Assignment 4
AE-706/320

March 25, 2019

Problem no.1:

Consider 2-D general diffusion problem

$$\frac{\partial u}{\partial t} = \nu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] \tag{1}$$

With initial condition as:

$$u(x, y) = A * Exp\left(- \left(\frac{(x-10)^2}{2} + \frac{(y-10)^2}{2} \right) \right) \tag{2}$$

Take $A = 2$ with domain size as $X = [0, 20]$, $Y = [0, 20]$

Solve this problem with Finite Volume Method with 101 $\times$ 101 cells , $dt = 0.01$, $\nu = 0.1$. Run this problem for $t = 40$ secs. Plot a 3D projection in python for 0 ,10, 20 ,30 ,40 secs. Compare the diagonal plots of all the timesteps.

Problem no.2:

Consider 2-D advection problem

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{3}$$

With initial condition as:

$$u(x, y) = A * Exp\left(- \left(\frac{(x-5)^2}{2} + \frac{(y-5)^2}{2} \right) \right) \tag{4}$$

Take $A = 2$ with domain size as $X = [0, 20]$, $Y = [0, 20]$

Solve this problem with Finite Volume Method with 101 $\times$ 101 cells , $dt = 0.001$. Apply Lax-Friedrichs , Upwind and Lax-Wendroff (FTCS2) scheme and run it for $t = 15$ secs. Plot a 3D projection in python for $t = 0, 5, 10, 15$ secs. Plot the diagonal for $t = 5$ secs when the profile is in center. Compare the plots with all the schemes.
Comment on the dispersion and diffusion in the plots.

Problem no.3:

Consider 2-D Burger's problem

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial y} = \nu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] \tag{5}$$

With initial condition as:

$$u(x, y) = A * Exp\left(-\left(\frac{(x-5)^2}{2} + \frac{(y-5)^2}{2}\right)\right) \tag{6}$$

Take $A = 2$ with domain size as $X = [0, 15]$, $Y = [0, 15]$ and $\nu = 0.0$

Solve this problem with Finite Volume Method with 101 $\times$ 101 cells , $dt = 0.001$. Apply Lax-Friedrichs , Upwind, Lax-Wendroff(FTCS2) scheme and run it for $t = 25$ secs.

Plot a 3D projection in python for $t = 0, 5, 15, 25$ secs.
Plot the variation of the **both** diagonal values for $t = 20$ secs .

Compare the diagonal plots by changing the ν with $\nu = 0.02, 0.04$ for all schemes for $t = 20$
Comment on the dispersion and diffusion in the plots.