CS 663 Assignment 5

Question 1

October, 2018

Solution goes as follows:

Given:
$g_1 = f_1 + h_2 * f_2$
$g_2 = h_1 * f_1 + f_2$

Taking Discrete Fourier Transform of the above two equations we get:

$G_1 = F_1 + H_2F_2$
$G_2 = H_1F_1 + F_2$

Solving for F_1 and F_2 from the above two equations:

$F_1 = \frac{G_1 - H_2G_2}{1 - H_1H_2}$, $F_2 = \frac{G_2 - H_1G_1}{1 - H_1H_2}$

As g_1, g_2, h_1 and h_2 are known to us, we can solve for f_1 and f_2 by taking the Inverse Discrete Fourier Transform of F_1 and F_2.

$f_1 = F^{-1}(F_1)$
$\implies f_1 = F^{-1}(\frac{G_1 - H_2G_2}{1 - H_1H_2})$

$f_2 = F^{-1}(F_2)$
$\implies f_2 = F^{-1}(\frac{G_2 - H_1G_1}{1 - H_1H_2})$

The problem with the solution obtained for f_1 and f_2 is that the denominator of the expressions $(1 - H_1H_2)$ can go to zero. As h_1 and h_2 are blur kernels, H_1 and H_2 are low pass filters. Low pass filters allow lower frequencies to remain intact while eliminating the higher frequencies. An ideal low pass filter is defined as follows:

$H(u, v) = 1$, if $u^2 + v^2 \leq D^2$
$= 0$ otherwise

This results in ringing artifacts which is eliminated by weakening the low pass filters to not totally eliminate higher frequencies. For example:

(Butterworth filter) $H(u, v) = \frac{1}{1 + (\frac{\sqrt{u^2+v^2}}{D})^{2N}}$ (n, D) : filter parameters

(Gaussian filter) $H(u, v) = exp(-(u^2 + v^2)/(2\sigma^2))$ (σ) : filter parameters

So the point is that the way the low pass filters are defined, their values is approximately 1 for lower frequencies and close to 0 for higher frequencies.

Hence for lower frequencies, $H_1 \approx 1$ and $H_2 \approx 1$.
$\implies 1 - H_1 H_2 \approx 0$ because of which estimates of F_1 and F_2 shoots up to very high value (tending to infinity) and will be hugely erroneous even for a very small amount of noise leading lead to improper restoration of f_1 and f_2.

This issue can be addressed by adding an ϵ term to the denominator to prevent it from going to zero at lower frequencies.