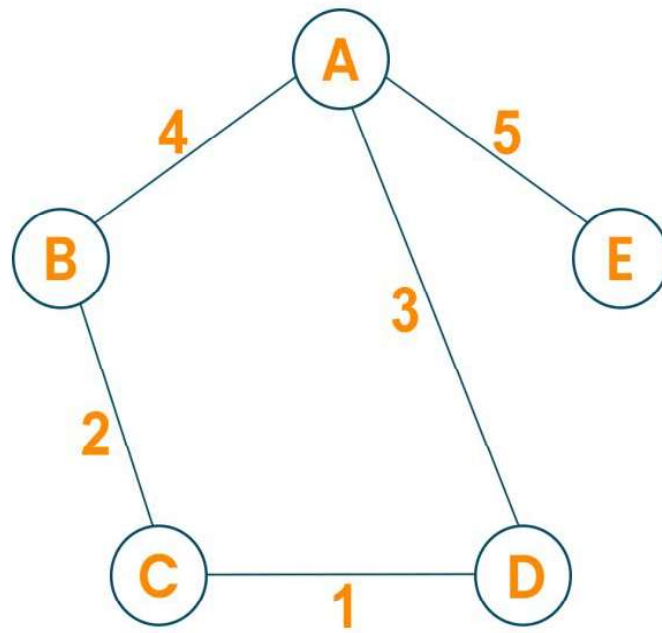M = { **1** };

while (M ≠ N)

　　let (m, n) be the lowest cost edge such that m
　　∈ M **and** n ∈ N - M; T = T ∪ {(m, n)}

　　M = M ∪ {n}

Here we create two sets of nodes i.e M and M-N. M set contains the list of nodes that have been visited and the M-N set contains the nodes that haven't been visited. Later, we will move each node from M to M-N after each step by connecting the least weight edge.
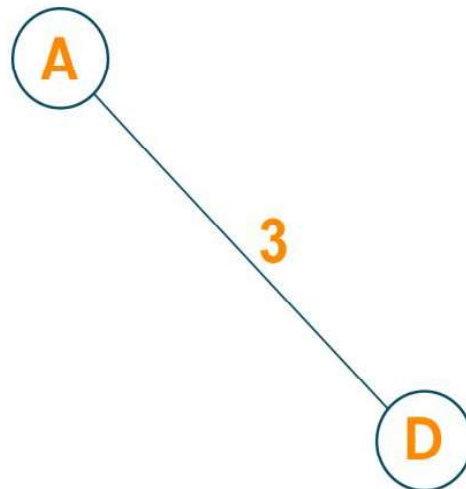
Example

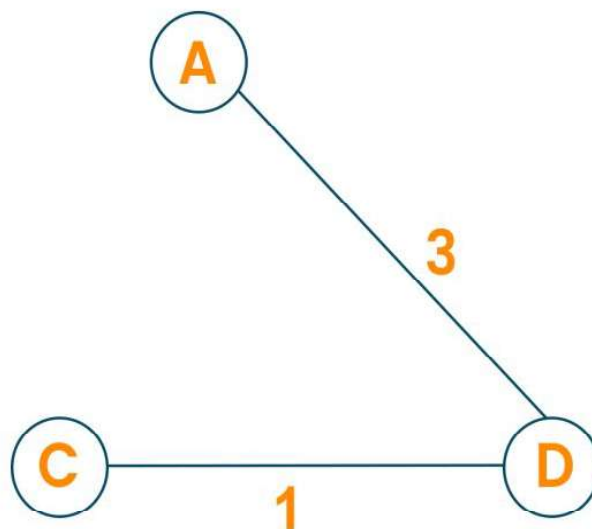Let us consider the below-weighted graph

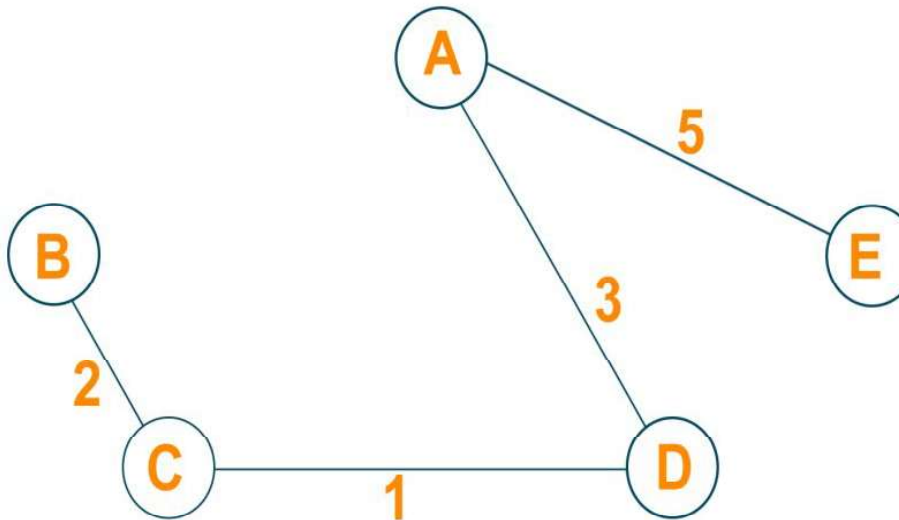Later we will consider the source vertex to initialize the algorithm

A

Now, we will choose the shortest weight edge from the source vertex and add it to finding the spanning tree.

A
3
D

Then, choose the next nearest node connected with the minimum edge and add it to the solution. If there are multiple choices then choose anyone.

A
3
C ——1—— D

Continue the steps until all nodes are included and we find the minimum spanning tree.

Time Complexity:

The running time for prim's algorithm is O(VlogV + ElogV) which is equal to O(ElogV) because every insertion of a node in the solution takes logarithmic time. Here, E is the number of edges and V is the number of vertices/nodes. However, we can improve the running time complexity to O(E + logV) of prim's algorithm using Fibonacci Heaps.

```python
# Prim's Algorithm in Python

INF = 9999999
# number of vertices in graph
N = 5
#creating graph by adjacency matrix method
G = [[0, 19, 5, 0, 0],
     [19, 0, 5, 9, 2],
     [5, 5, 0, 1, 6],
     [0, 9, 1, 0, 1],
     [0, 2, 6, 1, 0]]

selected_node = [0, 0, 0, 0, 0]

no_edge = 0

selected_node[0] = True

# printing for edge and weight
print("Edge : Weight\n")
while (no_edge < N - 1):
```

```python
minimum = INF
a = 0
b = 0
for m in range(N):
    if selected_node[m]:
        for n in range(N):
            if ((not selected_node[n]) and G[m][n]):
                # not in selected and there is an edge
                if minimum > G[m][n]:
                    minimum = G[m][n]
                    a = m
                    b = n
print(str(a) + "-" + str(b) + ":" + str(G[a][b]))
selected_node[b] = True
no_edge += 1
```

Applications:

•   Prim's algorithm is used in network design

•   It is used in network cycles and rail tracks connecting all the cities

•   Prim's algorithm is used in laying cables of electrical wiring

•   Prim's algorithm is used in irrigation channels and placing microwave towers

•   It is used in cluster analysis

•   Prim's algorithm is used in gaming development and cognitive science

•   Pathfinding algorithms in artificial intelligence and traveling salesman problems make use of prim's algorithm.

Input: `g.graph = [[0, 2, 0, 6, 0],`

```
            [2, 0, 3, 8, 5],
            [0, 3, 0, 0, 7],
            [6, 8, 0, 0, 9],
            [0, 5, 7, 9, 0]]
Or
G = [[0, 19, 5, 0, 0],
     [19, 0, 5, 9, 2],
```

```
[5, 5, 0, 1, 6],
[0, 9, 1, 0, 1],
[0, 2, 6, 1, 0]]
```

Output:

```
Edge       Weight
0 - 1        2
1 - 2        3
0 - 3        6
1 - 4        5
Or

Edge : Weight


0-2:5
2-3:1
3-4:1
4-1:2
```

Conclusion: Successfully able to implement Greedy search Algorithm for Prims Minimum Spanning tree.

Outcome: The minimum spanning tree has its own importance to learn the prim's algorithm which leads to find the solution to many problems. When it comes to finding the minimum spanning tree for the dense graphs, prim's algorithm is the first choice.

Questions:

1. How does Prim's Algorithm Work?
2. What Is a Minimum Spanning Tree?
3. What Is Prim's Minimum Spanning Tree Algorithm?
4. How to Implement the Prim's Algorithm
5. Does Prim's algorithm run on a negative edge weight graph?

6. Can there be multiple MST for a graph?