Assignment 3
AE-706/320

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1. Advection equation:

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0 \tag{1}$$

Problem no.1:

Consider the linear advection equation with a=1. Implement the FTFS, FTCS and FTBS schemes for this problem. 1. Consider the unit domain [0,1] with IC and BC's as

$$\begin{aligned} u(0,t) &= 1, t > 0 \\ u(x,0) &= 0, \end{aligned} \tag{2}$$

Solve this problem using the different schemes using 50 grid points but with different CFL values 0.4, 0.9 and 1.2 . Report your results systematically. Simulate the problem for 0.2,0.5,0.7,1.0 second.

Problem no.2:

Solve this problem with FTCS , FTBS , FTFS with CFL 0.5

$$\begin{aligned} u(x,0) &= sin(2\pi x), \\ u(0,t) &= 0.0 \end{aligned} \tag{3}$$

For time 0 ,1,1.5 , 3 and 4 secs. with domain of [0,1] and a=1 with 100 grid points. Apply periodic boundary conditions for this problem

Change the domain to [-1,1] Solve this problem with FTBS , FTCS2 with CFL 0.7 and 0.4 respectively.

$$\begin{aligned} u(x) &= cos^2(\pi x), \mid x \mid < 0.5 \\ u(x) &= 0, else \end{aligned} \tag{4}$$

with periodic condition for time 2 , 4, 6 secs with 80 grid points.

Comment on whats really happening with in these 2 problems.

Problem no.3: Consider a periodic domain [-1,1] and solve the following test cases introduced by Laney but using the stable FTBS scheme and using the FTCS2 scheme which was discussed and used by Prof. Ramakrishna in the demo and class.

Find u(x, 4) where:

$$\begin{aligned} u(x,0) &= 1, \mid x \mid < 1/3, \\ u(x,0) &= 0, \frac{1}{3} < \mid x \mid < 1 \end{aligned} \tag{5}$$

CFL = 0.8, 40 points
Again find: Find u(x, 4) and u(x,40) with same IC use 600 grid points.
Comment on the plots what you have observed.