Game-Theoretic Models of Information Overload in Social Networks

A Presentation for CS886

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Background

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- Increasing irrelevant updates on social media newsfeeds, or information overload.

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- Symmetric: requires consent from both sides to maintain tie - eg., Facebook.
- Asymmetric: requires consent from only one side to maintain tie - eg., Twitter.
- Authors mainly look at asymmetric social networks.

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- Makeup of newsfeed becomes very important to user.
- Mix of newsfeed is determined by the activity level of user's friends.
- *How much one hears from one particular friend is not in user's control.*

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  - Updates from friends are useful, but excessive updates have diminishing value.
  - Users can be partitioned as producers and consumers of information (80 - 20 rule).

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- Engagement: Users get frustrated by high update rate of followees and leave the social network.

Graph Model

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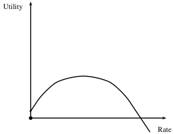
- Producer i updates at a frequency (rate) of r_i.
- Payoff for producer i is r_i times the number of followers he/she has.

Nash equilibrium

- In Nash equilibrium, each player knows the strategy of other players, and no player has anything to gain by changing their strategy.

Figure : Utility of Consumer for a Specific Producer



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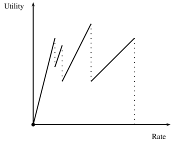

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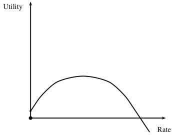

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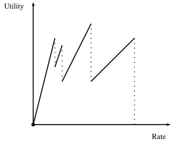

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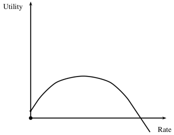

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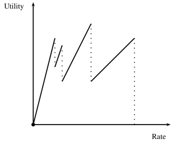

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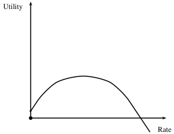

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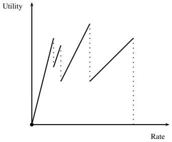

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- For producers, discontinuities at points where consumers stopped following.

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- $U_j = \sum_i x_i r_i q_{ij} - \lambda(\sum_i x_i r_i)$
- Very hard to solve if $x_i \in \{0, 1\}$, so simplify it to $x_i \in [0, 1]$.

User strategy

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- If there exists a lower valued producer l, with $q_{il} < q_{ij}$, utility of i strictly increases.
- Suggests that producers will tweet at a very high rate, and consumer will follow only one producer.
- Not very realistic...

Greedy Users

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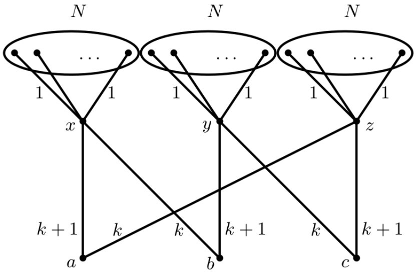
Definition

Consider consumer j and let $q_1 \geq \ldots \geq q_n$ be the sorted order of $q_{1j}, \ldots, q_{nj}$, and k be the largest index such that $\sum_{i=1}^{k} r_i \leq q_k$. Under the greedy model, consumer j follows the k producers for who he has the highest quality and no one else.

Example for Nash Equilibrium not existing



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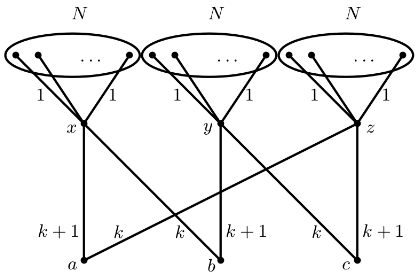

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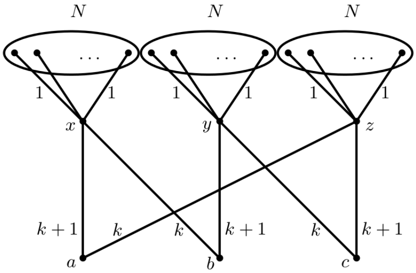

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- Create a dependency graph for a game, where a directed edge is drawn between producers x and y if $q_{ix} > q_{iy}$.
  - If resultant graph is acyclic, Nash equilibrium exists.
  - Similar argument as previously - nodes are topologically ordered and same induction argument.

Matchings Characterizes Pure Rate Equilibrium

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- A consumer i is said to be *critical* to a producer j, if j drops i if r_i is increased.
- Find a matching from all subsets of producers to consumers that are critical for them.
- Check matching to see if it is a possible Nash equilibrium.

Engagement Model - some notations

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- C_j be the set of producers that consumer i follows.
- Let S be a function such that, $S(\sum_{i \in C_j} r_i)$ is the probability that consumer j stays in the social network.
- Expected utility of producer i is $U_i(r) = r_i(\sum_{j \in F_i} S(\sum_{i' \in C_j} r_{i'}))$.

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Cons

- In my opinion, not enough motivation as to why the particular models of social networks has been chosen.
- Matching Characterization not elaborated upon, perhaps had to be shortened?
- Understandable why fractional flow is considered in Followership model, but may not be realistic.
- Could look at Mixed State Nash equilibrium?

Pros

- Takes into account rate of information flow in social networks.
- Characterizations of Nash equilibrium shown (in Followership model).
- Empirical evidence to support rate of updates as a parameter.

References

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