CS 663 Assignment 5

Question 2

October, 2018

For 1D Images:

Given:

$$g = h * f$$

Taking Discrete Fourier Transform of the above equation, we get:

$$G = HF$$

$$\implies F = \frac{G}{H}$$

$$\implies f = F^{-1}(\frac{G}{H})$$

As h represents gradient operation, in 1D h can be assumed as $h = [-1\ 0\ 1]$. So we can express g as:

$$g(x) = f(x+1) - f(x)$$

Taking DFT of the above equation, we get:

$$\implies G(u) = F(u)exp(-j2\pi u/M) - F(u)$$

$$\implies F(u) = \frac{G(u)}{(exp(-j2\pi u/M) - 1)}$$

So for lower frequencies ($u \approx 0$), the denominator of the above expression tends to zero and hence f cannot be properly recovered.

For 2D Images:

For gradient in X-direction: $h_x = [-1\ 0\ 1; -2\ 0\ 2; -1\ 0\ 1]$

$$G_x = H_x F$$

$$\implies F = \frac{G_x}{H}$$

$$\implies f = F^{-1}(\frac{G_x}{H})$$

Again we can represent,

$$g_x(x,y) = [f(x+1,y+1) + 2f(x+1,y) + f(x+1,y-1)] - [f(x-1,y+1) + 2f(x-1,y) + f(x-1,y-1)]$$

Taking Discrete Fourier Transform of the above equation, we get:

$$G_x(u,v) = F(u,v)[exp(-j2\pi u/M_u) - 1][exp(j2\pi v/M_v) + exp(-j2\pi v/M_v) + 1]$$
$$\implies F(u,v) = \frac{G_x(u,v)}{[exp(-j2\pi u/M_u) - 1][exp(j2\pi v/M_v) + exp(-j2\pi v/M_v) + 1]}$$

Similarly for gradient in Y-direction: $h_y = [-1\ \ -2\ \ -1; 0\ 0\ 0; 1\ 2\ 1]$

$$F(u,v) = \frac{G_y(u,v)}{[exp(j2\pi u/M_u) + exp(-j2\pi u/M_u) + 1][exp(-j2\pi v/M_v) - 1]}$$

So for both X and Y direction, the denominator tends to zero for lower frequencies ($u \approx 0$ and $v \approx 0$)