Consider a unit square with the bottom left corner at the origin. Discretize the square into an N x N grid. Set the boundary conditions to

a).x^2 - y^2 ,

b).exp(x-y),

Implement the Jacobi, Gauss-Seidel, and SOR schemes. At each iteration, calculate the difference between the previous solution and the next, i.e.

$$(err = \frac{\sqrt{\Sigma_{i,j}(\phi_{i,j}^{n+1} - \phi_{i,j}^{n})^2}}{N} \backslash)$$

Also calculate the residue instead of the error above and see how that behaves.

Iterate until the difference (error or the residue) is less than the machine epsilon*2.

1) A). Solve this problem for N=11, Plot the error versus the iteration count on a semi-log plot. Do this for the Jacobi and Gauss-Seidel schemes for all boundary conditions

   B). Solve this problem for N=21, 51, and 101. Plot the error versus the iteration count on a semi-log plot. Do this for the Jacobi and Gauss-Seidel schemes. For 1^{st} boundary condition only

Solve all questions for 1^{st} functions only from Q2.

2. For the SOR scheme, choose N=50. Fix the number of iterations to 20,40,50,60,100 and hunt for a suitable w value between 0 to 2 in steps of 0.1.Check if the plot changes.

3. In above question change N = 100 and fix no of iteration to 20 and check is value of w getting change. show the difference in the plot.

4. Once an approximate w_{opt} is found, hunt for a better estimate of w for the optimal in the range, (w_opt - 0.1, w_opt + 0.1) with steps of w in 0.01. Use 50 iterations and N = 20.

5. Does the w_opt change if N=101 and with a total of 100 iterations.

6. Repeat the hunt for an optimal w_opt (with w = linspace(1, 2, 11)) but this time calculate the number of iterations it takes to converge to machine epsilon instead of the error. Plot the number of iterations vs. w.

7. Having found the optimal w, take N=101 and solve this using all the three schemes and plot the error versus the number of iterations in a semi-log plot.

Use Python to code and submit your source code along with a PDF that shows the results of all your code. Create a ZIP file with the following structure for your submission:

a. Create a directory called a2-rollnumber (for example a2-170010031).

b. Put your source code and PDF report in this directory.(report should be in PDF only don't use ask for user input)

c. ZIP up the directory into a file called a2-rollnumber.zip (for example a2-170010031.zip).

Submit this zip file. Please follow these instructions carefully, marks will be deducted if this is not followed.