Objective

The objective of the assignment was to solve 1D Euler equation subject to different boundary conditions and analyse the convergence of solution with number of grid cells.

Brief about the codes

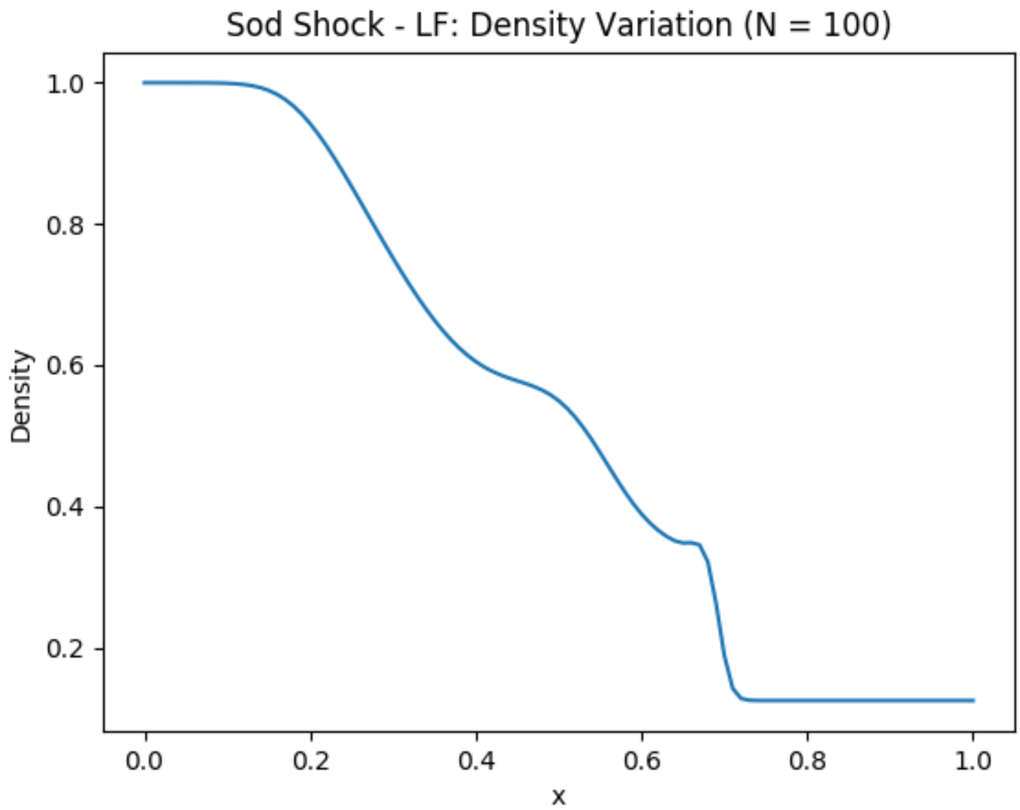
The submission contains the following codes:

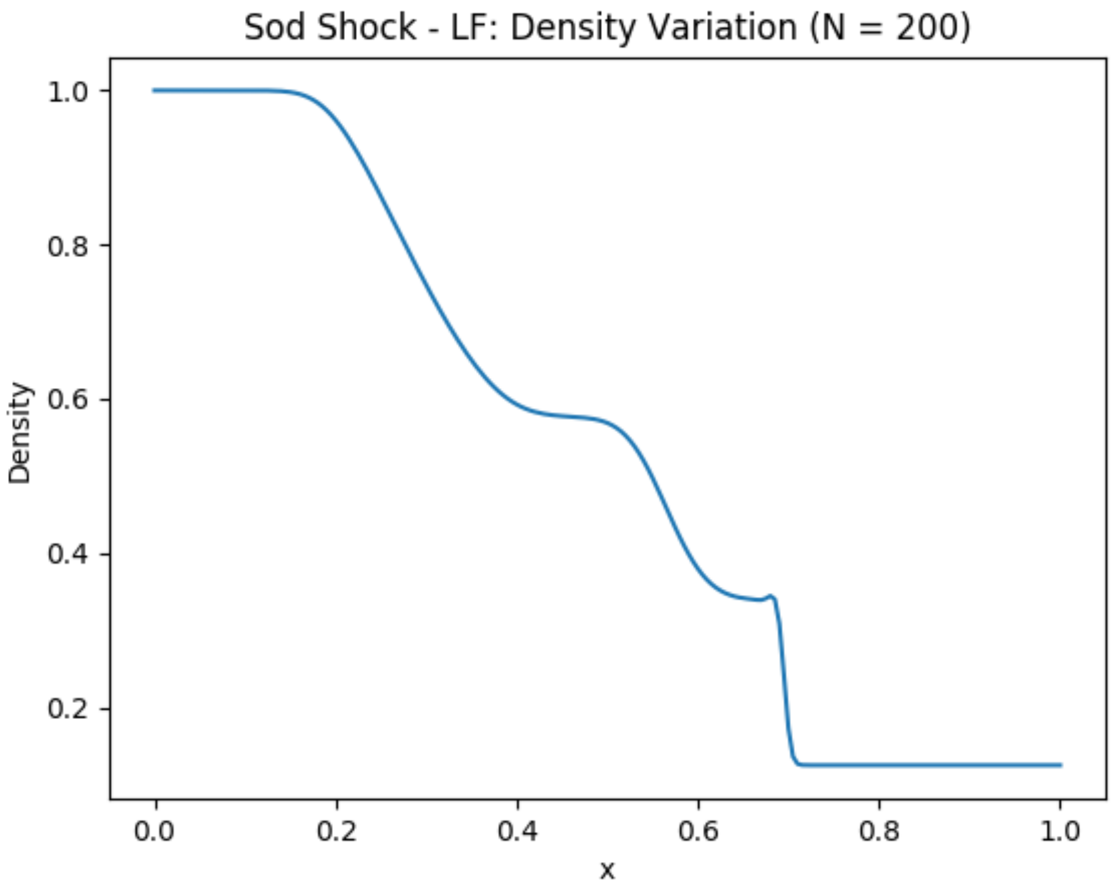
1. sod_shock.py
2. test_case_2.py
3. Noh.py
4. shu-osher.py

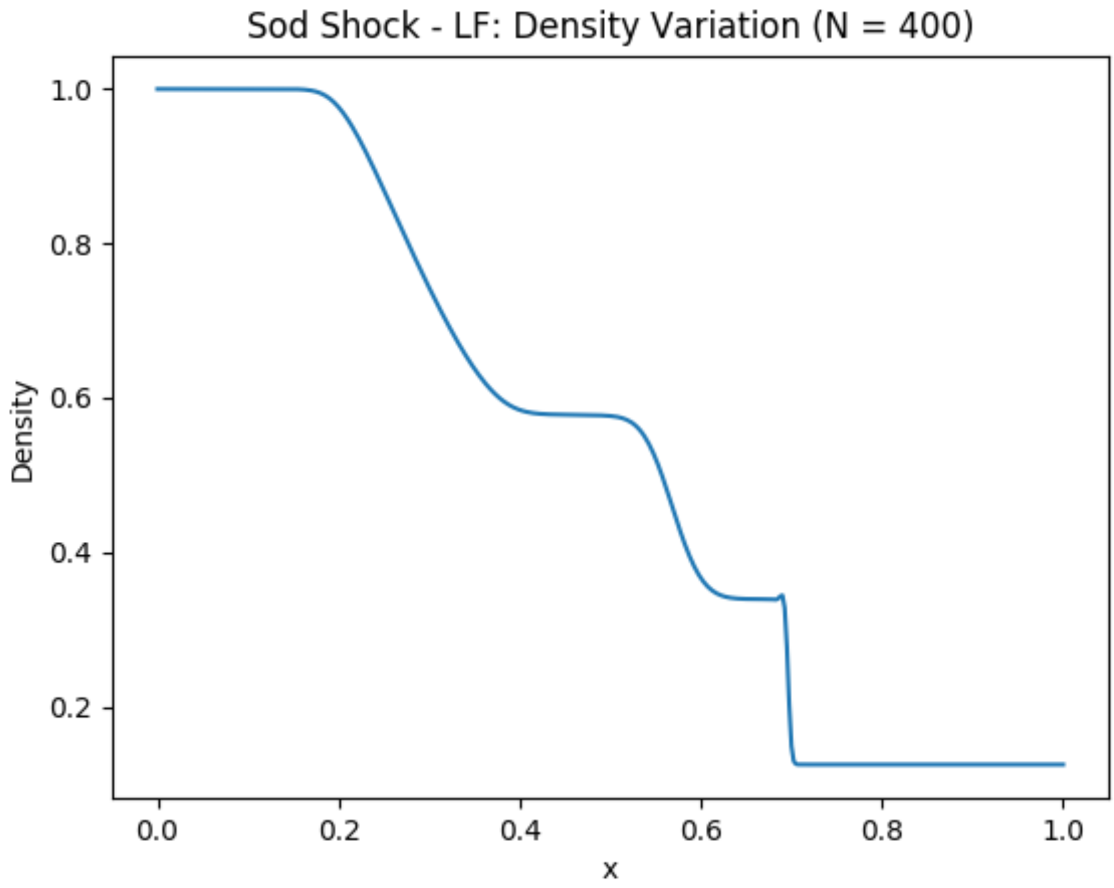
All these codes solve Lax-Friedrichs and Rusanov iteration scheme for euler equation subject to different boundary conditions. The implementation is vectorised for better performance. All the generated graphs are displayed and saved in a folder (created at run time).

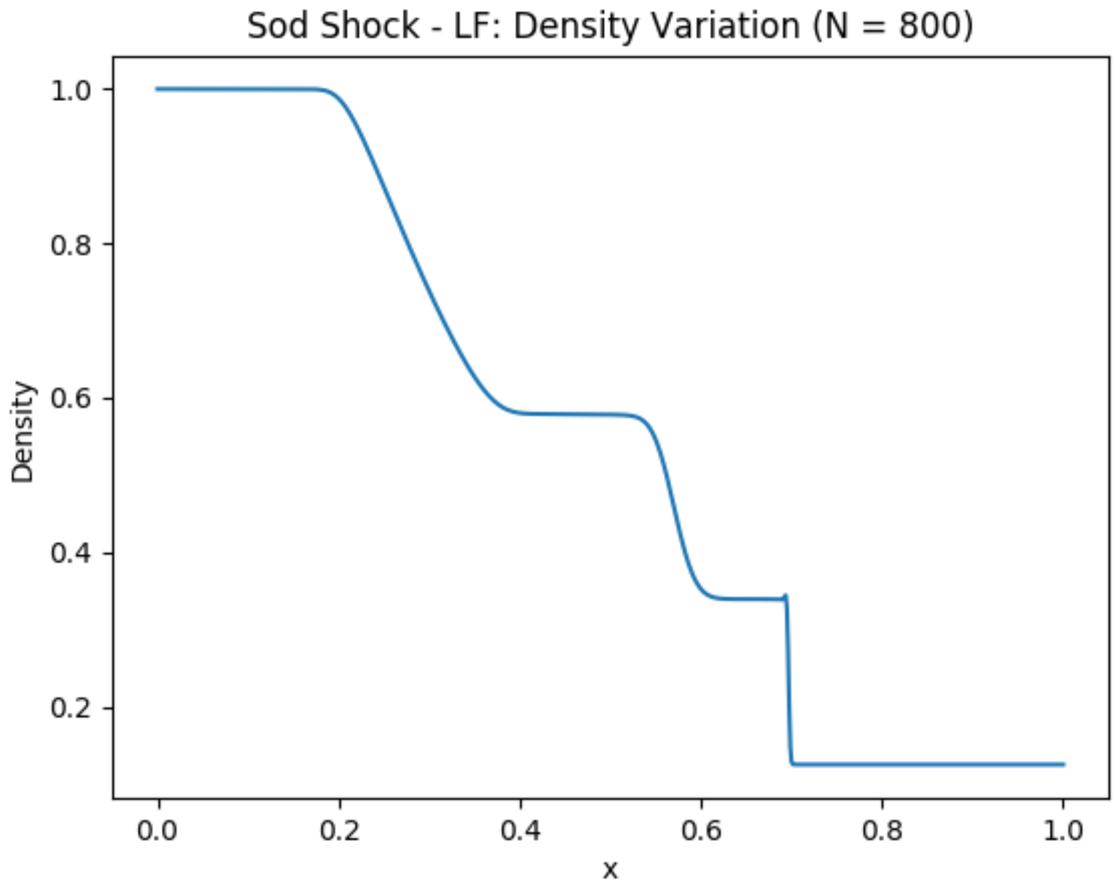
Plots: Sod Shock Tube Problem

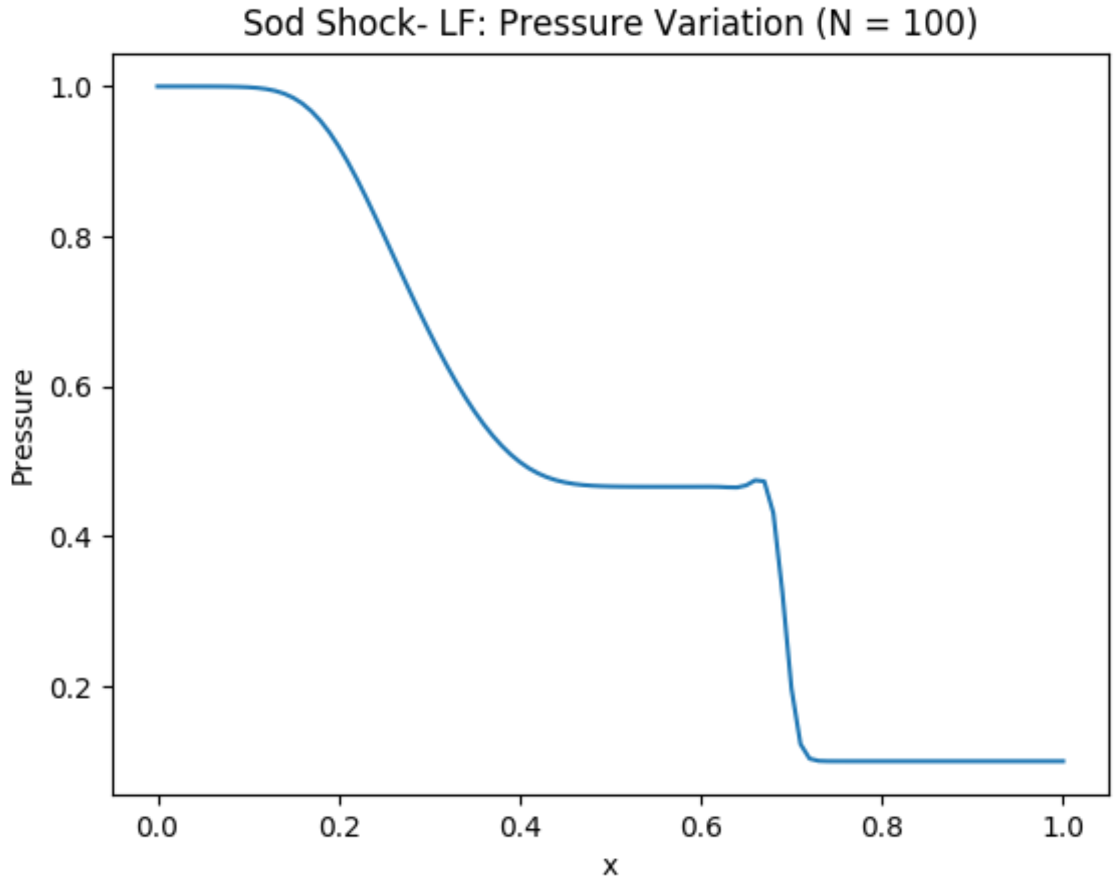
Lax-Friedrich Scheme

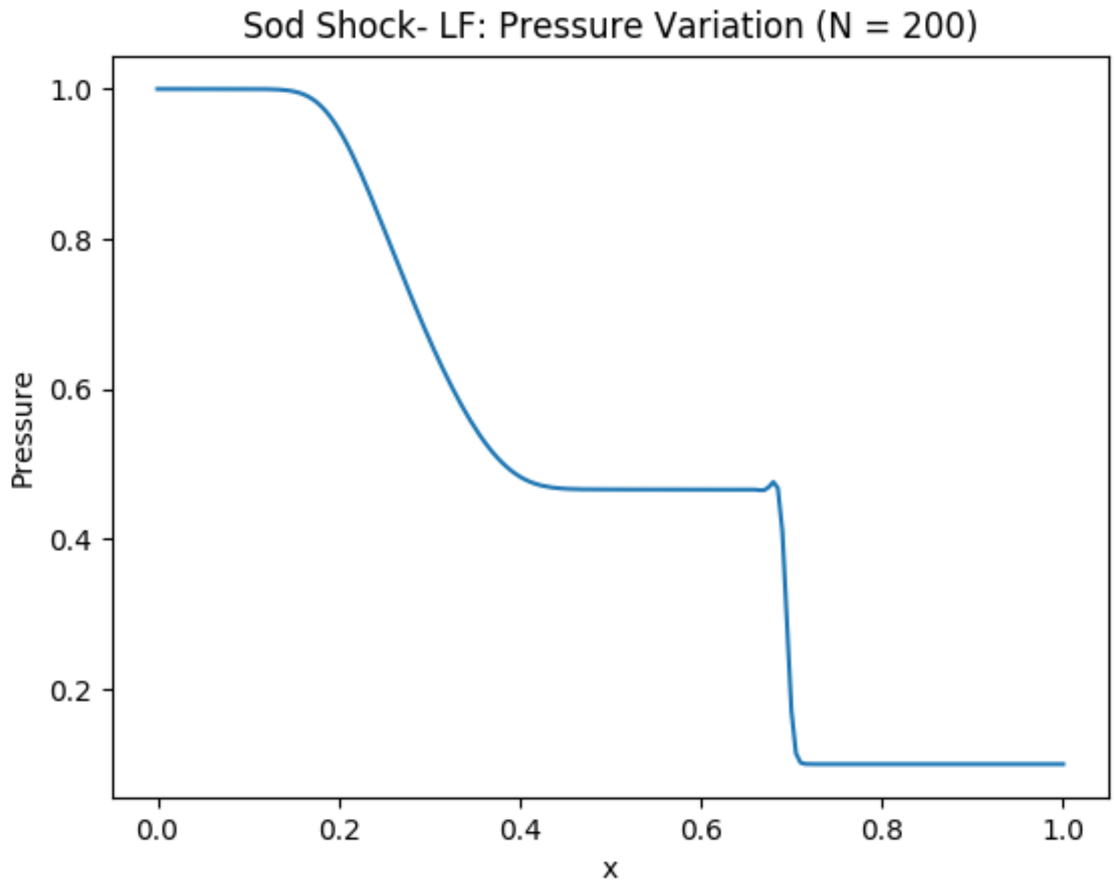


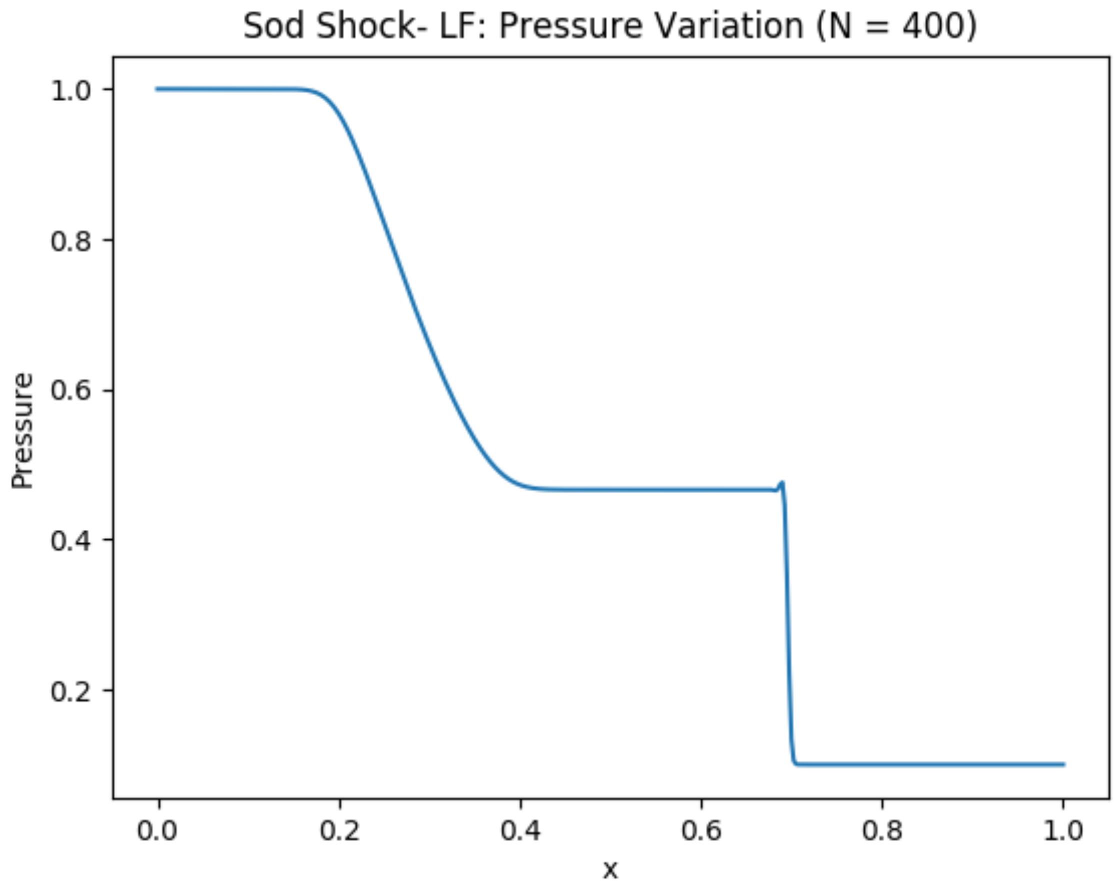


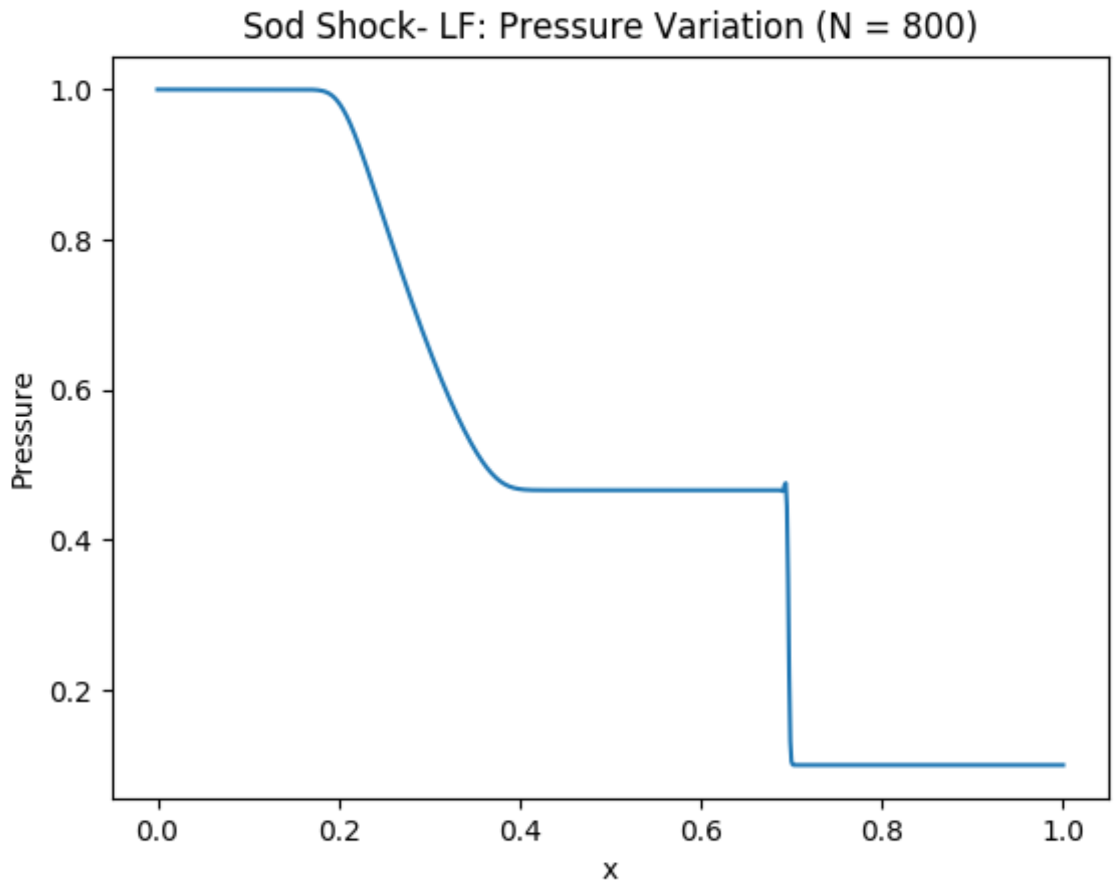

Sod Shock - LF: Density Variation (N = 400)
Density
x
0.0
0.2
0.4
0.6
0.8
1.0

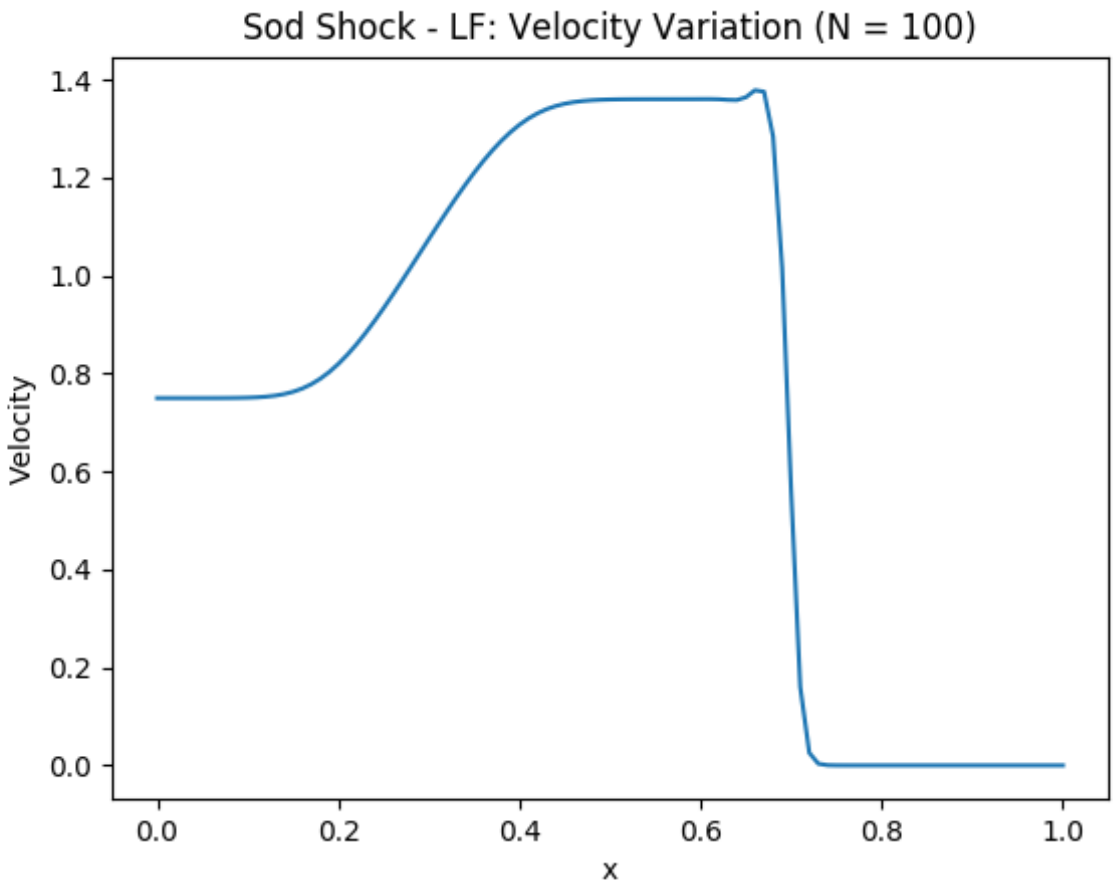

Sod Shock - LF: Density Variation (N = 800)
Density
x
0.0
0.2
0.4
0.6
0.8
1.0


Sod Shock- LF: Pressure Variation (N = 100)
Pressure
x
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

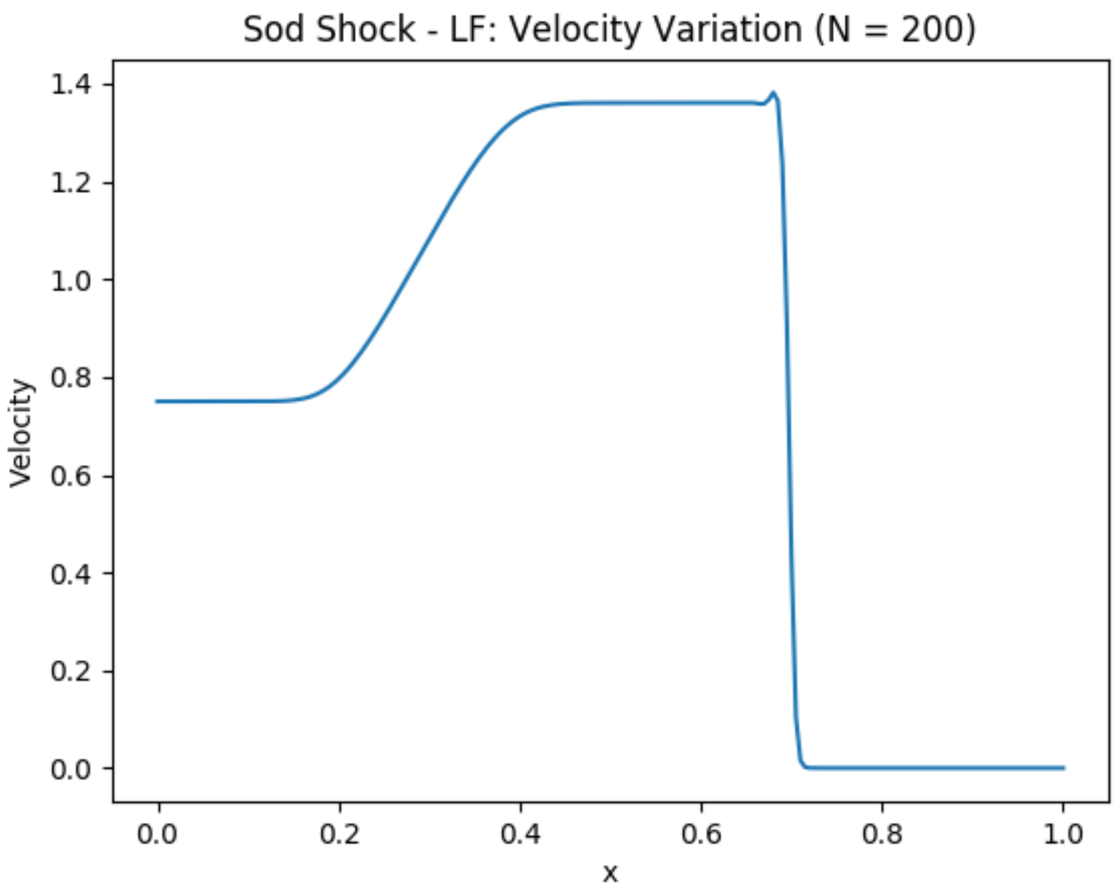

Sod Shock- LF: Pressure Variation (N = 200)
Pressure
x
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0


Sod Shock- LF: Pressure Variation (N = 400)
Pressure
x
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

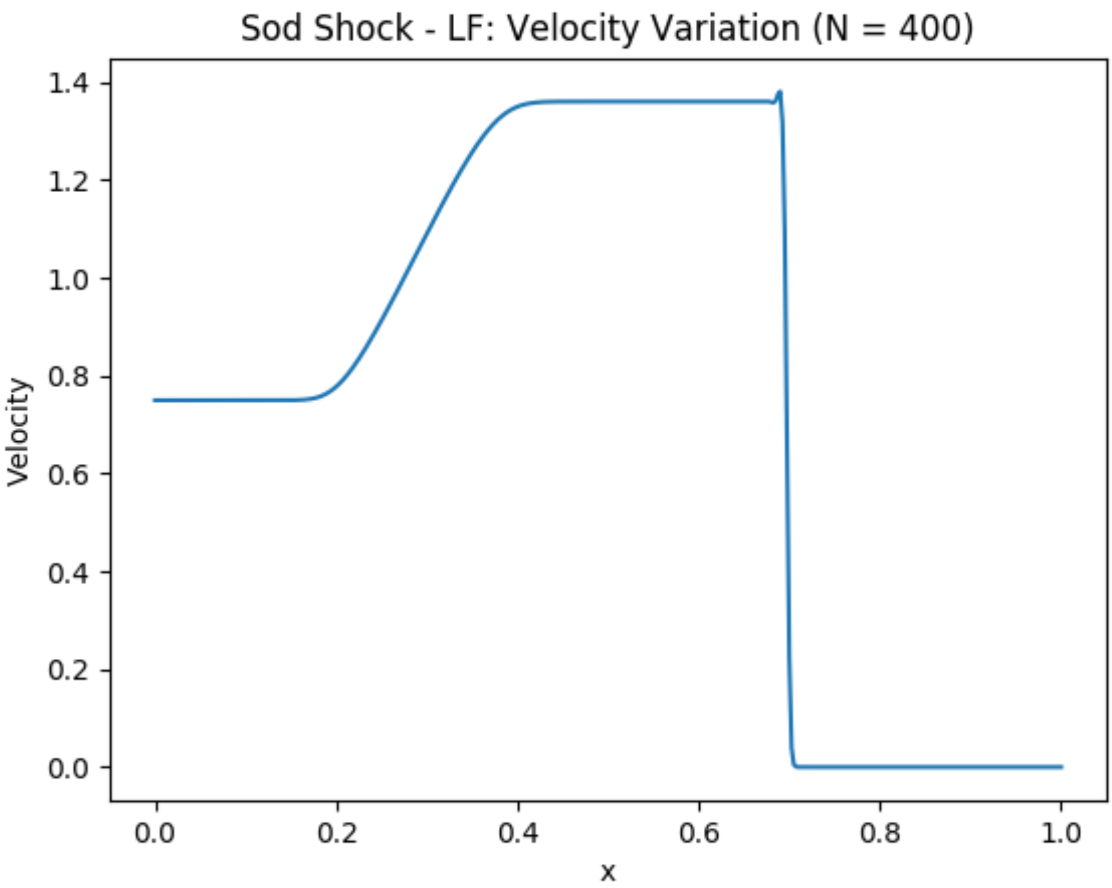

Sod Shock- LF: Pressure Variation (N = 800)
Pressure
x
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

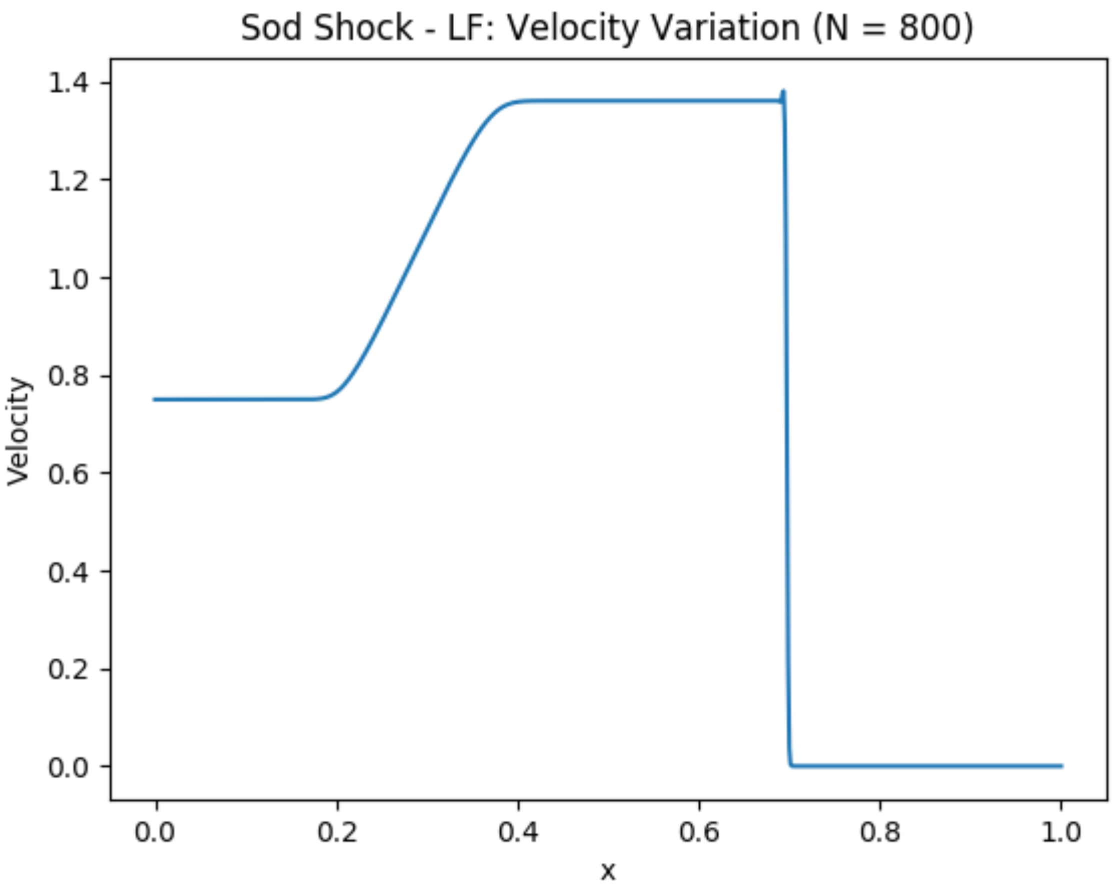


Sod Shock - LF: Velocity Variation (N = 100)
Velocity
x
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

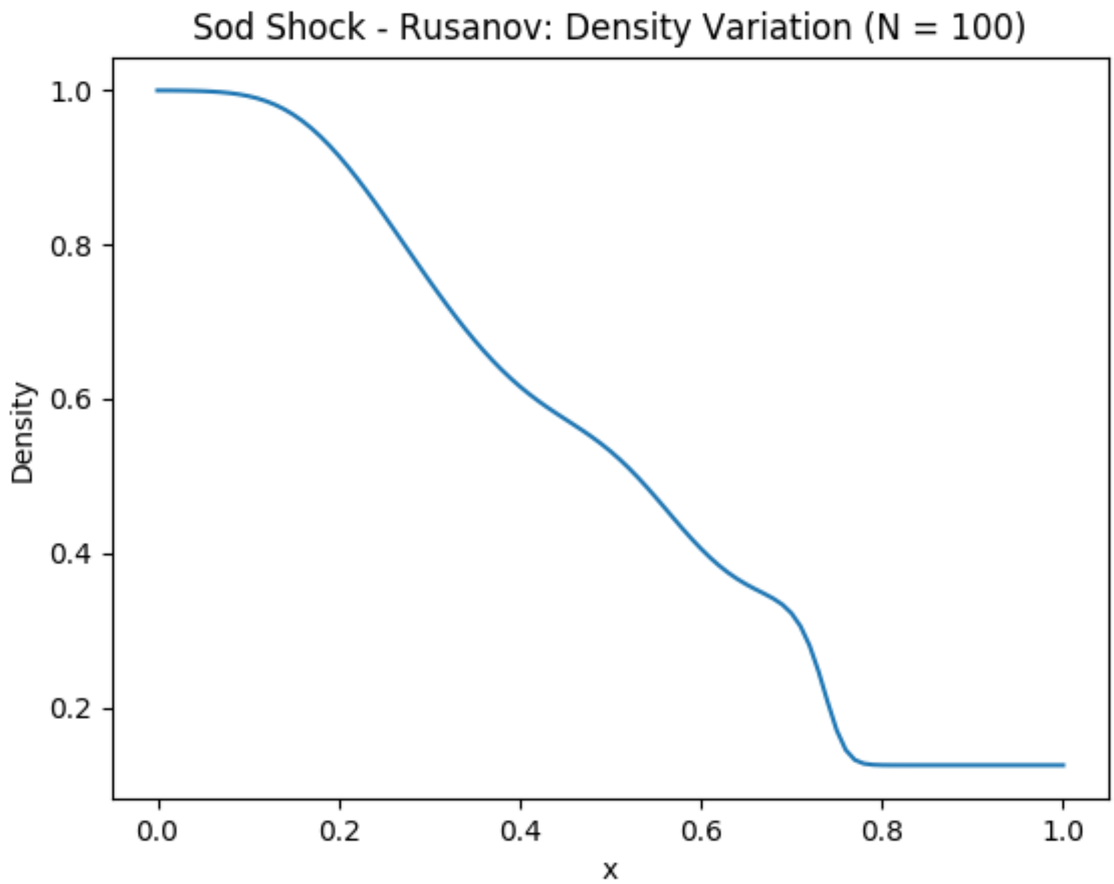


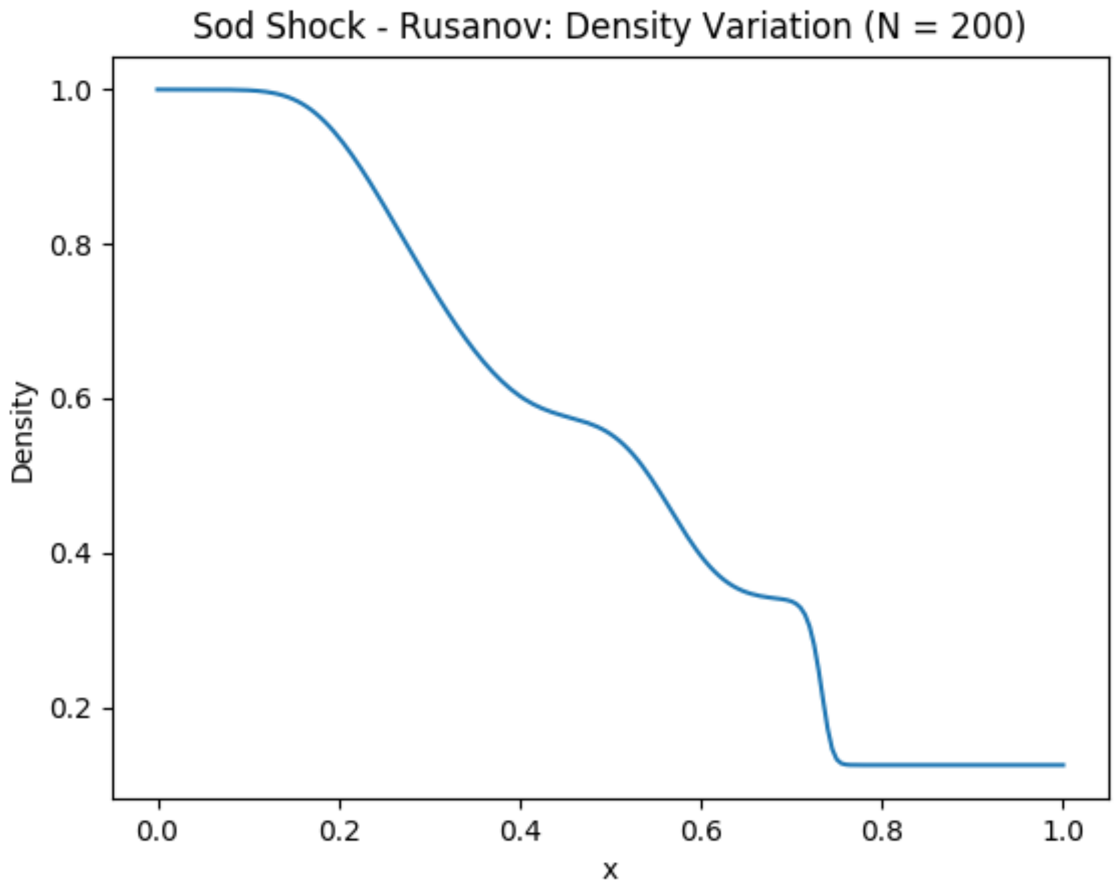
Sod Shock - LF: Velocity Variation (N = 200)
Velocity
x
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

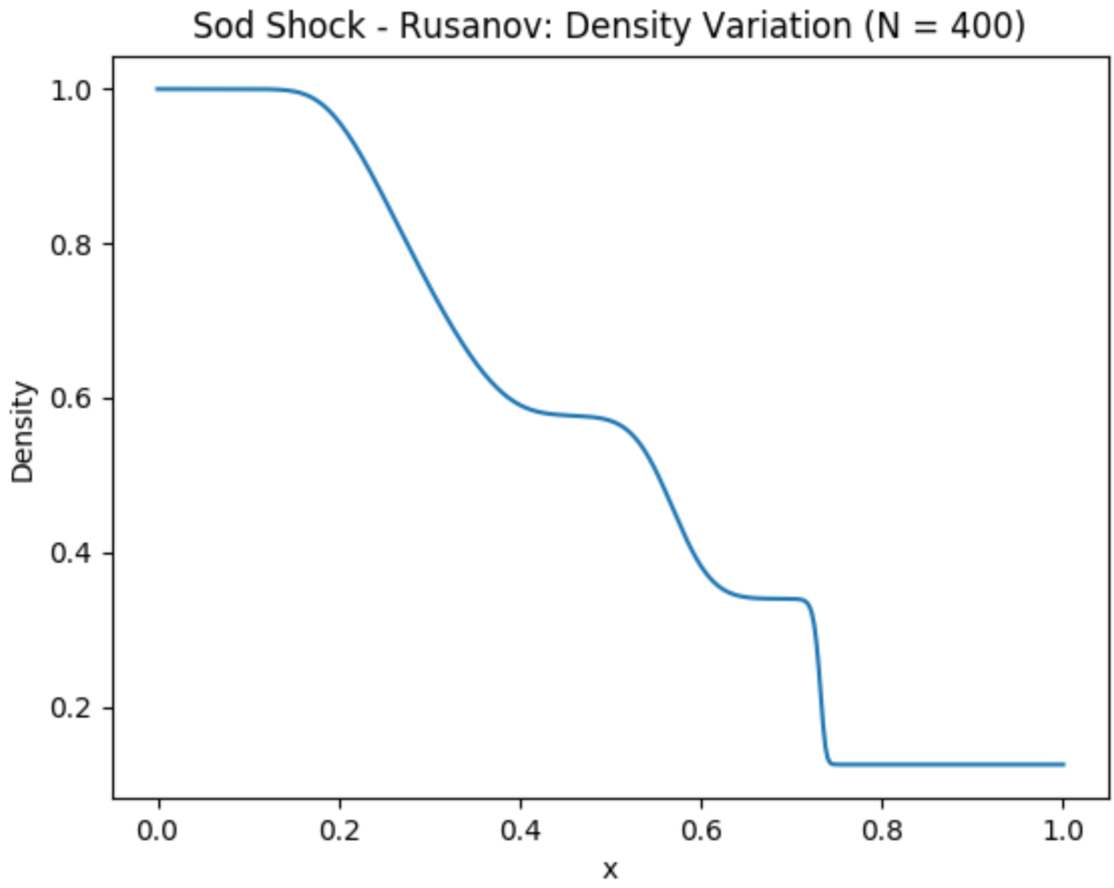

Sod Shock - LF: Velocity Variation (N = 400)
Velocity
x
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

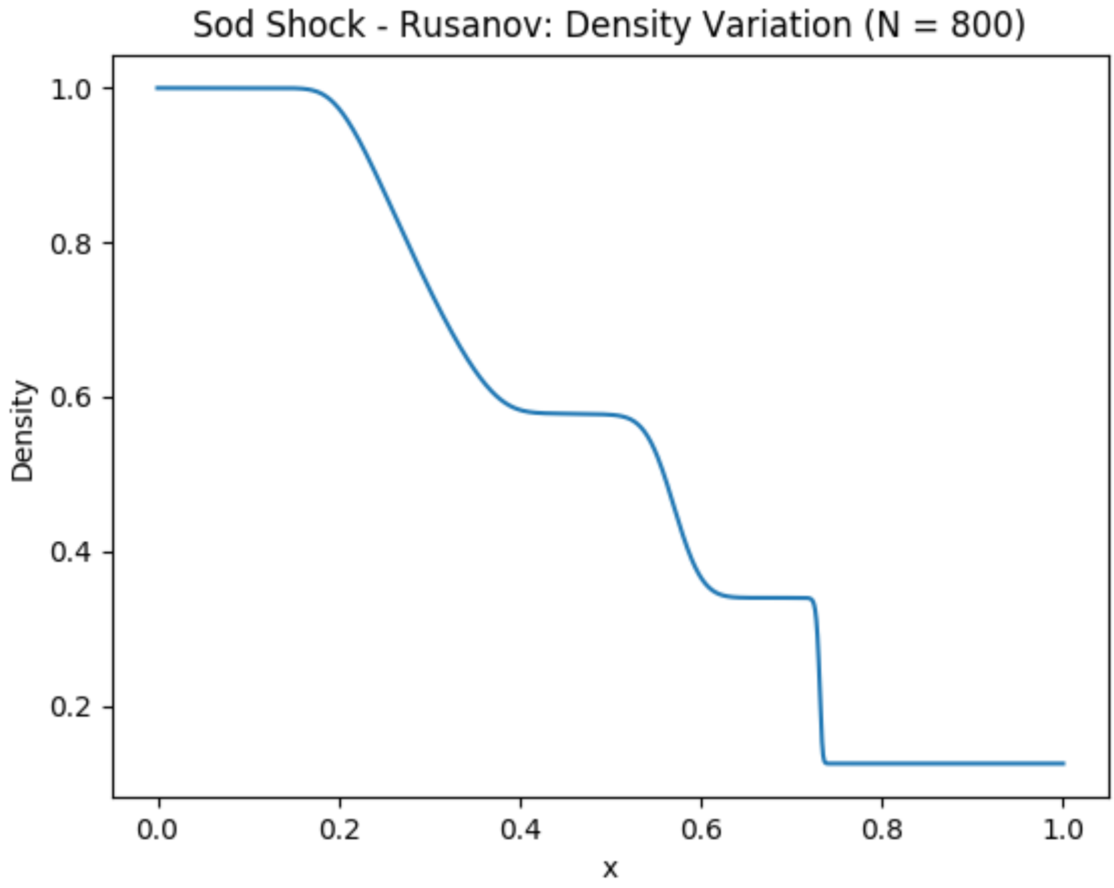

Sod Shock - LF: Velocity Variation (N = 800)
Velocity
x
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

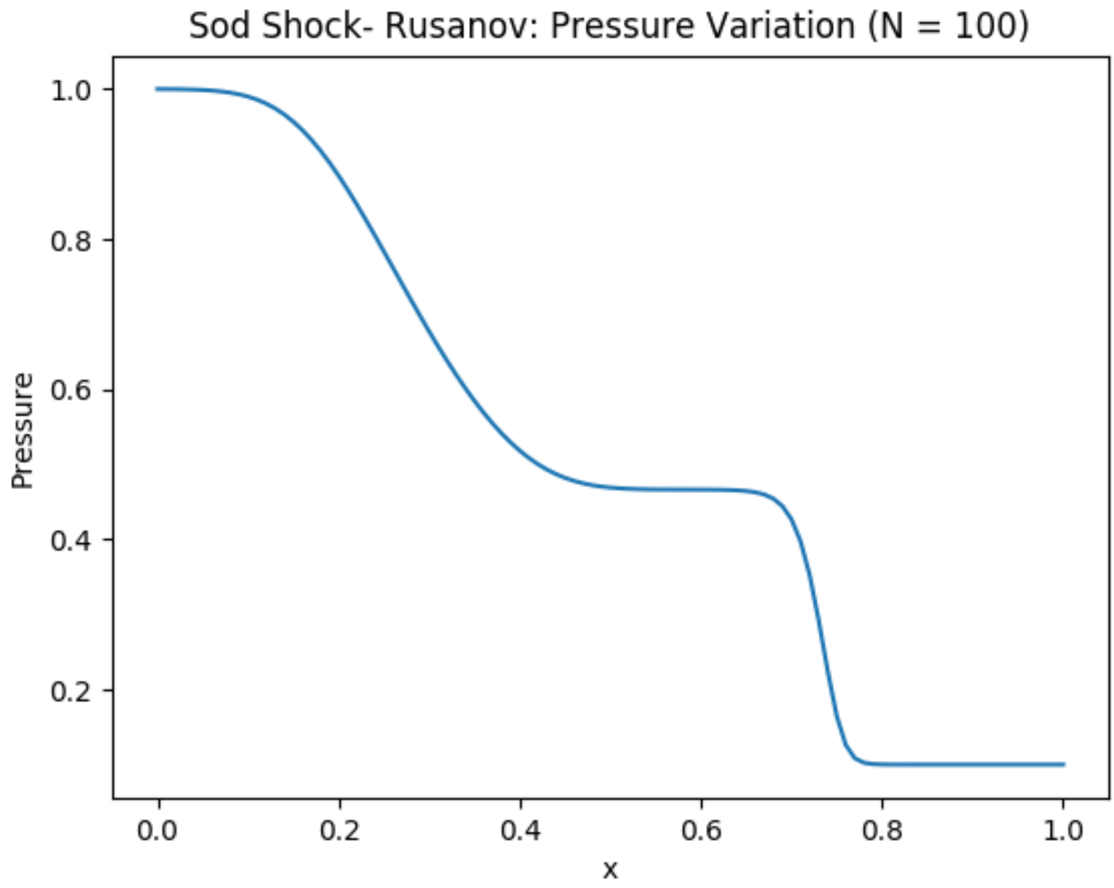
Rusanov Scheme

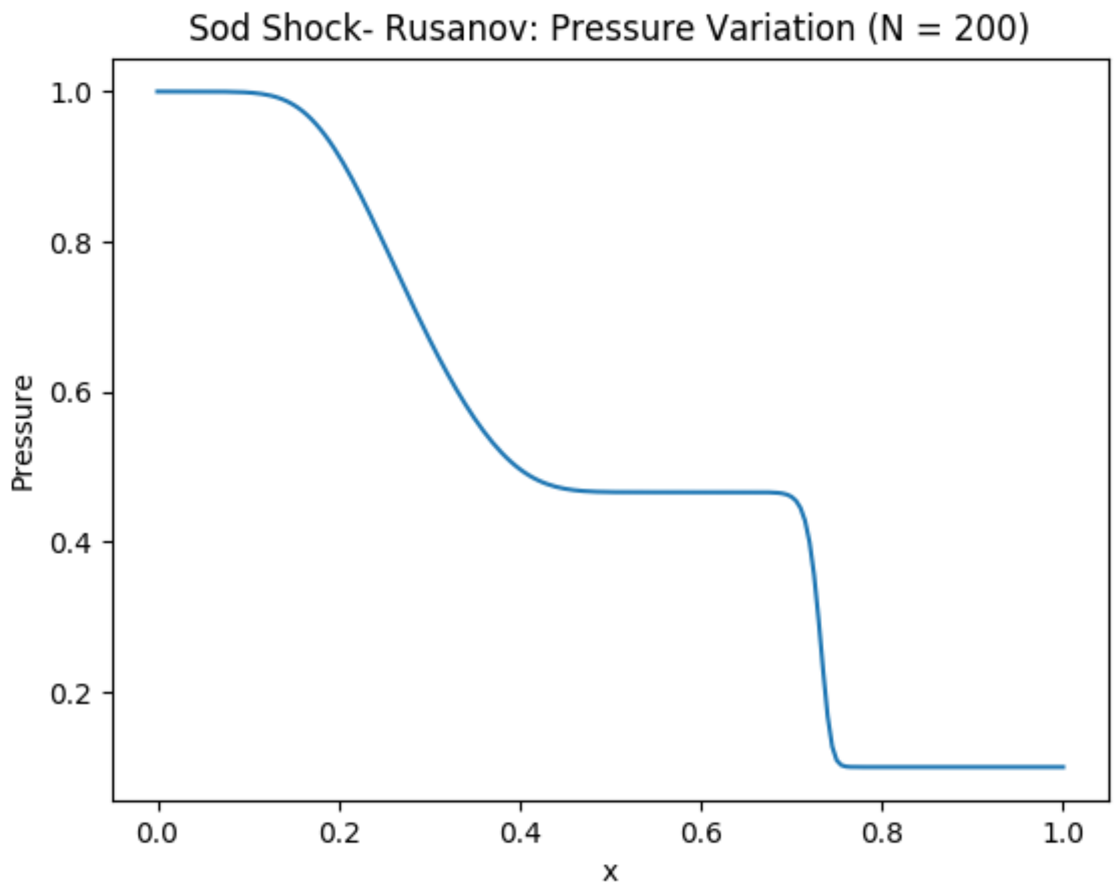


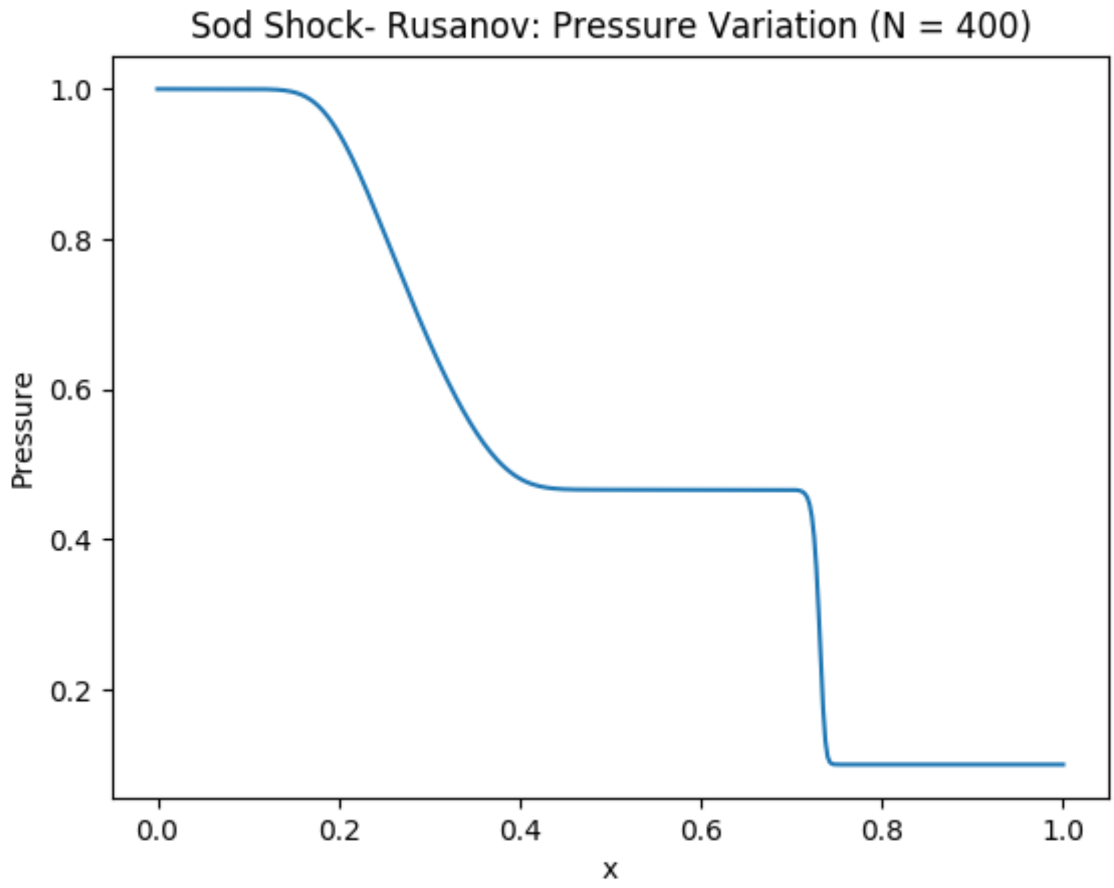


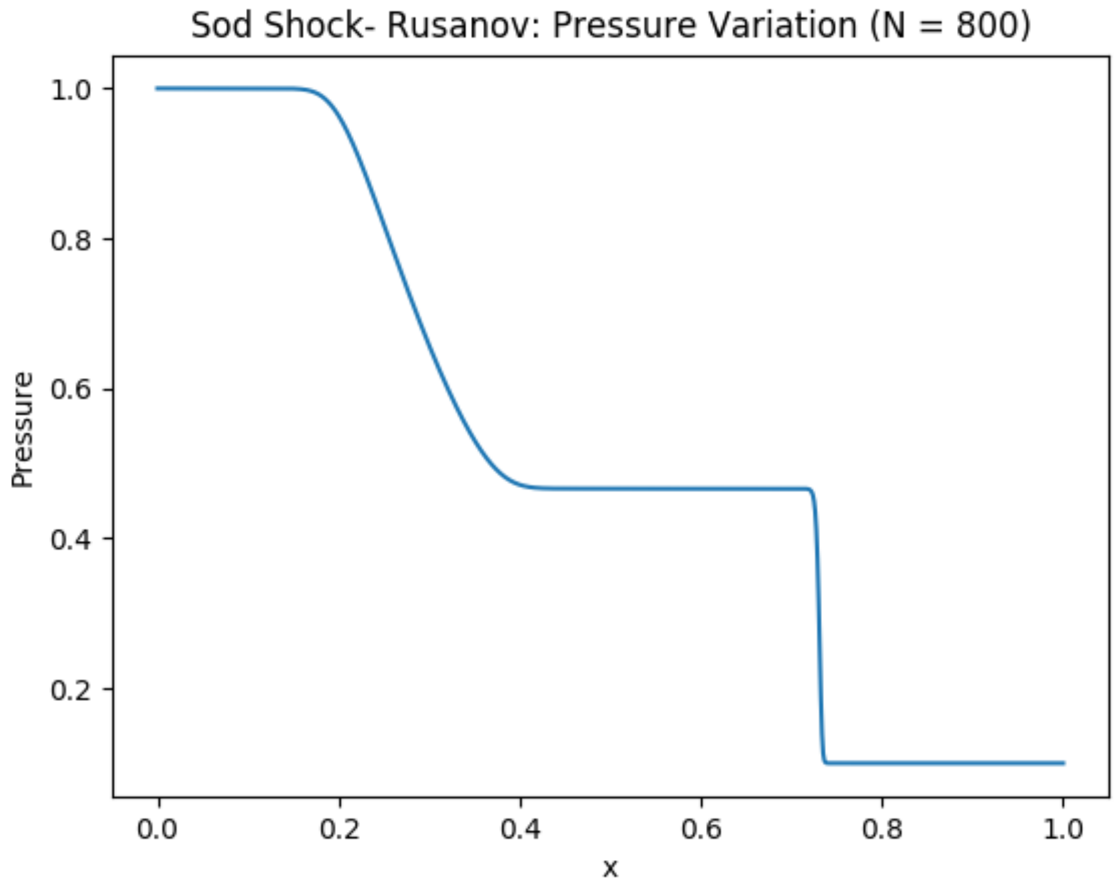

Sod Shock - Rusanov: Density Variation (N = 400)
Density
x
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

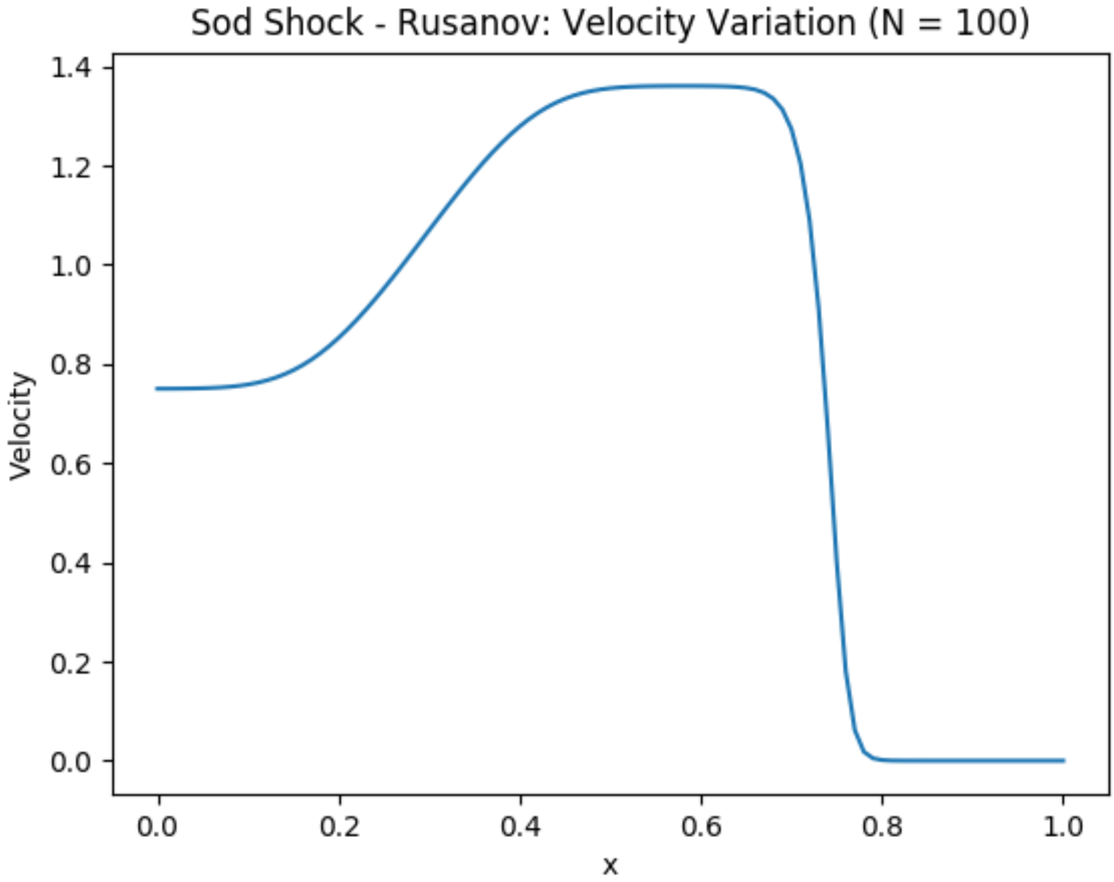

Sod Shock - Rusanov: Density Variation (N = 800)
Density
x
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

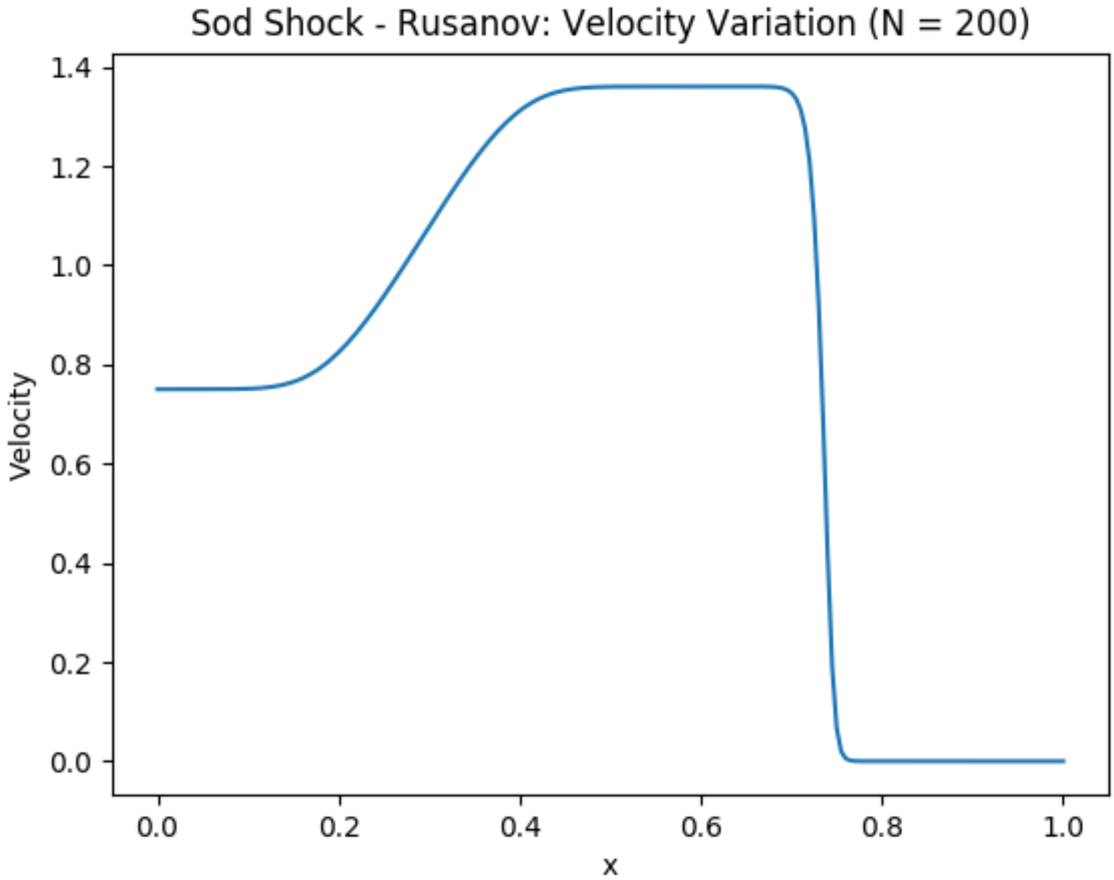

Sod Shock- Rusanov: Pressure Variation (N = 100)
Pressure
x
0.2
0.4
0.6
0.8
1.0
0.0

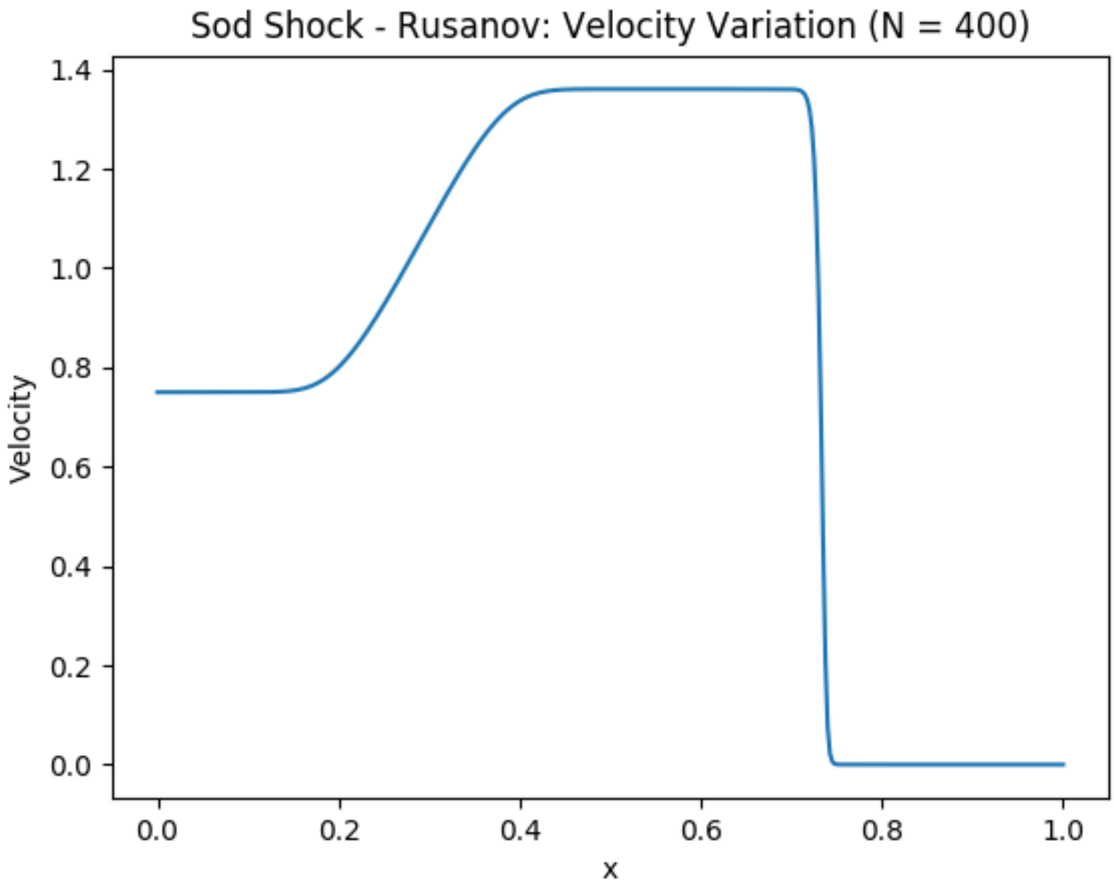

Sod Shock- Rusanov: Pressure Variation (N = 200)
Pressure
x
0.2
0.4
0.6
0.8
1.0
0.0

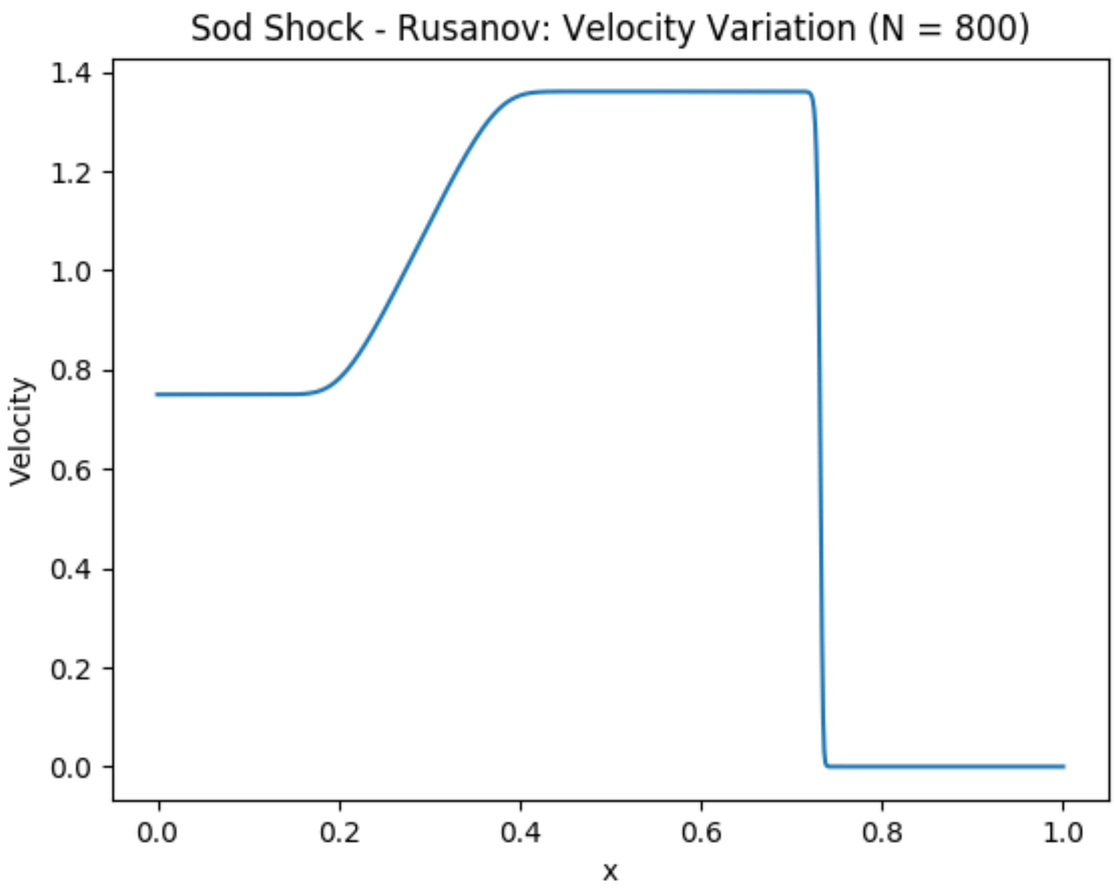

Sod Shock- Rusanov: Pressure Variation (N = 400)
Pressure
x
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0


Sod Shock- Rusanov: Pressure Variation (N = 800)
Pressure
x
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

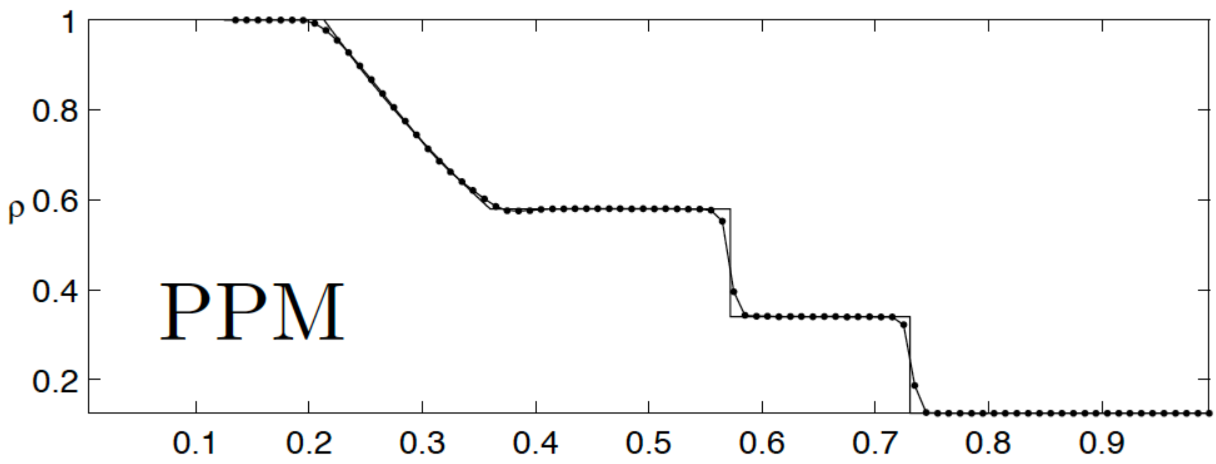

Sod Shock - Rusanov: Velocity Variation (N = 100)
Velocity
x
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4


Sod Shock - Rusanov: Velocity Variation (N = 200)
Velocity
x
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4


Sod Shock - Rusanov: Velocity Variation (N = 400)
Velocity
x
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4


Sod Shock - Rusanov: Velocity Variation (N = 800)
Velocity
x
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Exact Solution



Conclusions

As can be seen from the exact solution from the given research paper, as number of grid cells increase, the solution gets closer to the actual solution. Due to greater effects of dispersion, we see sharper discontinuities in Lax-Friedrich scheme whereas the Rusanov solution shows much more smoother variation. Both the schemes resolve shock well without oscillations.

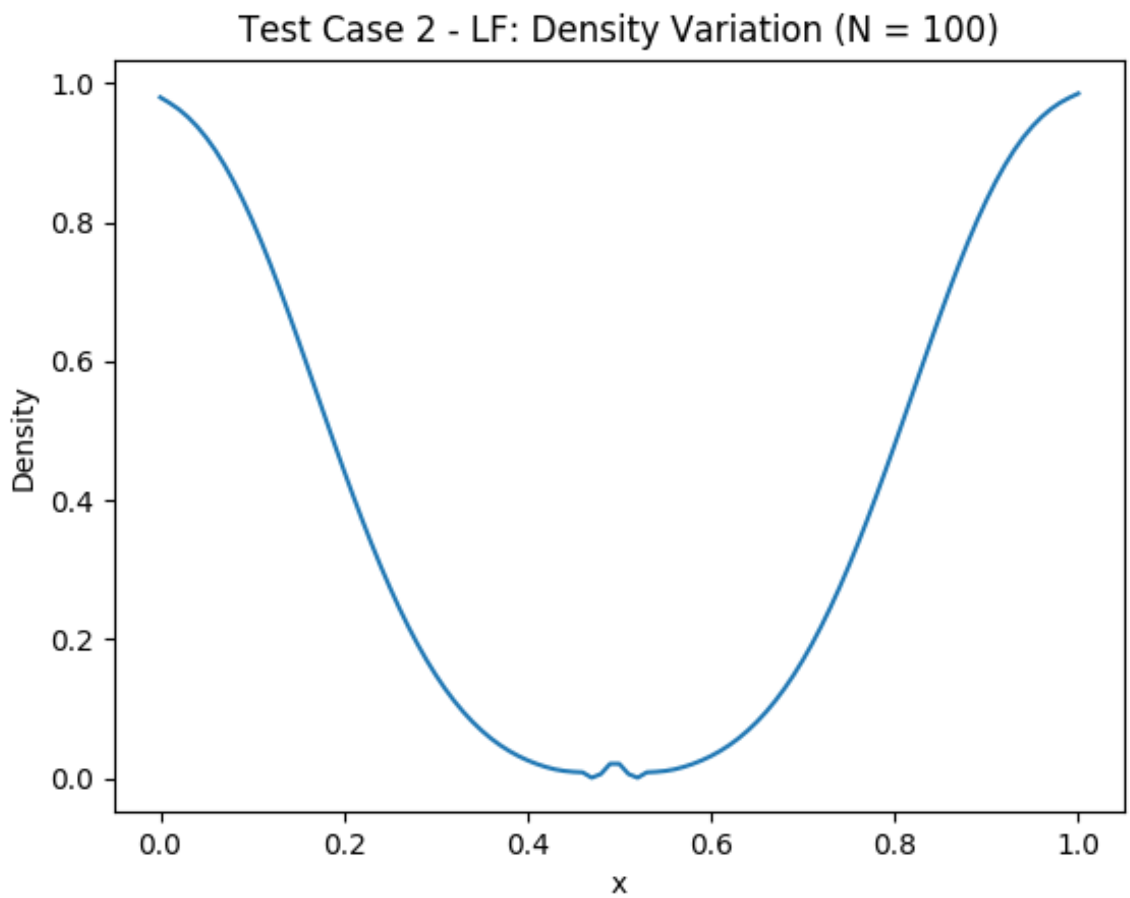
From the analytical solution, we can see 3 dips in density trends which are also seen in my solution. The convergence of velocity and pressure graphs with number of grid cells is faster as compared to the density variation.

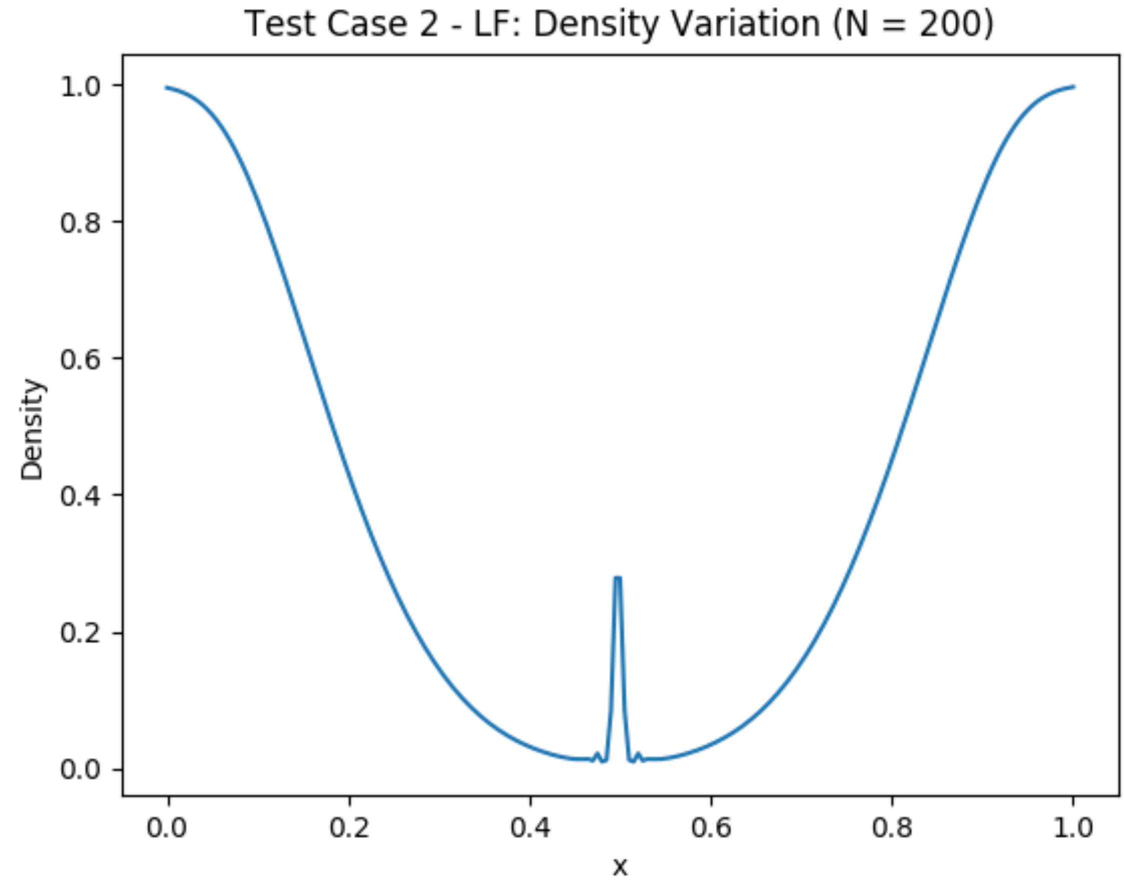
On increasing grid cells, solution tends to converge to actual solution.

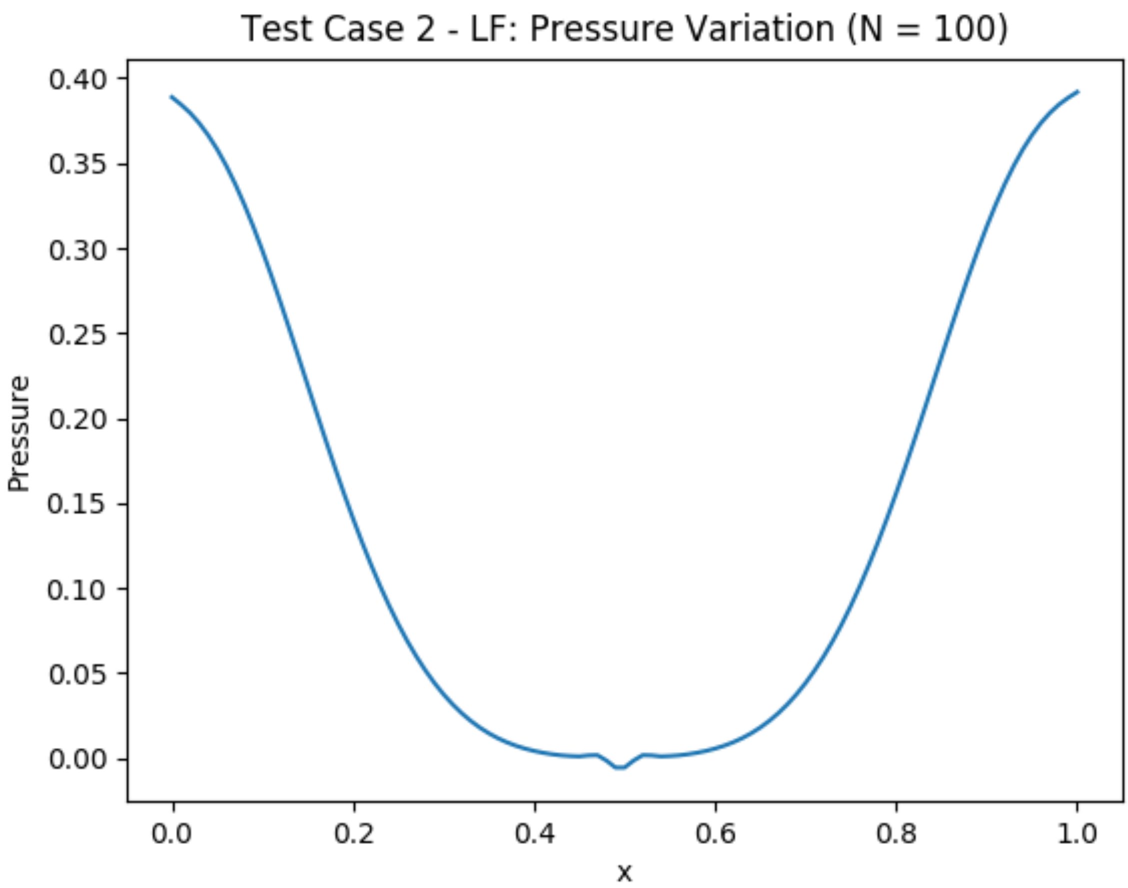
CFD Assignment 5 - Bonus Questions

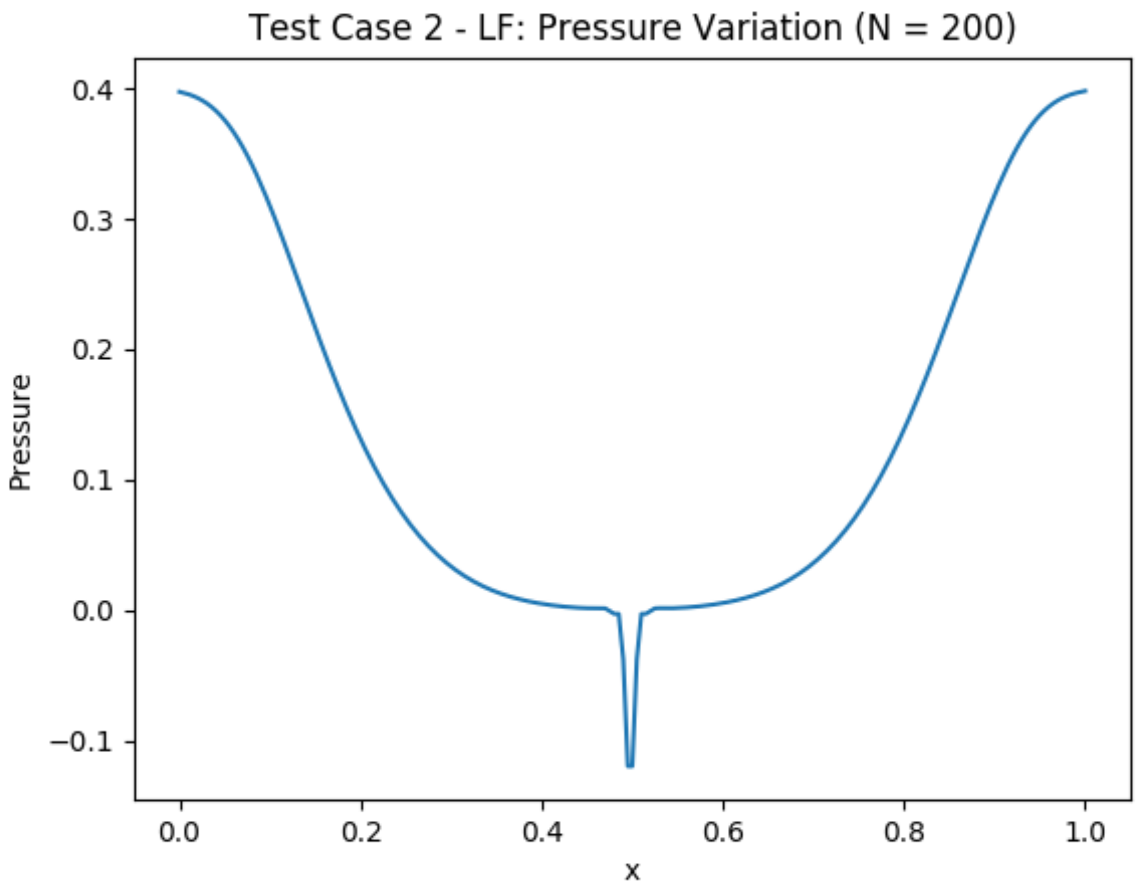
Plots and Results

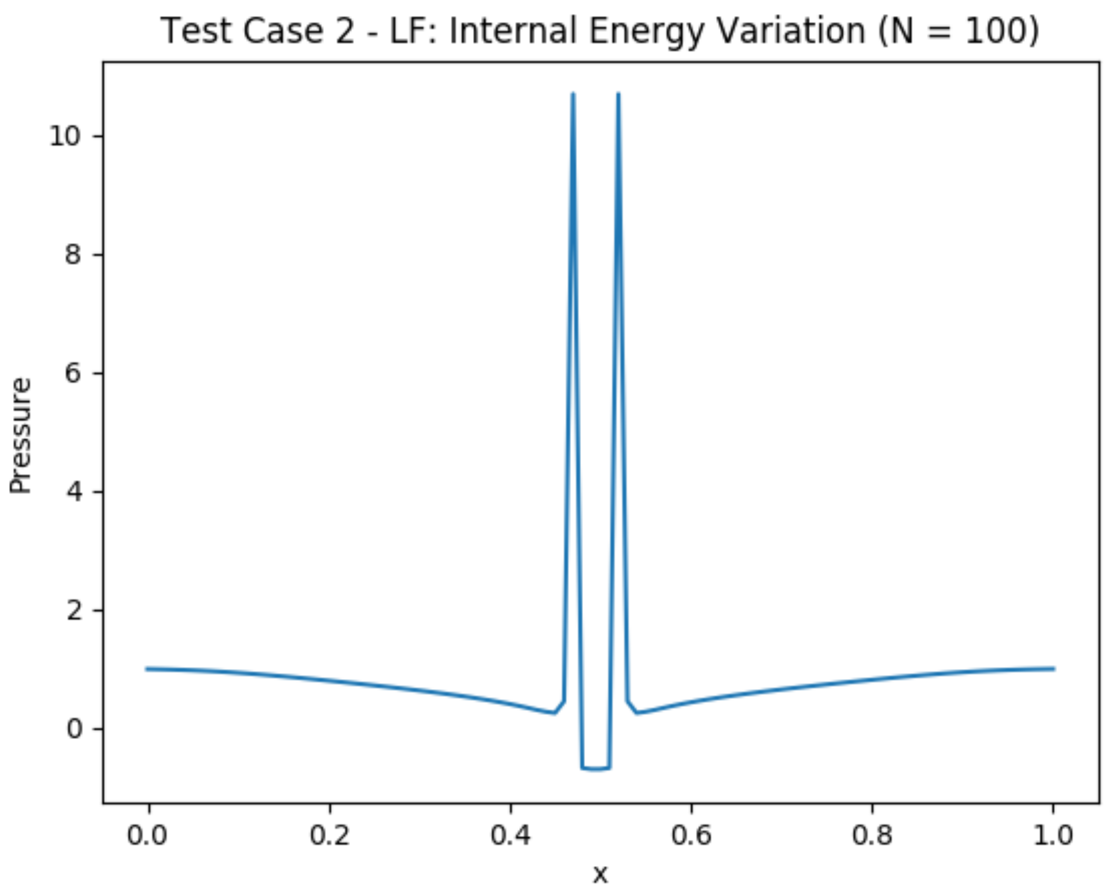
Test Case 2

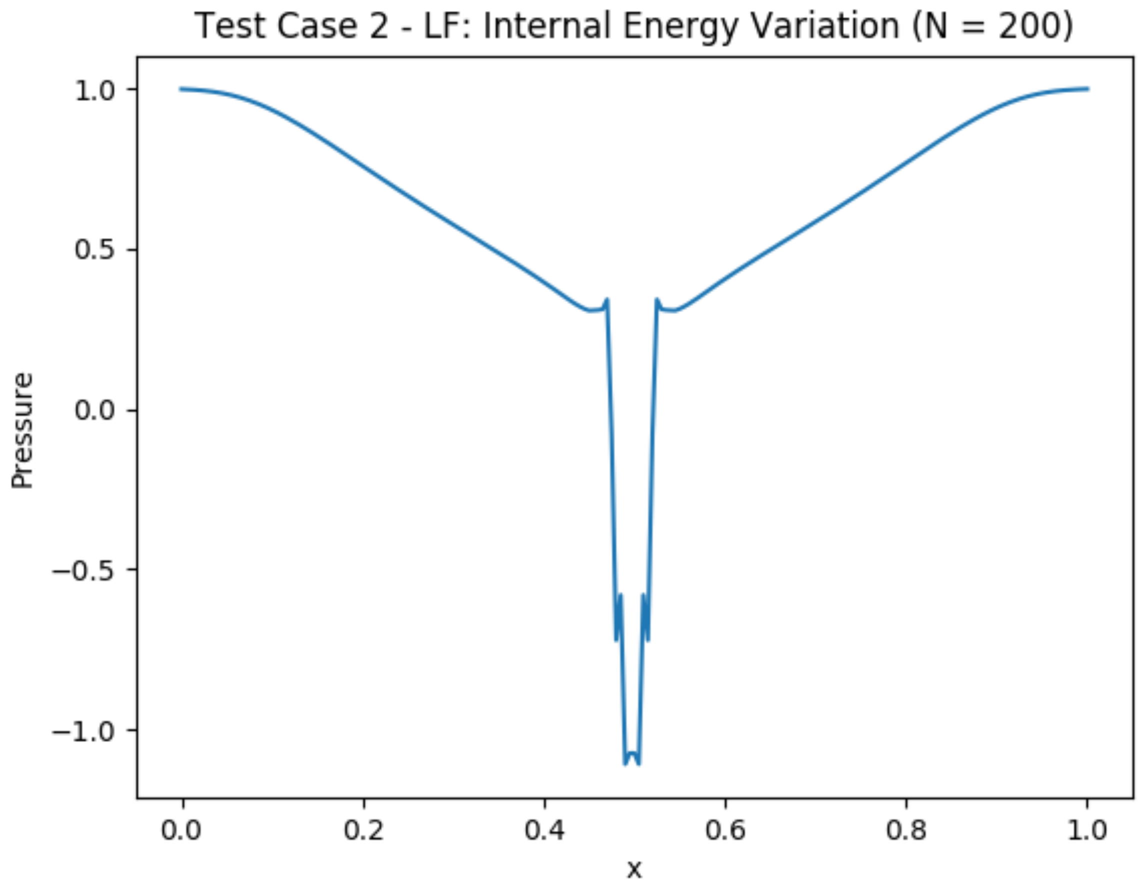



Test Case 2 - LF: Density Variation (N = 200)
Density
x
0.0
0.2
0.4
0.6
0.8
1.0

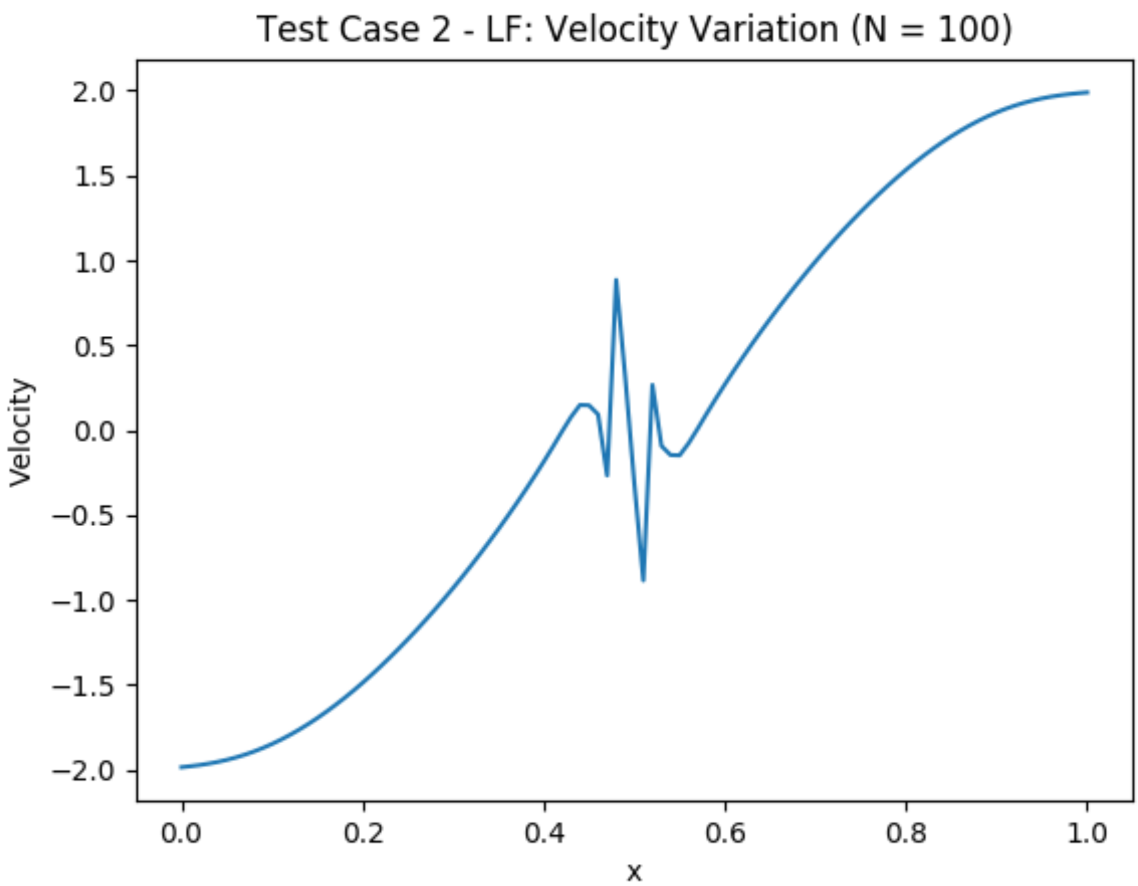

Test Case 2 - LF: Pressure Variation (N = 100)
Pressure
x
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.0
0.2
0.4
0.6
0.8
1.0


Test Case 2 - LF: Pressure Variation (N = 200)
Pressure
x
0.4
0.3
0.2
0.1
0.0
−0.1
0.0
0.2
0.4
0.6
0.8
1.0

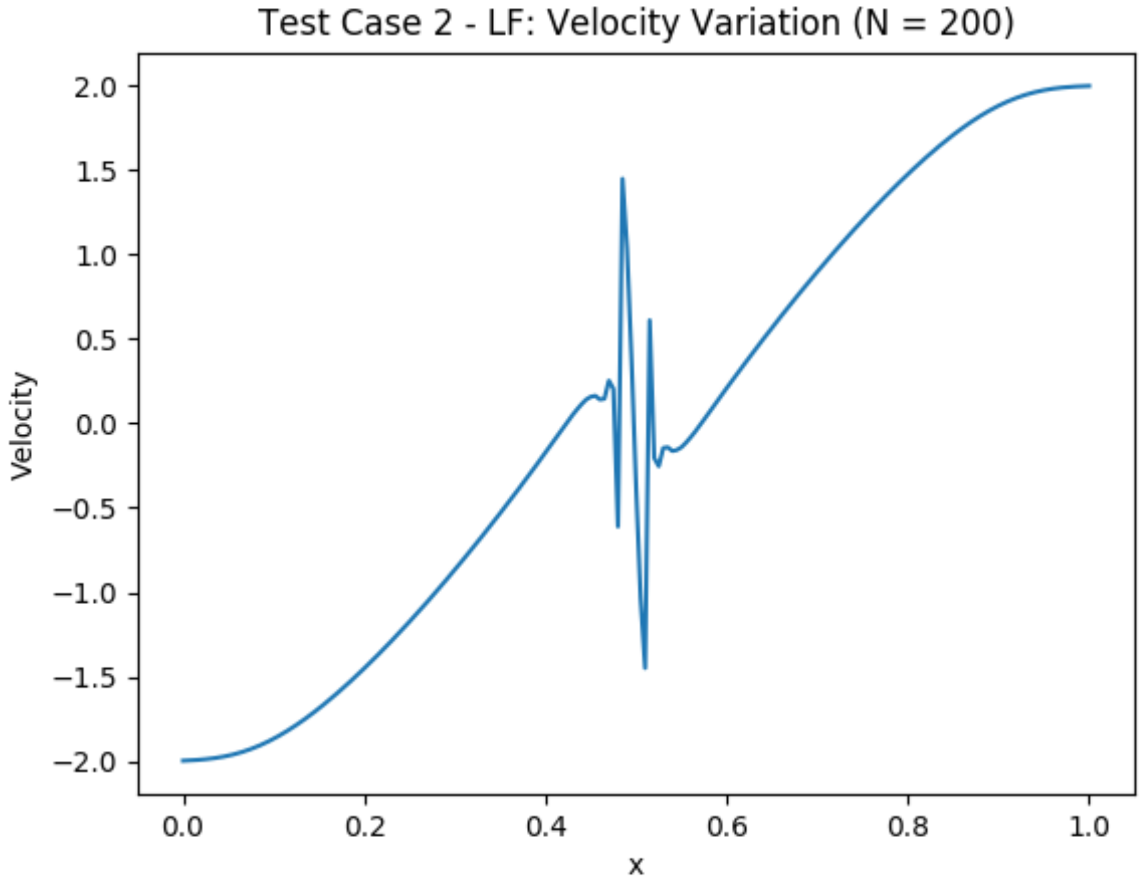

Test Case 2 - LF: Internal Energy Variation (N = 100)
Pressure
x
10
8
6
4
2
0
0.0
0.2
0.4
0.6
0.8
1.0



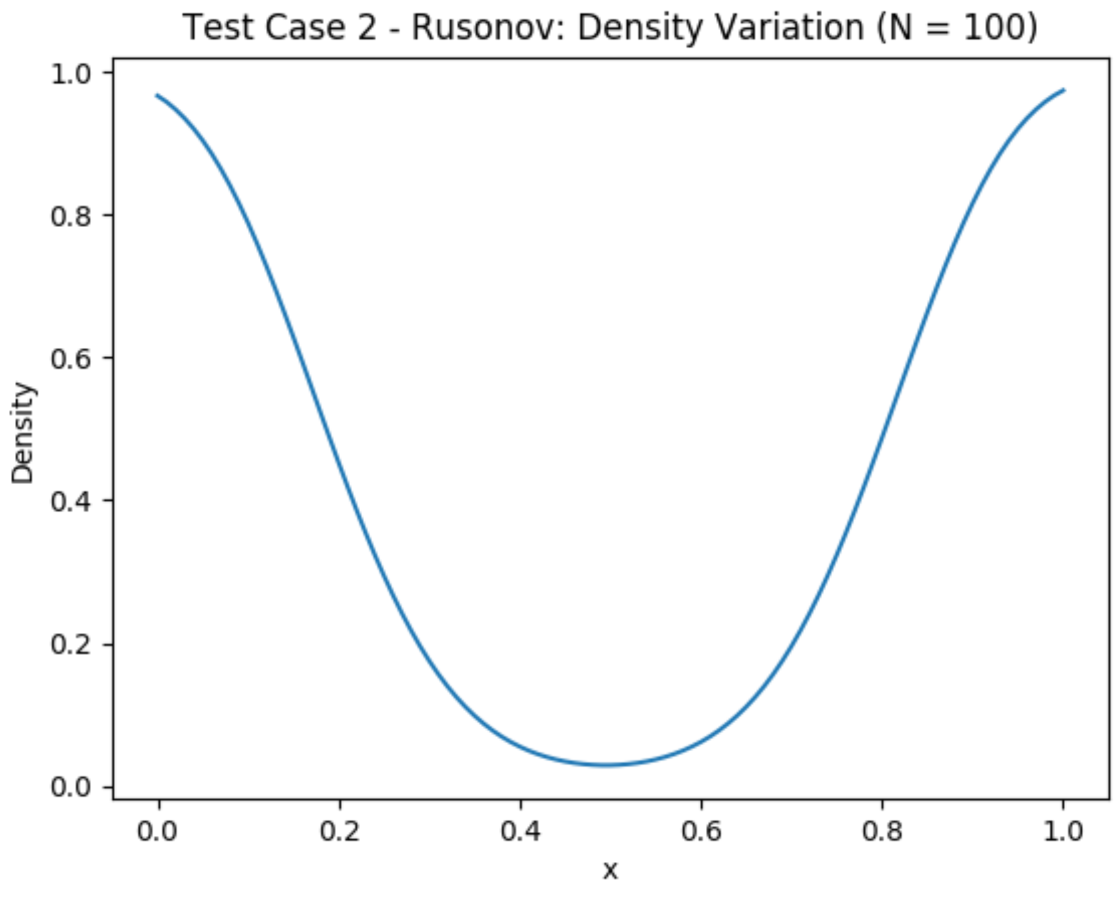
Test Case 2 - LF: Internal Energy Variation (N = 200)
Pressure
x
1.0
0.5
0.0
−0.5
−1.0
0.0
0.2
0.4
0.6
0.8
1.0

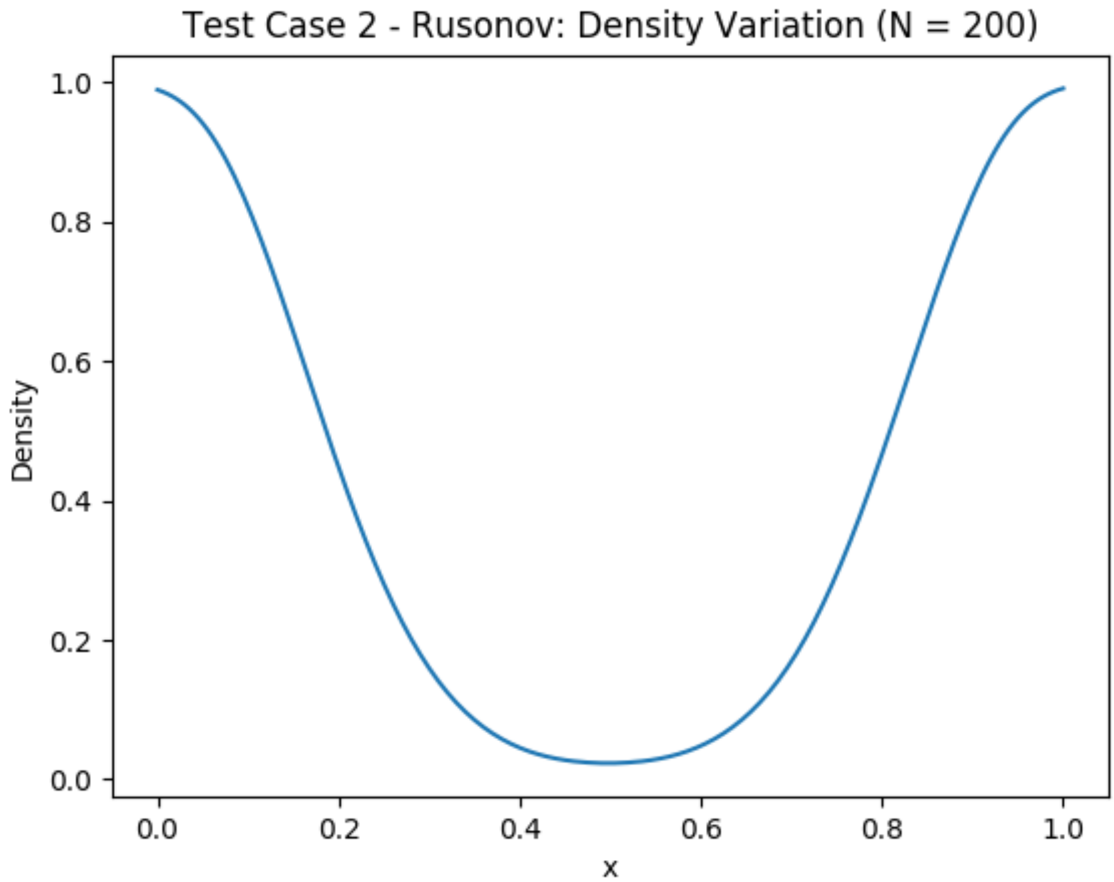


Test Case 2 - LF: Velocity Variation (N = 100)
Velocity
x
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0.0
0.2
0.4
0.6
0.8
1.0

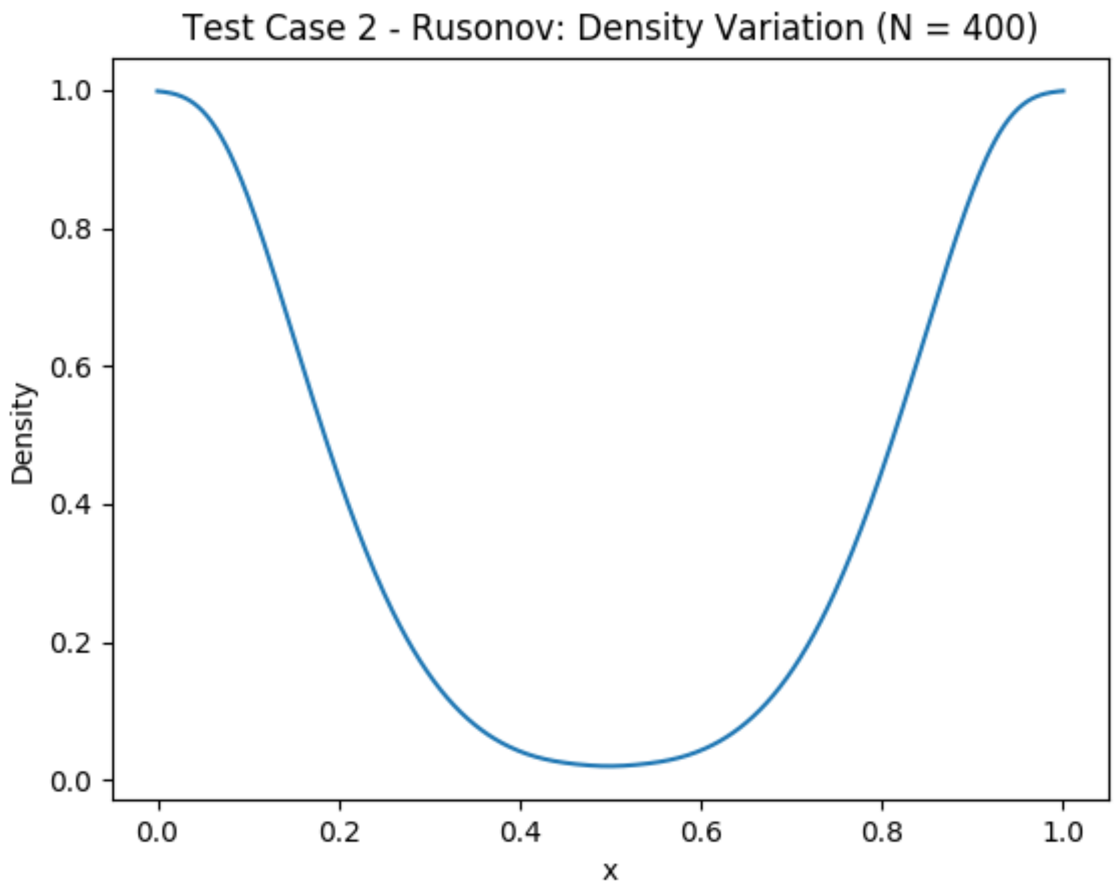


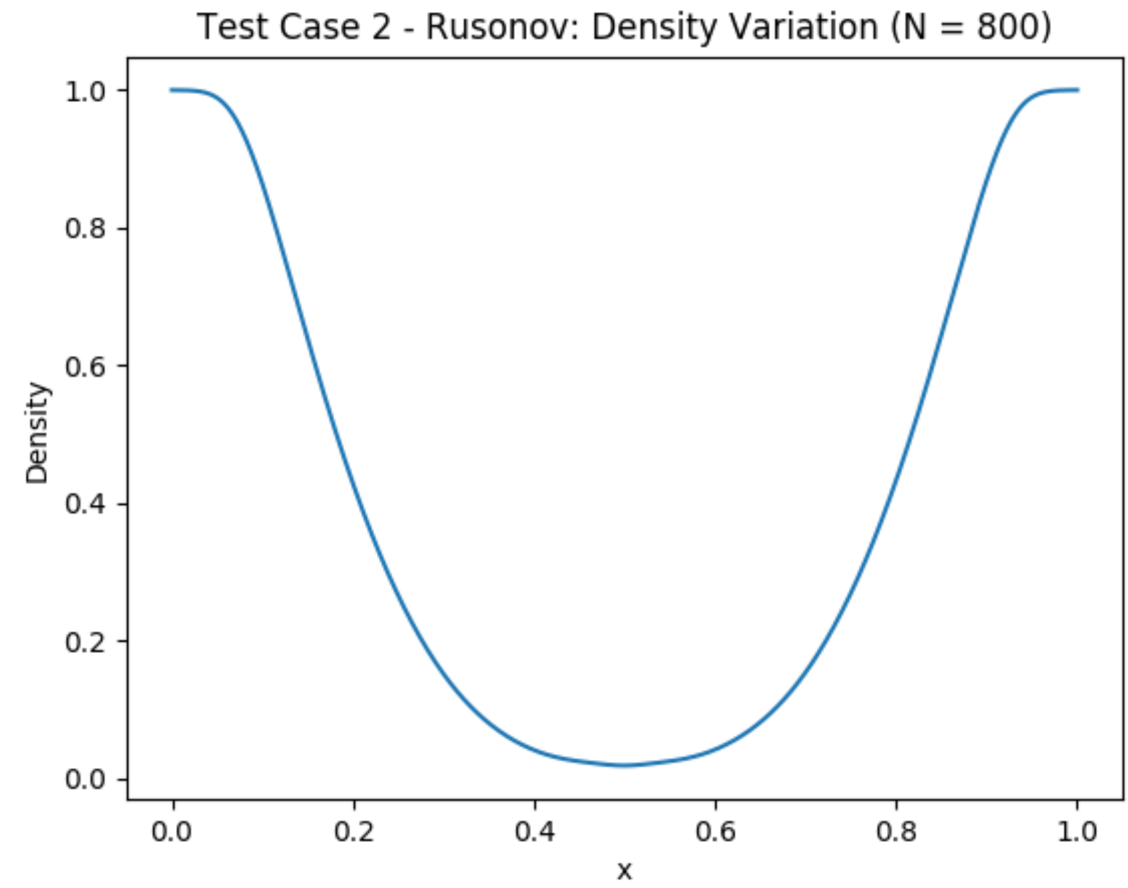
For Rusanov scheme:

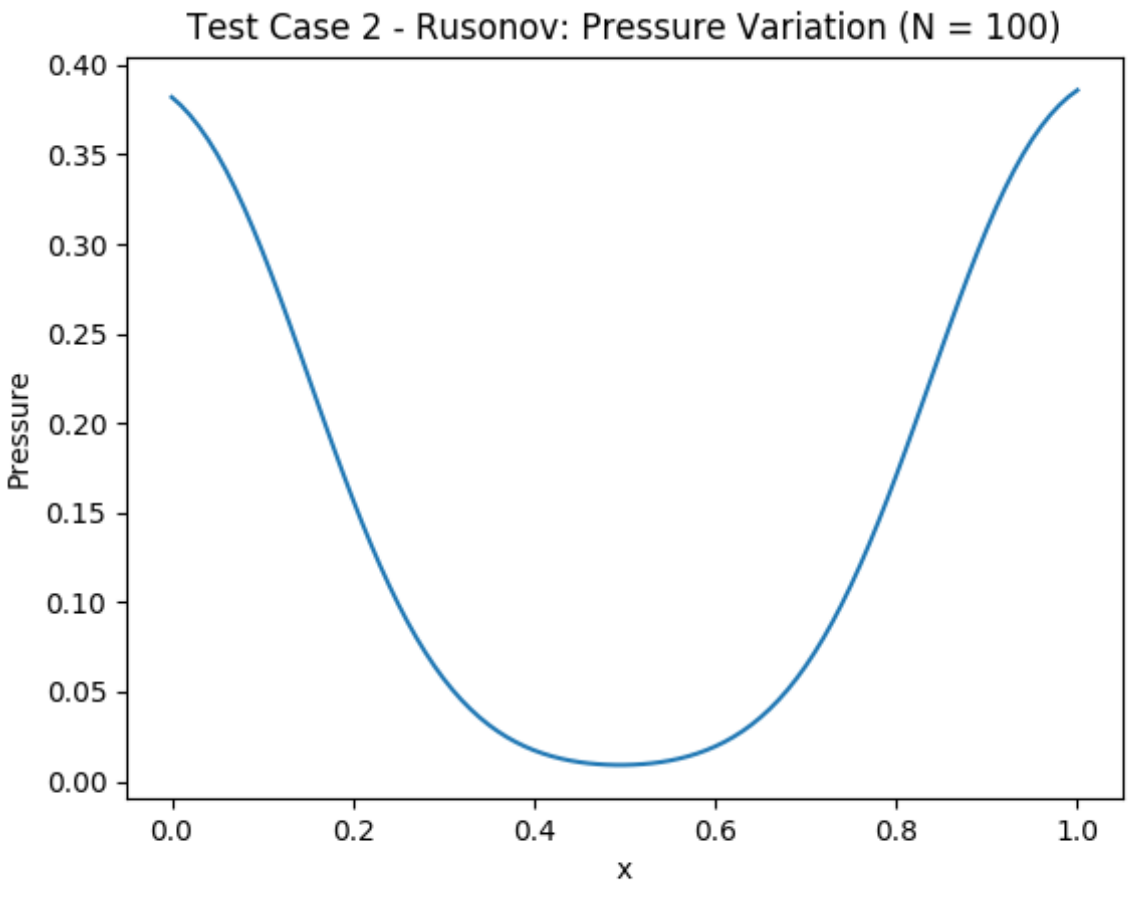


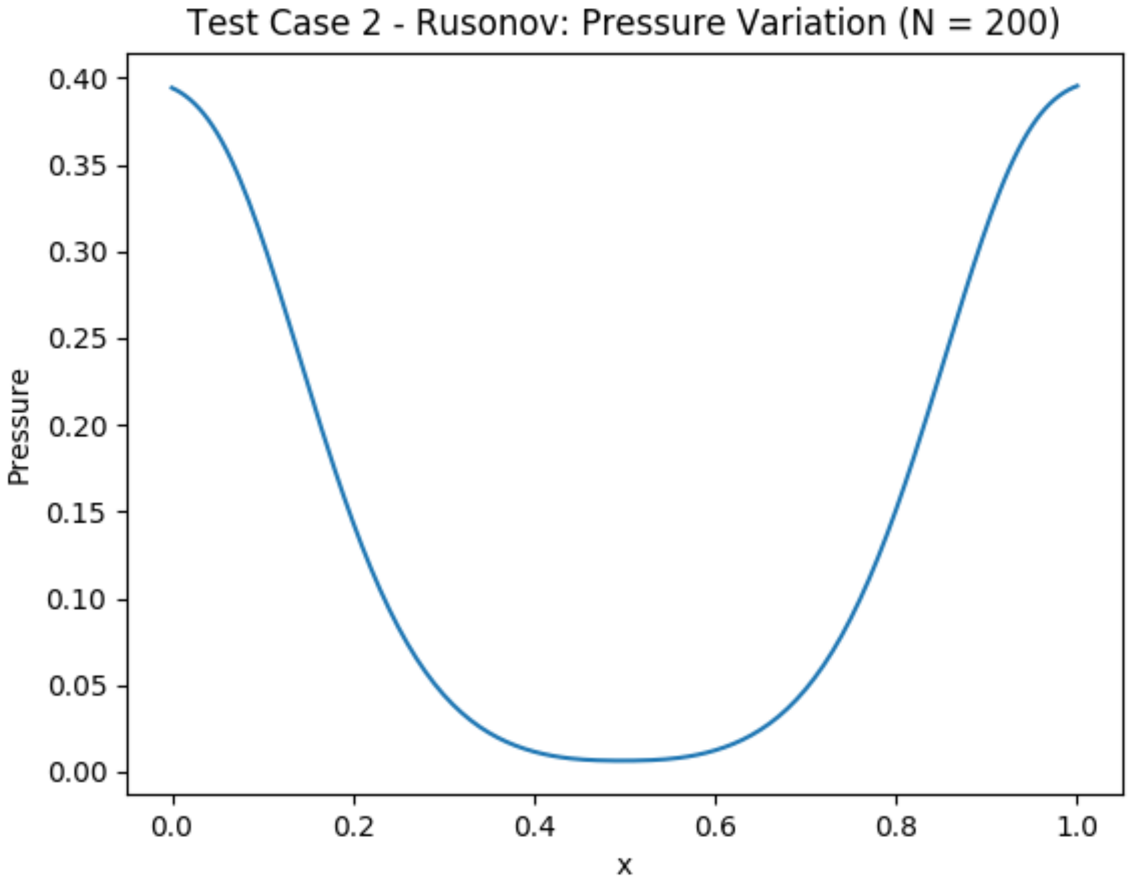


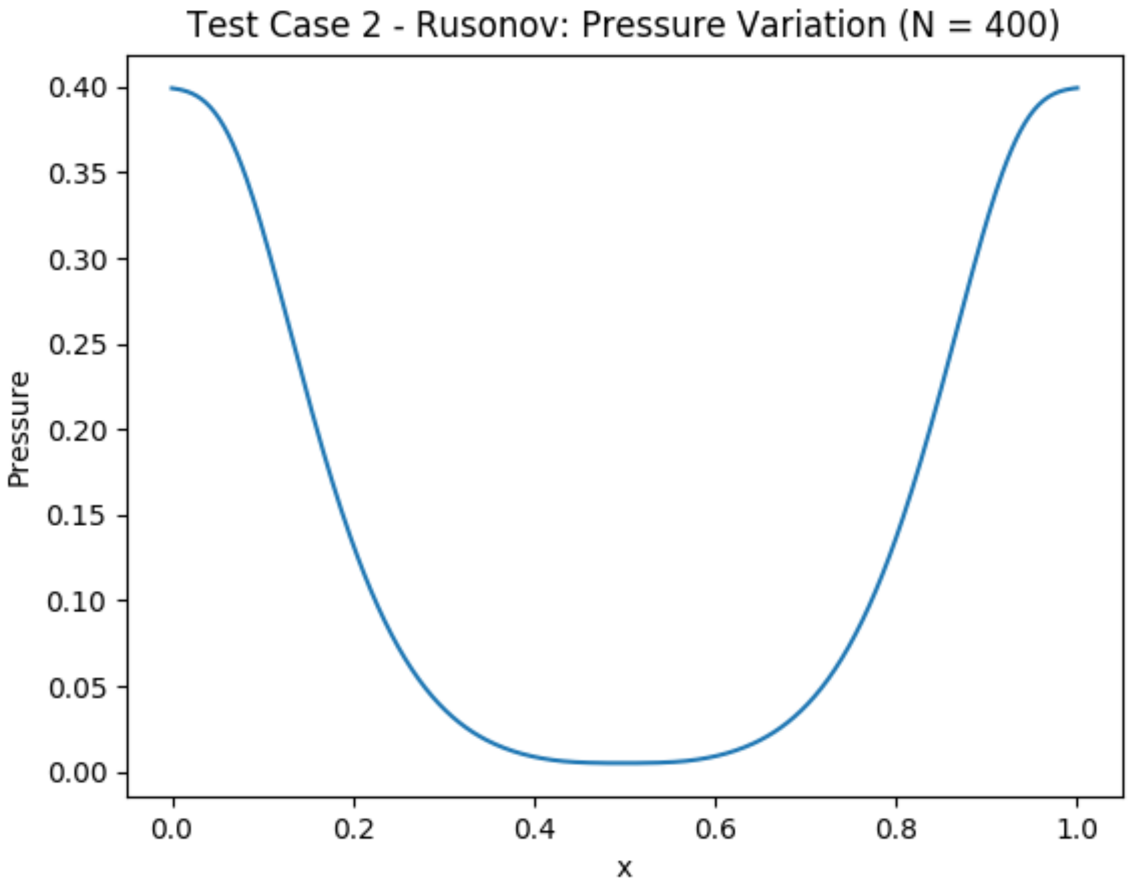
Rusanov Scheme

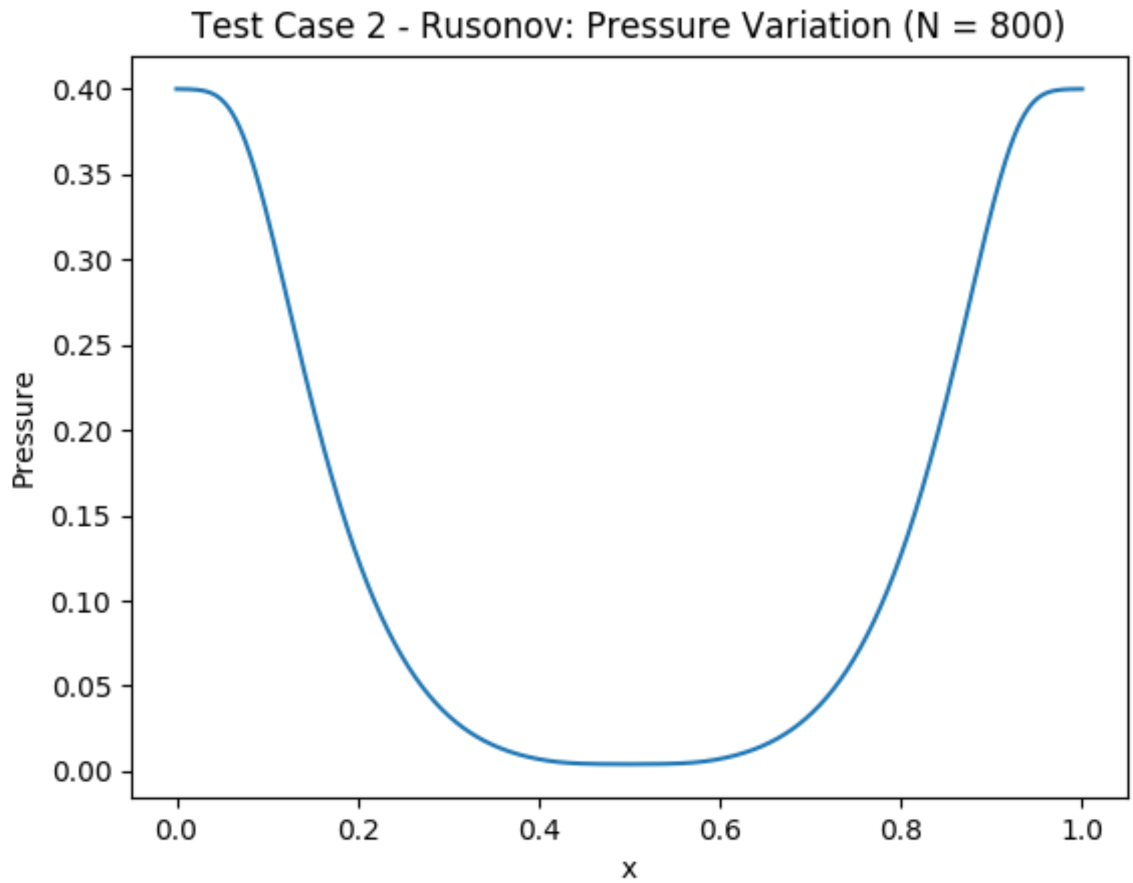


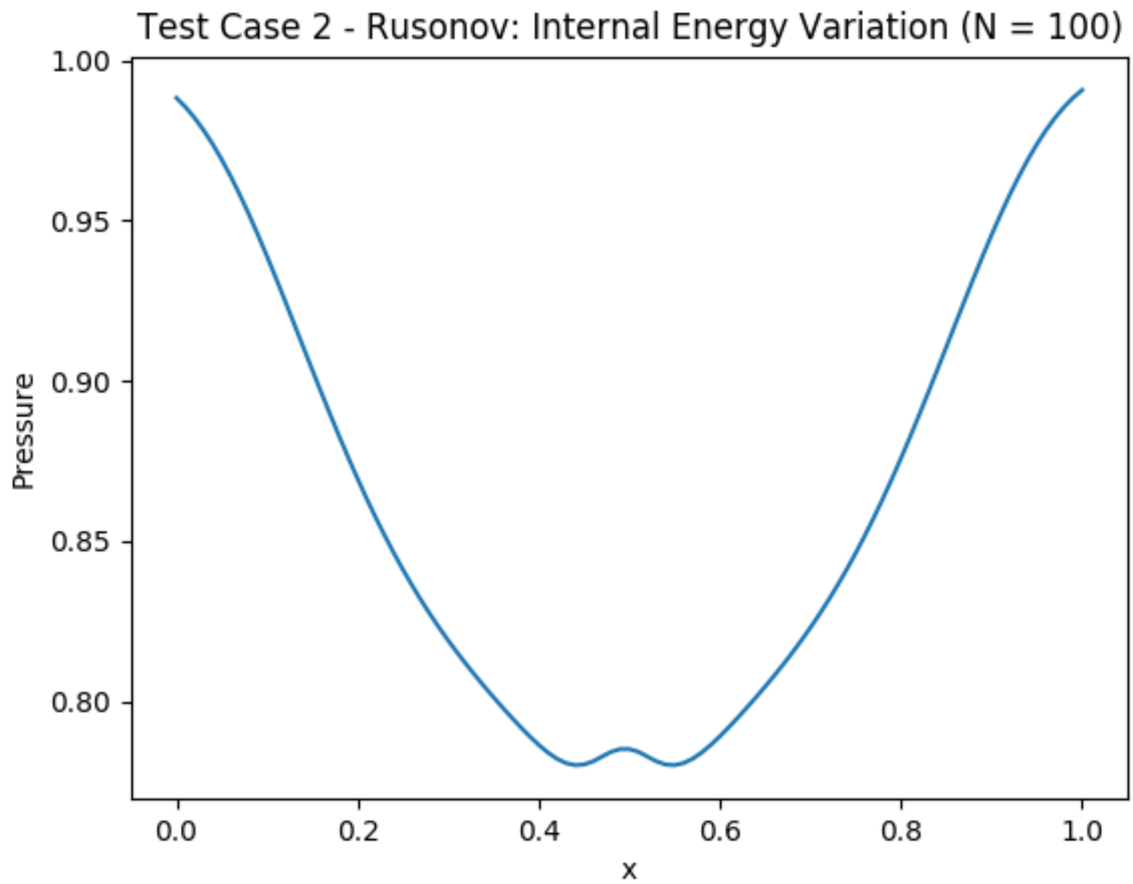

Test Case 2 - Rusonov: Density Variation (N = 800)
Density
x
0.0
0.2
0.4
0.6
0.8
1.0

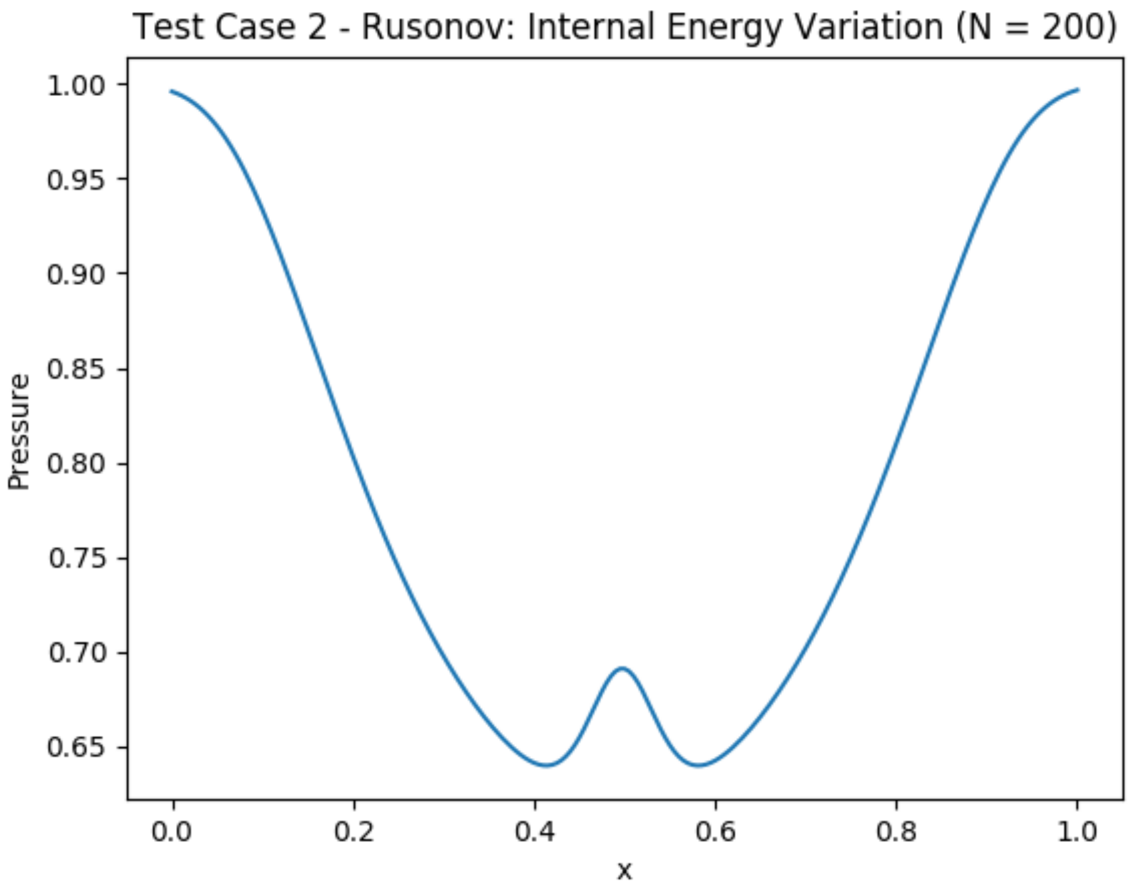

Test Case 2 - Rusonov: Pressure Variation (N = 100)
Pressure
x
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.0
0.2
0.4
0.6
0.8
1.0

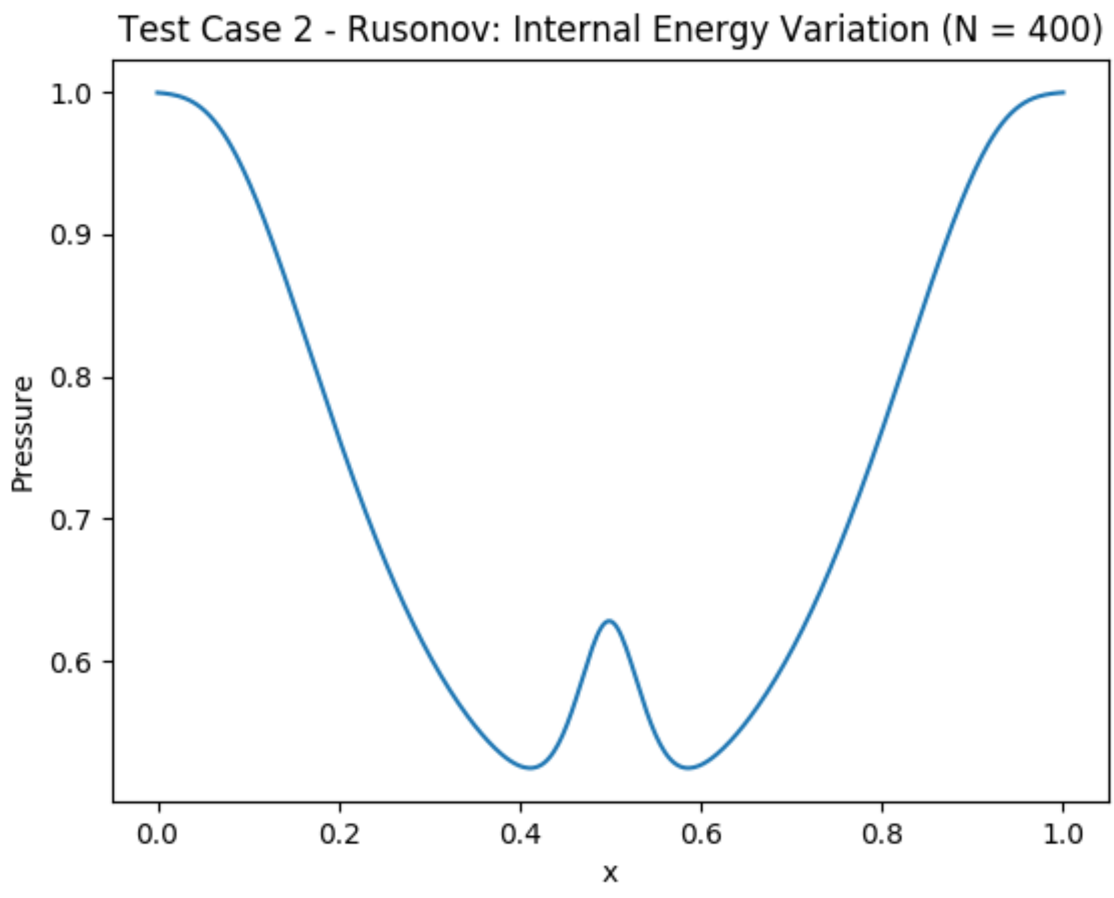

Test Case 2 - Rusonov: Pressure Variation (N = 200)
Pressure
x
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.0
0.2
0.4
0.6
0.8
1.0

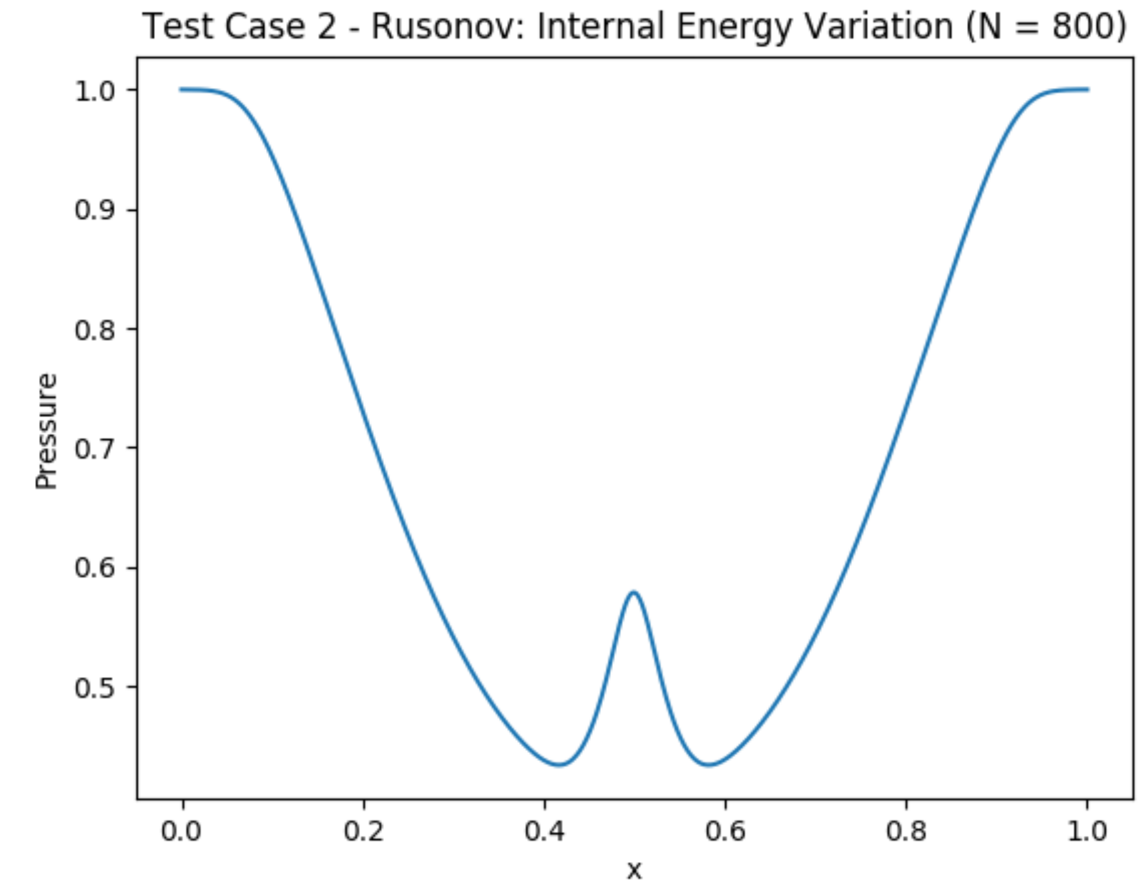

Test Case 2 - Rusonov: Pressure Variation (N = 400)
Pressure
x
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.0
0.2
0.4
0.6
0.8
1.0

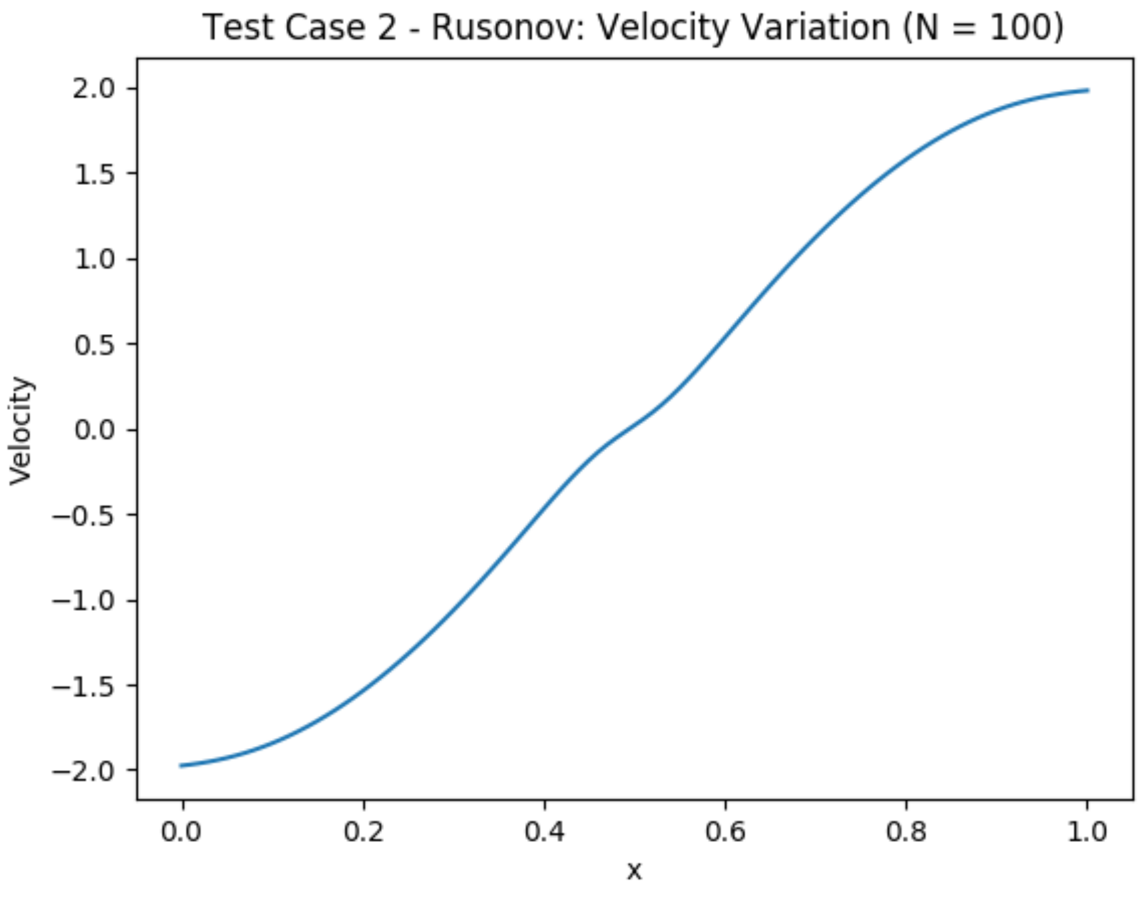

Test Case 2 - Rusonov: Pressure Variation (N = 800)
Pressure
x
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0
0.2
0.4
0.6
0.8
1.0


Test Case 2 - Rusonov: Internal Energy Variation (N = 100)
Pressure
x
1.00
0.95
0.90
0.85
0.80
0.0
0.2
0.4
0.6
0.8
1.0

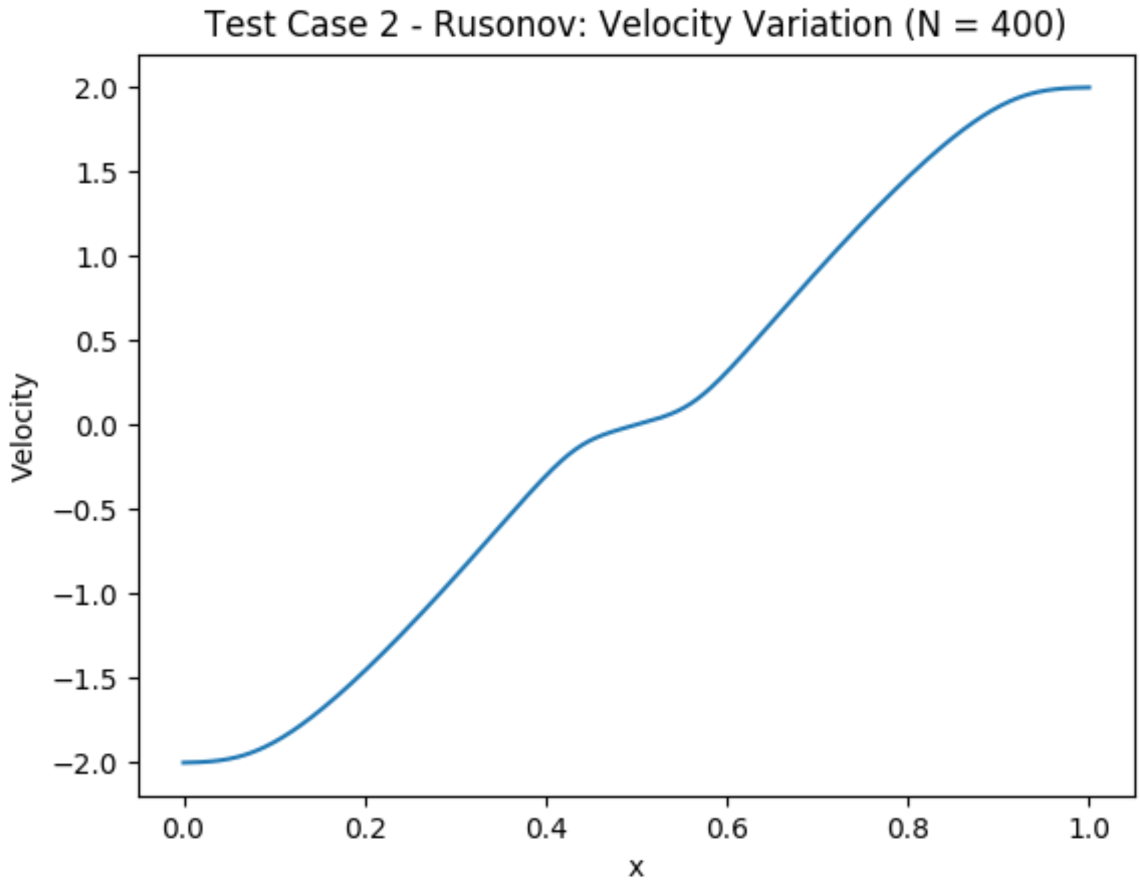

Test Case 2 - Rusonov: Internal Energy Variation (N = 200)
Pressure
x
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.0
0.2
0.4
0.6
0.8
1.0

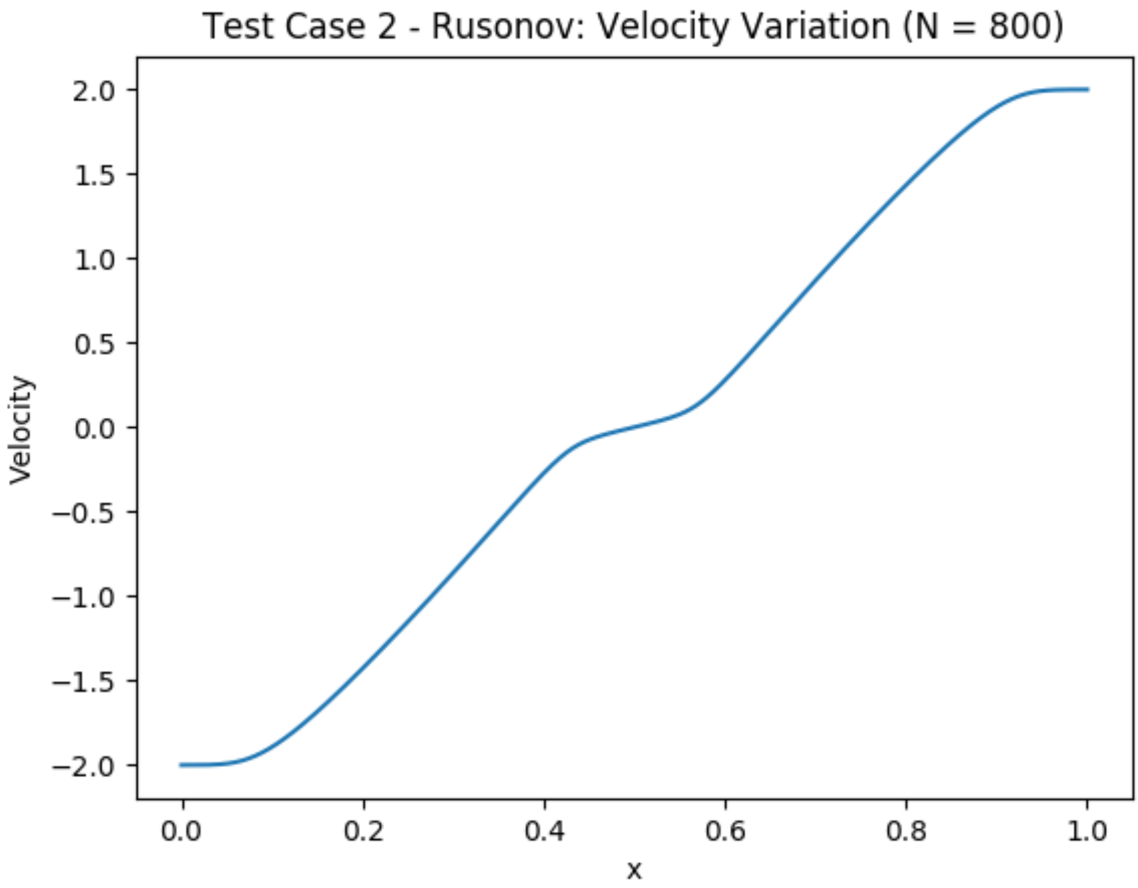

Test Case 2 - Rusonov: Internal Energy Variation (N = 400)
Pressure
x
1.0
0.9
0.8
0.7
0.6
0.0
0.2
0.4
0.6
0.8
1.0


Test Case 2 - Rusonov: Internal Energy Variation (N = 800)
Pressure
x
1.0
0.9
0.8
0.7
0.6
0.5
0.0
0.2
0.4
0.6
0.8
1.0

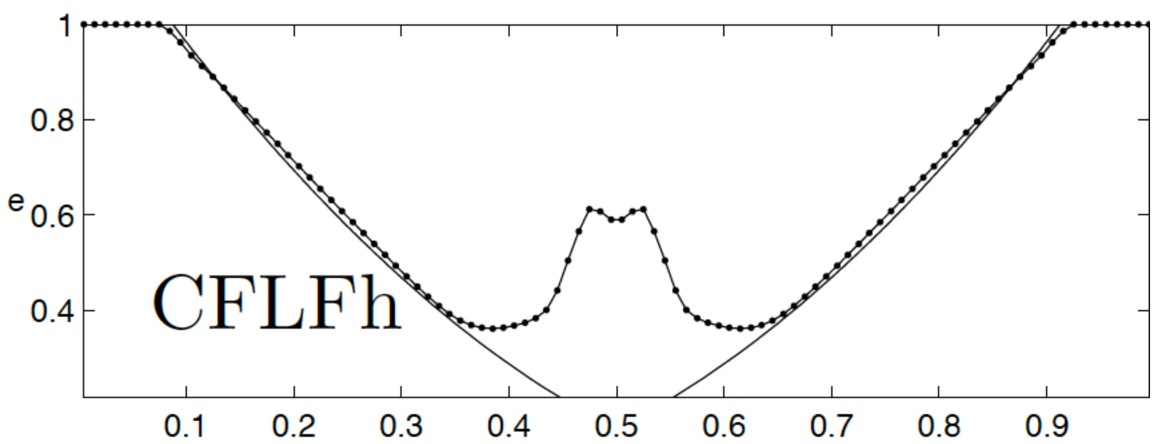

Test Case 2 - Rusonov: Velocity Variation (N = 100)
Velocity
x
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0.0
0.2
0.4
0.6
0.8
1.0


Test Case 2 - Rusonov: Velocity Variation (N = 200)
Velocity
x
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0.0
0.2
0.4
0.6
0.8
1.0

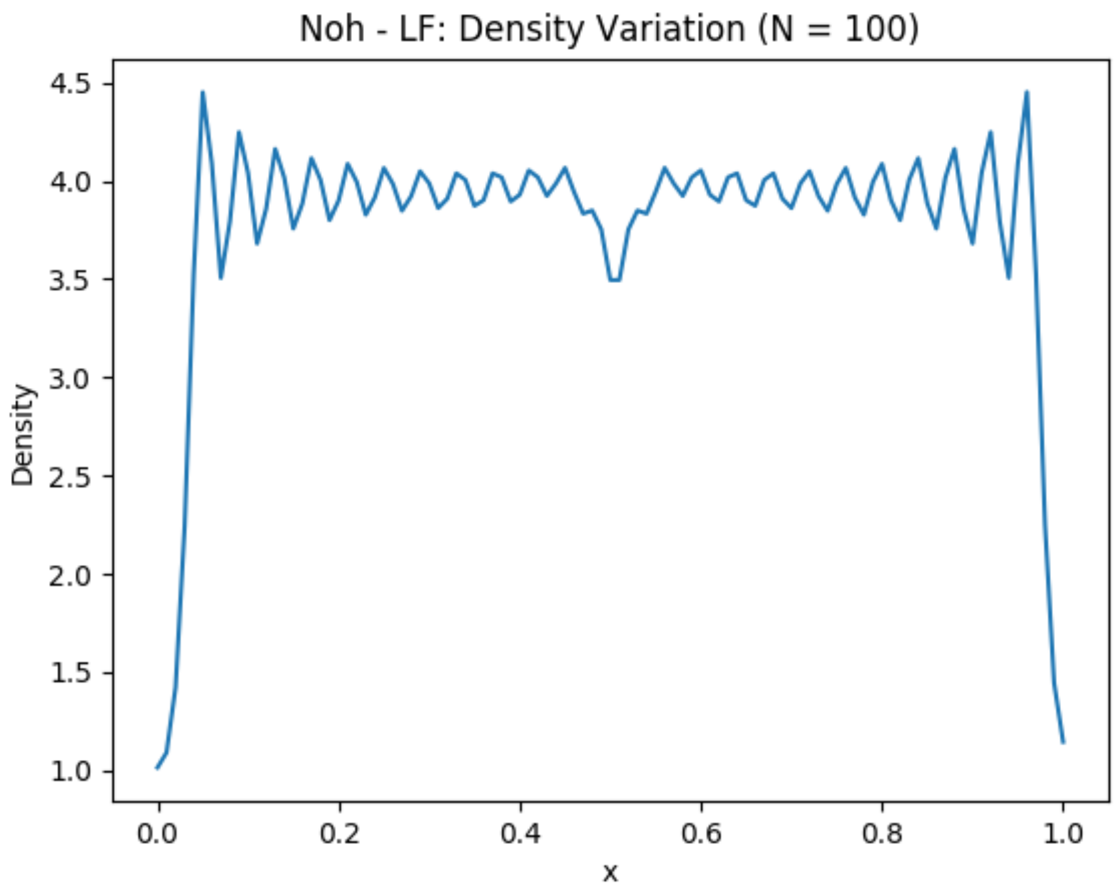

Test Case 2 - Rusonov: Velocity Variation (N = 400)
Velocity
x
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0.0
0.2
0.4
0.6
0.8
1.0



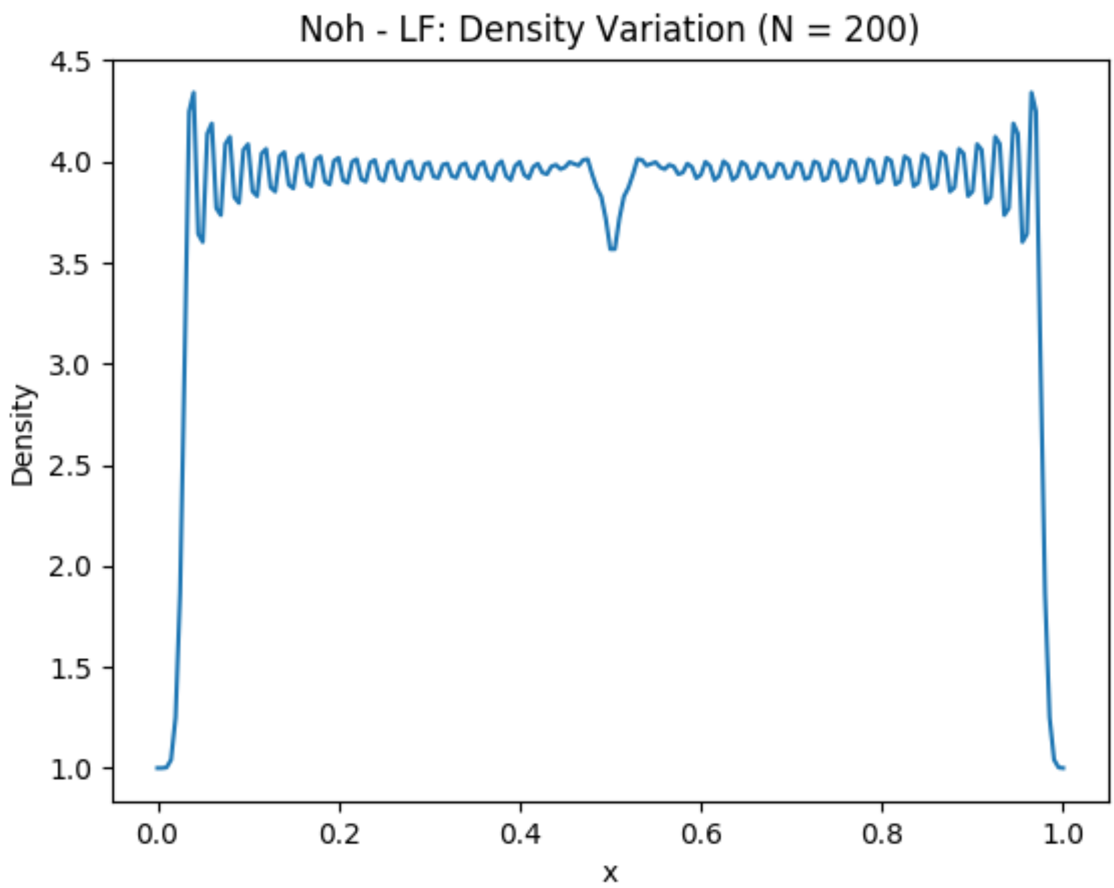
Exact Solution



Noh Problem

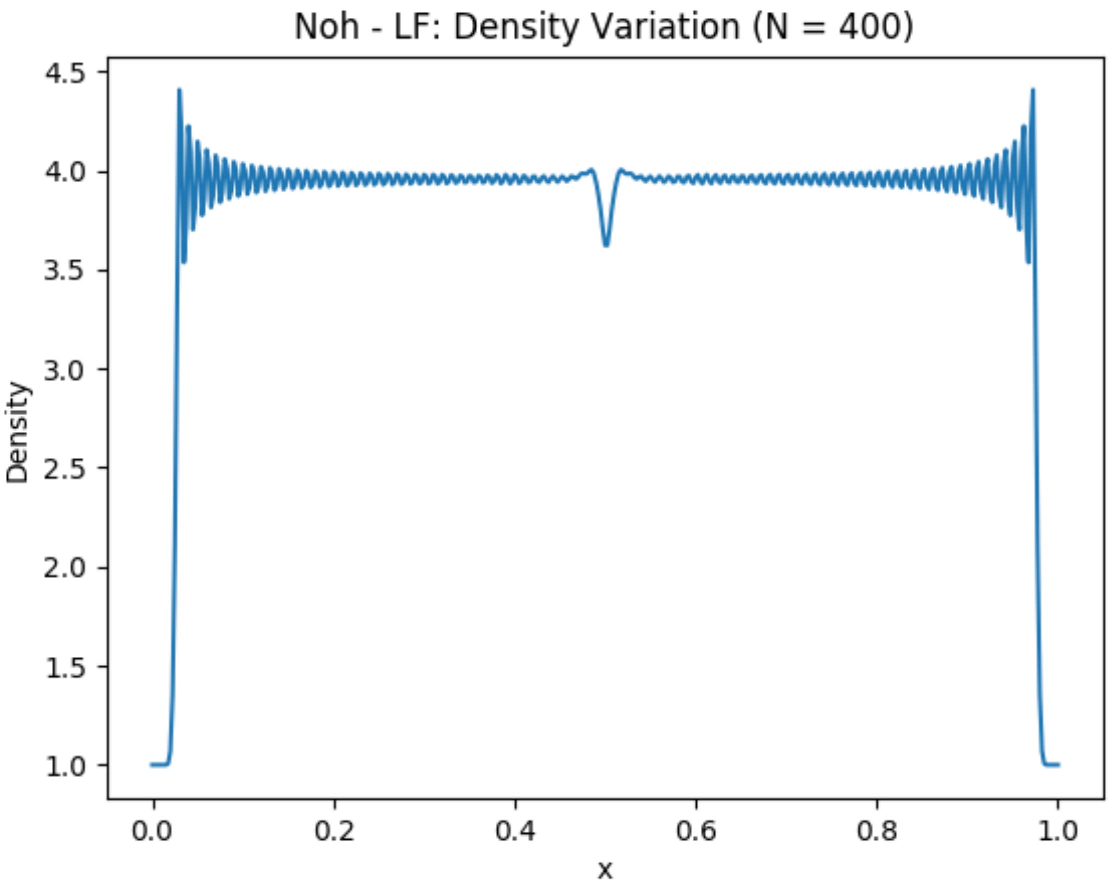


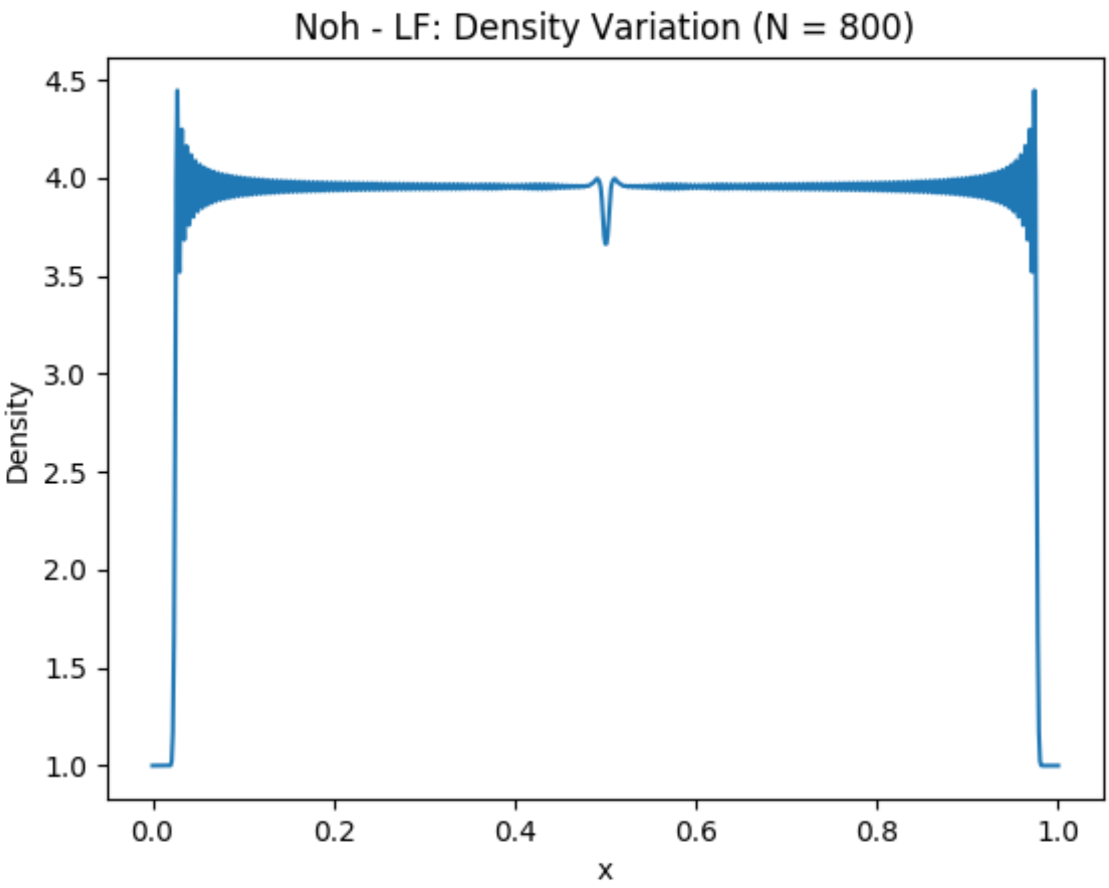
Noh - LF: Density Variation (N = 100)
Density
x

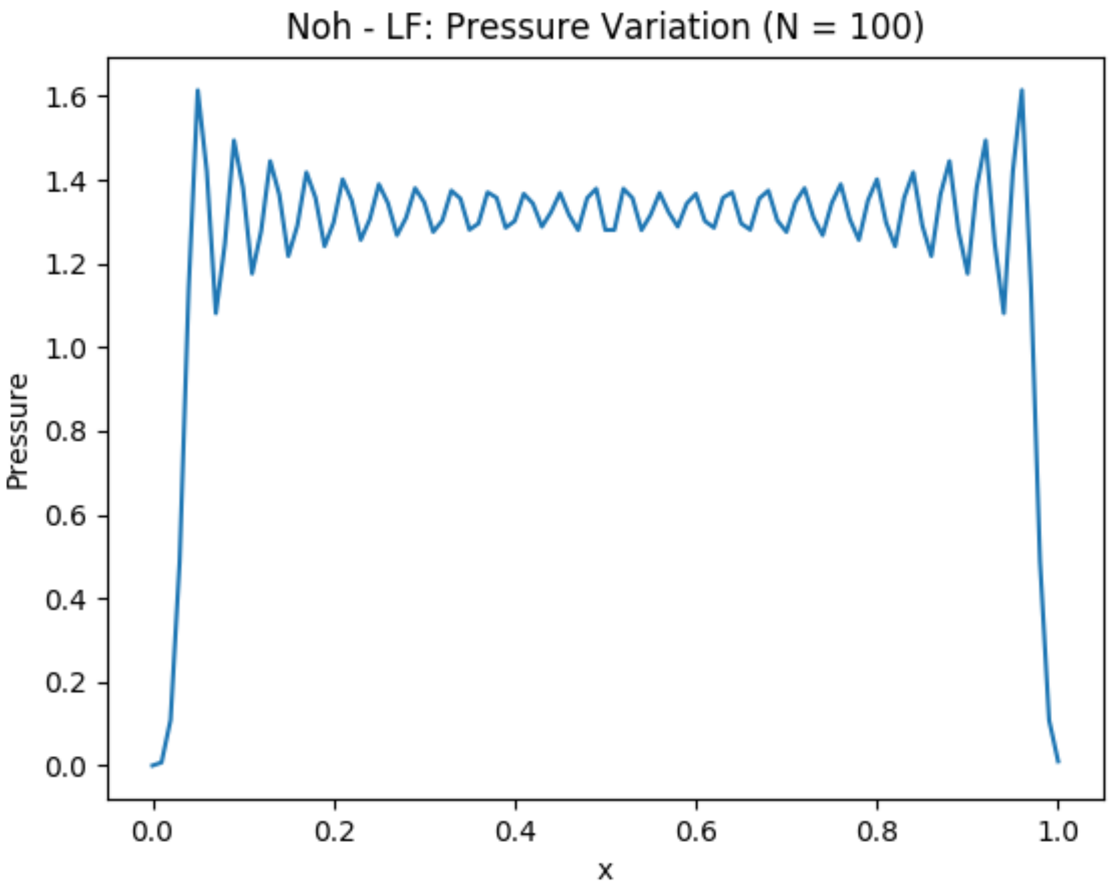


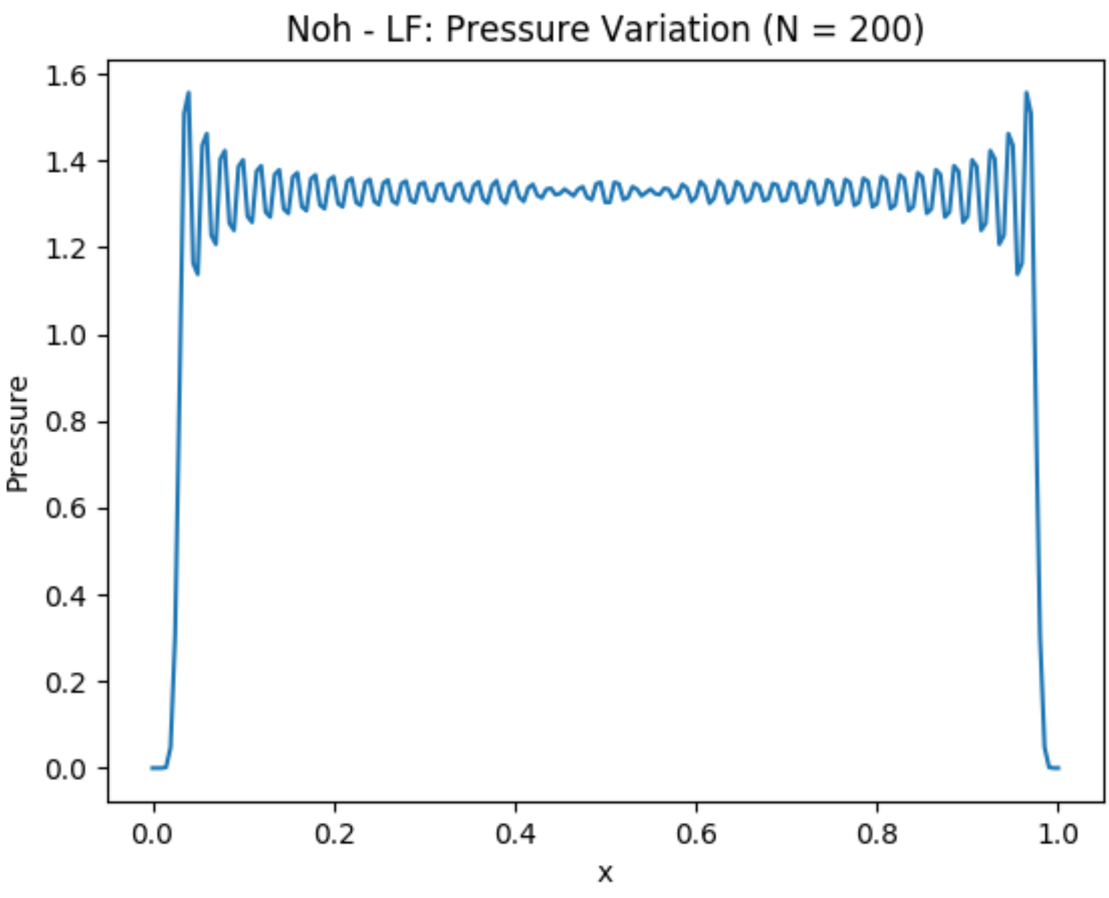
Noh - LF: Density Variation (N = 200)
Density
x

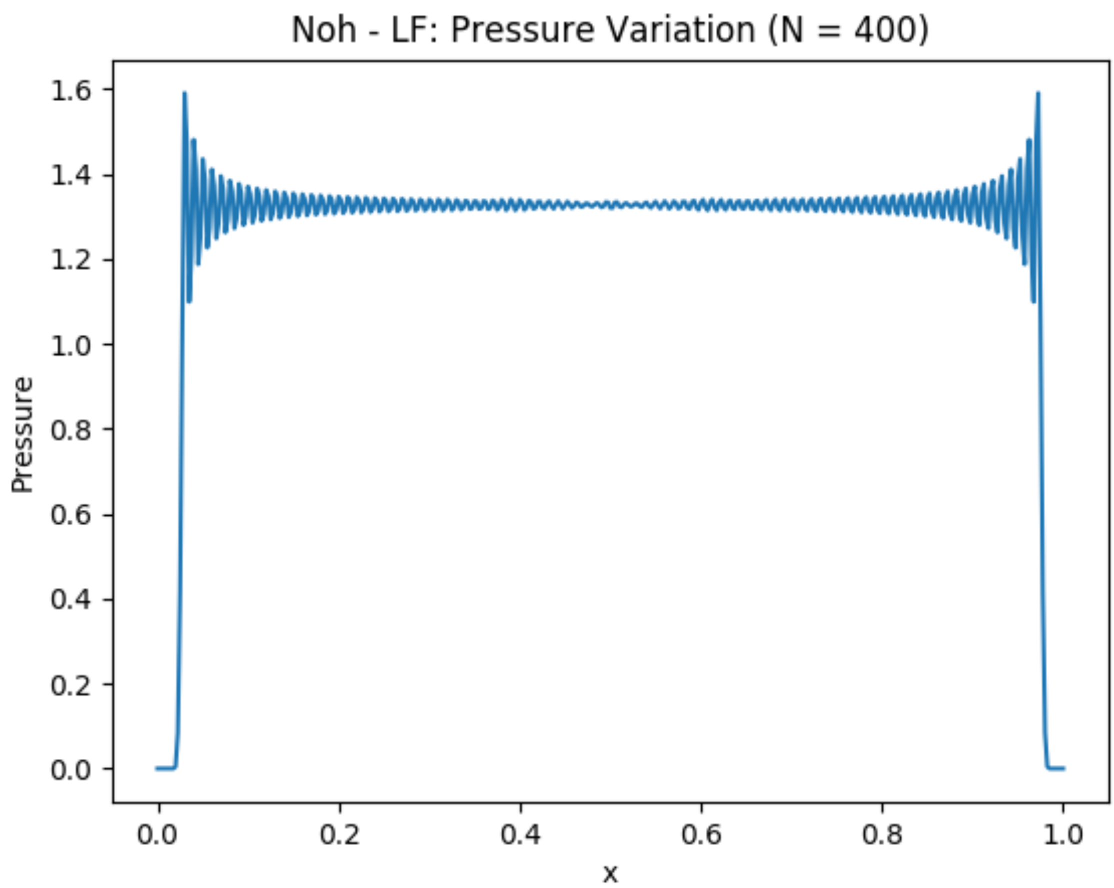
Lax-Friedrich Scheme

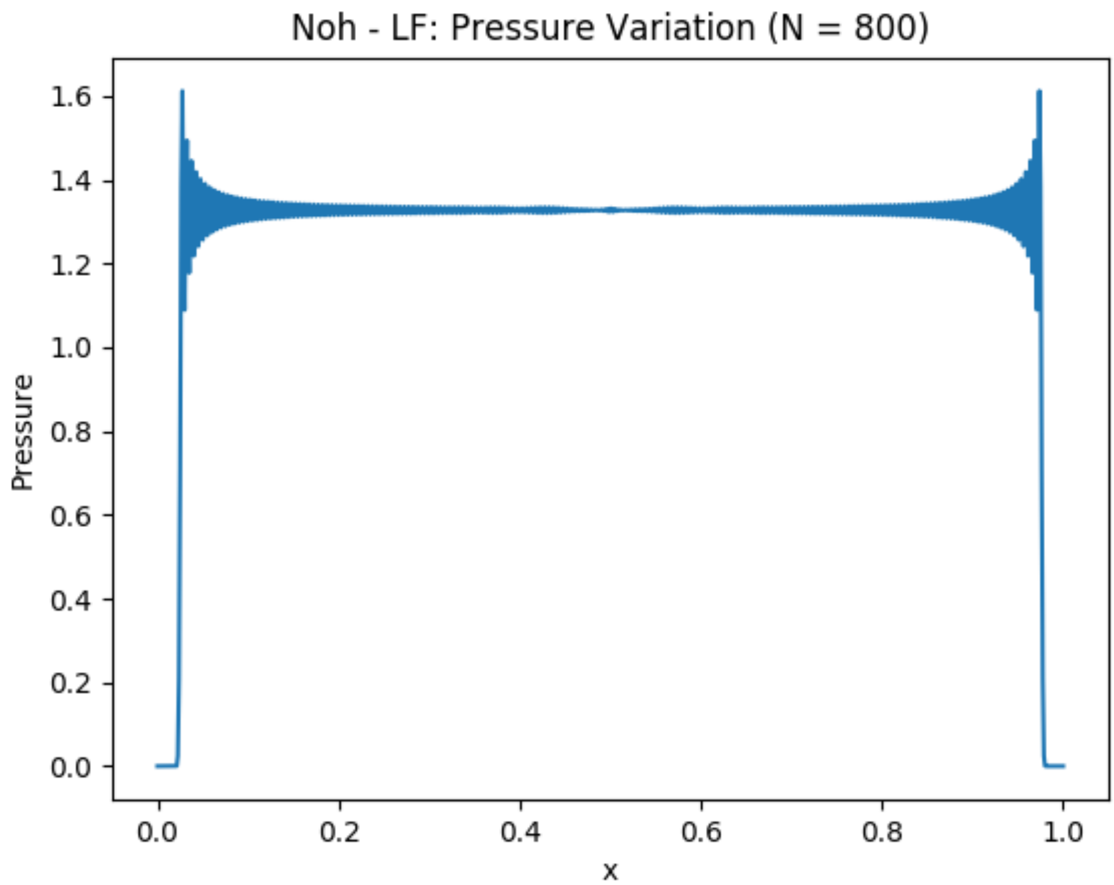


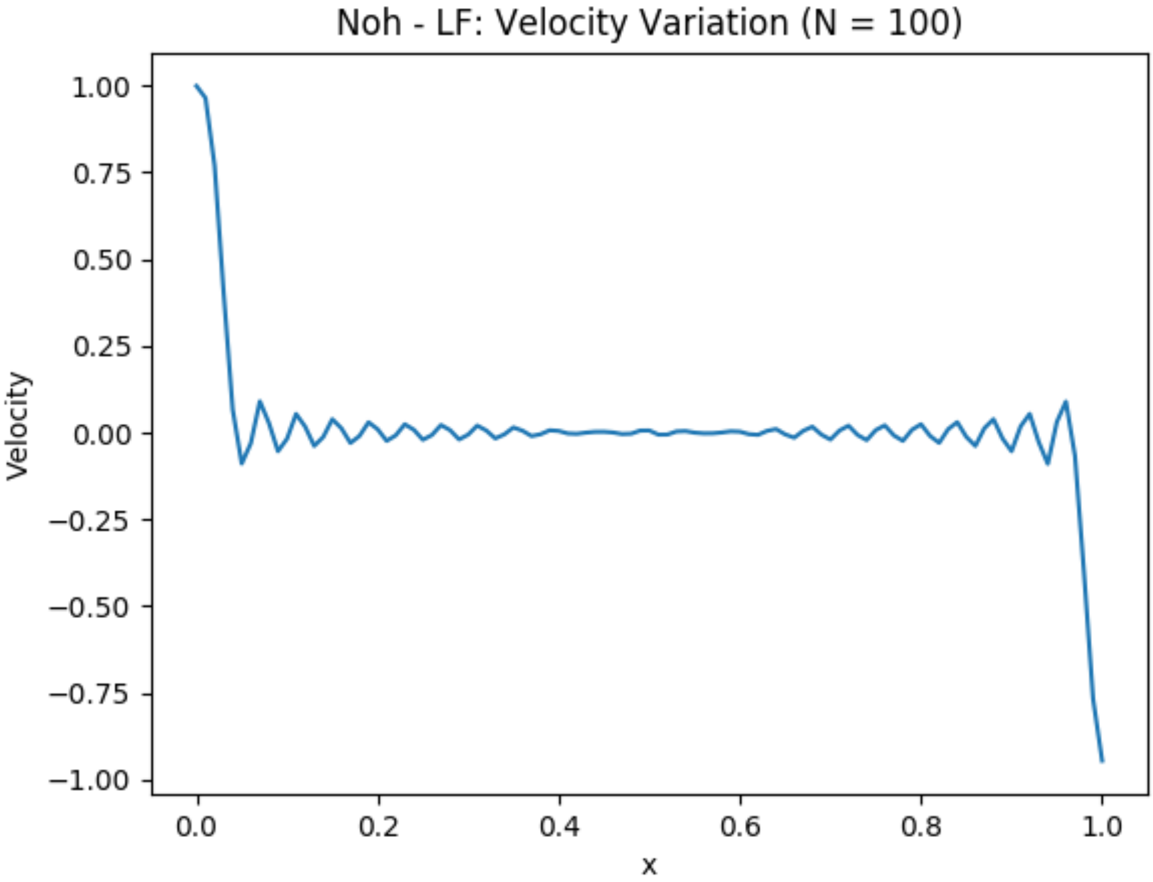


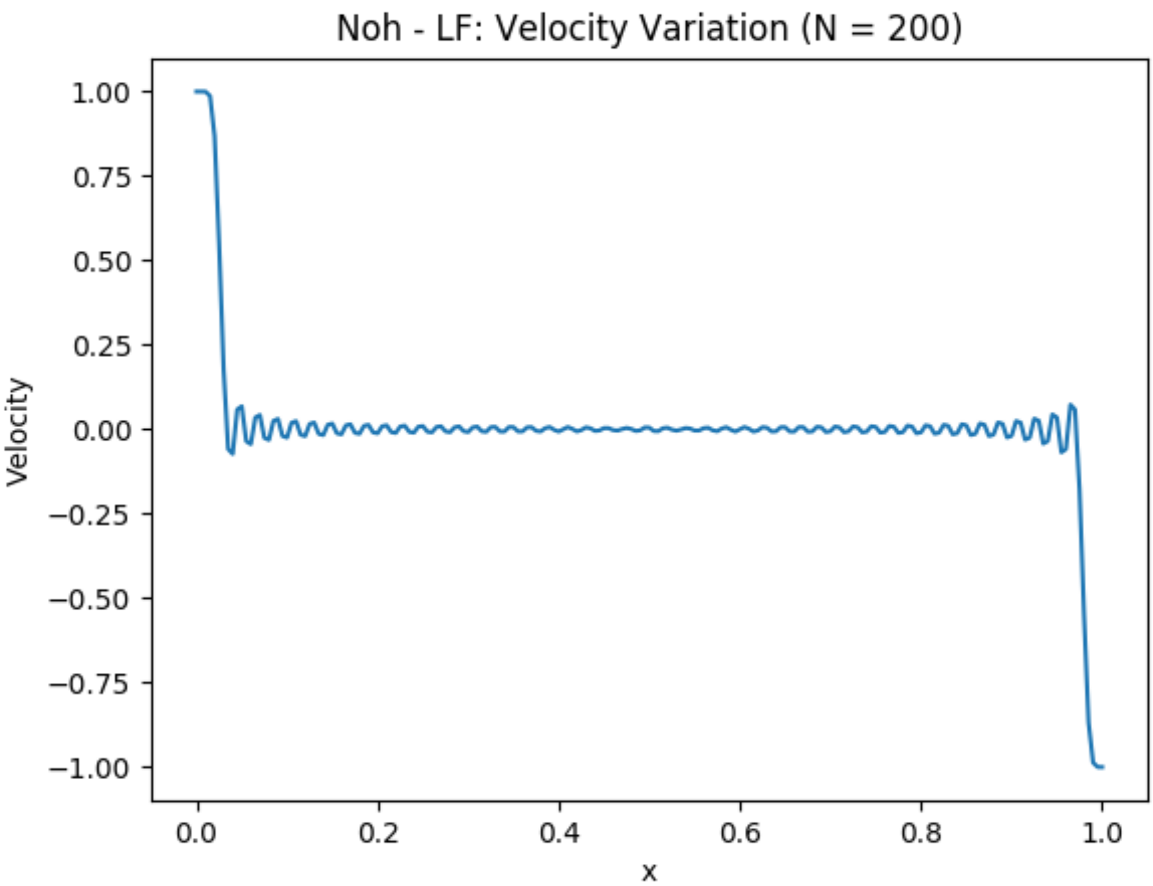

Noh - LF: Pressure Variation (N = 100)
Pressure
x
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6

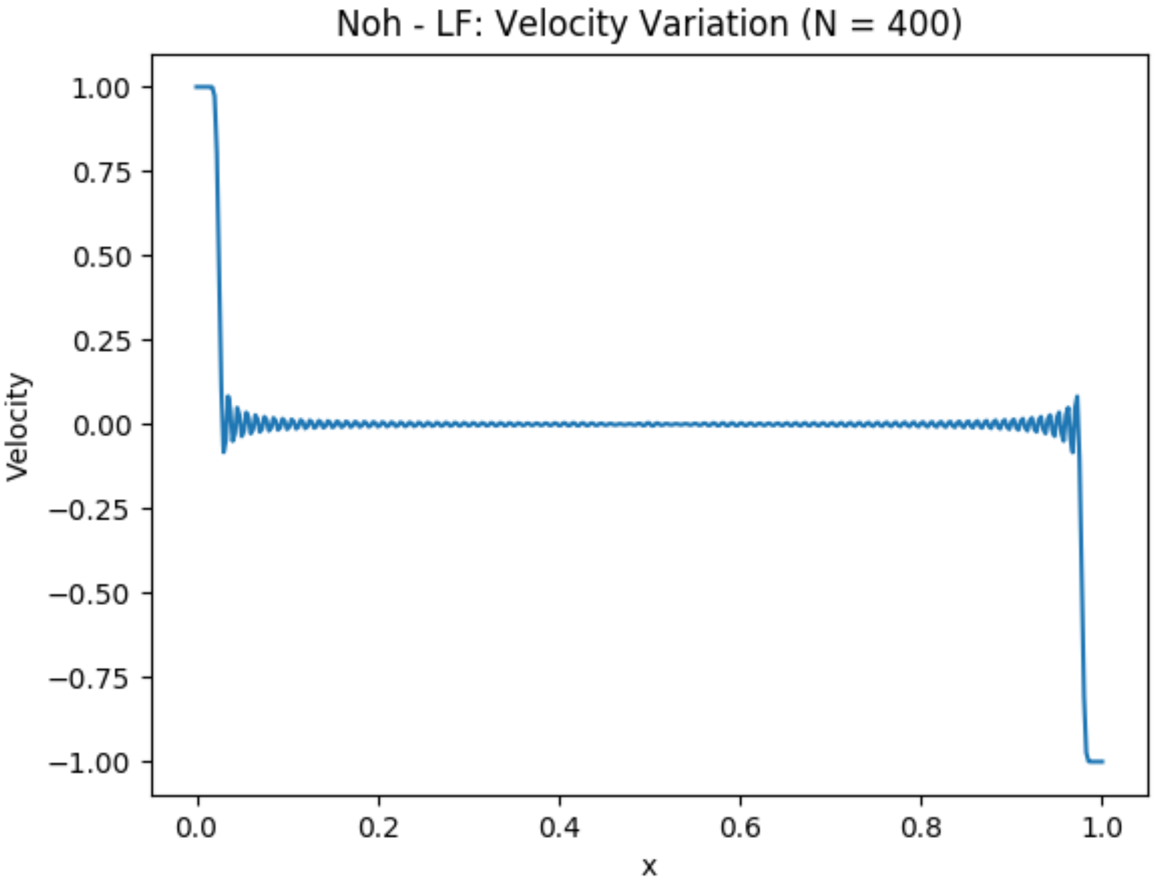

Noh - LF: Pressure Variation (N = 200)
Pressure
x
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6

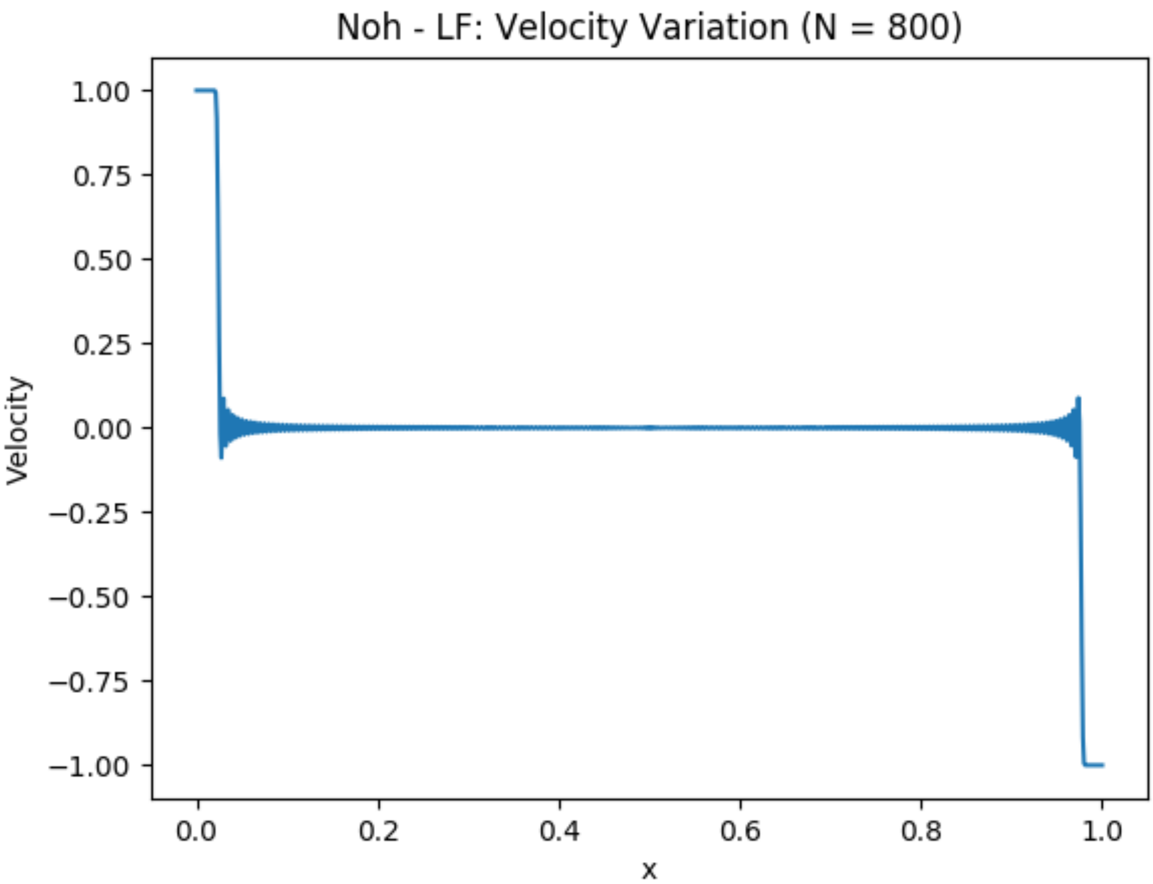

Noh - LF: Pressure Variation (N = 400)
Pressure
x
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6


Noh - LF: Pressure Variation (N = 800)
Pressure
x
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6

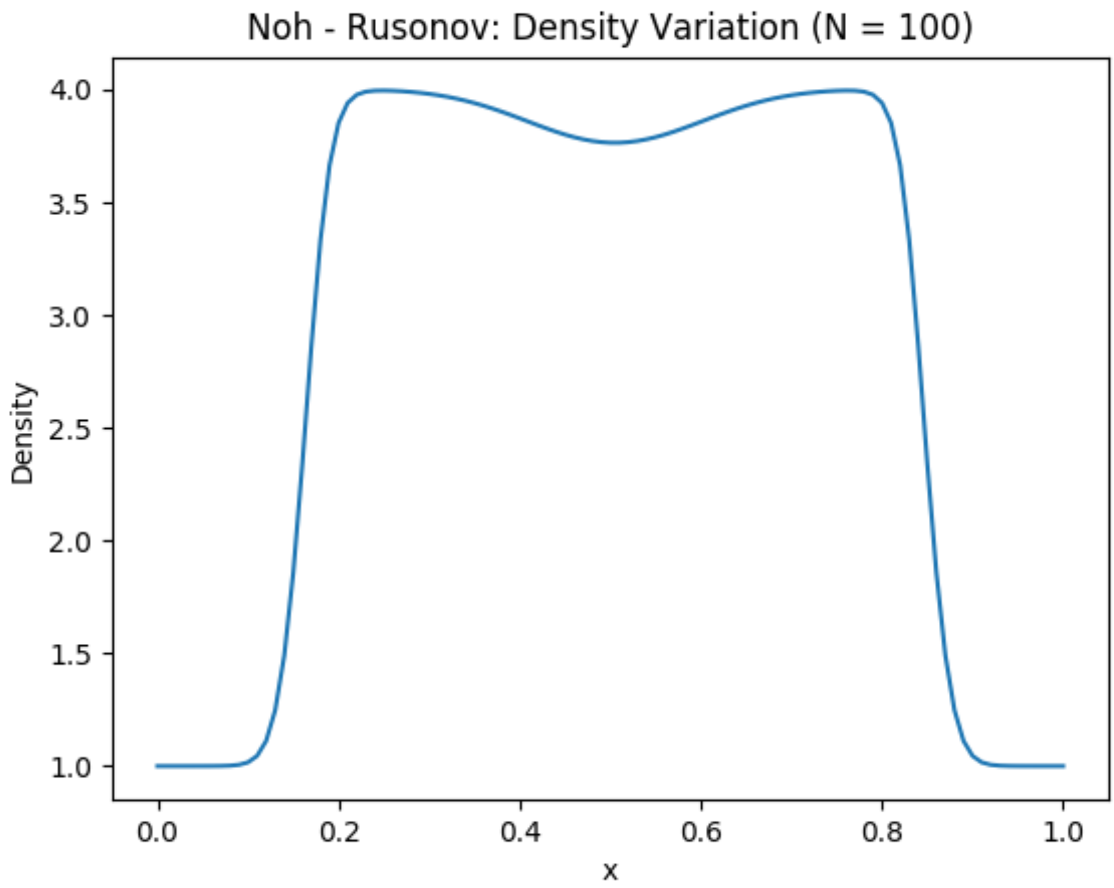

Noh - LF: Velocity Variation (N = 100)
Velocity
x

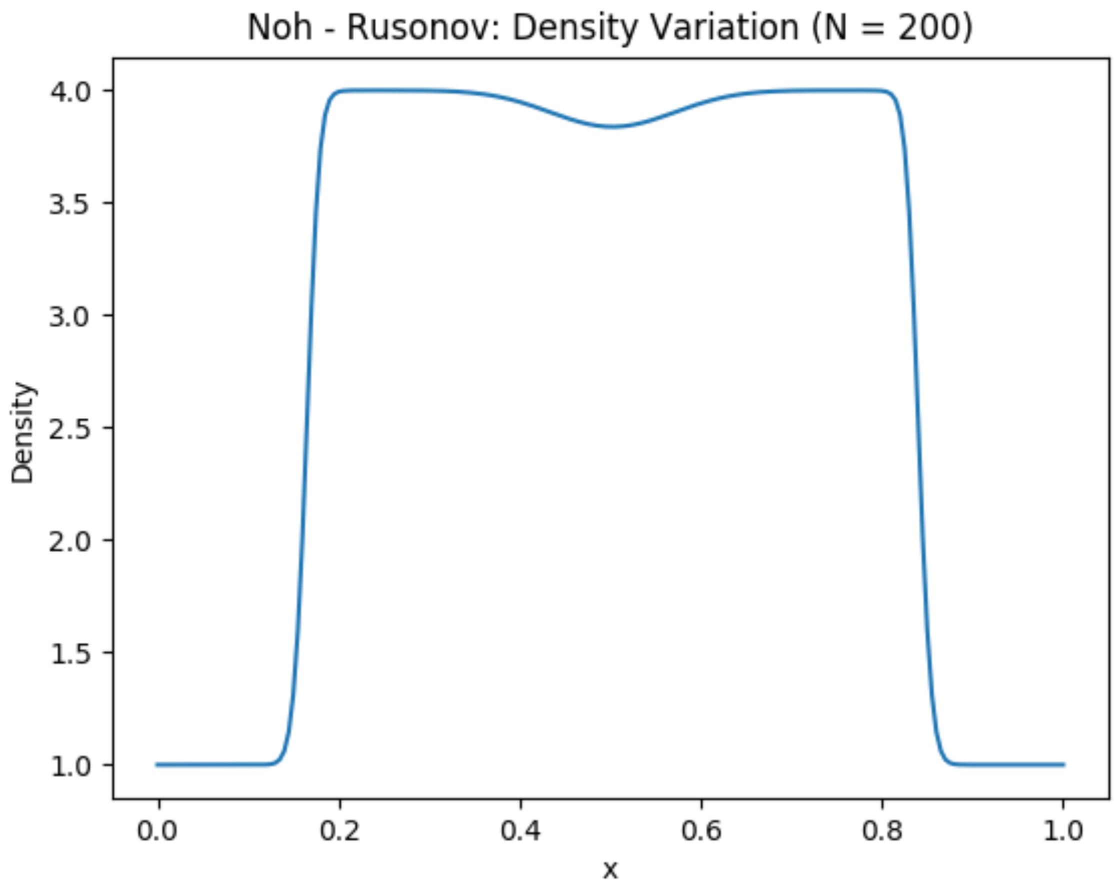

Noh - LF: Velocity Variation (N = 200)
Velocity
x

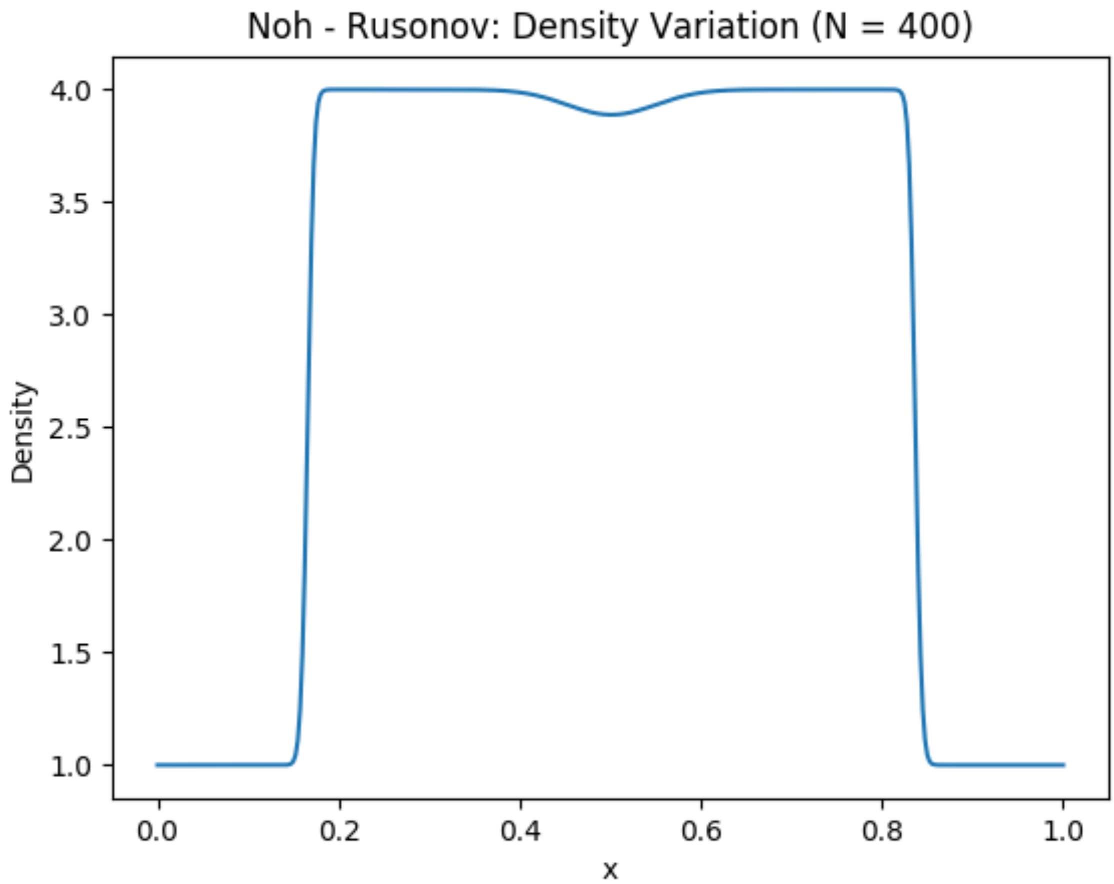

Noh - LF: Velocity Variation (N = 400)
Velocity
x
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
0.2
0.4
0.6
0.8
1.0

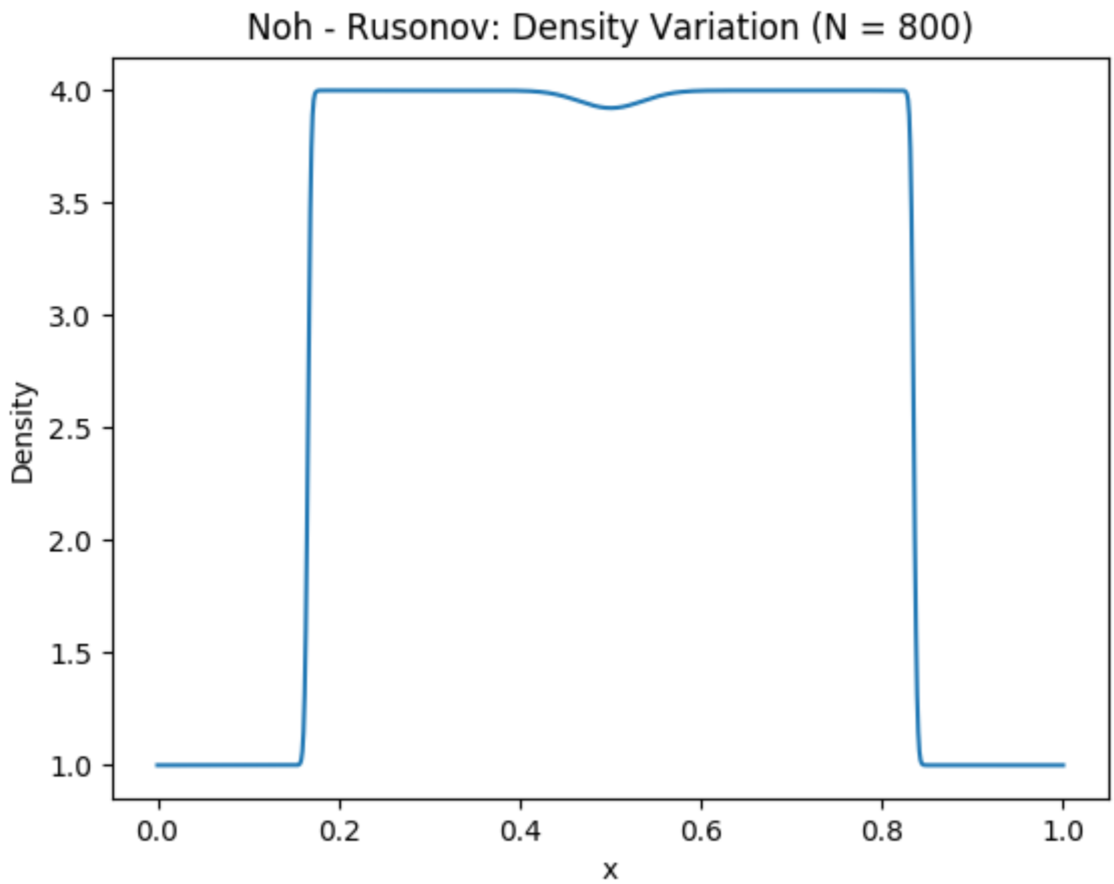

Noh - LF: Velocity Variation (N = 800)
Velocity
x
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
0.2
0.4
0.6
0.8
1.0

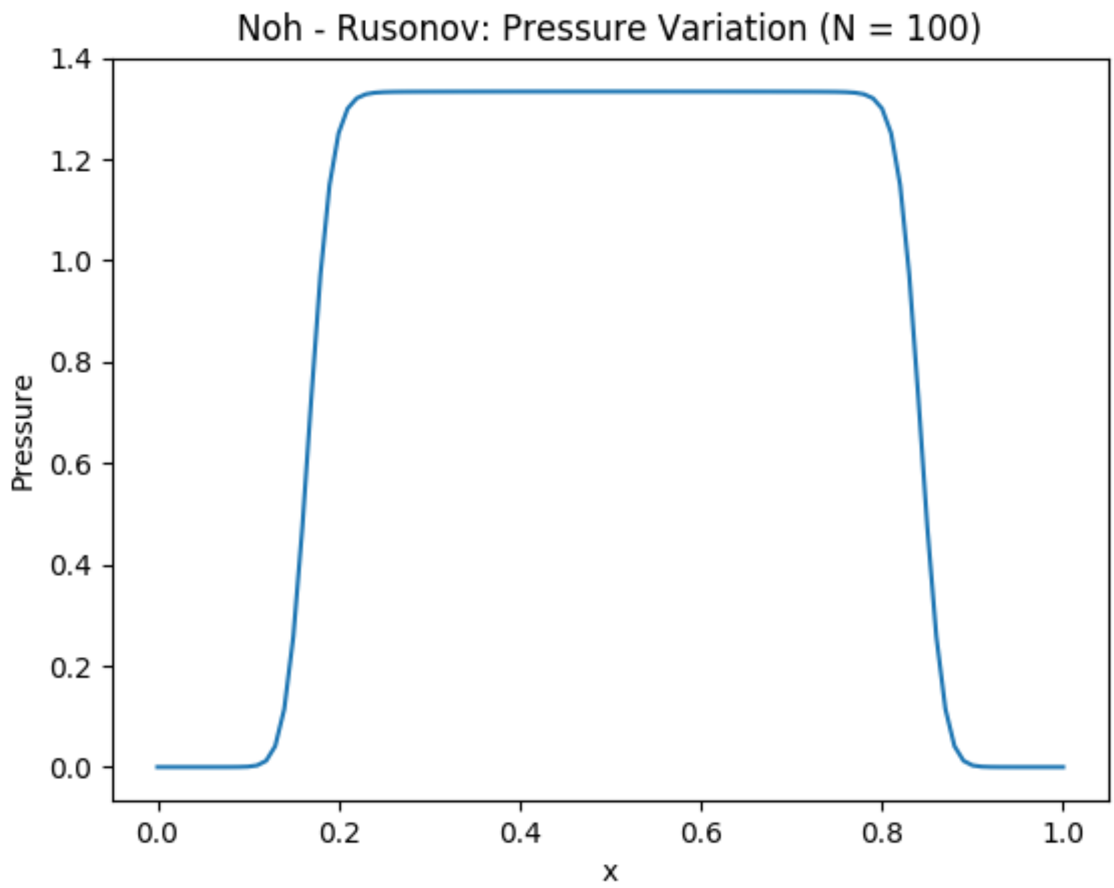
Rusanov Scheme:

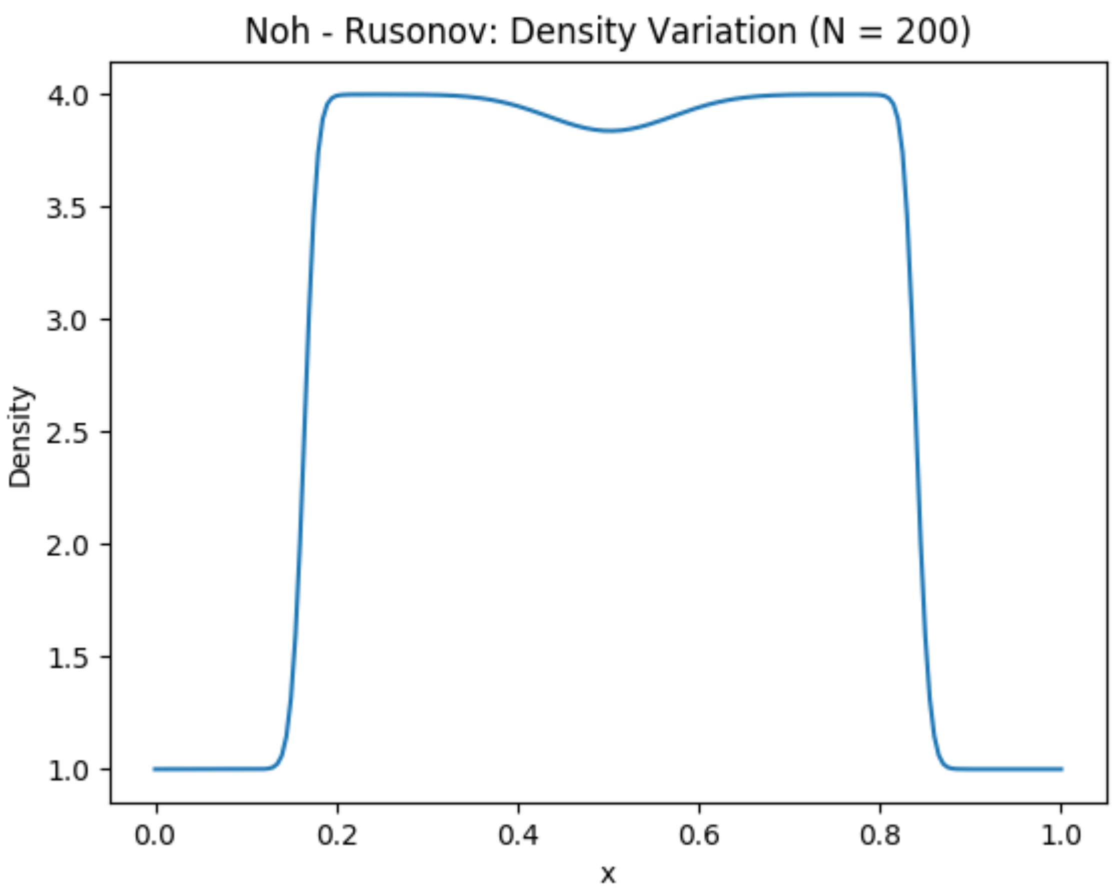


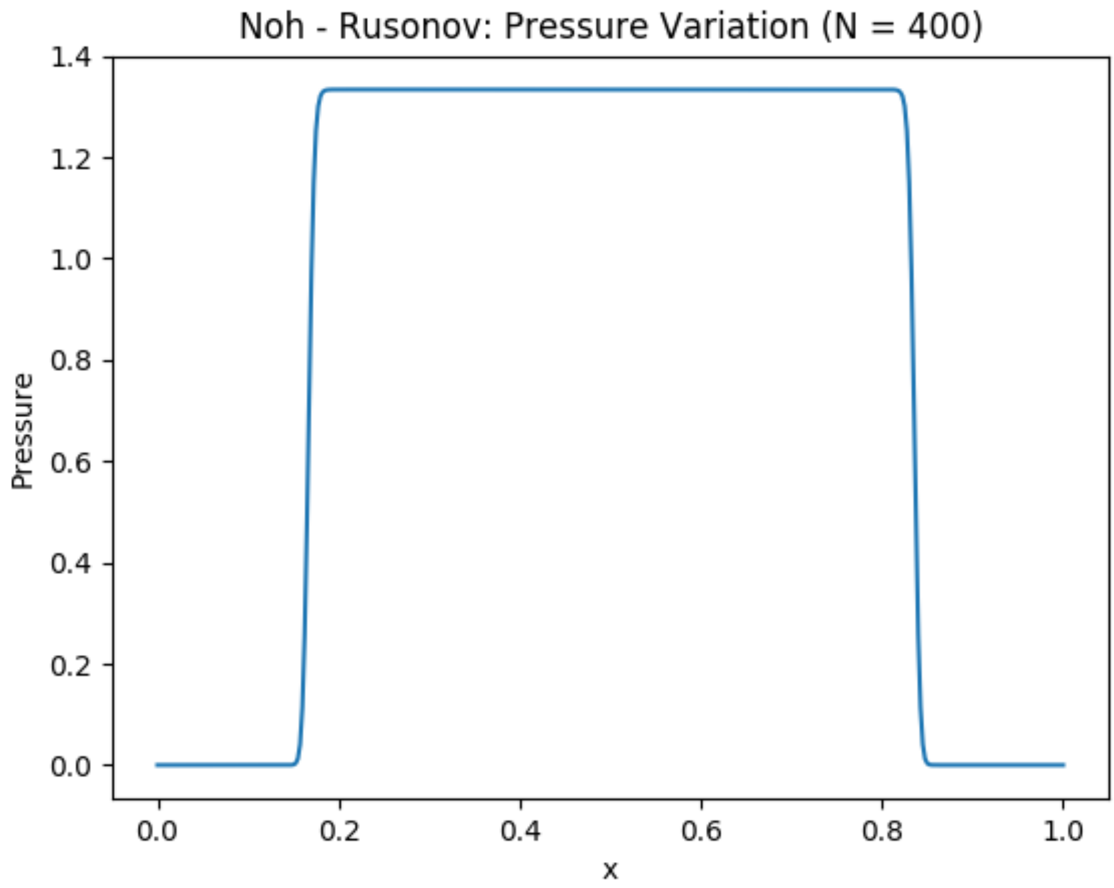


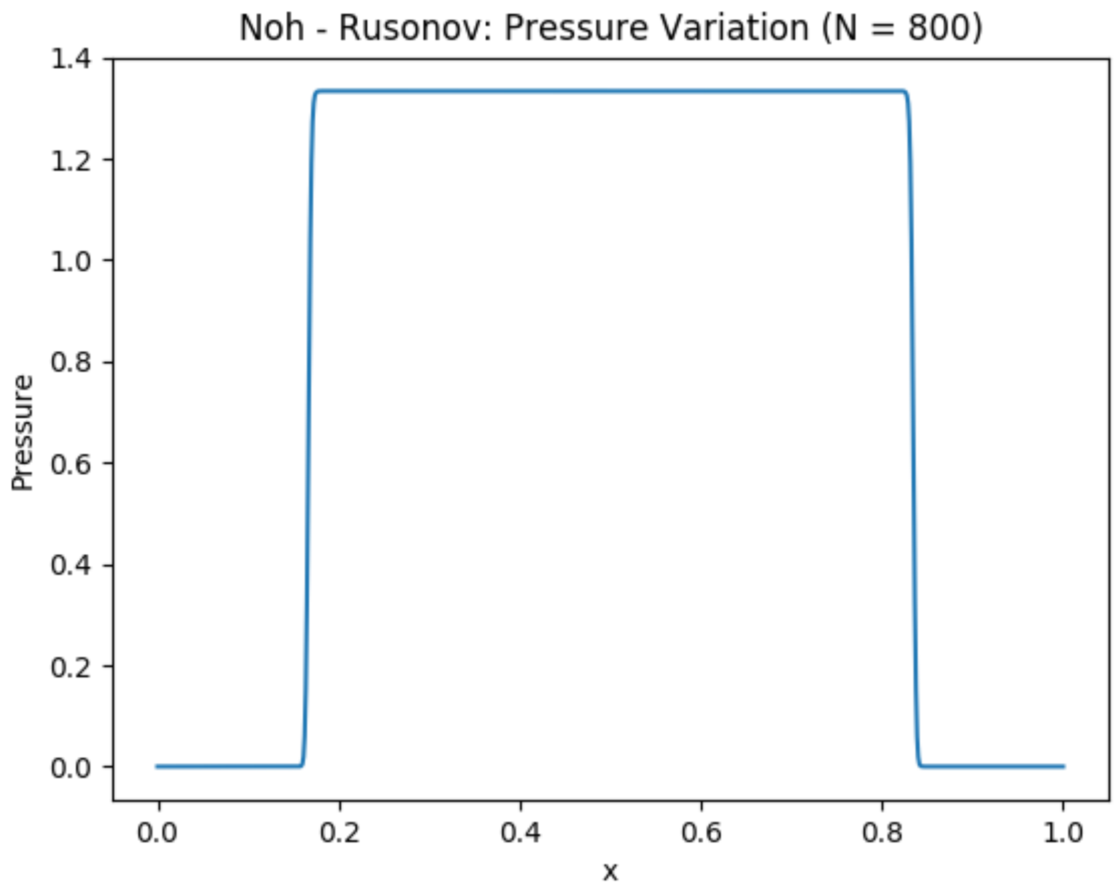

Noh - Rusonov: Density Variation (N = 400)
Density
x
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.0
0.2
0.4
0.6
0.8
1.0

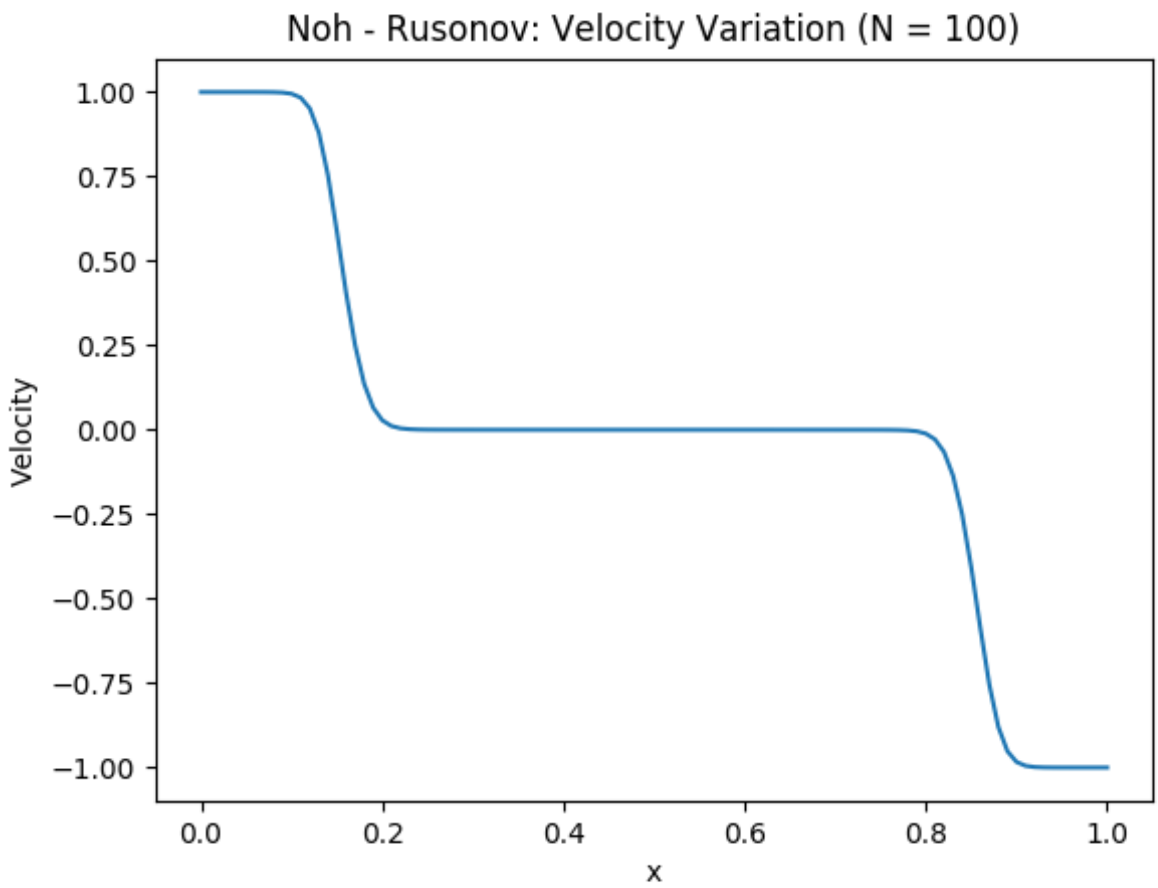

Noh - Rusonov: Density Variation (N = 800)
Density
x
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.0
0.2
0.4
0.6
0.8
1.0

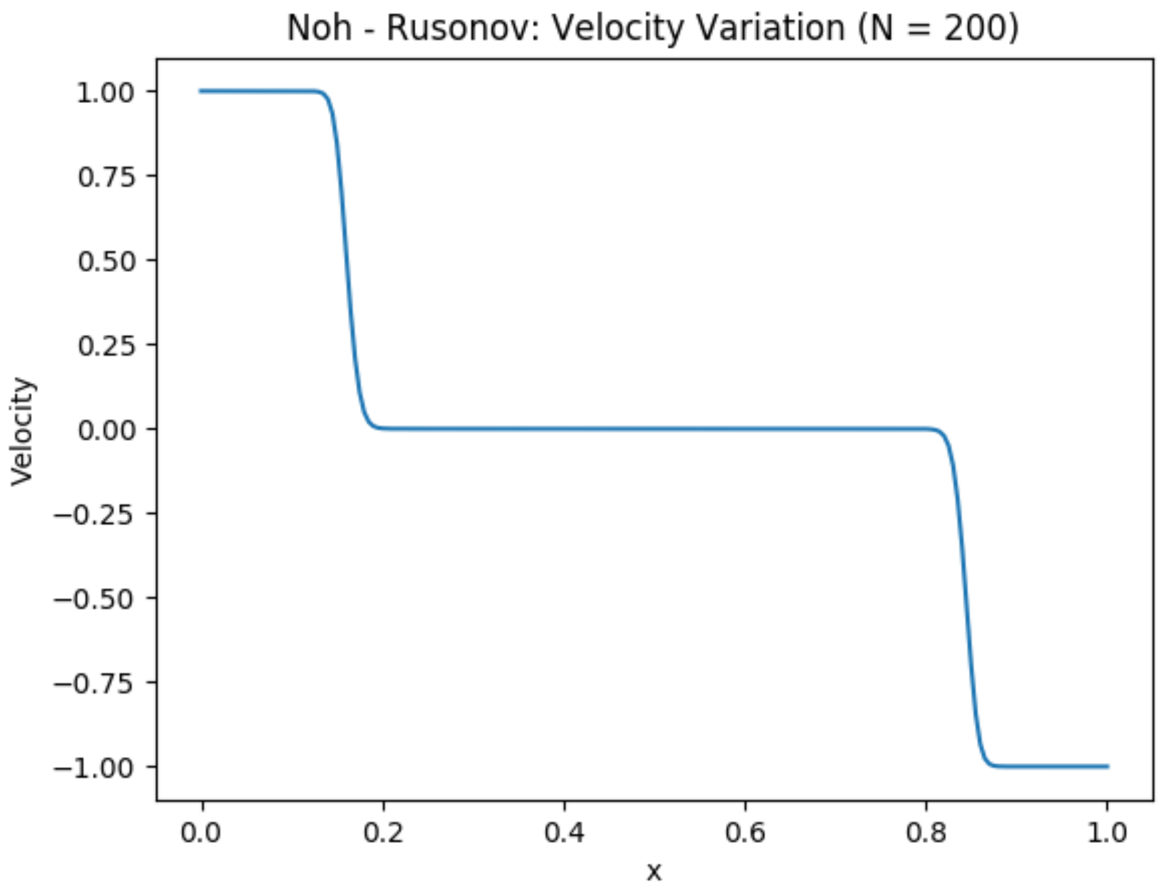

Noh - Rusonov: Pressure Variation (N = 100)
Pressure
x
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

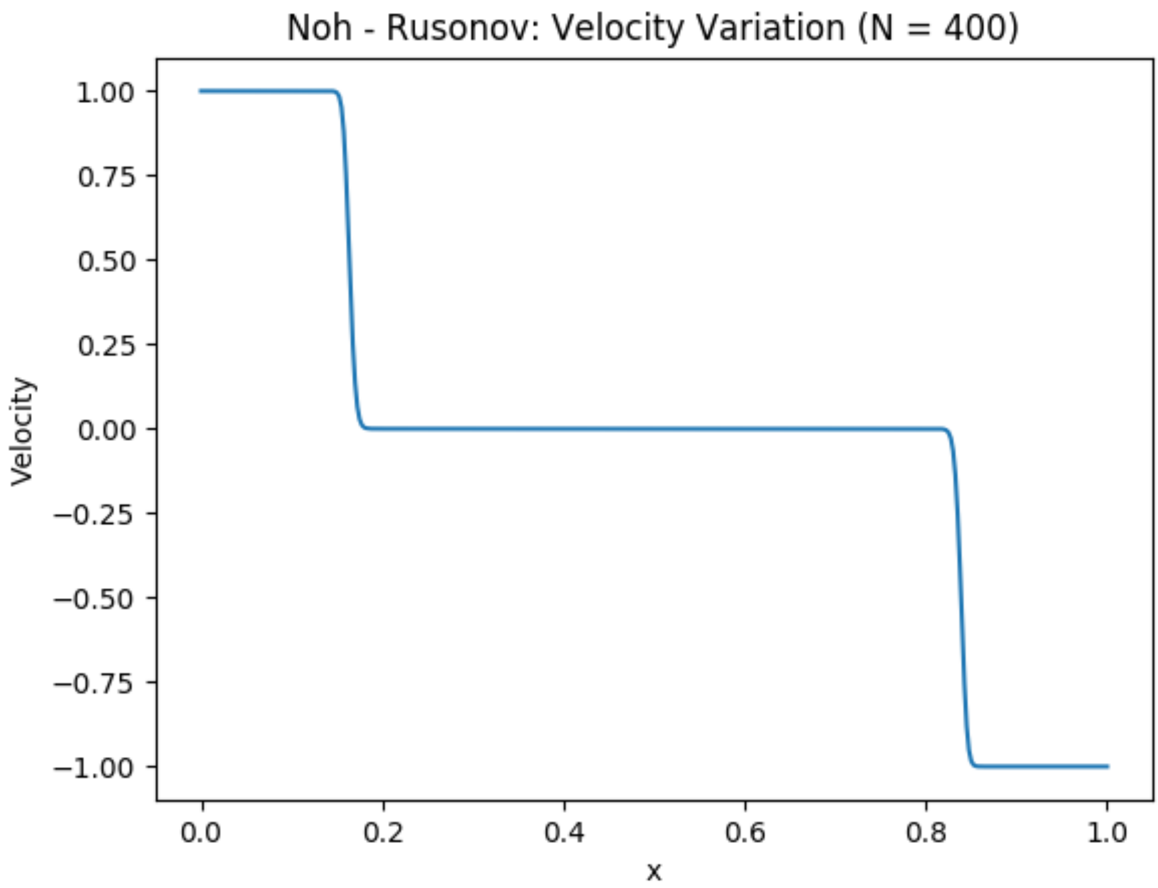

Noh - Rusonov: Density Variation (N = 200)
Density
x
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0.0
0.2
0.4
0.6
0.8
1.0

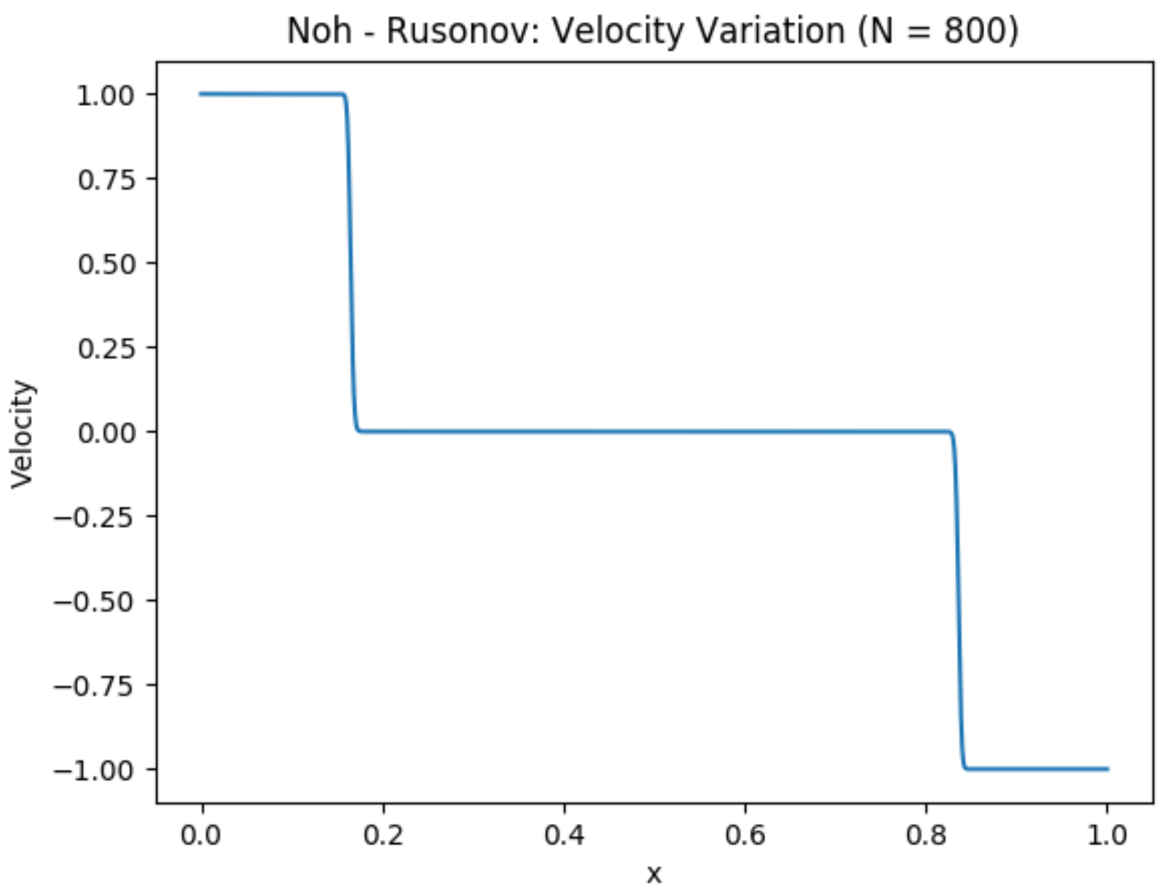

Noh - Rusonov: Pressure Variation (N = 400)
Pressure
x


Noh - Rusonov: Pressure Variation (N = 800)
Pressure
x


Noh - Rusonov: Velocity Variation (N = 100)
Velocity
x
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
0.2
0.4
0.6
0.8
1.0

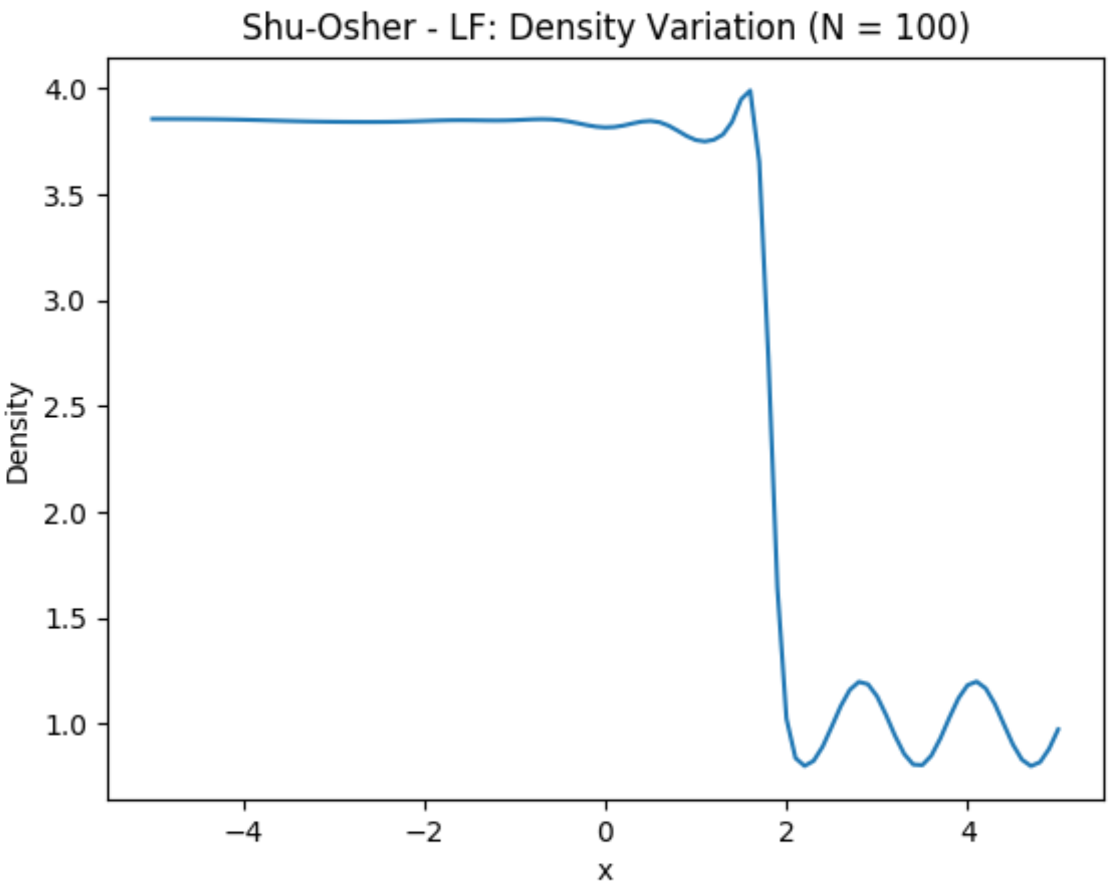

Noh - Rusonov: Velocity Variation (N = 200)
Velocity
x
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
0.2
0.4
0.6
0.8
1.0

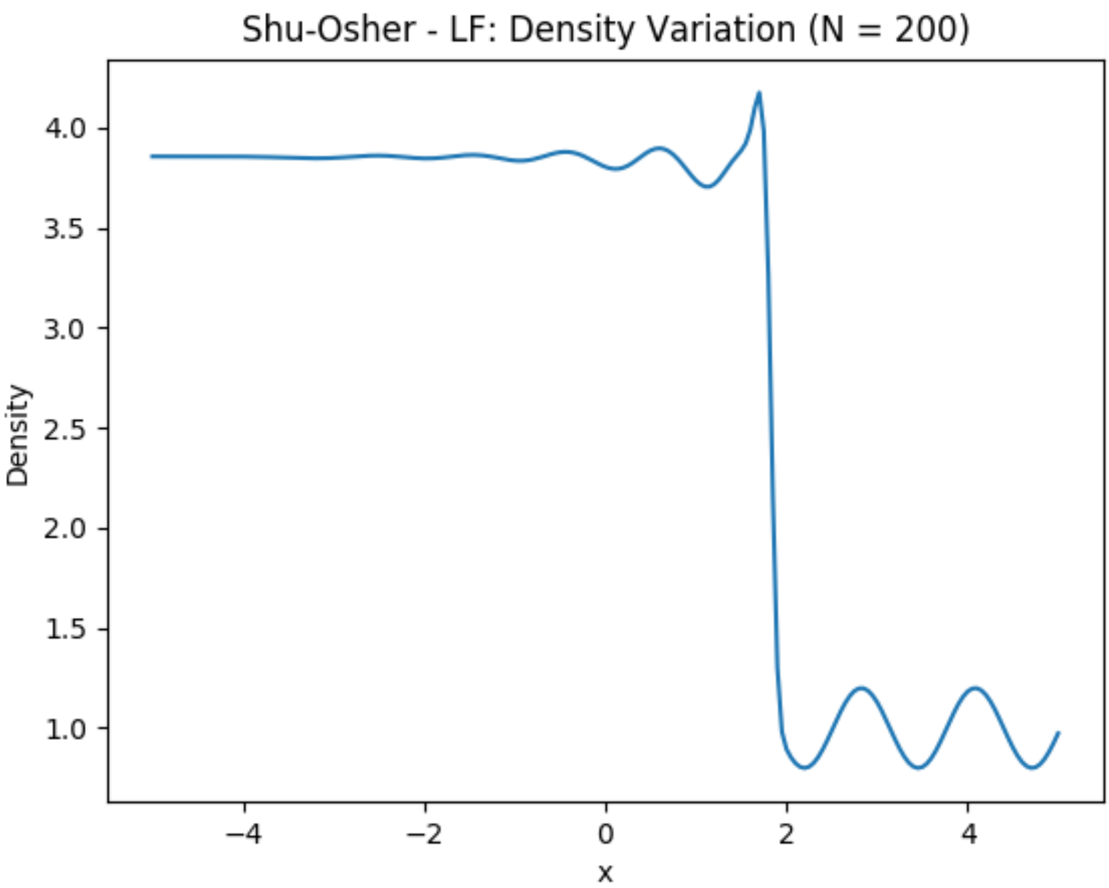

Noh - Rusonov: Velocity Variation (N = 400)
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
Velocity
0.0
0.2
0.4
0.6
0.8
1.0
x

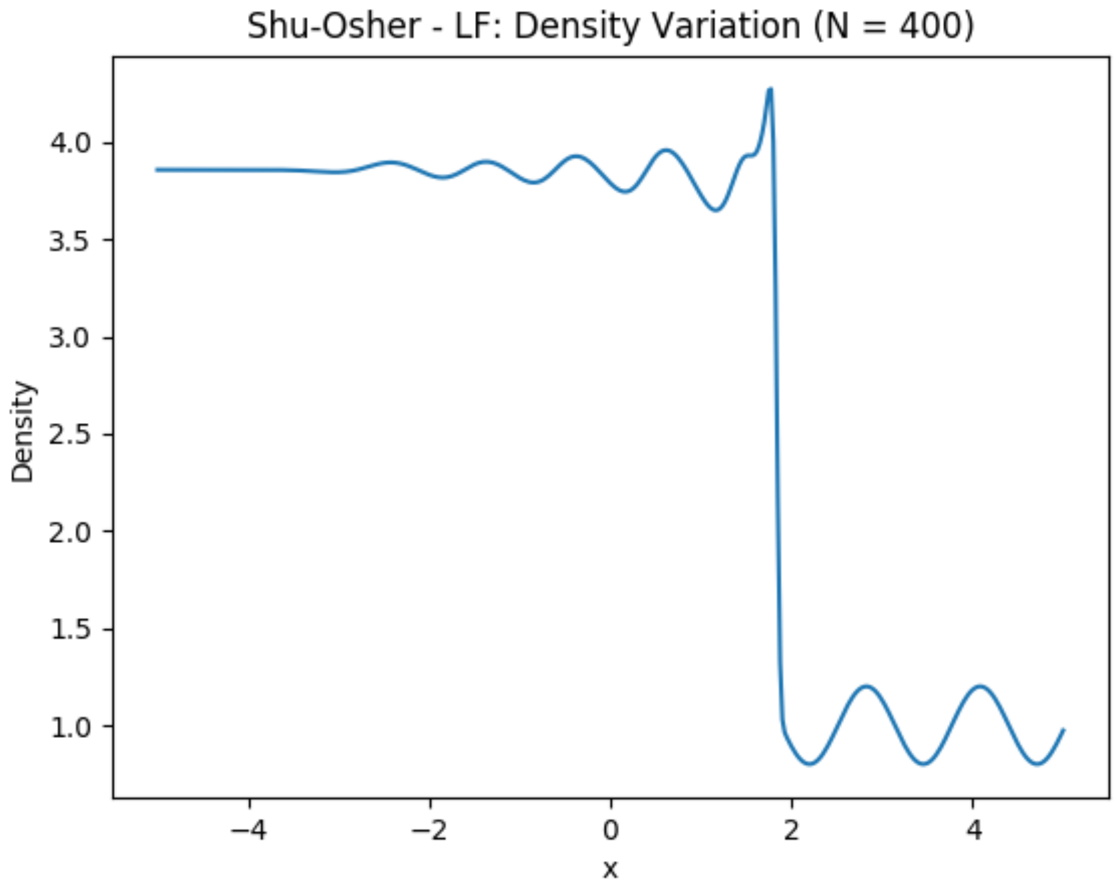

Noh - Rusonov: Velocity Variation (N = 800)
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
Velocity
0.0
0.2
0.4
0.6
0.8
1.0
x

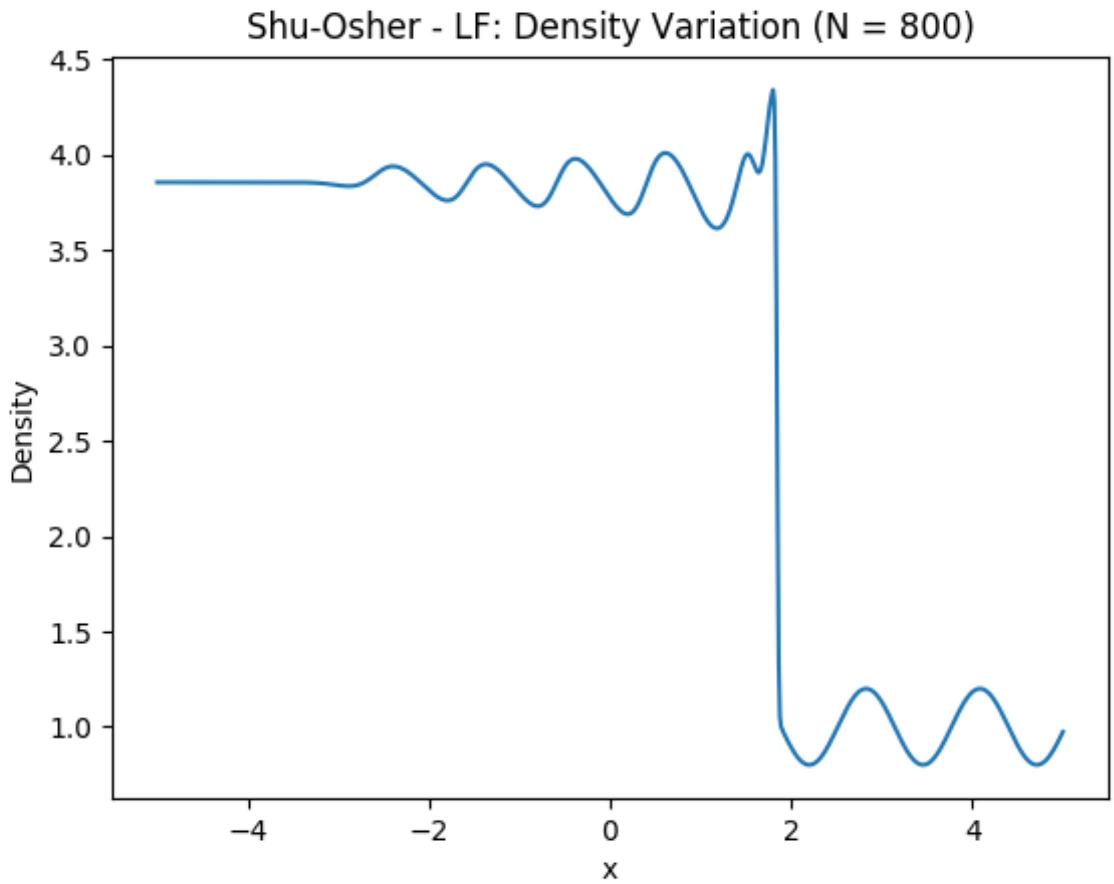
Shu-Osher Problem

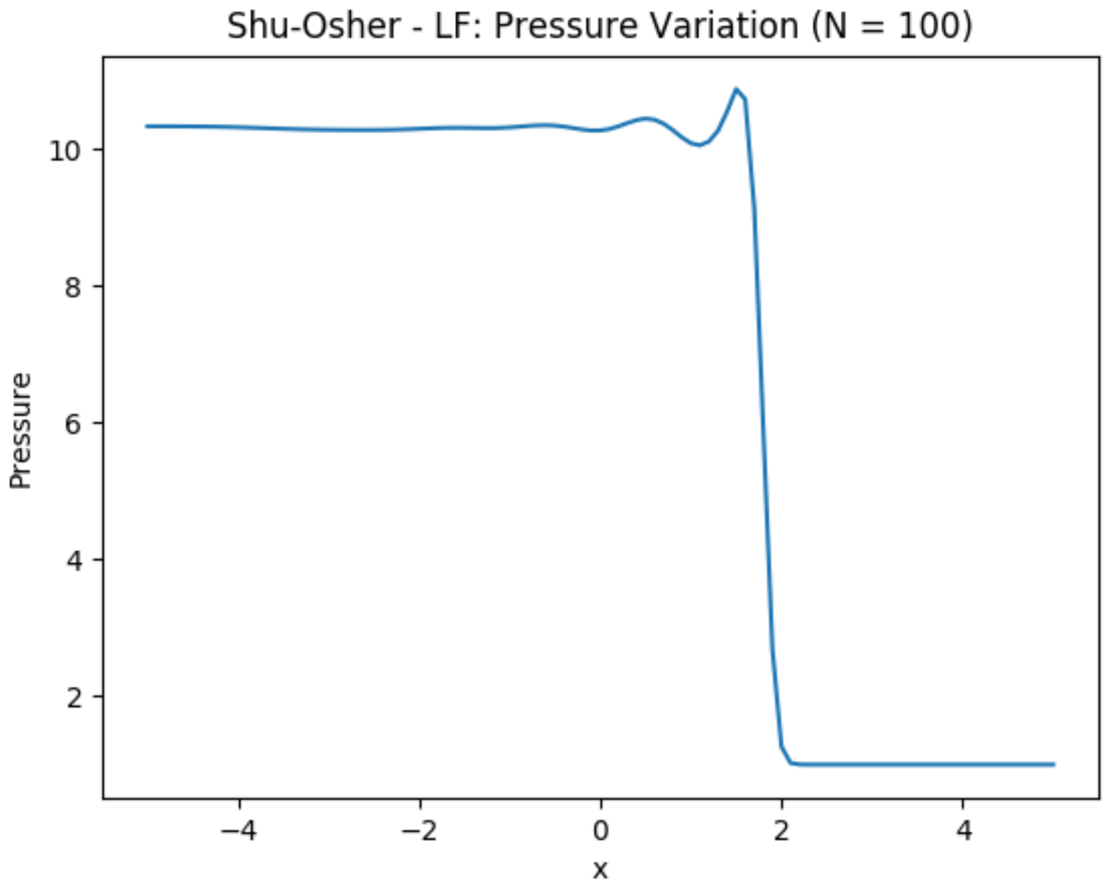
Lax-Friedrich

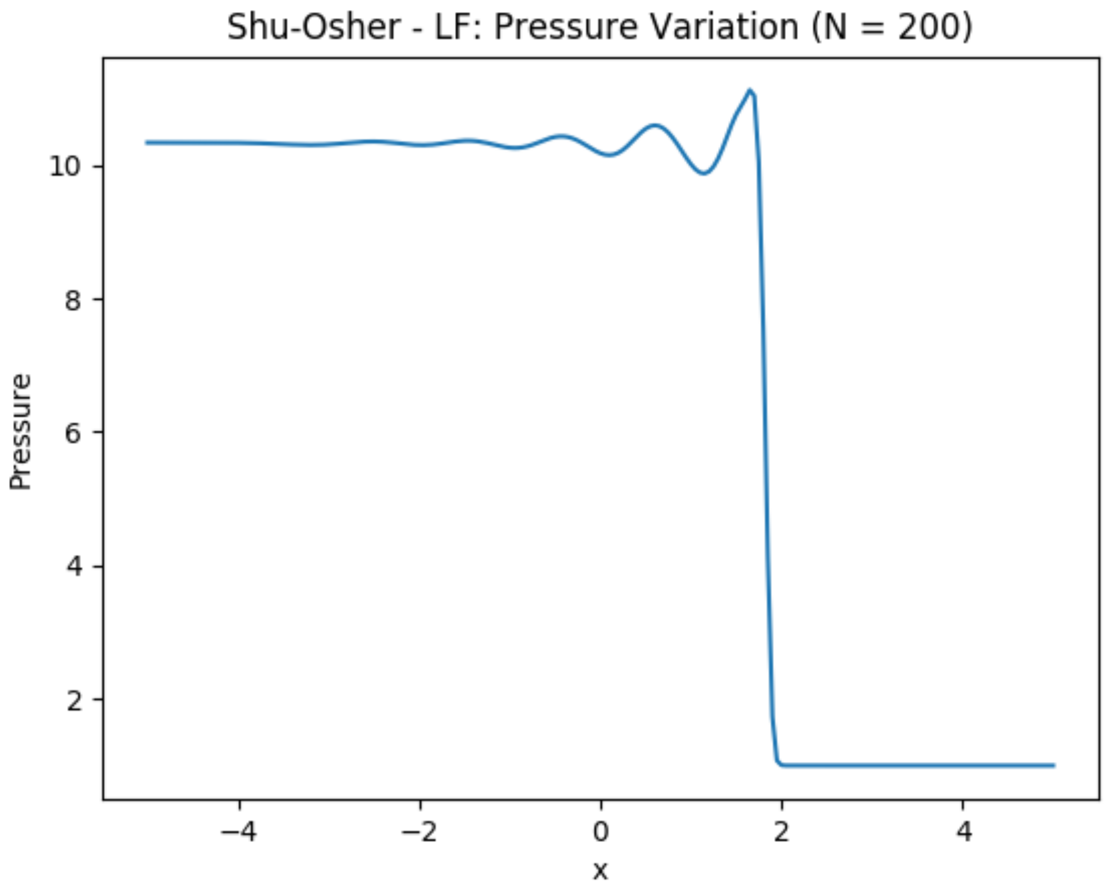


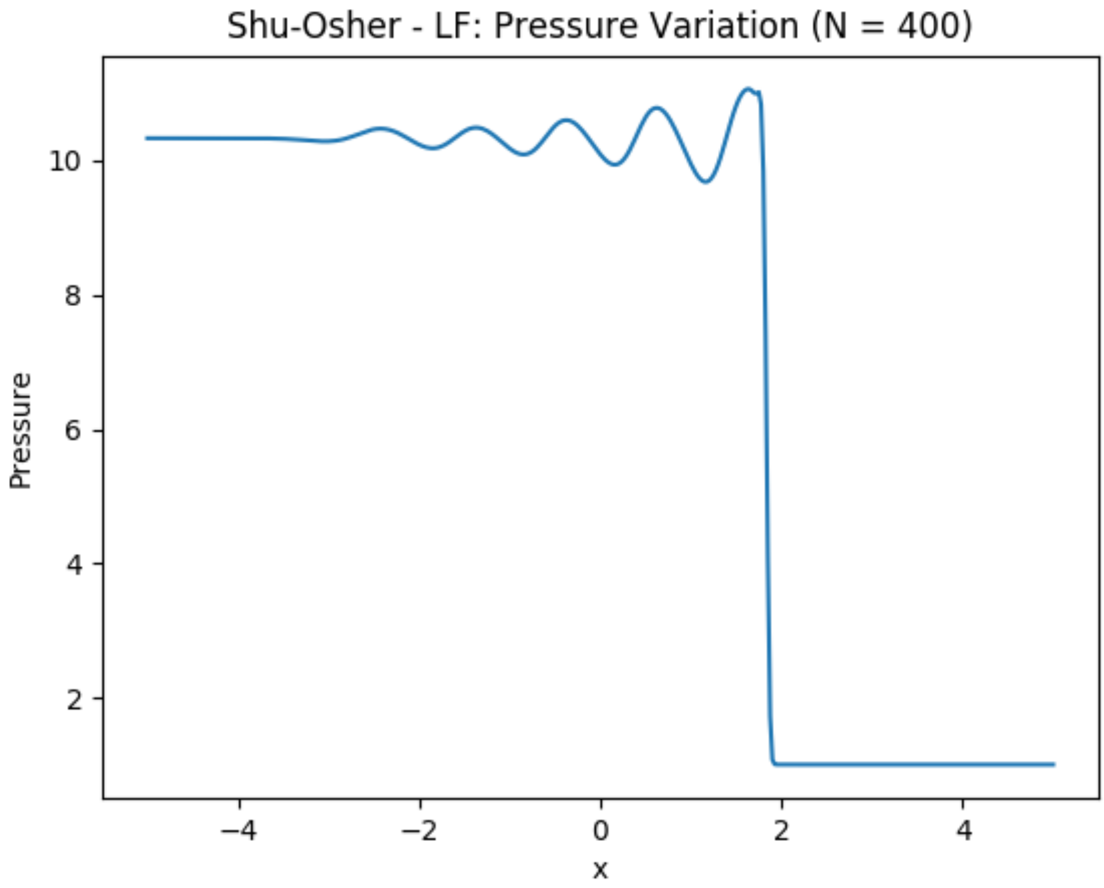



Shu-Osher - LF: Density Variation (N = 400)
Density
x
4.0
3.5
3.0
2.5
2.0
1.5
1.0
−4
−2
0
2
4

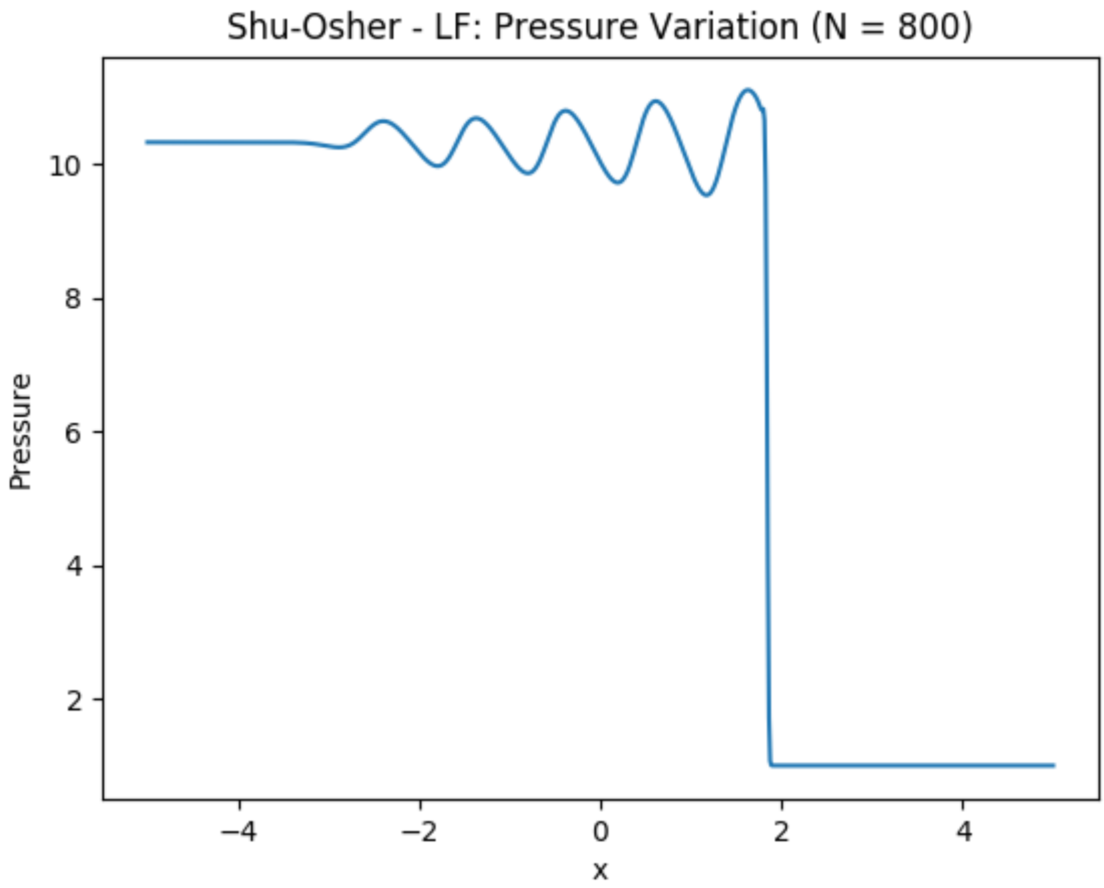

Shu-Osher - LF: Density Variation (N = 800)
Density
x
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
−4
−2
0
2
4


Shu-Osher - LF: Pressure Variation (N = 100)
Pressure
x
10
8
6
4
2
−4
−2
0
2
4

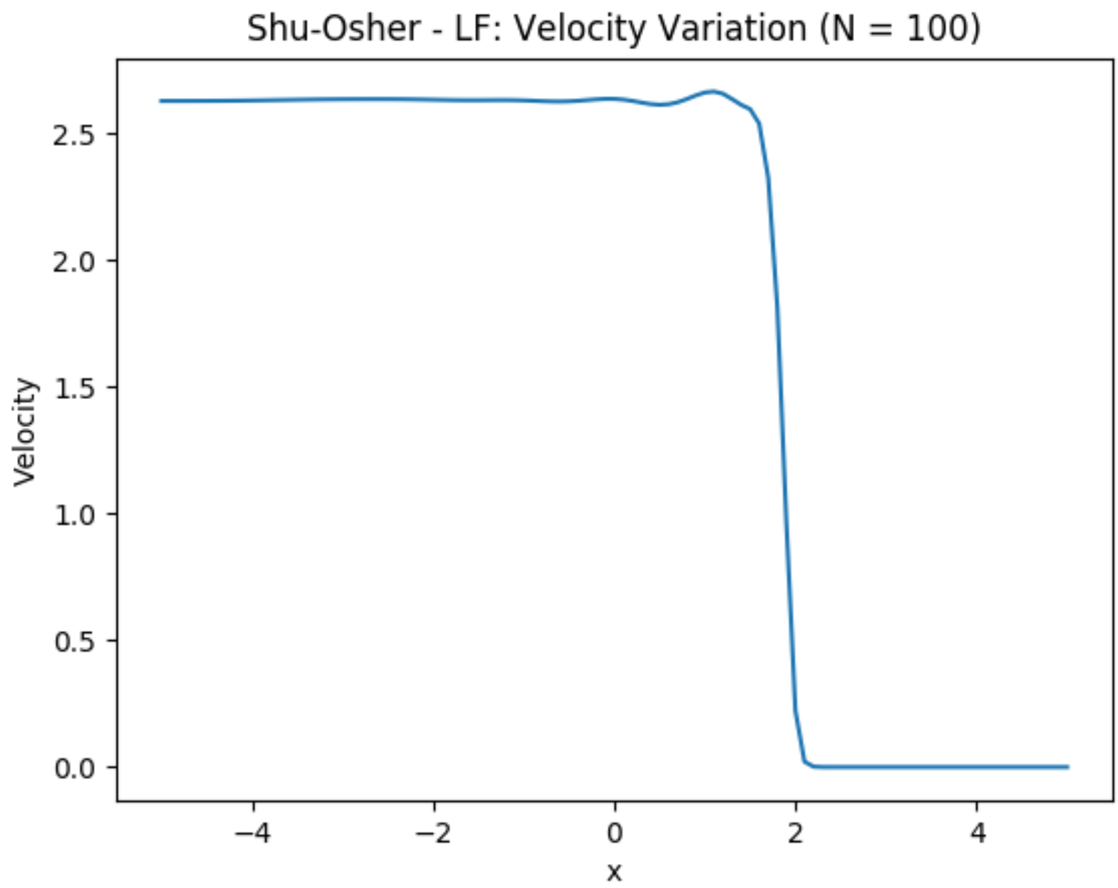

Shu-Osher - LF: Pressure Variation (N = 200)
Pressure
x
10
8
6
4
2
−4
−2
0
2
4



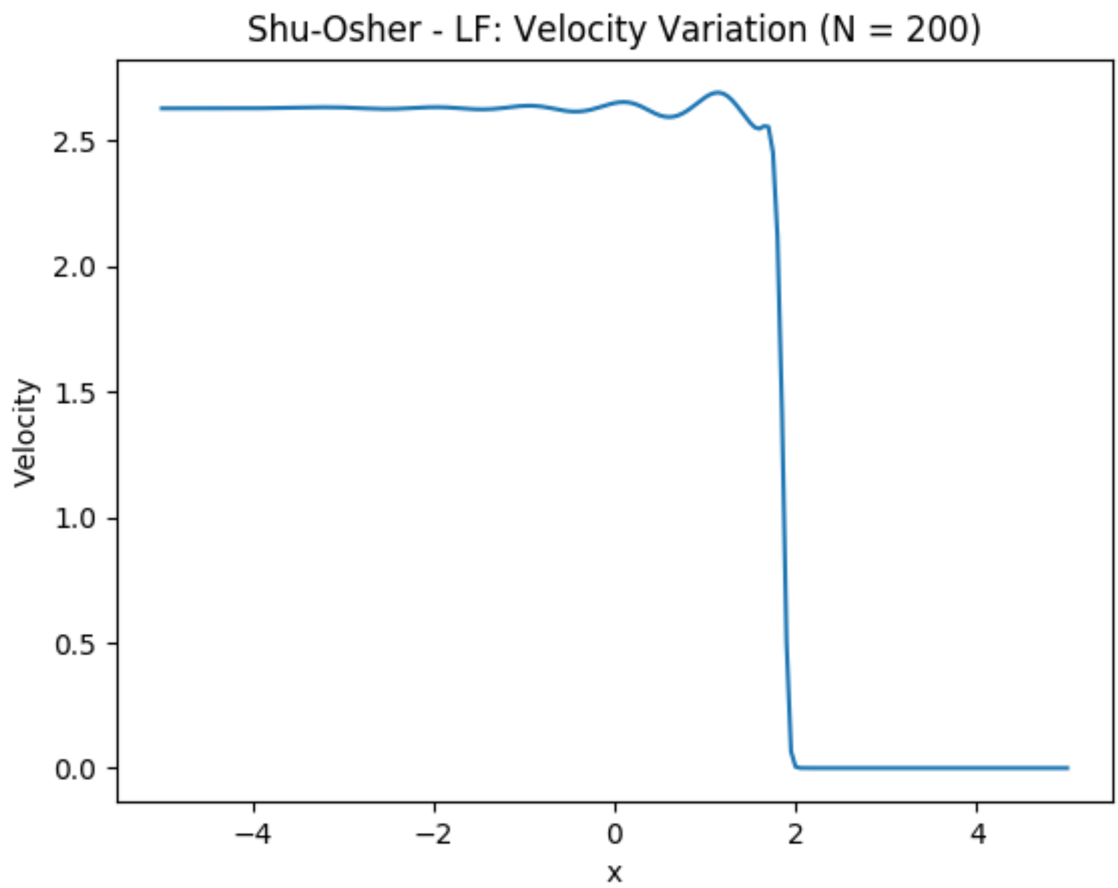
Shu-Osher - LF: Pressure Variation (N = 400)
10
8
6
4
2
Pressure
−4
−2
0
2
4
x



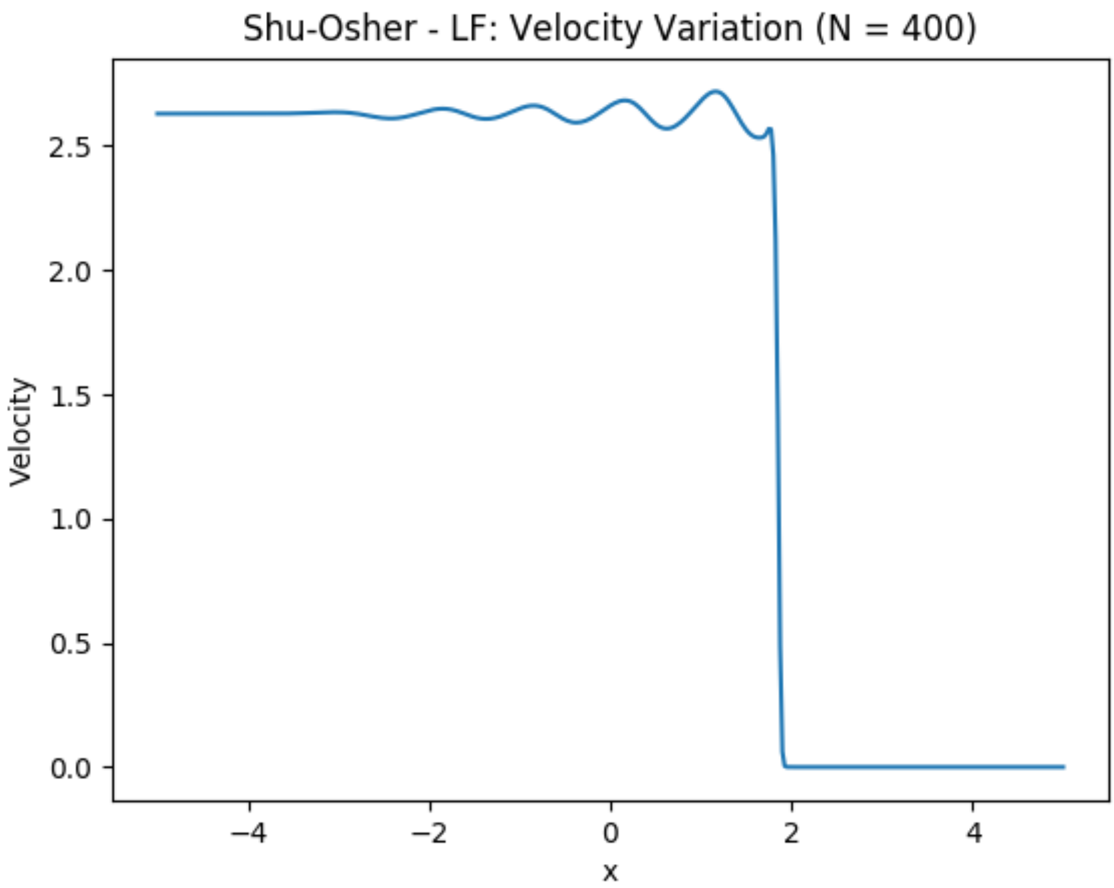
Shu-Osher - LF: Pressure Variation (N = 800)
10
8
6
4
2
Pressure
−4
−2
0
2
4
x

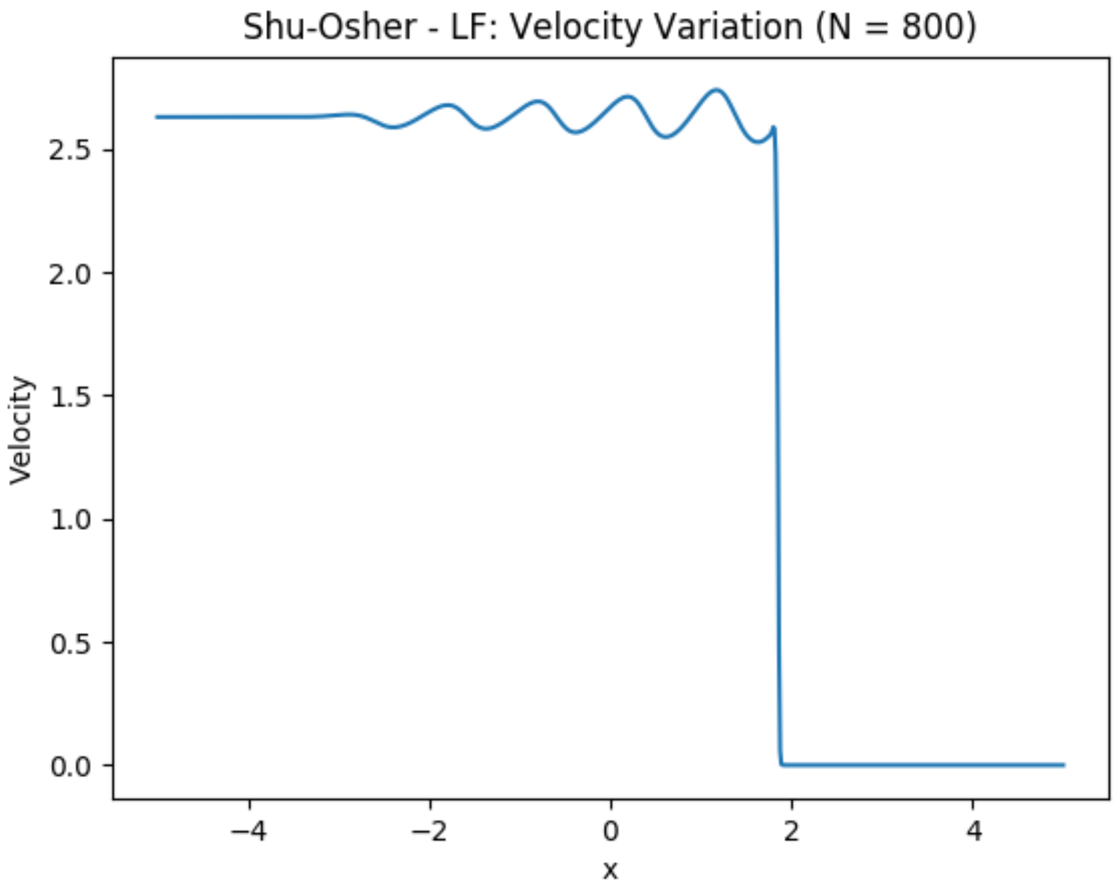


Shu-Osher - LF: Velocity Variation (N = 100)
Velocity
2.5
2.0
1.5
1.0
0.5
0.0
−4
−2
0
2
4
x

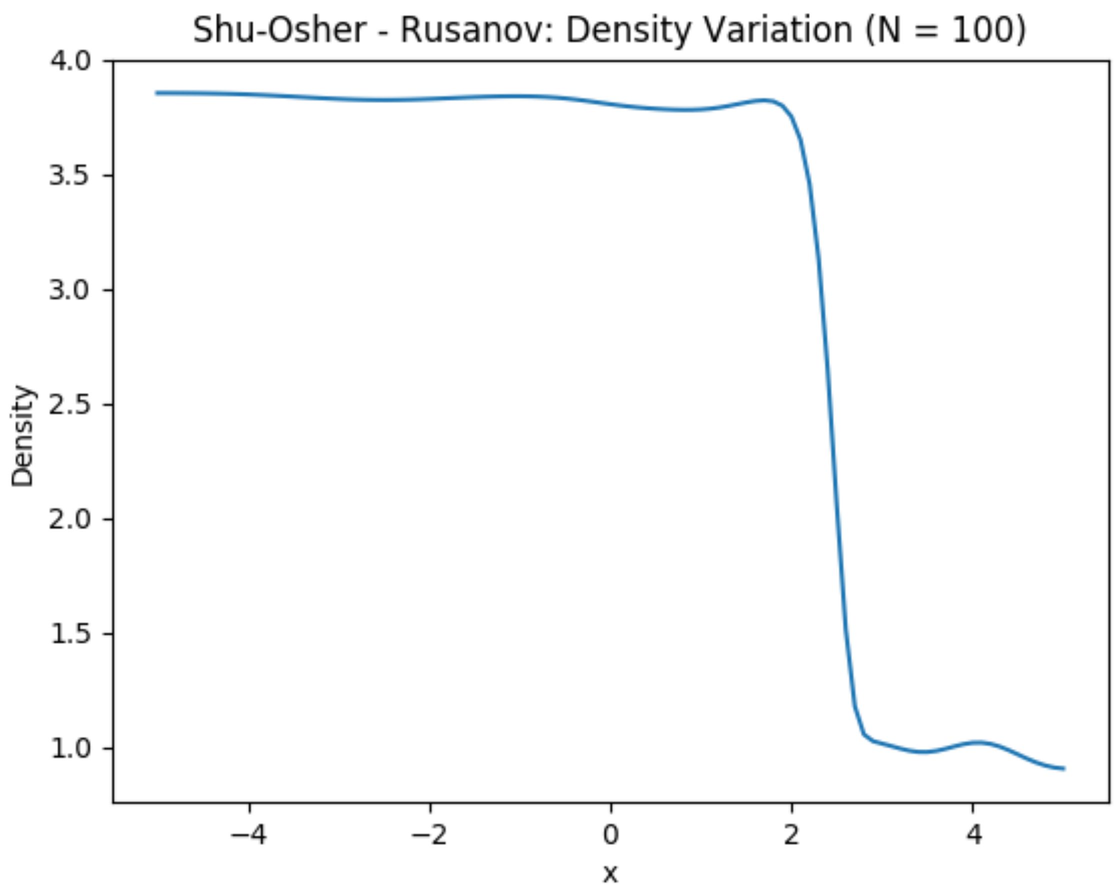


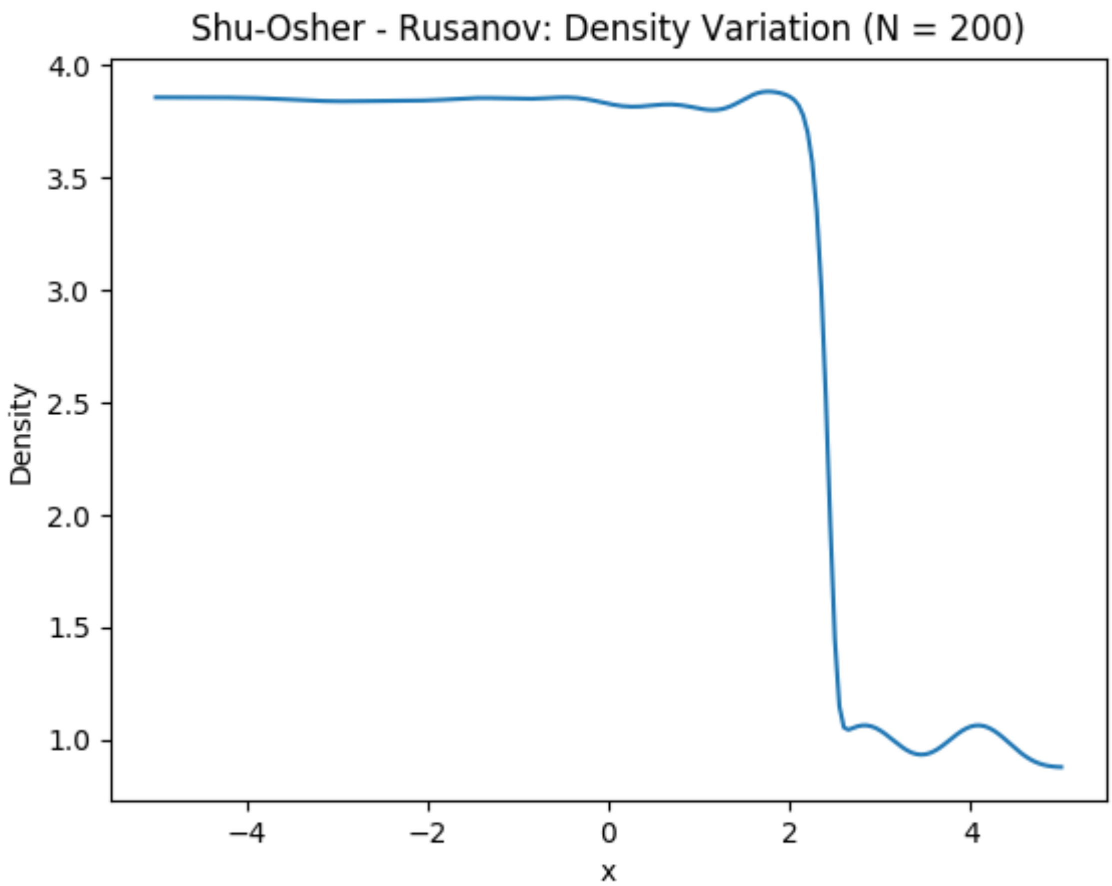
Shu-Osher - LF: Velocity Variation (N = 200)
Velocity
2.5
2.0
1.5
1.0
0.5
0.0
−4
−2
0
2
4
x

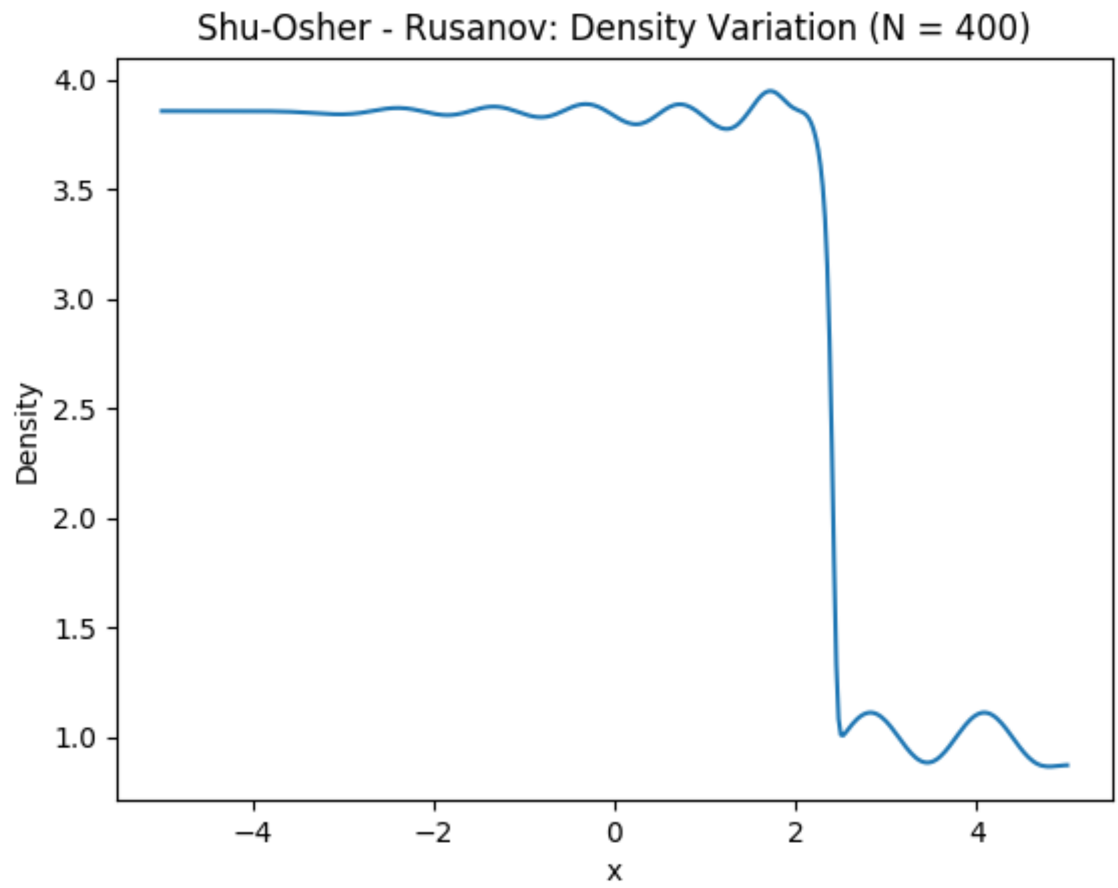

Shu-Osher - LF: Velocity Variation (N = 400)
Velocity
x
2.5
2.0
1.5
1.0
0.5
0.0
−4
−2
0
2
4

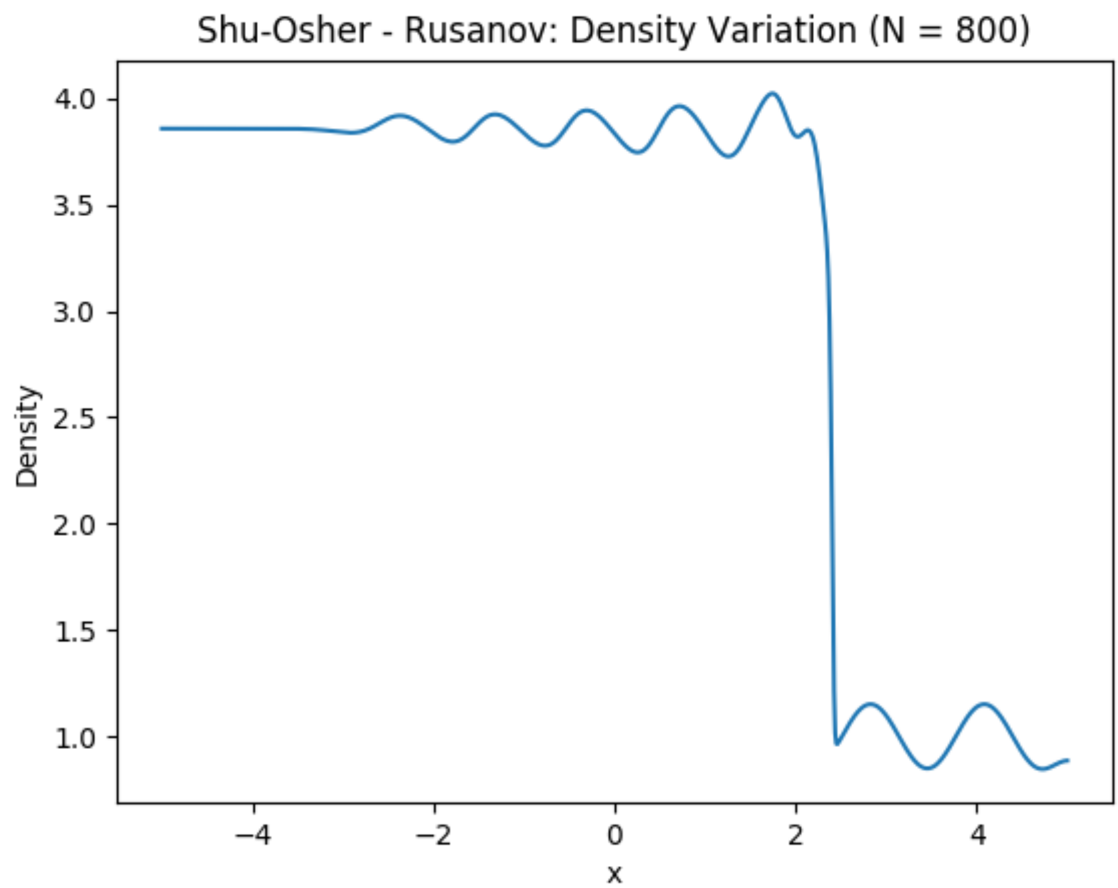

Shu-Osher - LF: Velocity Variation (N = 800)
Velocity
x
2.5
2.0
1.5
1.0
0.5
0.0
−4
−2
0
2
4

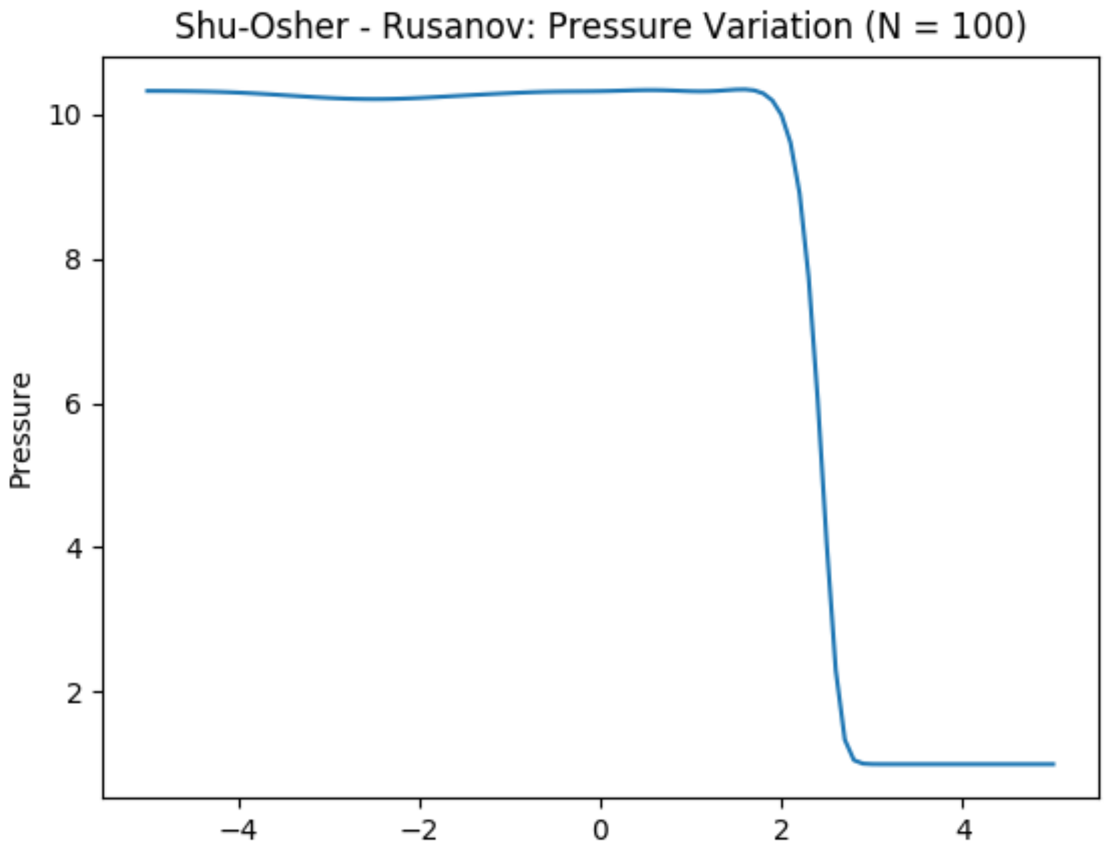
Rusanov Scheme:

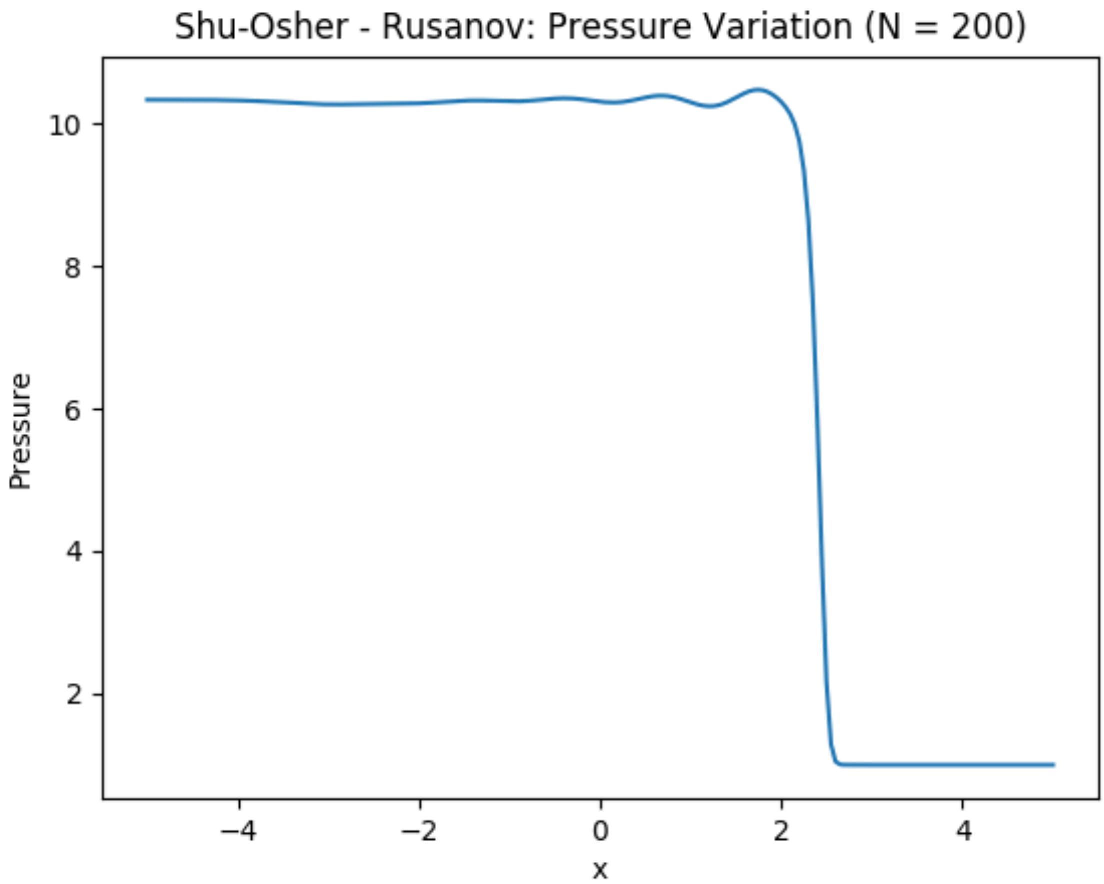


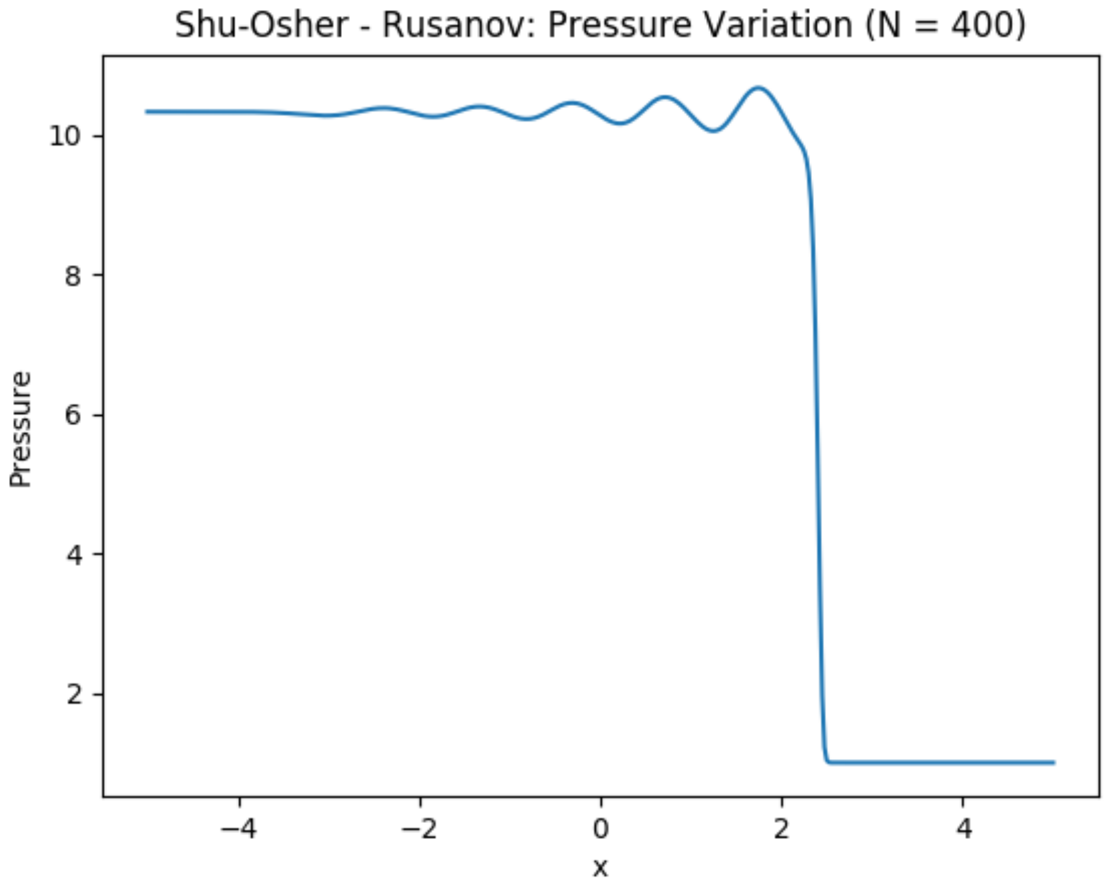


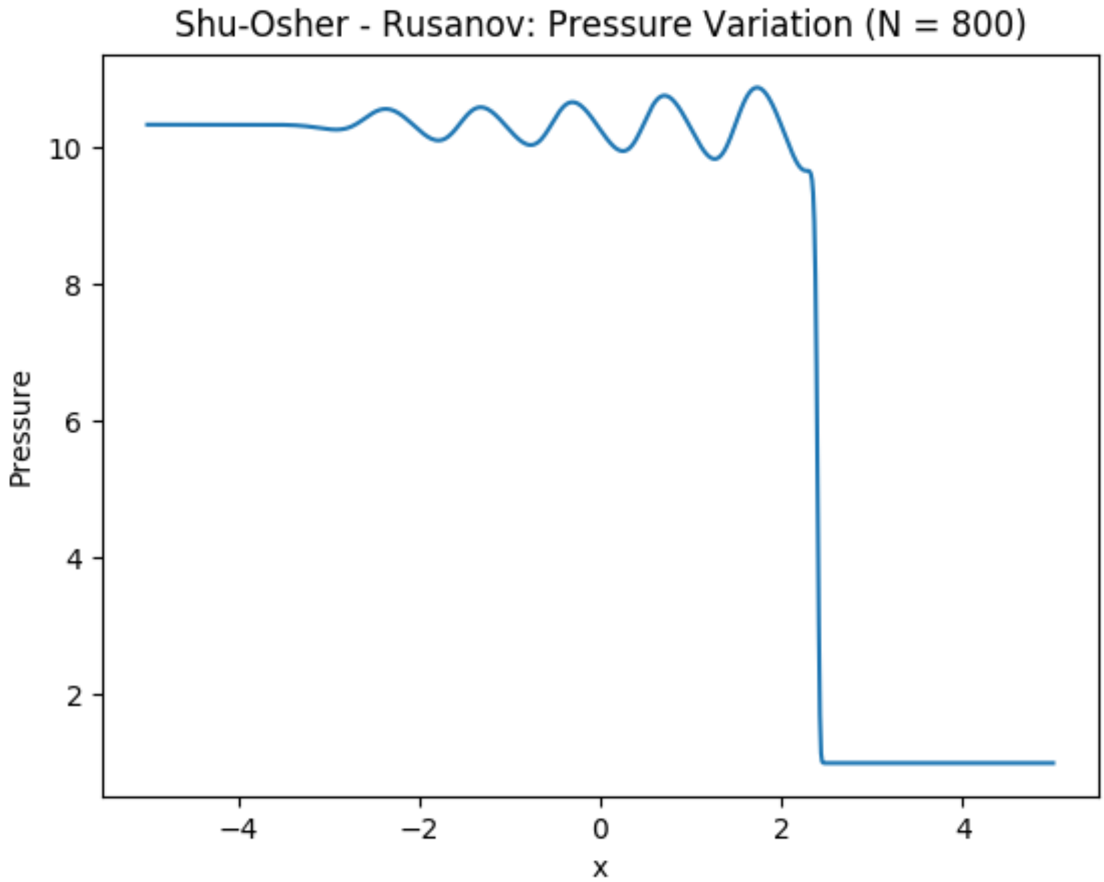

Shu-Osher - Rusanov: Density Variation (N = 400)
Density
x
4.0
3.5
3.0
2.5
2.0
1.5
1.0
−4
−2
0
2
4

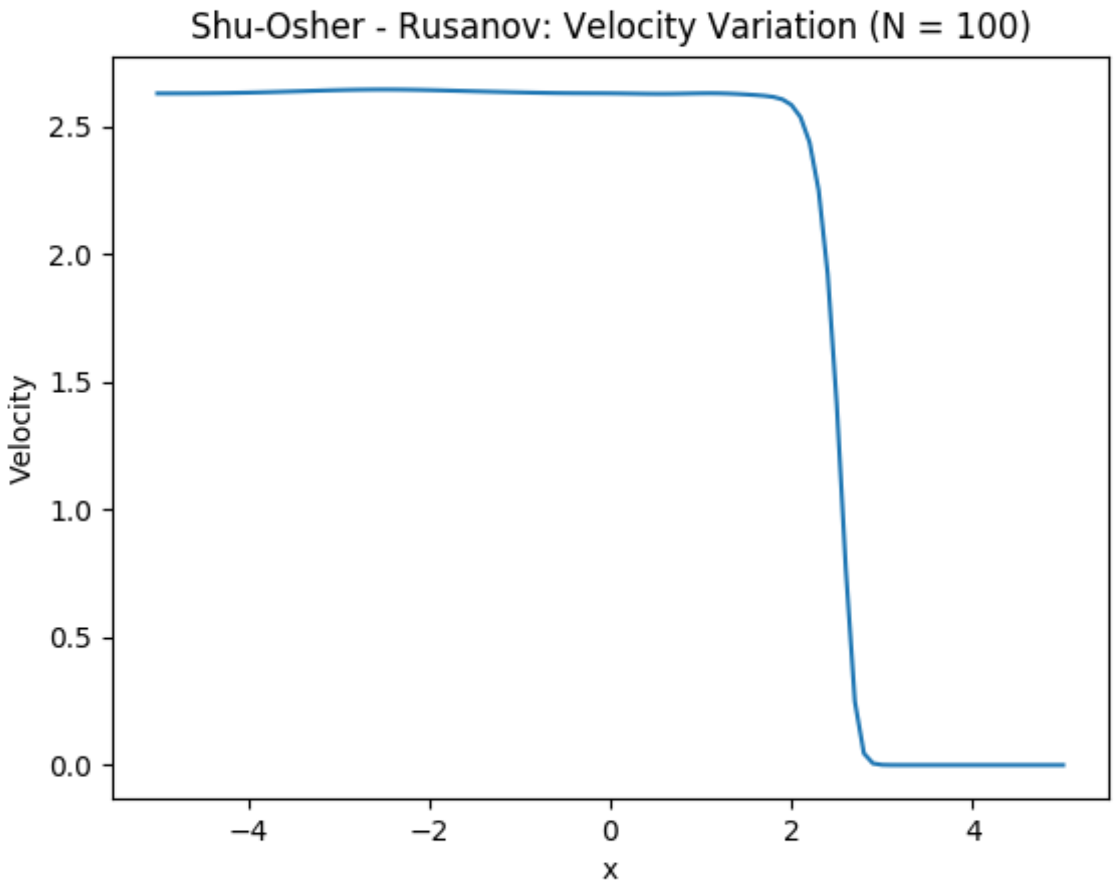

Shu-Osher - Rusanov: Density Variation (N = 800)
Density
x
4.0
3.5
3.0
2.5
2.0
1.5
1.0
−4
−2
0
2
4

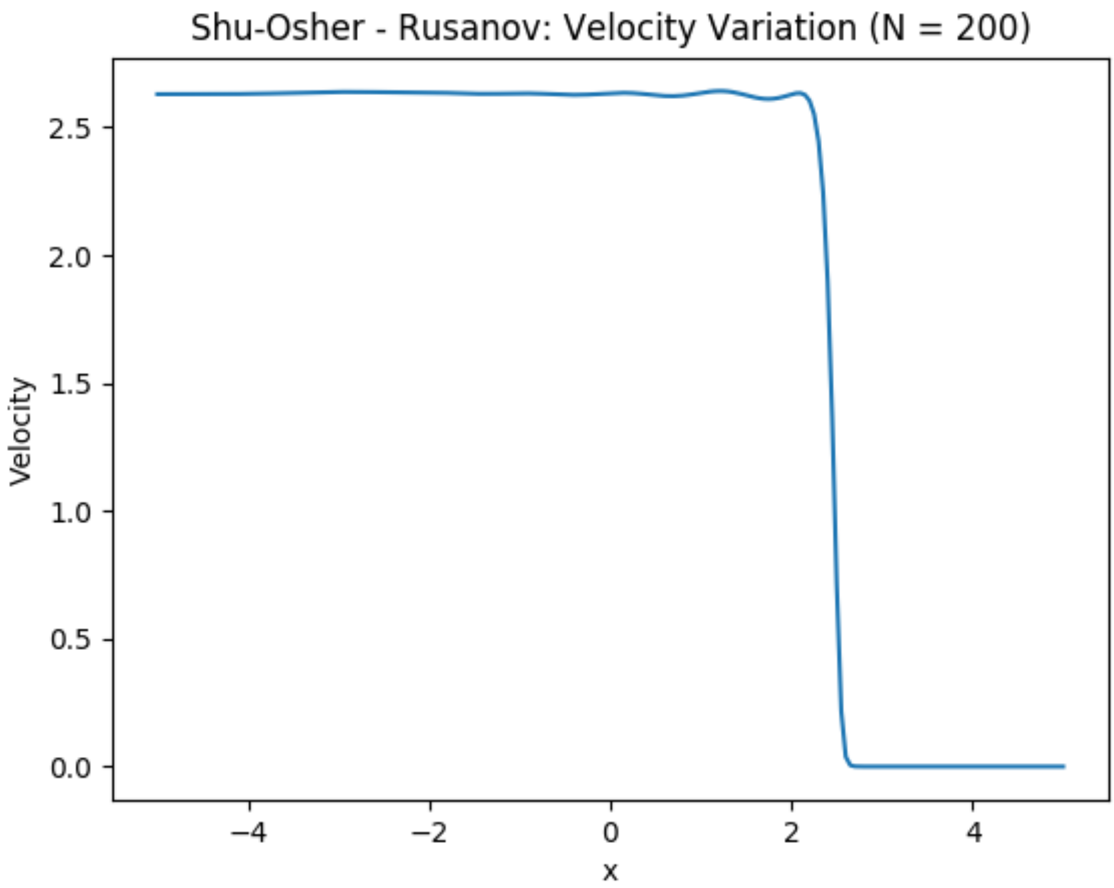

Shu-Osher - Rusanov: Pressure Variation (N = 100)
Pressure
10
8
6
4
2
−4
−2
0
2
4

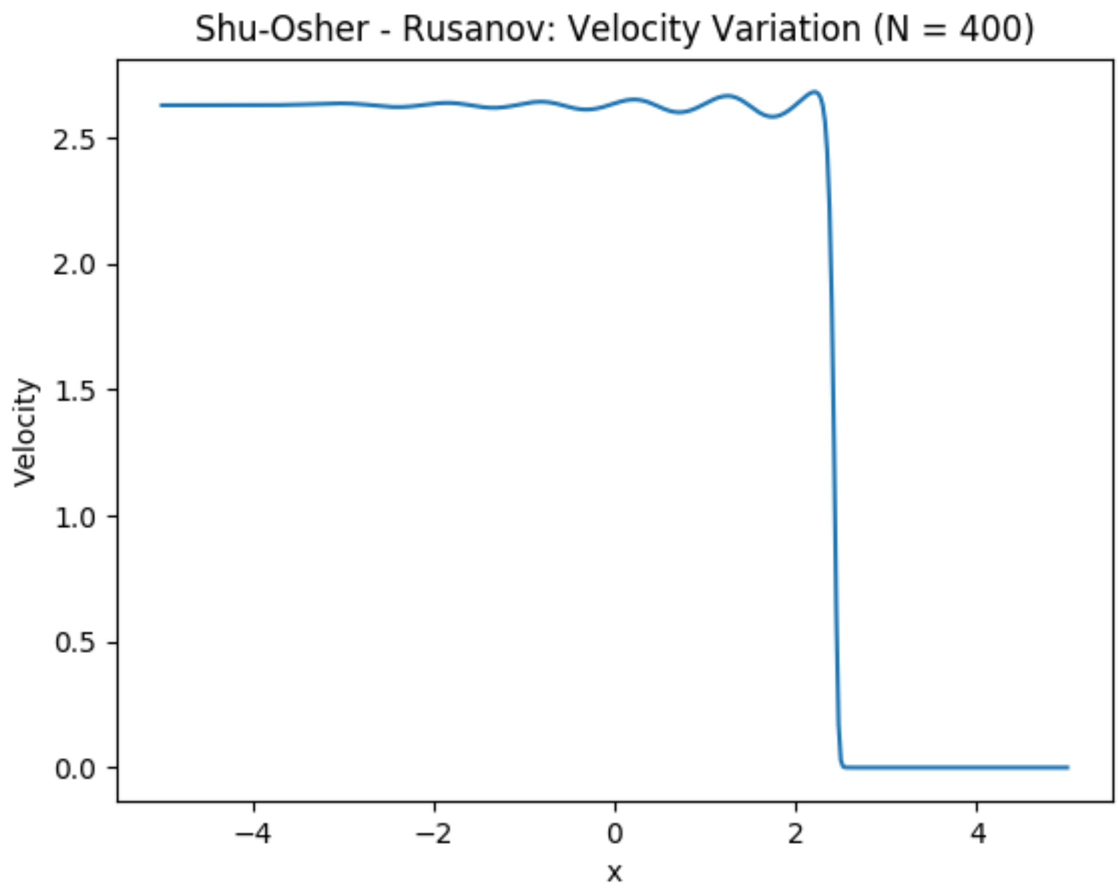

Shu-Osher - Rusanov: Pressure Variation (N = 200)
Pressure
10
8
6
4
2
−4
−2
0
2
4
x


Shu-Osher - Rusanov: Pressure Variation (N = 400)
Pressure
x
10
8
6
4
2
−4
−2
0
2
4

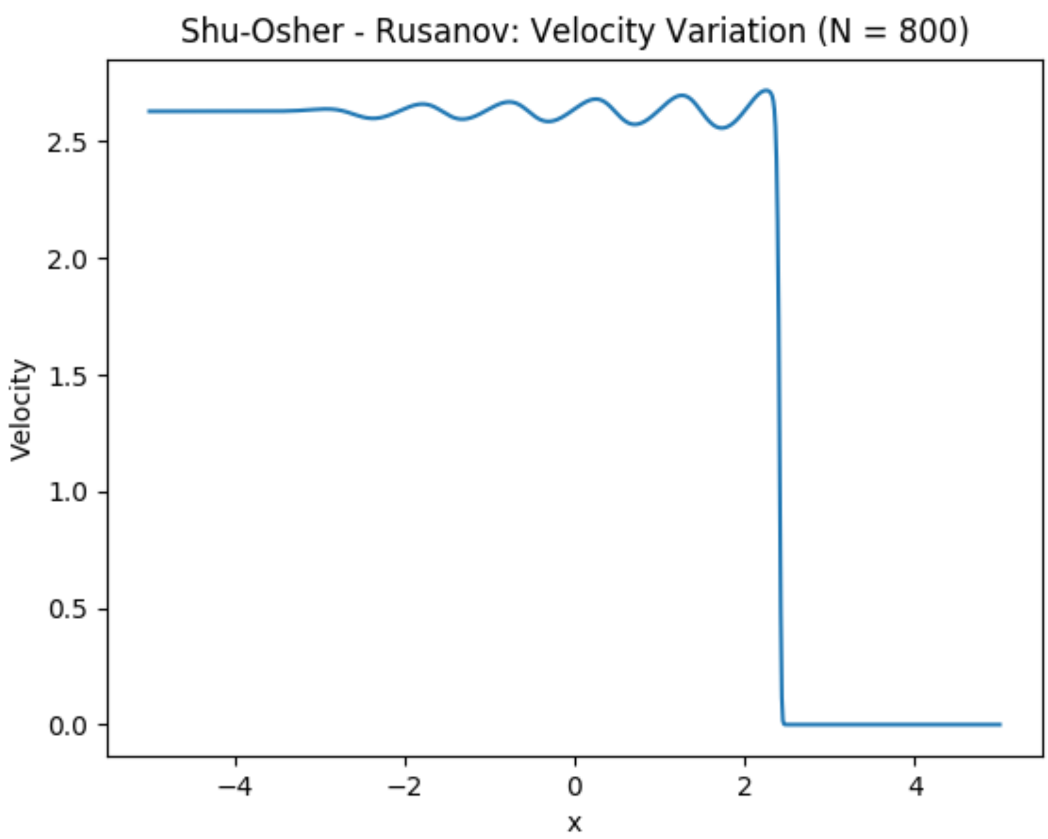

Shu-Osher - Rusanov: Pressure Variation (N = 800)
Pressure
x
10
8
6
4
2
−4
−2
0
2
4


Shu-Osher - Rusanov: Velocity Variation (N = 100)
Velocity
x
2.5
2.0
1.5
1.0
0.5
0.0
−4
−2
0
2
4


Shu-Osher - Rusanov: Velocity Variation (N = 200)
Velocity
x
2.5
2.0
1.5
1.0
0.5
0.0
−4
−2
0
2
4



Shu-Osher - Rusanov: Velocity Variation (N = 400)
Velocity
2.5
2.0
1.5
1.0
0.5
0.0
−4
−2
0
2
4
x



Shu-Osher - Rusanov: Velocity Variation (N = 800)
Velocity
2.5
2.0
1.5
1.0
0.5
0.0
−4
−2
0
2
4
x

Conclusions

1. Lax-Friedrich Scheme is unstable for TestCase2 and we get overflow error for 400 and 800 grid cells. We do not get diverging oscillations or blow up for Rusanov as we did for Lax-Friedrich scheme. The convergence of Rusanov is quite fast as we get similar output on increasing the grid cells. On comparing the actual solution of internal energy, we can see that we do not get a peak in the middle like the ones obtained by the authors in the reference paper so maybe, Rusanov seems to be working betters than those schemes.
2. We took gamma = 1.66 for Noh's problem which is unlike other problems. Although Lax-=Friedrich in this case is not as divergent as in test case 2, it is still not able to resolve the shock wave well as we are getting oscillations. The dispersion effect of Lax-Friedrich can be seen more clearly in the form of ripple like irregularities. Rusanov solution is much smoother and we do not see dispersive effect. Both the solution resemble the actual solution with Rusanov scheme getting a much better solution in terms of resemblance to the actual.
3. Both the schemes in this case are able to resolve the shock wave with some oscillations due to oscillatory boundary conditions. The oscillations become much more apparent on increasing the grid cells. Solution obtained matches the one in the paper (given below).
4. In test case 2, we get peak on increasing grid cells whereas in other cases solution converges to reference solution on increasing grid cells.