Introduction to Navigation & Guidance

(Course Code: AE 410/641)

Department of Aerospace Engineering
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Assignment - 2

Total Marks: 60

Instructions

- Marks for each portion of a question are given separately.
- In cases where the answers of two students are found to be copied, both of them will be awarded zero marks for that particular question.
- Symbols have their usual meanings, unless stated otherwise.
- Assignment needs to be submitted online on Moodle. Please adhere to the deadline.
- For programming questions, codes for generating all the necessary plots also need to be submitted. Enclose all your scripts within a compressed folder, and attach along with your submission.
- The attachment and the file names must be meaningful for evaluation. For instance, you can call your script as `Q3b.m` and the folder as `rollnumber.zip`.
- Generated plots must be labeled properly, it is recommended that the plots should also be included in your assignment.

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1. A missile's desired navigation law is given as $\dot{\psi}_M = 3\,\dot{\psi}_T$. However, the navigation law of the missile changes due to Radome effect, and becomes $\dot{\psi}_M = 1.8\,\dot{\psi}_T$, as shown in Figure 1. If $\dot{\psi}_M = 0.15$ rad/s, calculate the value of α and $\dot{\epsilon}$. [5]

2. Consider the interceptor-target geometry shown in Figure 2, for which the following data is given:

$$V_M = 400 \text{ m/s},\ \theta = 30^\circ,\ \gamma_T = 60^\circ,\ r = 7 \text{ km},\ V_T = 0.5V_M$$

   (a) Find the set of values of initial heading angles for which target capture can be guaranteed if the interceptor uses *realistic true proportional navigation* guidance with the navigation constant, $N = 3$.

   (b) Plot the corresponding initial conditions in (V_{θ_0}, V_{R_0}) space.

   (c) Tabulate and plot the initial heading angles that led to capture versus the interception time.

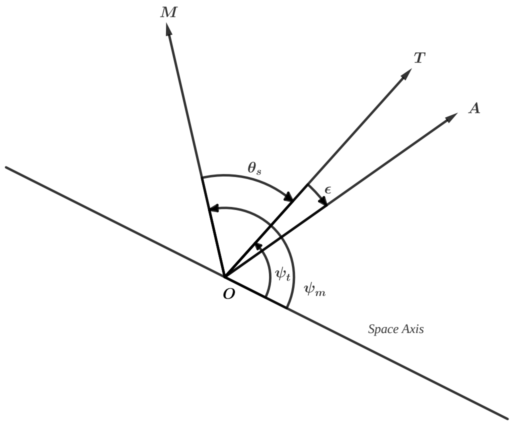


Figure 1: Change in navigation law due to Radome effect.

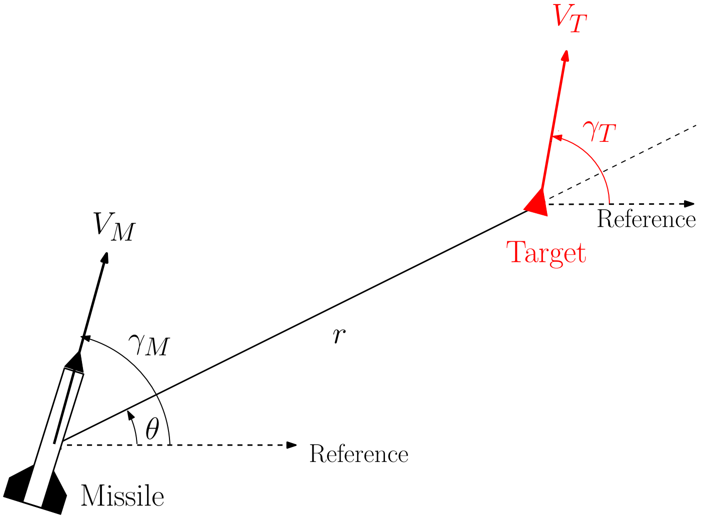


Figure 2: Planar engagement geometry between an interceptor and a target.

[2+2+1]

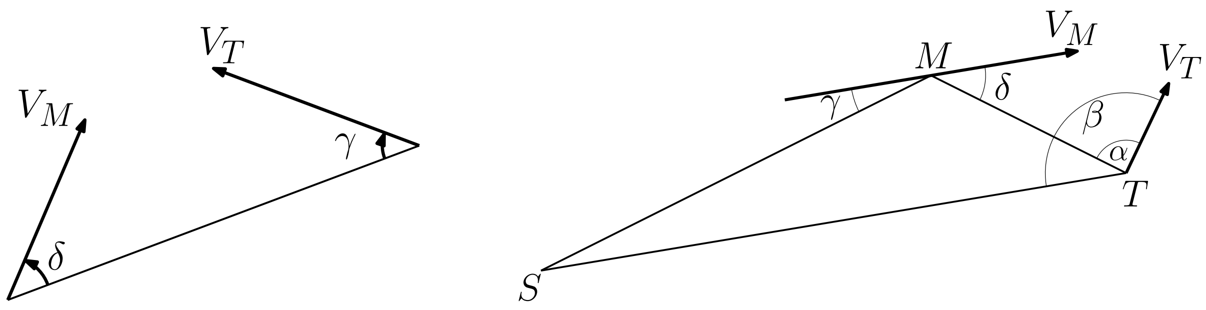
3. An interceptor is said to perform *active homing* if the source of energy to illuminate the target, and the receiver to detect the energy reflected from the target, are contained with the interceptor. However, if the source of energy is external to the interceptor, such as a ground station, but the receiver is still carried with it, then the interceptor is said to be in *semi-active homing*.

   (a) The velocity vectors of the interceptor and the target are denoted by V_M and V_T, respectively. What would be the Doppler shift, f_D, on the target echo if the interceptor is directly pointing towards the target, as shown in Figure 3a?

   (b) Determine the Doppler shift in the case of semi-active homing, as shown in Figure 3b.

Note that S, M and T, respectively represent the ground station, the interceptor and the target. The respective angles are shown in Figure 3b.

(c) In both the cases, determine the range of Doppler frequency shift if the velocities of the participants are given as $V_M = 600$ m/s and $V_T = 300$ m/s. Take the wavelength of the illumination, $\lambda = 30$ mm.



(a) Active homing. (b) Semi-active homing.

Figure 3: Various homing scenarios.

[3+3+4]

4. (a) For a missile following a pursuit course, derive the expression for the lateral acceleration, a_M, in terms of LOS angle, θ. Show that for small magnitudes of $K = \dfrac{V_M}{V_T}$ and deviation angle, δ, the deviated pure pursuit demands lesser lateral acceleration than ordinary pure pursuit, if $K > 0$ and $0 < K \sin\delta < \sin\theta_T$, where θ_T is the angle that the target's velocity vector submits with the LOS.

   (b) Plot the trajectories and lateral accelerations for an interceptor guided by pure pursuit for various speed ratios of the participants, $K = 0.5, 1, 2$. Take the interceptor's and the target's flight path angle to be 45° and 120°, respectively. Assume that the LOS subtends 0° with the reference.

   (c) What can you infer about the interception from the speed ratios of the participants.

[3+6+1]

5. (a) If T_f denotes the time of interception, then the *time-to-go*, t_{go}, at any moment $t \in [0, T_f]$ is defined as the time remaining till interception, i.e., $t_{\text{go}}(t) = T_f - t$.

   For an interceptor guided by deviated pure pursuit, the time-to-go is given as

$$t_{\text{go}} = \frac{r\left[V_r(t) + 2V_m \cos\delta(t) - V_\theta \tan\delta(t)\right]}{V_M^2 - V_T^2},$$

   where $V_r = \dot{r}$ and $V_\theta = r\dot{\theta}$ are the relative velocity components of the interceptor-target engagement, and δ is the constant deviation angle from the LOS. Prove that $t_{\text{go}} = 0$ is both necessary and sufficient condition for a successful interception. In other words, the target is captured if and only if $t_{\text{go}} = 0$.

(b) Assuming the speeds of the interceptor and the target as 400 m/s and 200 m/s, respectively, plot the trajectories and the lateral accelerations of the interceptor guided by deviated pursuit for deviation angles, $\delta = 0°, 15°, 60°$. Take $\theta = 0°, \gamma_M = 45°, \gamma_T = 120°$.

(c) What changes do you observe when the deviation angle is increased?

[3+6+1]

6. (a) For an interceptor is guided by *true proportional navigation*, the target is captured if the navigation constant, $N > \frac{1}{2}$. Justify this claim by plotting the trajectories of interceptor for various N, indicating what values of N lead to interception, and otherwise.

To intercept a stationary target, take the speed and the flight path angle of the interceptor to be 400 m/s and 45°, respectively. Assume initial radial separation between the participants as 10 km, and the LOS to be 0°. Take $N = 0.25, 0.5, 1, 2, 3, 50$ for plotting the trajectories.

(b) What happens if $N \to \infty$?

[8+2]

7. (a) Assuming that the target is stationary, for an interceptor guided by *pure proportional navigation*, show that the expression of relative range, r, as a function of lead angle, $\sigma = \gamma_M - \theta$, is given by

$$r = \frac{r_0}{(\sin\sigma_0)^{1/(N-1)}}(\sin\sigma)^{1/(N-1)} ,$$

where r_0, σ_0 are the initial values of relative range and the lead angle, respectively.

(b) Consider an engagement scenario where missile and target are launched from ground and have a separation of 15 km. Assume that the speed of missile and target are 500 m/s and 300 m/s, respectively. The target is moving at an angle of 90° from the ground. The missile's autopilot is assumed to be perfect and target is non-maneuvering. If the missile is launched with heading angle error of ±20°, then plot the trajectories of missile and target using both RTPN and PPN guidance laws.

[6+2+2]