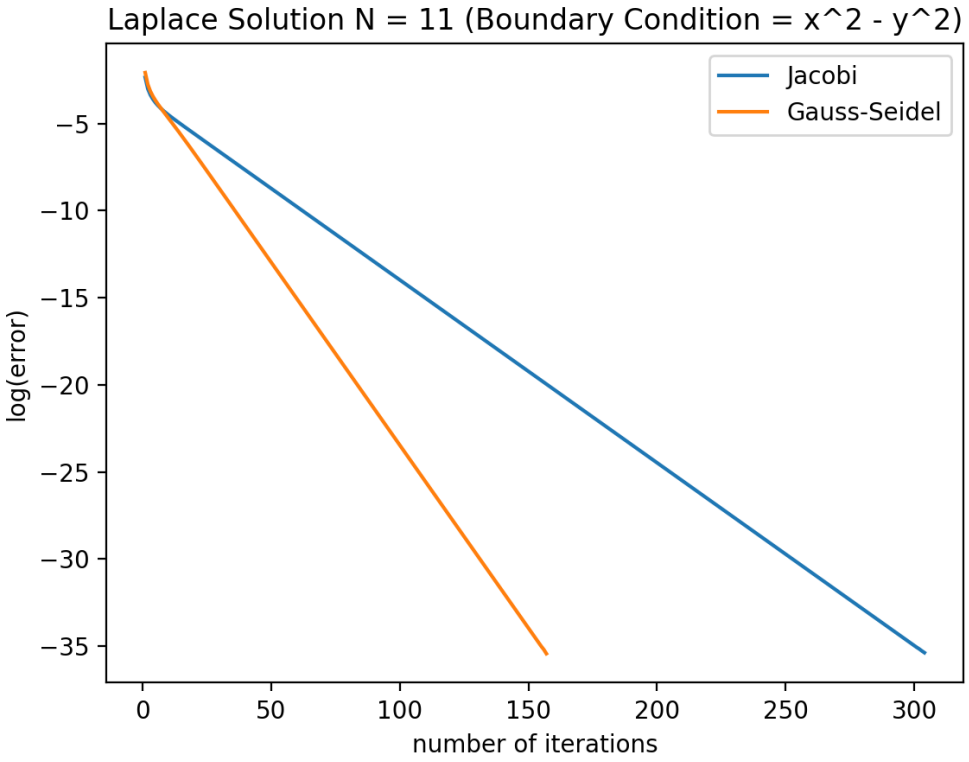
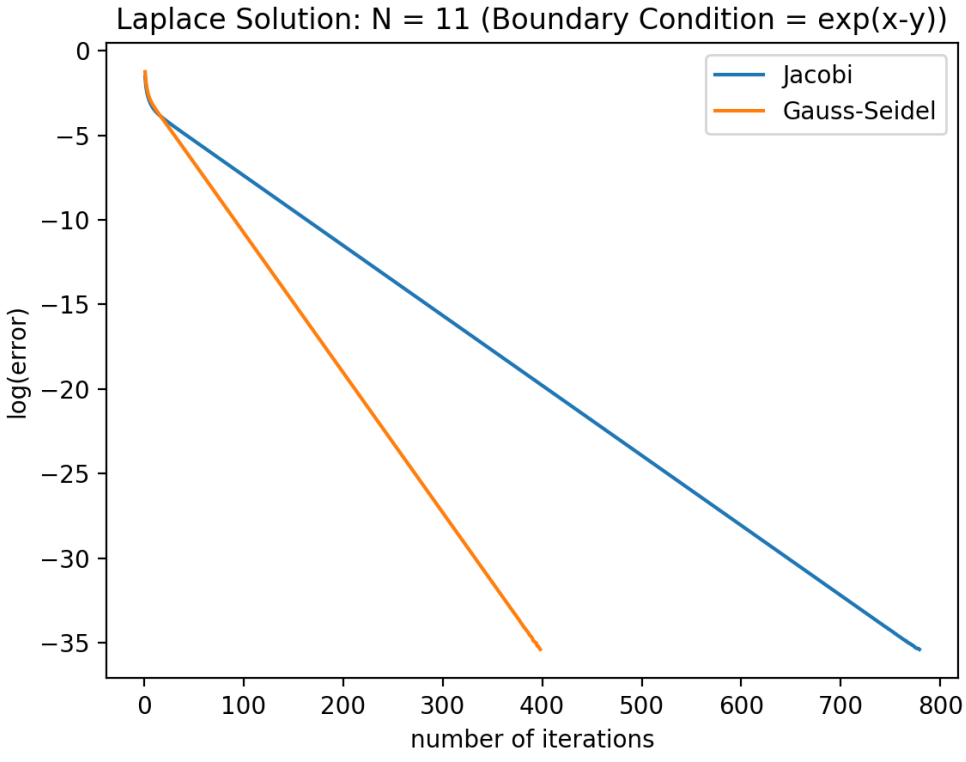
CFD HW 2

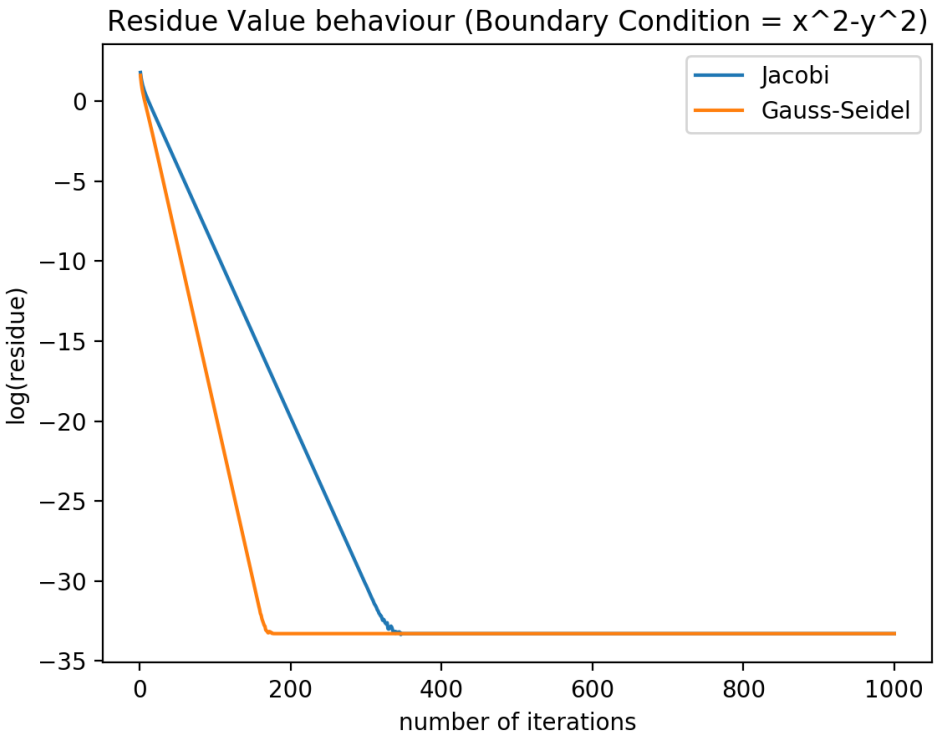
The assignment contains 4 python scripts - gauss_seidel.py, jacobi.py, solve_surface_plot.py and SOR.py. Values of N, boundary conditions and number of iterations were changed in this code itself for the various plots in this report.

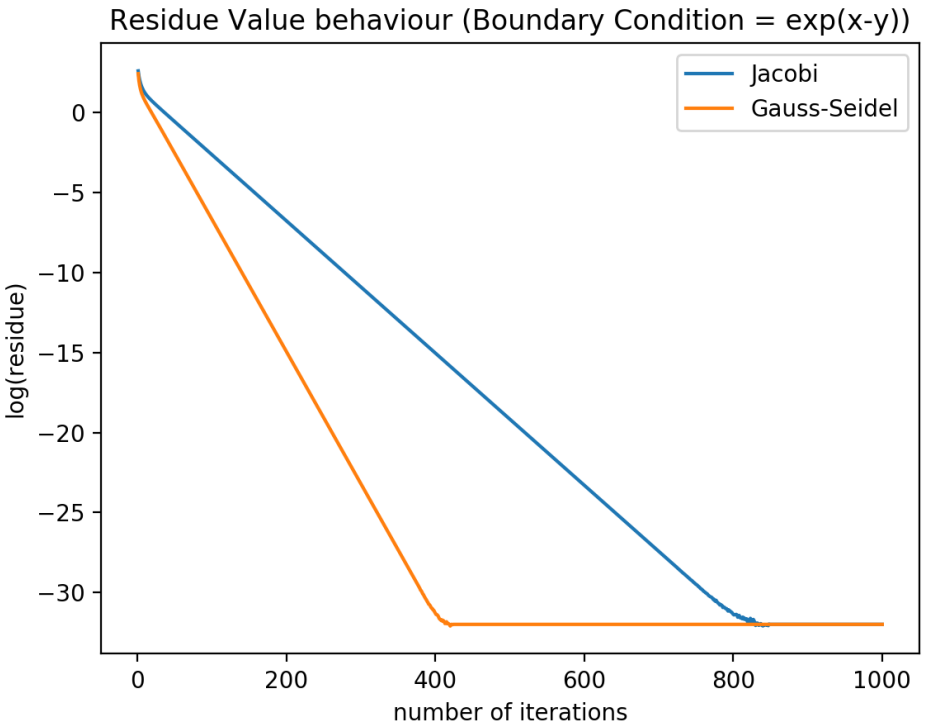
Question 1A





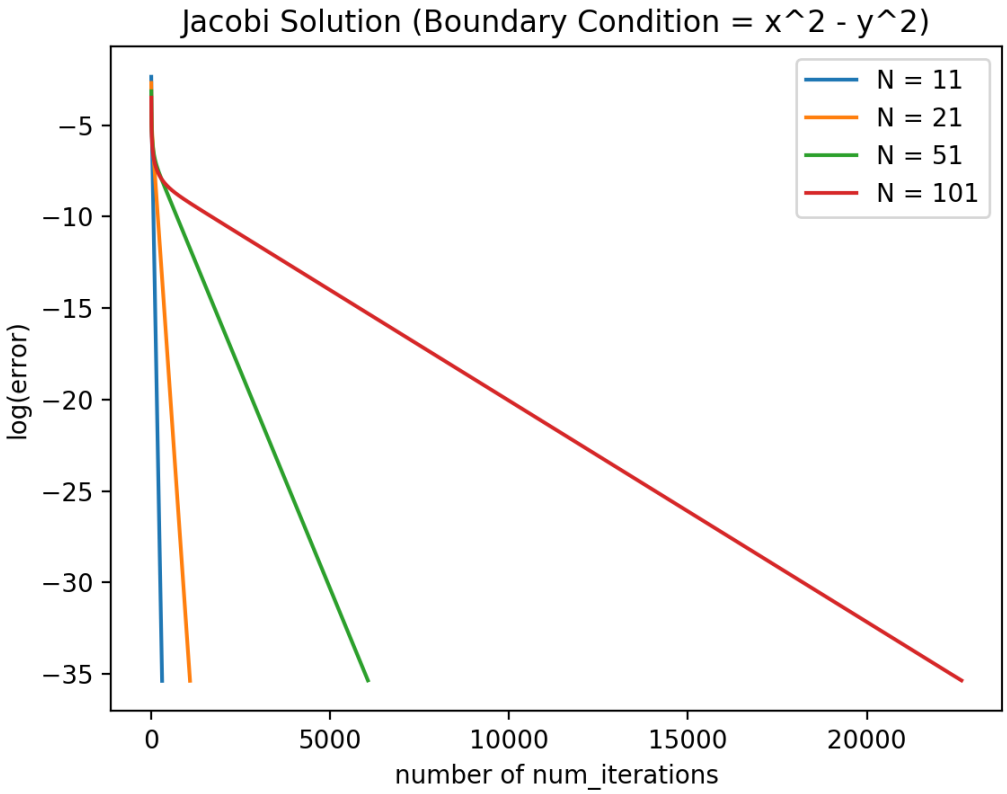
We can see that for the second boundary conditions, it takes more number of iterations to converge as compared to the first boundary condition which is due to faster decay of exponential function due to which it takes longer time for ϕ to propagate as compared to more uniform $x^2 - y^2$.

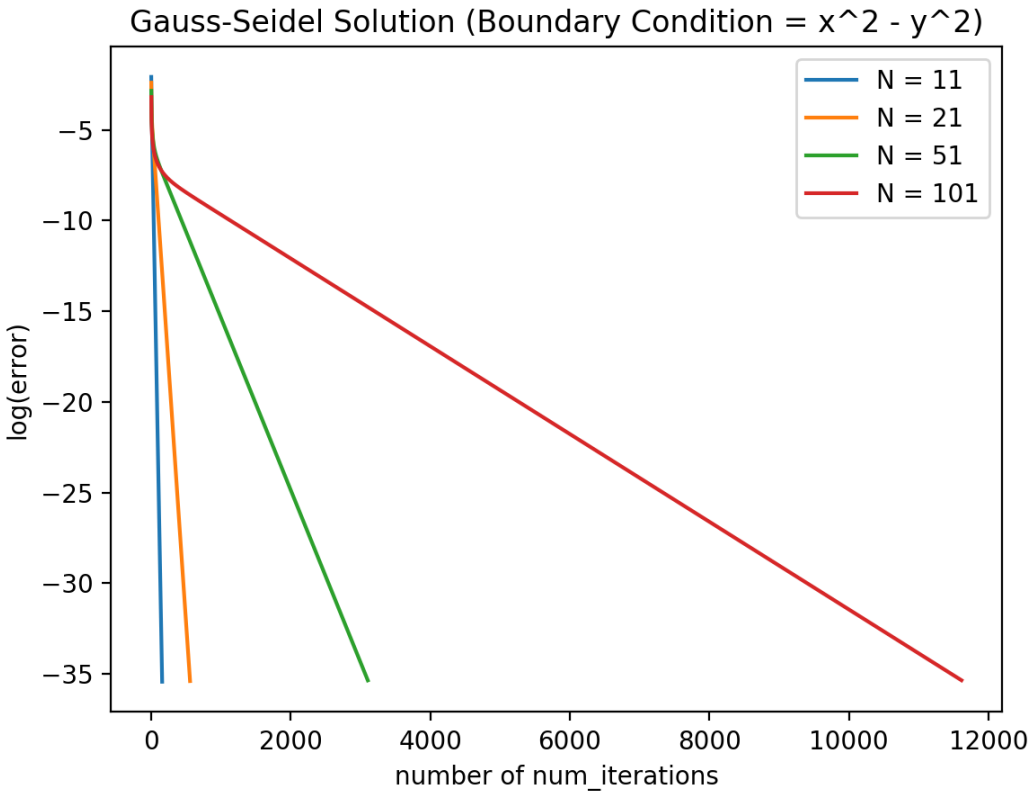




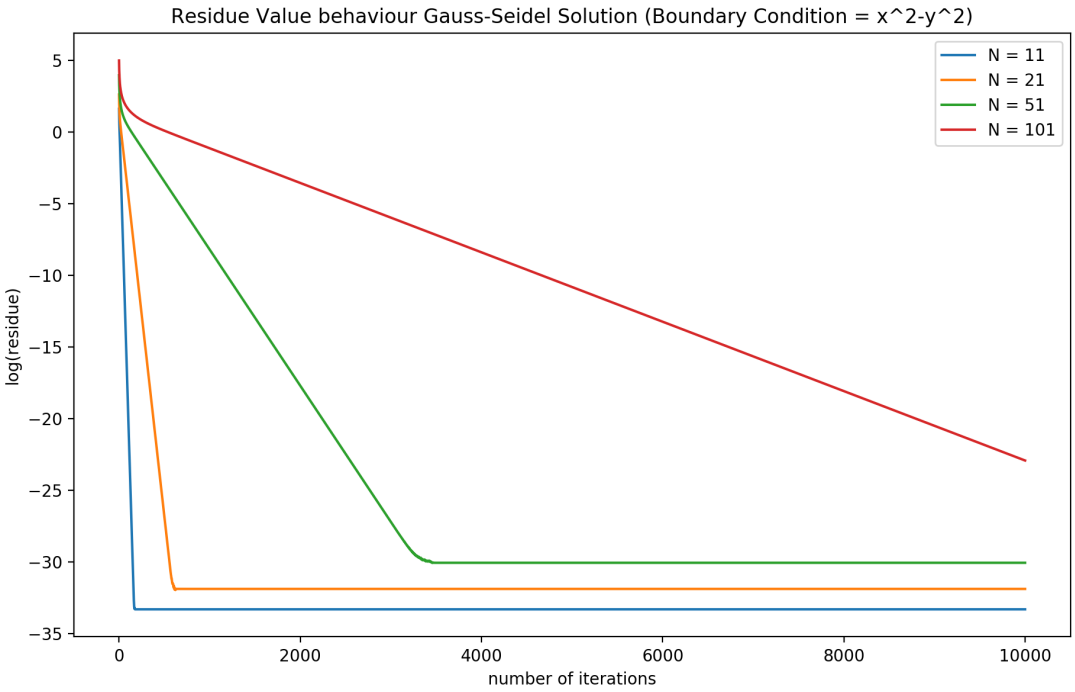
We see that residue converges after number of iterations where our error became below 2*(machine epsilon). There is only within some error that we can solve our equation. So, this makes sense.

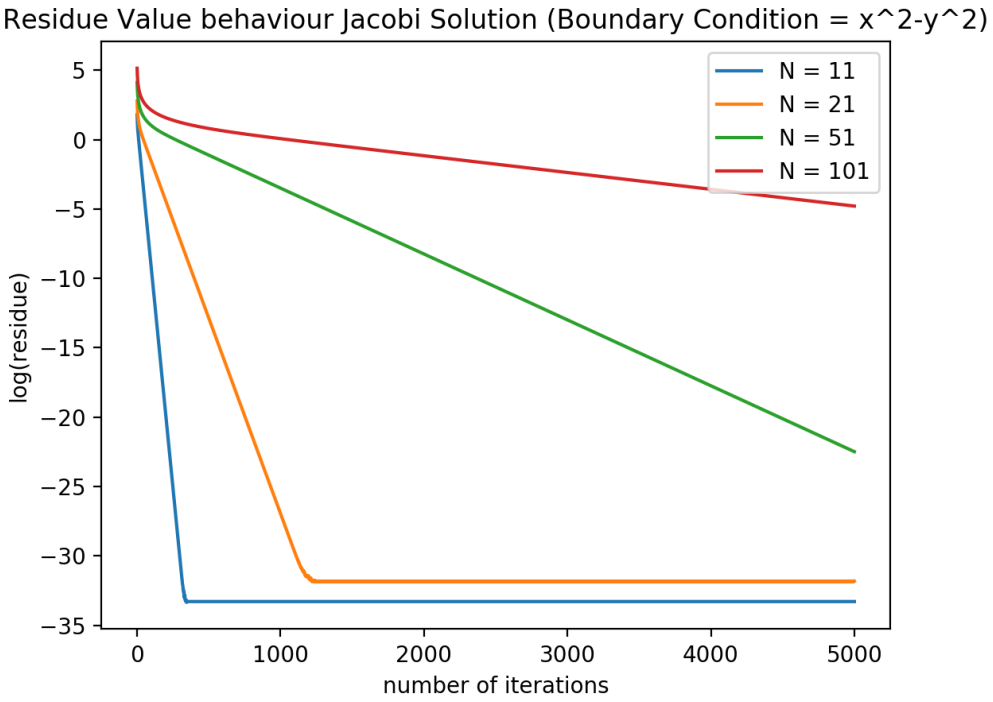
Question 1B



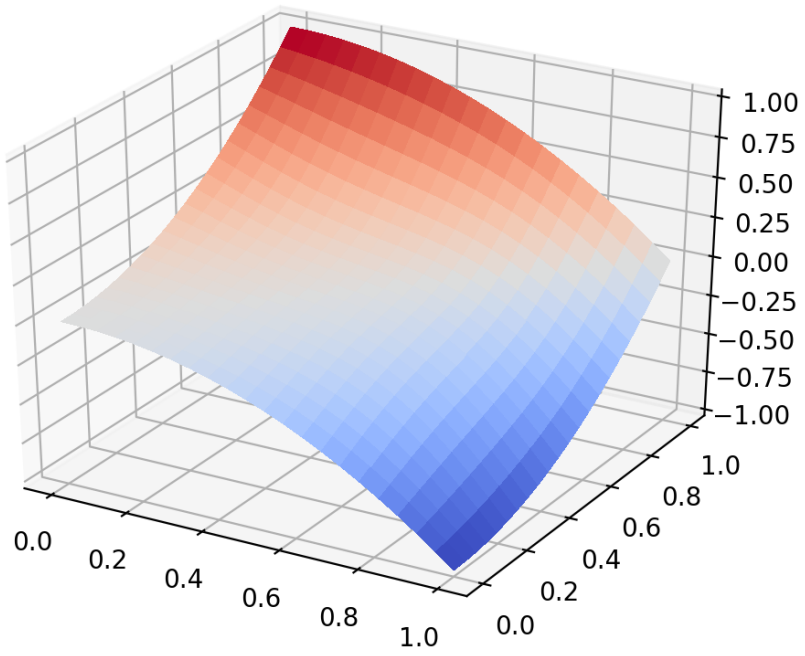


The above graph shows the variation of error using Jacobi and Gauss Seidel methods as a solution to Laplace equation. We can see that our solution scheme converges as the error reduces as number of iteration increases. We can also observe that as number of grid points increases, number of iterations required to reduce the error to 2(machine epsilon) also increases, which makes sense as for a larger grid it takes more time to propagate from boundary. Moreover number of iterations required is almost half in case of Gauss Seidel which is expected as Gauss-Seidel which is due to the fact that Gauss Seidel uses updated values for iteration and Jacobi uses values which are all from the previous iteration. In general, this does not ensure that the Gauss-Seidel's method would converge faster than Jacobi iterations but in case of Laplace function, this helps in faster propagation of ϕ values.

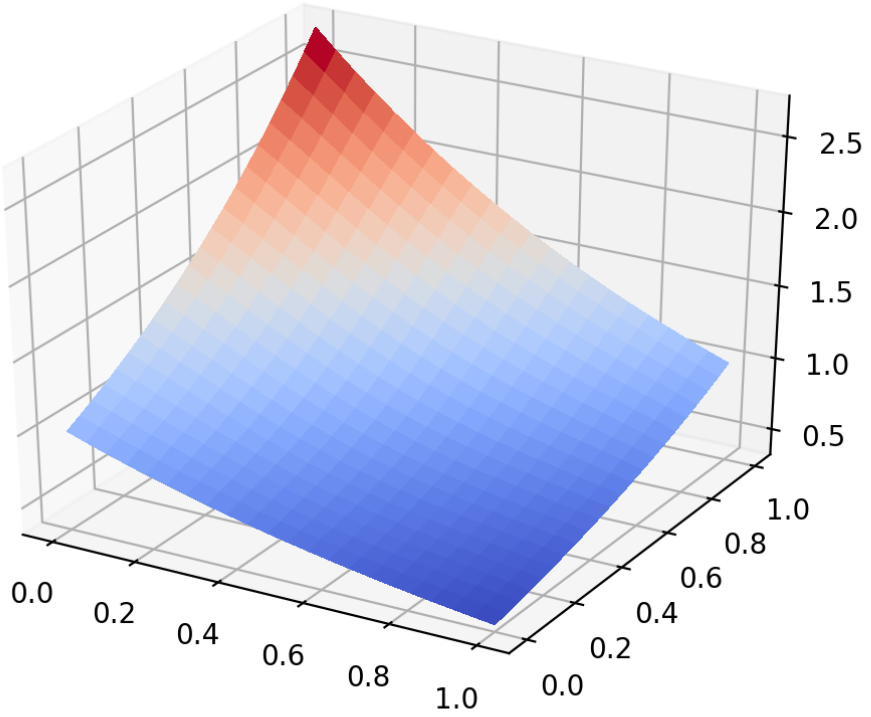




We can see a clear correlation between convergence of error to twice of machine episilon and saturation of residue value.

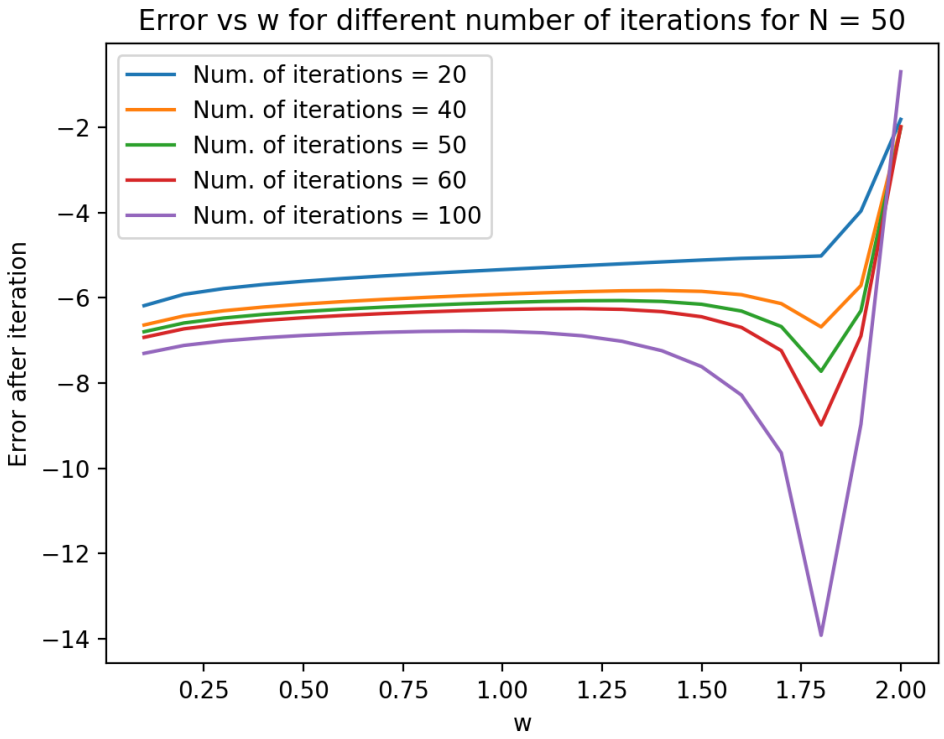


Surface plot for gauss seidel solution for N = 21 (Boundary Condition 1)



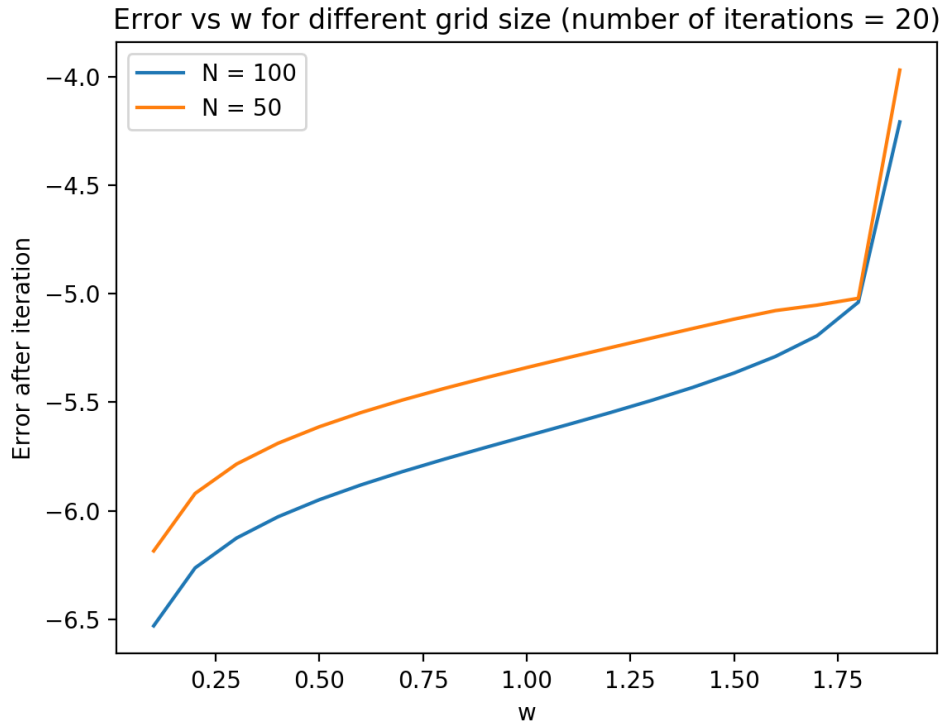
Surface plot for gauss seidel solution for N = 21 (Boundary Condition 2) - Here we can see that the drop is very steep and hence it takes more number of iterations for convergence in the 2nd boundary condition.

Question 2



I got W_opt = 1.8 for all the number of iterations values except when number of iterations is 20. W_opt for 20 iterations is 0.1. As we can see that all the graphs proceed in a similar manner with the error increasing slightly before suffering a major dip around w = 1.8. In case of 20 iterations, the number of iterations are too few for the dip to go below the initial low value. A consistent trend is that the error starts to diverge after w = 1.8.

Question 3



W_opt does not change. We get minimum error at w = 0.1 and in both the plots, we

Question 4

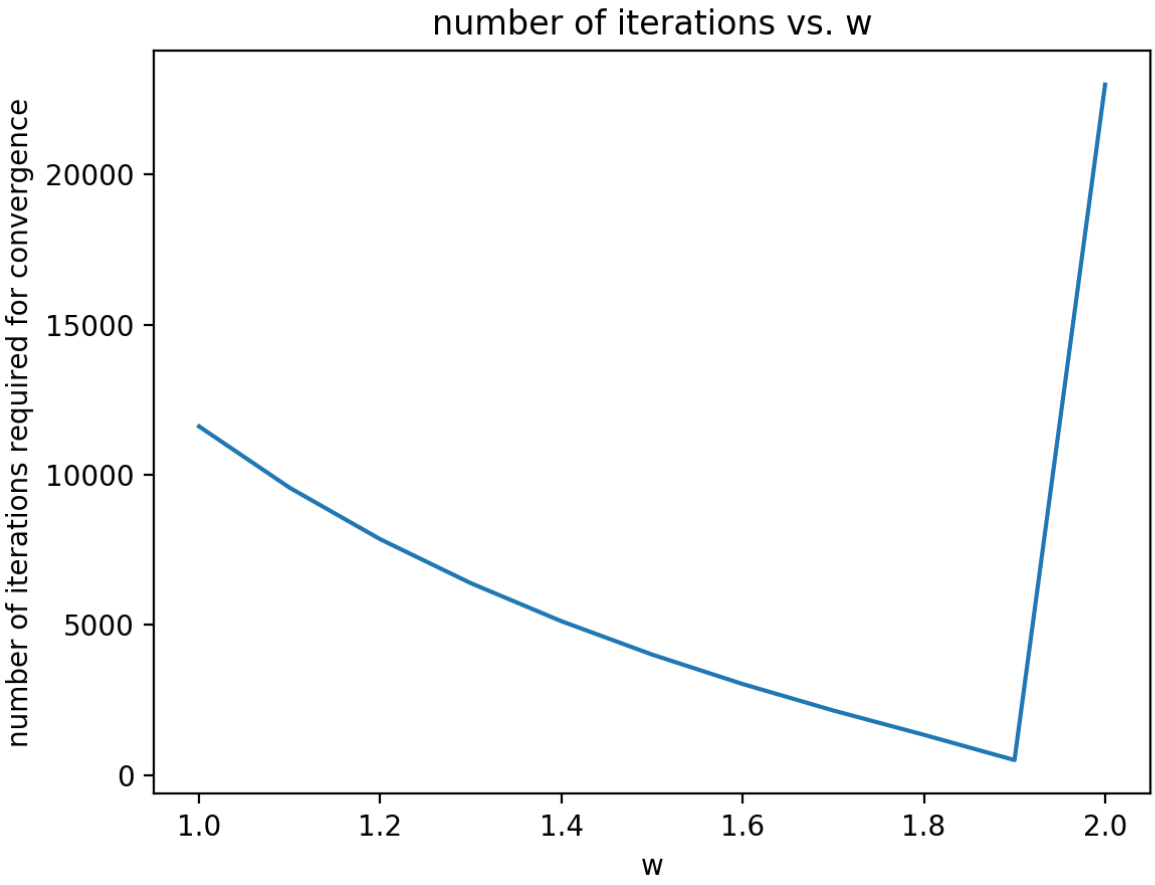
For N = 20 and total iterations of 50, I got the approximate of w_opt = 1.6. After iterating again in the interval of (1.5, 1.7), I got w_opt = 1.61.

Question 5

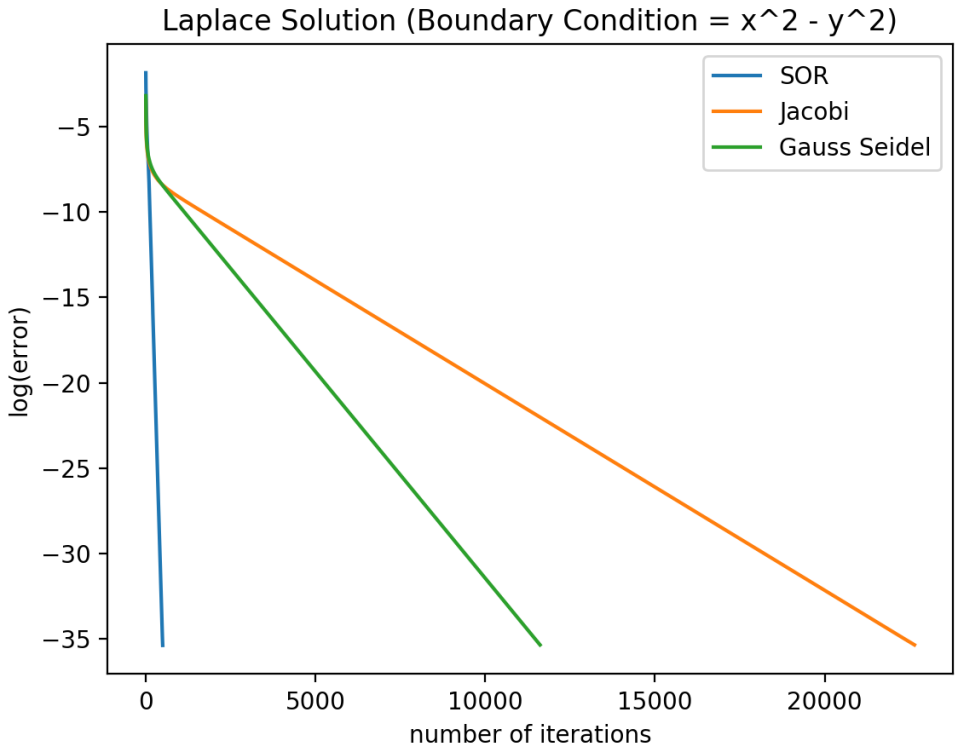
For N = 101 and total iterations of 100, I got w_opt = 1.9.

Question 6

Number of iterations required till convergence decreases steadily till 1.9. The solution does not converge for w = 2. For w = 2, $\phi^{n+1} \approx \phi^n$ so the error stops changing after certain iteration values. If we



Question 7



The above plot clearly shows that convergence of SOR is way faster that convergence of Gauss Seidel which is faster than that of Jacobi. As convergence depends on spectral radius of the numerical scheme and spectral radius of SOR is smallest, it was expected that it will converge the fastest and with the largest spectral radius, we expected Jacobi to converge the slowest.