



CS 736 Programming Assignment 2

Image Denoising with MRF Priors

Submitted By

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1 Objective

The objective of this assignment was to perform image denoising using MRF priors. We used three different MRF priors and fine tuned the hyper parameters to produce the best denoised image. We used gradient descent to minimise the cost function and dynamic step change for faster convergence.

2 Code Structure

We have used python (Version 3.7.4) for coding.

1. python scripts:

- i `functions.py`: Implementation of different MRF priors and RRMSE function calculation
- ii `denoise.py`: ICM Denoising algorithm for finding the MAP estimate of denoised image
- iii `main.py`: Runs the Denoising algorithm and reports and plots all the results for questions 1 and 2
- iv `grid_run.py`: Runs denoising algorithm over a grid of γ and α to find the optimum values of these parameters

2. Other files:

- i `requirements.txt`: Contains the libraries used along with their corresponding versions

Instructions to run (will generate results for both the questions with respective optimum parameter values):

```
$ python main.py
```

3 Analysis And Results: Brain MRI Images

3.1 Part A: Original RRMSE Values

3.1.1 Low Noise Image

Original RRMSE Values between Noiseless and Low Noise Image: 0.0519

3.1.2 Medium Noise Image

Original RRMSE Values between Noiseless and Medium Noise Image: 0.13125

3.1.3 High Noise Image

Original RRMSE Values between Noiseless and High Noise Image: 0.15553

3.2 Part B: RRMSE values for different parameter values

3.2.1 Low Noise Level

Quadratic Prior: Parameters: α

1. $\alpha^* = 0.0875$

2. $\text{RRMSE}(\alpha^*) = 0.04673$
3. $\text{RRMSE}(1.2\alpha^*) = 0.04689$
4. $\text{RRMSE}(0.8\alpha^*) = 0.0469$

Discontinuity Adaptive Huber Prior: Parameters: α, γ

1. $\alpha^* = 0.8748, \gamma^* = 0.00211$
2. $\text{RRMSE}(\alpha^*, \gamma^*) = 0.04295$
3. $\text{RRMSE}(1.2\alpha^*, \gamma^*) = 0.16522$
4. $\text{RRMSE}(0.8\alpha^*, \gamma^*) = 0.14732$
5. $\text{RRMSE}(\alpha^*, 1.2\gamma^*) = 0.13232$
6. $\text{RRMSE}(\alpha^*, 0.8\gamma^*) = 0.13818$

Discontinuity Adaptive Prior: Parameters: α, γ

1. $\alpha^* = 0.8748, \gamma^* = 0.0023$
2. $\text{RRMSE}(\alpha^*, \gamma^*) = 0.04300$
3. $\text{RRMSE}(1.2\alpha^*, \gamma^*) = 0.15544$
4. $\text{RRMSE}(0.8\alpha^*, \gamma^*) = 0.14697$
5. $\text{RRMSE}(\alpha^*, 1.2\gamma^*) = 0.13188$
6. $\text{RRMSE}(\alpha^*, 0.8\gamma^*) = 0.13749$

3.2.2 Medium Noise Level

Quadratic Prior: Parameters: α

1. $\alpha^* = 0.1773$
2. $\text{RRMSE}(\alpha^*) = 0.11610$
3. $\text{RRMSE}(1.2\alpha^*) = 0.11644$
4. $\text{RRMSE}(0.8\alpha^*) = 0.11649$

Discontinuity Adaptive Huber Prior: Parameters: α, γ

1. $\alpha^* = 0.8663, \gamma^* = 0.0048$
2. $\text{RRMSE}(\alpha^*, \gamma^*) = 0.11186$
3. $\text{RRMSE}(1.2\alpha^*, \gamma^*) = 0.407$
4. $\text{RRMSE}(0.8\alpha^*, \gamma^*) = 0.1393$
5. $\text{RRMSE}(\alpha^*, 1.2\gamma^*) = 0.12315$

6. $\text{RRMSE}(\alpha^*, 0.8\gamma^*) = 0.12736$

Discontinuity Adaptive Prior: Parameters: α, γ

1. $\alpha^* = 0.8862, \gamma^* = 0.00438$
2. $\text{RRMSE}(\alpha^*, \gamma^*) = 0.11200$
3. $\text{RRMSE}(1.2\alpha^*, \gamma^*) = 0.17137$
4. $\text{RRMSE}(0.8\alpha^*, \gamma^*) = 0.14047$
5. $\text{RRMSE}(\alpha^*, 1.2\gamma^*) = 0.12336$
6. $\text{RRMSE}(\alpha^*, 0.8\gamma^*) = 0.12716$

3.2.3 High Noise Level

Quadratic Prior: Parameters: α

1. $\alpha^* = 0.2407$
2. $\text{RRMSE}(\alpha^*) = 0.12708$
3. $\text{RRMSE}(1.2\alpha^*) = 0.12767$
4. $\text{RRMSE}(0.8\alpha^*) = 0.12775$

Discontinuity Adaptive Huber Prior: Parameters: α, γ

1. $\alpha^* = 0.7729, \gamma^* = 0.01389$
2. $\text{RRMSE}(\alpha^*, \gamma^*) = 0.12247$
3. $\text{RRMSE}(1.2\alpha^*, \gamma^*) = 0.15104$
4. $\text{RRMSE}(0.8\alpha^*, \gamma^*) = 0.12897$
5. $\text{RRMSE}(\alpha^*, 1.2\gamma^*) = 0.12339$
6. $\text{RRMSE}(\alpha^*, 0.8\gamma^*) = 0.12314$

Discontinuity Adaptive Prior: Parameters: α, γ

1. $\alpha^* = 0.9449, \gamma^* = 0.003$
2. $\text{RRMSE}(\alpha^*, \gamma^*) = 0.12238$
3. $\text{RRMSE}(1.2\alpha^*, \gamma^*) = 0.14183$
4. $\text{RRMSE}(0.8\alpha^*, \gamma^*) = 0.12775$
5. $\text{RRMSE}(\alpha^*, 1.2\gamma^*) = 0.12336$
6. $\text{RRMSE}(\alpha^*, 0.8\gamma^*) = 0.1229$

3.3 Part C: Noiseless, Noisy and Denoised Images

3.3.1 Low Noise Image

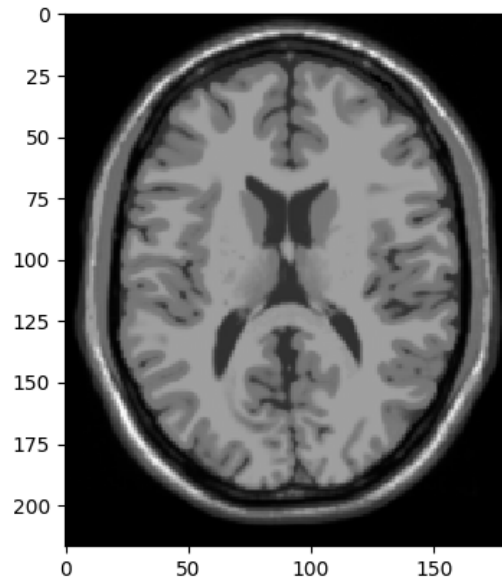


Figure 1: Ground Truth Noiseless Image

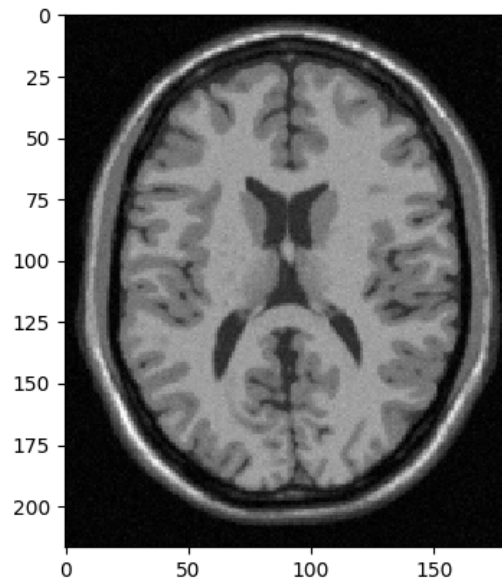


Figure 2: Noisy Image

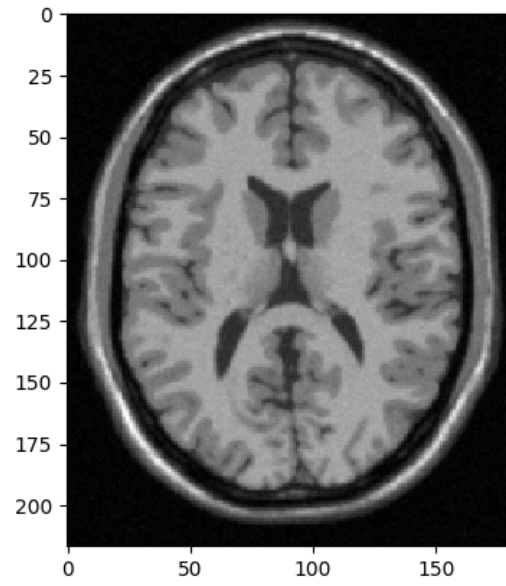


Figure 3: Image Denoised Using Quadratic Prior

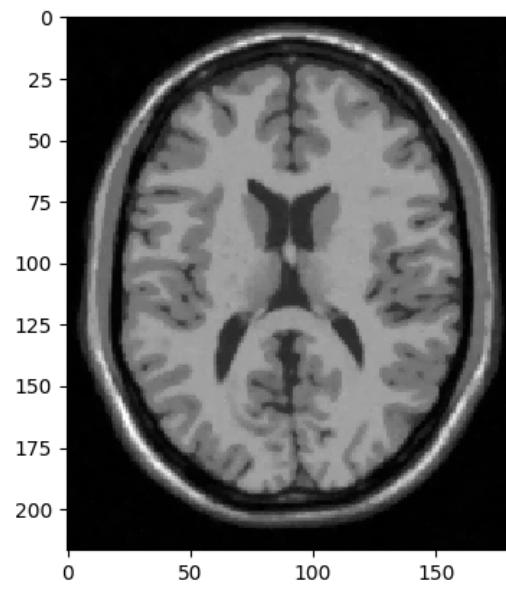


Figure 4: Image Denoised Using Discontinuity Adaptive Huber Prior

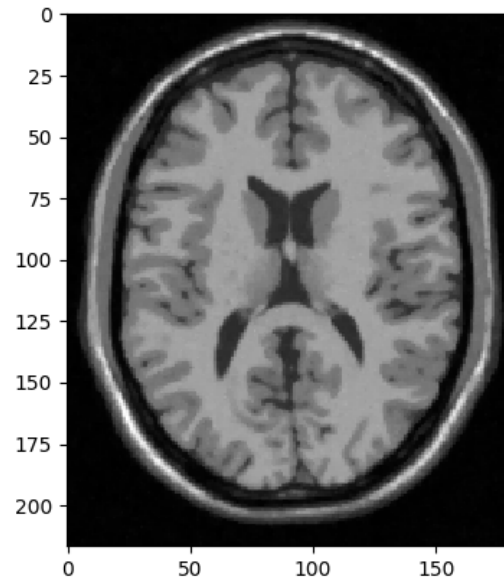


Figure 5: Image Denoised Using Discontinuity Adaptive Prior

3.3.2 Medium Noise Image

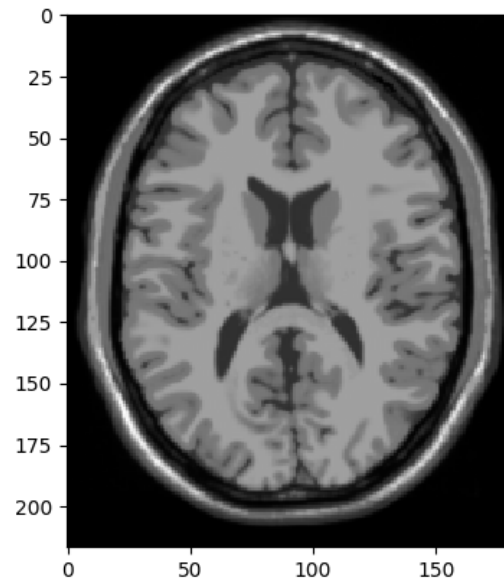


Figure 6: Ground Truth Noiseless Image

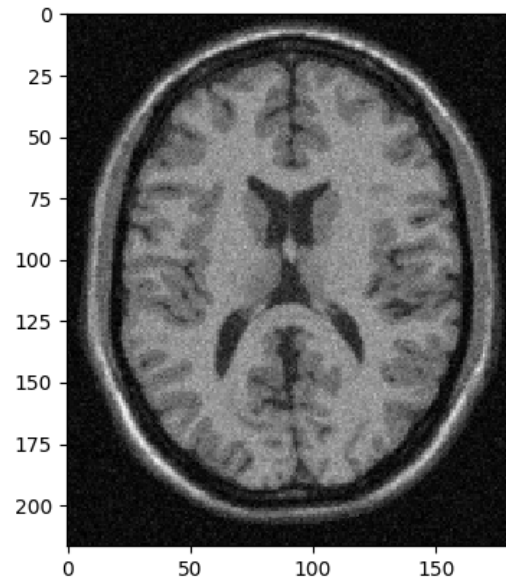


Figure 7: Noisy Image

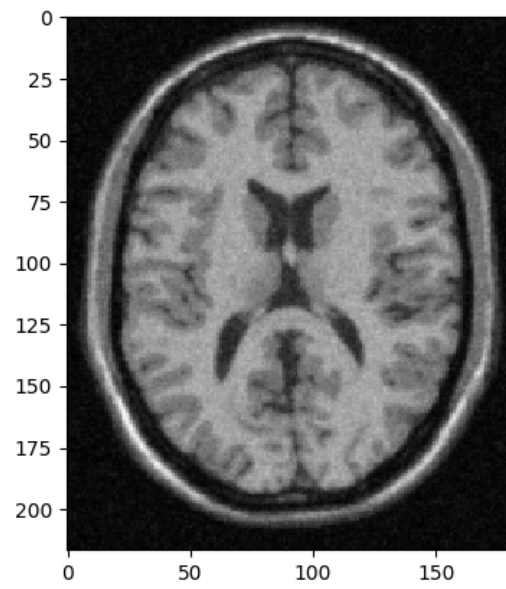


Figure 8: Image Denoised Using Quadratic Prior

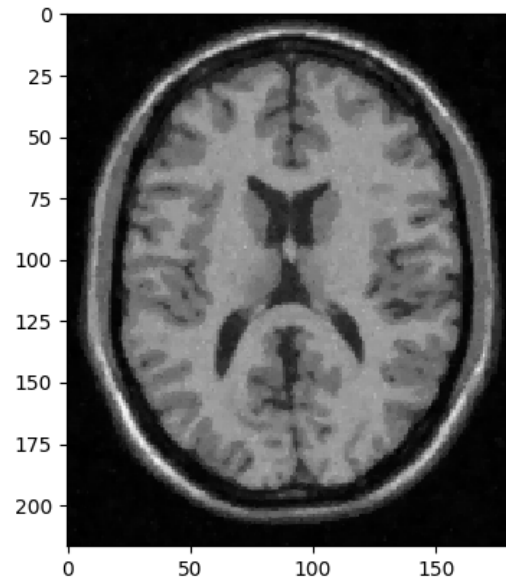


Figure 9: Image Denoised Using Discontinuity Adaptive Huber Prior

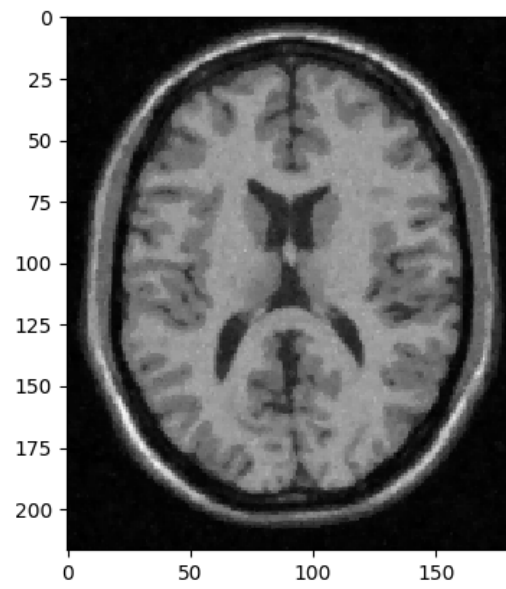


Figure 10: Image Denoised Using Discontinuity Adaptive Prior

3.3.3 High Noise Image

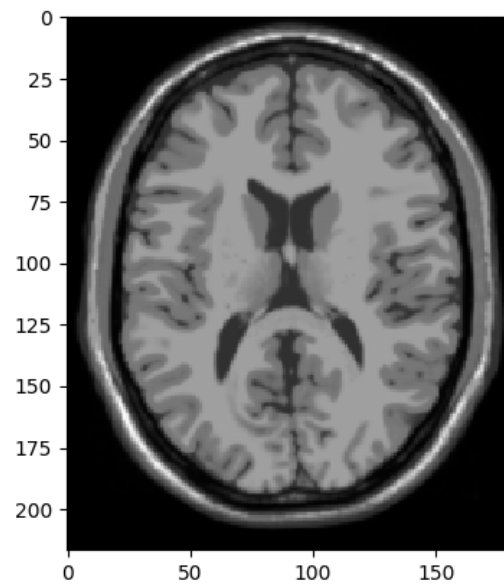


Figure 11: Ground Truth Noiseless Image

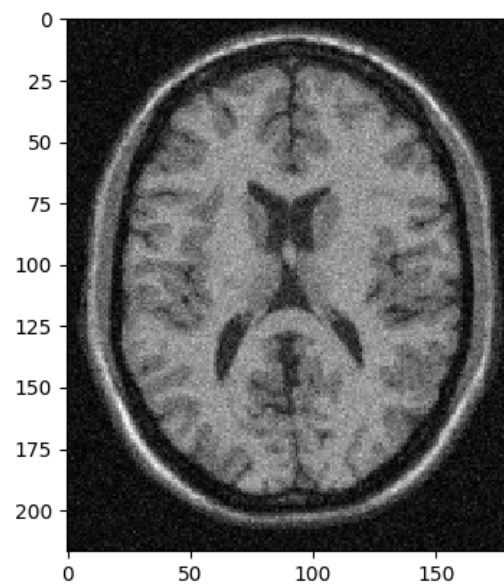


Figure 12: Noisy Image

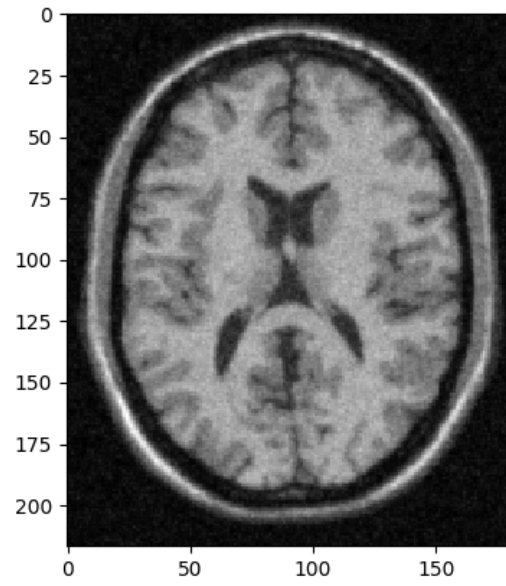


Figure 13: Image Denoised Using Quadratic Prior

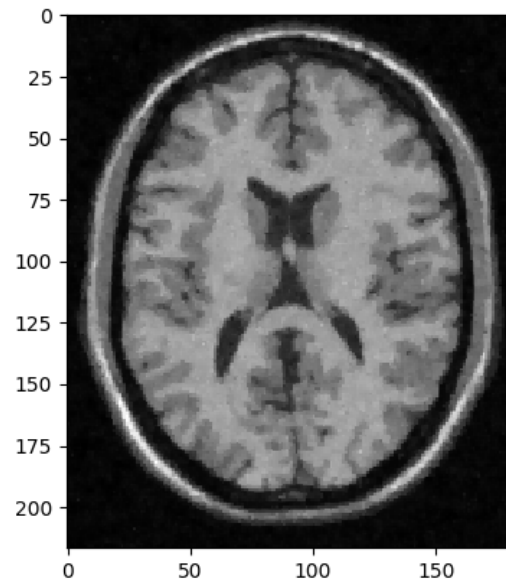


Figure 14: Image Denoised Using Discontinuity Adaptive Huber Prior

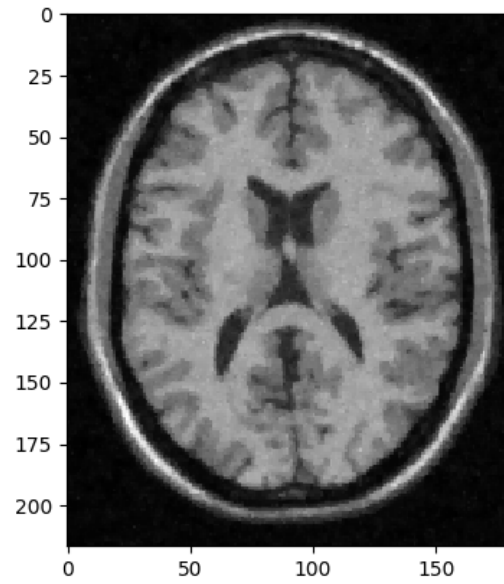


Figure 15: Image Denoised Using Discontinuity Adaptive Prior

3.4 Part D: Objective Function Plots

3.4.1 Low Noise Image

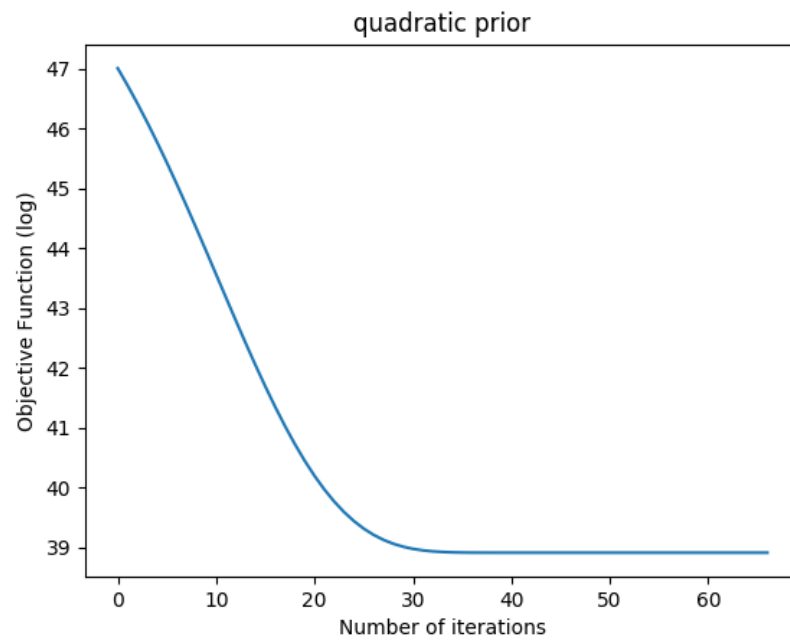


Figure 16: Objective function variation: Low Noise Image, Quadratic Prior

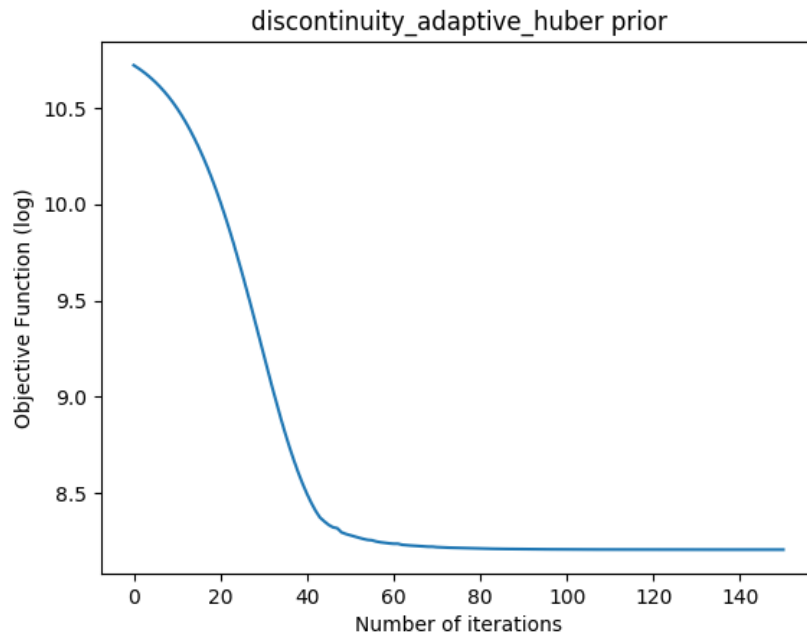


Figure 17: Objective function variation: Low Noise Image, Discontinuity Adaptive Huber Prior

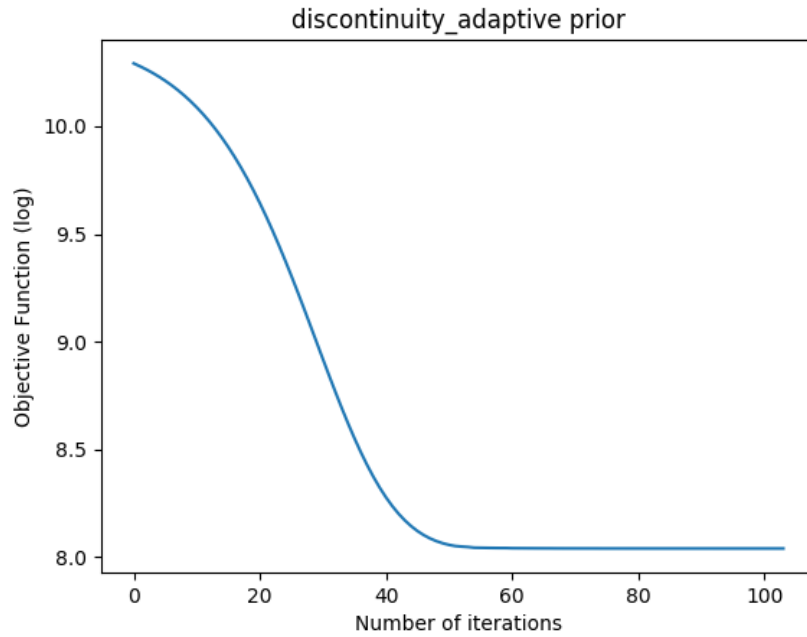


Figure 18: Objective function variation: Low Noise Image, Discontinuity Adaptive Huber Prior

3.4.2 Medium Noise Image

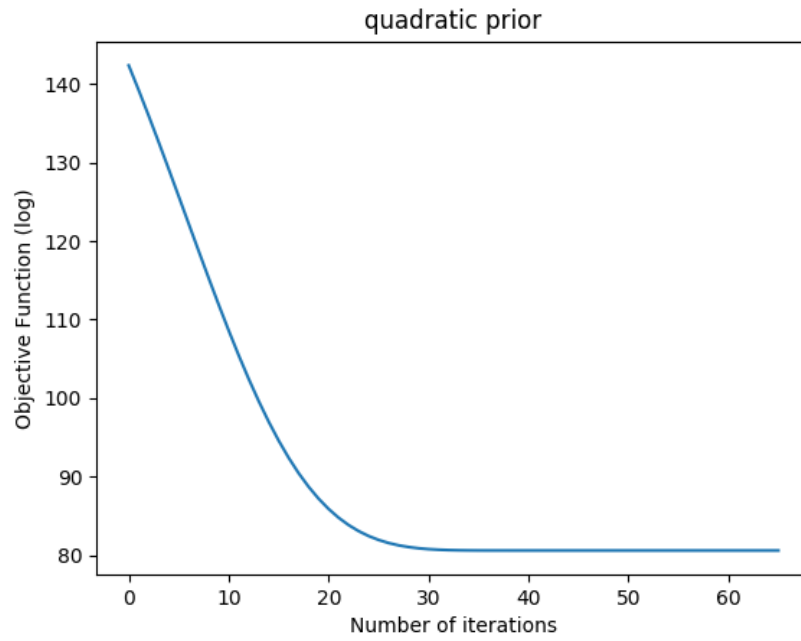


Figure 19: Objective function variation: Medium Noise Image, Quadratic Prior

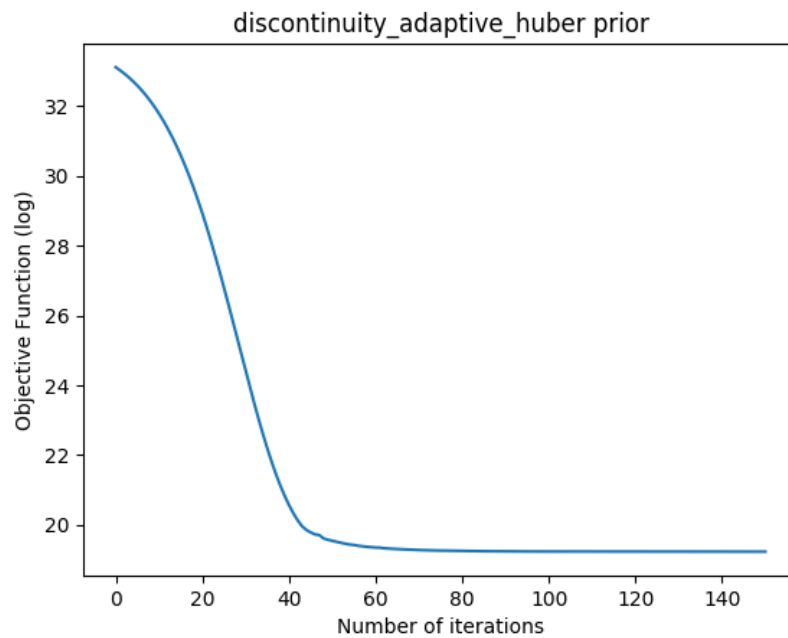


Figure 20: Objective function variation: Medium Noise Image, Discontinuity Adaptive Huber Prior

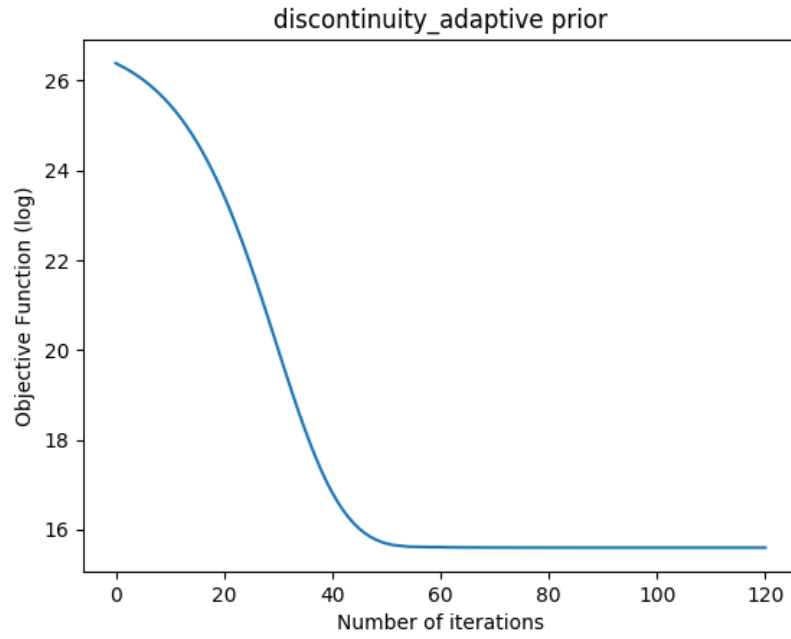


Figure 21: Objective function variation: Medium Noise Image, Discontinuity Adaptive Huber Prior

3.4.3 High Noise Image

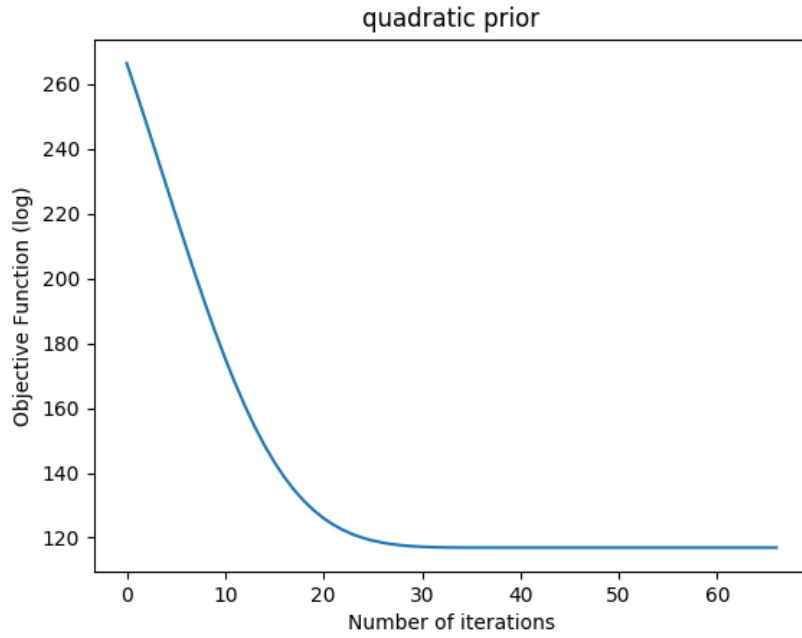


Figure 22: Objective function variation: High Noise Image, Quadratic Prior

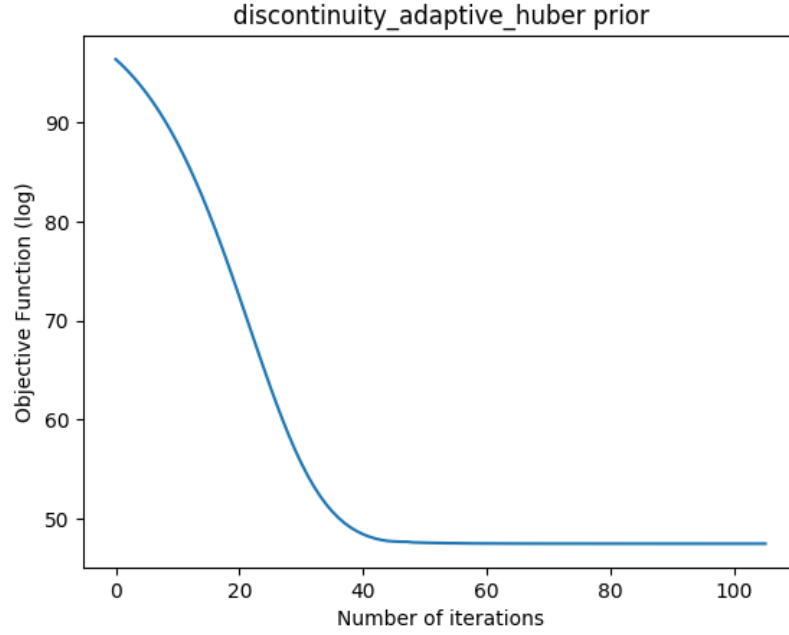


Figure 23: Objective function variation: High Noise Image, Discontinuity Adaptive Huber Prior

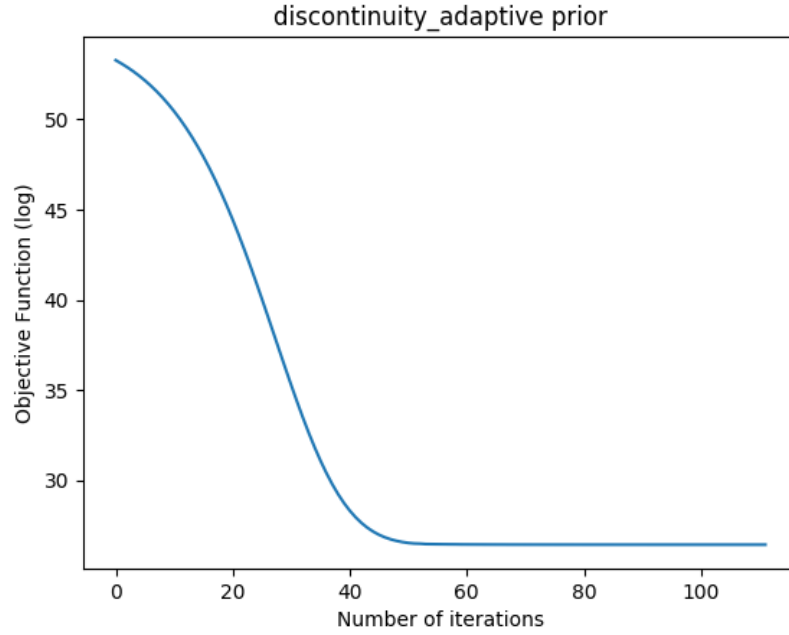


Figure 24: Objective function variation: High Noise Image, Discontinuity Adaptive Huber Prior

4 Analysis And Results: Coloured Digital Pathology Images

4.1 Proposed Strategy

We initially tried to do channel-wise denoising in the RGB space but the results were not that good which we believe is due to us trying to independently optimize different channels which have high correlation between them.

So we converted our image to HSV color-space. Although the three channels are not strictly independent in HSV, there is only one channel that controls the color. We independently find the optimal parameter values for each of channel of the image and finally combine the three denoised channels to get our final image.

4.2 Original RRMSE Values

For the complete image, Original RRMSE: 0.28595

4.2.1 H channel

Original RRMSE value for H channel: 0.28595

4.2.2 S channel

Original RRMSE value for S channel: 0.44423

4.2.3 V channel

Original RRMSE value for V channel: 0.17661

4.3 RRMSE Values for different parameter values

RRMSE for Quadratic prior denoised Image = 0.094847 RRMSE for Discontinuity Adaptive Huber prior denoised Image = 0.22235 RRMSE for Discontinuity Adaptive prior denoised Image = 0.09598

4.3.1 H channel

Quadratic Prior: Parameters: α

1. $\alpha^* = 0.9919$
2. $\text{RRMSE}(\alpha^*) = 0.09485$
3. $\text{RRMSE}(1.2\alpha^*) = 0.12077$
4. $\text{RRMSE}(0.8\alpha^*) = 0.1043$

Discontinuity Adaptive Huber Prior: Parameters: α, γ

1. $\alpha^* = 1, \gamma^* = 1$
2. $\text{RRMSE}(\alpha^*, \gamma^*) = 0.09714$
3. $\text{RRMSE}(1.2\alpha^*, \gamma^*) = 0.22235$
4. $\text{RRMSE}(0.8\alpha^*, \gamma^*) = 0.27431$
5. $\text{RRMSE}(\alpha^*, 1.2\gamma^*) = 0.09651$
6. $\text{RRMSE}(\alpha^*, 0.8\gamma^*) = 0.09776$

Discontinuity Adaptive Prior: Parameters: α, γ

1. $\alpha^* = 1, \gamma^* = 1$

2. $\text{RRMSE}(\alpha^*, \gamma^*) = 0.09599$
3. $\text{RRMSE}(1.2\alpha^*, \gamma^*) = 0.22274$
4. $\text{RRMSE}(0.8\alpha^*, \gamma^*) = 0.27446$
5. $\text{RRMSE}(\alpha^*, 1.2\gamma^*) = 0.09615$
6. $\text{RRMSE}(\alpha^*, 0.8\gamma^*) = 0.09647$

4.3.2 S channel

Quadratic Prior: Parameters: α

1. $\alpha^* = 0.75275$
2. $\text{RRMSE}(\alpha^*) = 0.20901$
3. $\text{RRMSE}(1.2\alpha^*) = 0.21847$
4. $\text{RRMSE}(0.8\alpha^*) = 0.21425$

Discontinuity Adaptive Huber Prior: Parameters: α, γ

1. $\alpha^* = 0.8974, \gamma^* = 0.02662$
2. $\text{RRMSE}(\alpha^*, \gamma^*) = 0.20841$
3. $\text{RRMSE}(1.2\alpha^*, \gamma^*) = 0.21497$
4. $\text{RRMSE}(0.8\alpha^*, \gamma^*) = 0.29182$
5. $\text{RRMSE}(\alpha^*, 1.2\gamma^*) = 0.2088$
6. $\text{RRMSE}(\alpha^*, 0.8\gamma^*) = 0.2098$

Discontinuity Adaptive Prior: Parameters: α, γ

1. $\alpha^* = 0.93878, \gamma^* = 0.02139$
2. $\text{RRMSE}(\alpha^*, \gamma^*) = 0.20846$
3. $\text{RRMSE}(1.2\alpha^*, \gamma^*) = 0.22976$
4. $\text{RRMSE}(0.8\alpha^*, \gamma^*) = 0.31367$
5. $\text{RRMSE}(\alpha^*, 1.2\gamma^*) = 0.20862$
6. $\text{RRMSE}(\alpha^*, 0.8\gamma^*) = 0.20929$

4.3.3 V channel

Quadratic Prior: Parameters: α

1. $\alpha^* = 0.7137$
2. $\text{RRMSE}(\alpha^*) = 0.09516$
3. $\text{RRMSE}(1.2\alpha^*) = 0.09796$
4. $\text{RRMSE}(0.8\alpha^*) = 0.09689$

Discontinuity Adaptive Huber Prior: Parameters: α, γ

1. $\alpha^* = 0.8718, \gamma^* = 0.02662$
2. $\text{RRMSE}(\alpha^*, \gamma^*) = 0.09341$
3. $\text{RRMSE}(1.2\alpha^*, \gamma^*) = 0.09463$
4. $\text{RRMSE}(0.8\alpha^*, \gamma^*) = 0.11408$
5. $\text{RRMSE}(\alpha^*, 1.2\gamma^*) = 0.0948$
6. $\text{RRMSE}(\alpha^*, 0.8\gamma^*) = 0.09495$

Discontinuity Adaptive Prior: Parameters: α, γ

1. $\alpha^* = 0.9184, \gamma^* = 0.02138$
2. $\text{RRMSE}(\alpha^*, \gamma^*) = 0.09473$
3. $\text{RRMSE}(1.2\alpha^*, \gamma^*) = 0.09693$
4. $\text{RRMSE}(0.8\alpha^*, \gamma^*) = 0.1233$
5. $\text{RRMSE}(\alpha^*, 1.2\gamma^*) = 0.09468$
6. $\text{RRMSE}(\alpha^*, 0.8\gamma^*) = 0.0952$

4.4 Noiseless, Noisy and Denoised Images

Note: The images were converted to HSV format for denoising and were saved as is and that is why they look different from the original images. The denoising is much more evident in HSV format and the denoised image looks to have slightly different colors when converted back to RGB space, so that is why the images have not been converted back to RGB space for saving and have been saved as is in the HSV space.

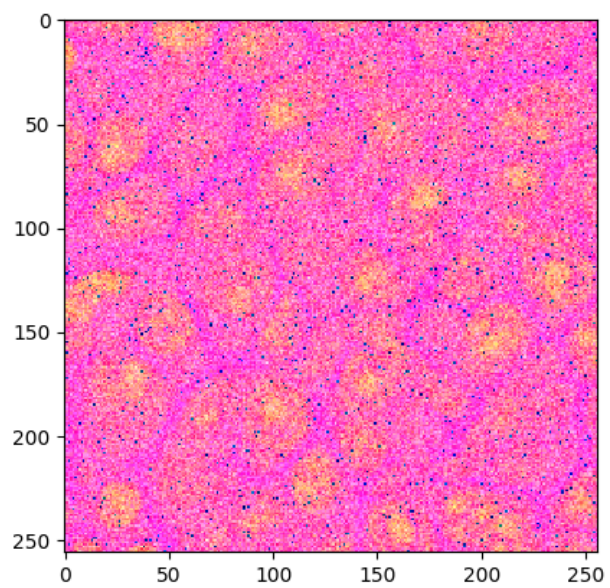


Figure 25: Original Noisy Image.

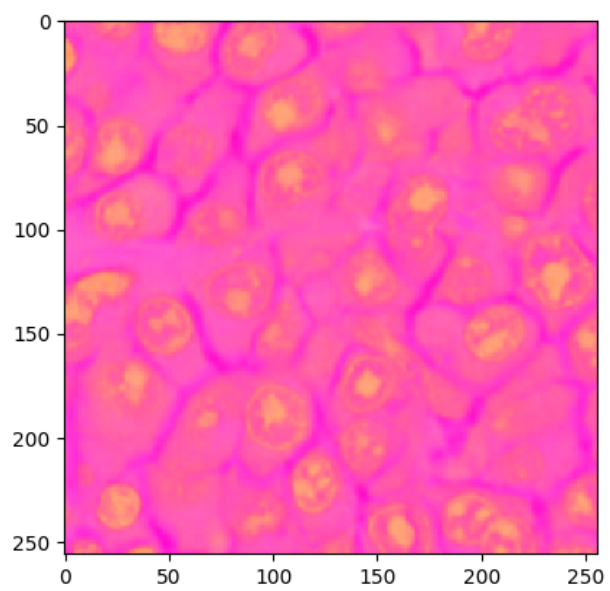


Figure 26: Original Ground truth noiseless Image.

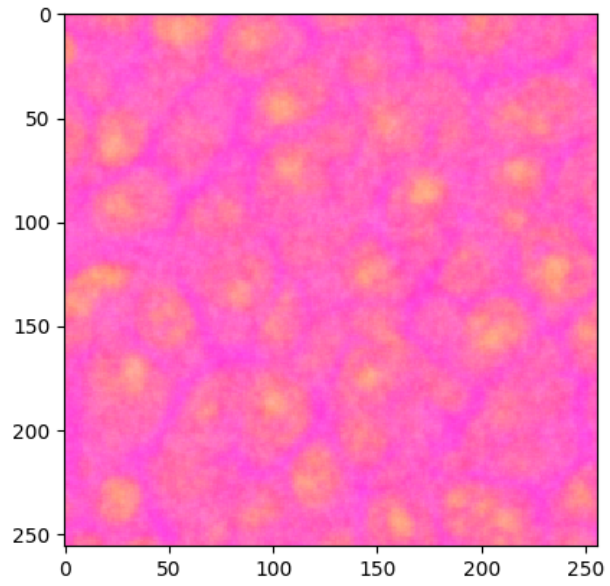


Figure 27: Quadratic Prior Denoised Image

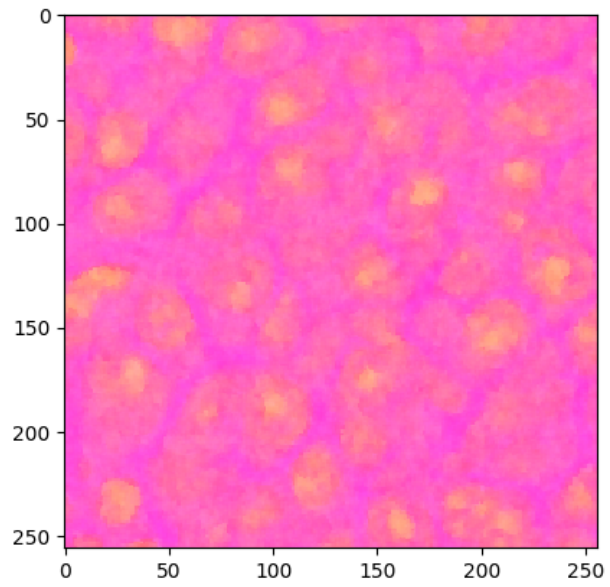


Figure 28: Discontinuity Adaptive Huber Prior Denoised Image.

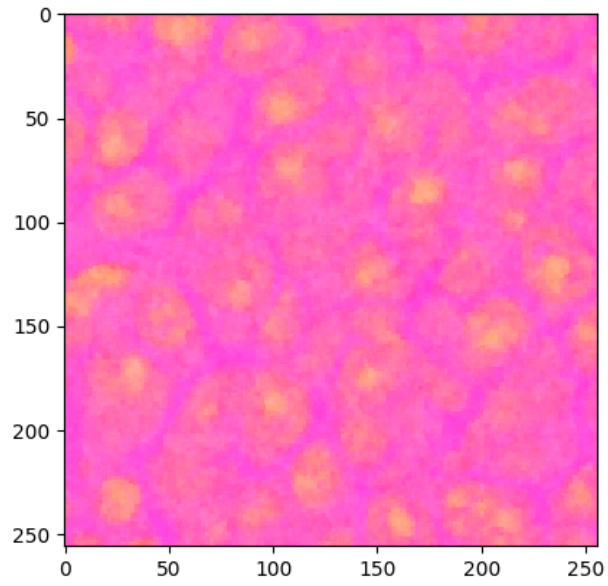


Figure 29: Discontinuity Adaptive Prior Denoised Image.

4.4.1 H channel

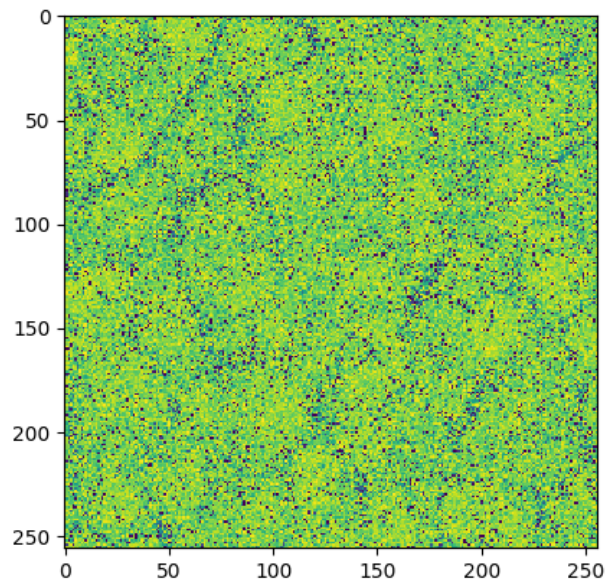


Figure 30: Noisy H channel

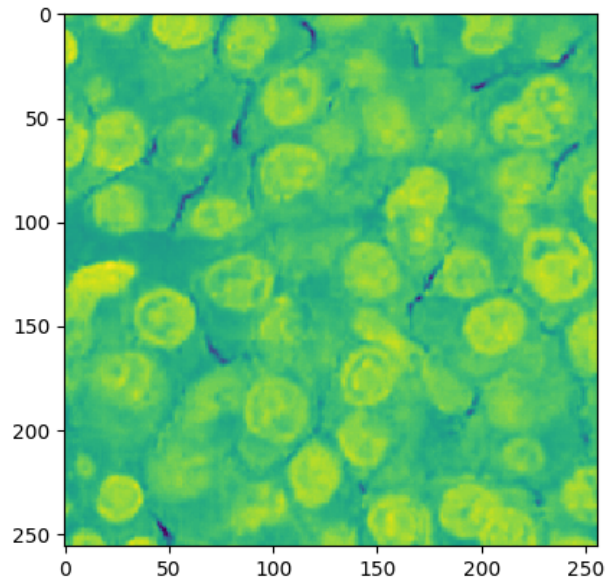


Figure 31: Noiseless H channel

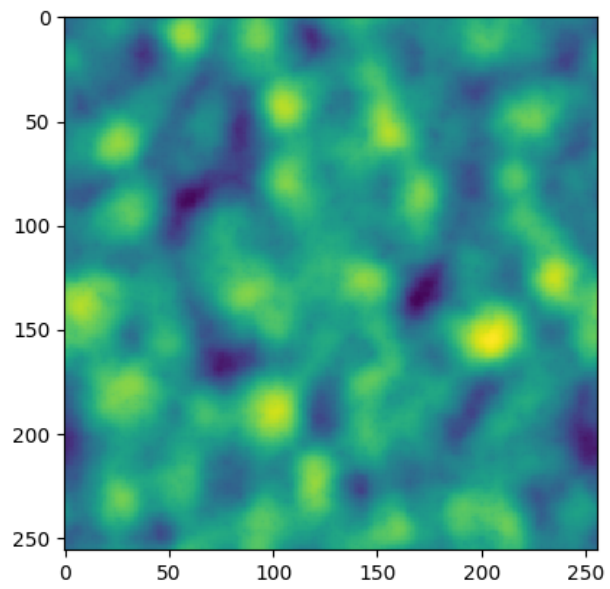


Figure 32: Quadratic Denoised H channel

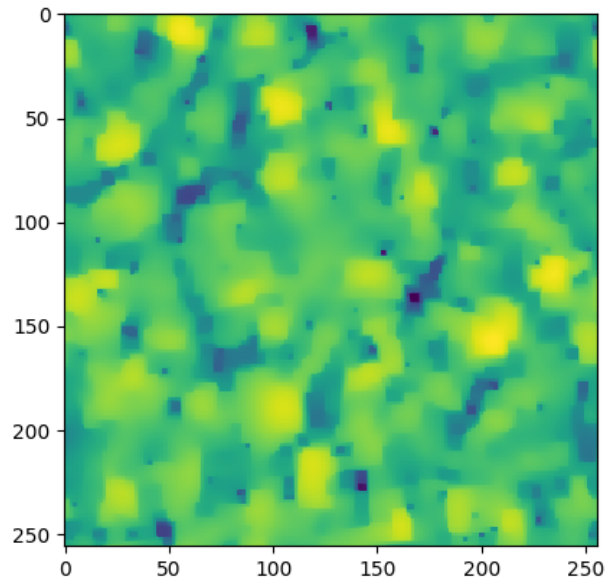


Figure 33: Discontinuity Adaptive Huber Denoised H channel

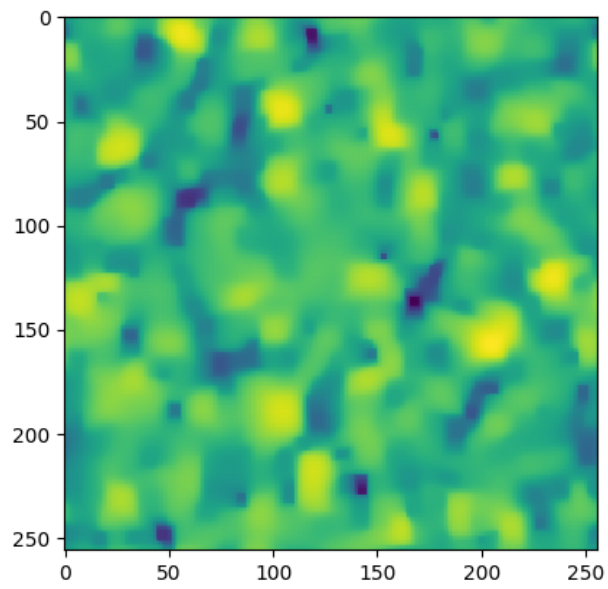


Figure 34: Discontinuity Adaptive H channel

4.4.2 S channel

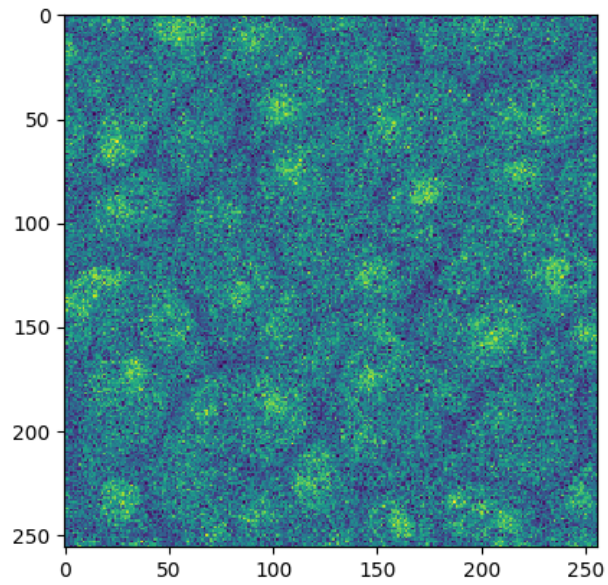


Figure 35: Noisy S channel

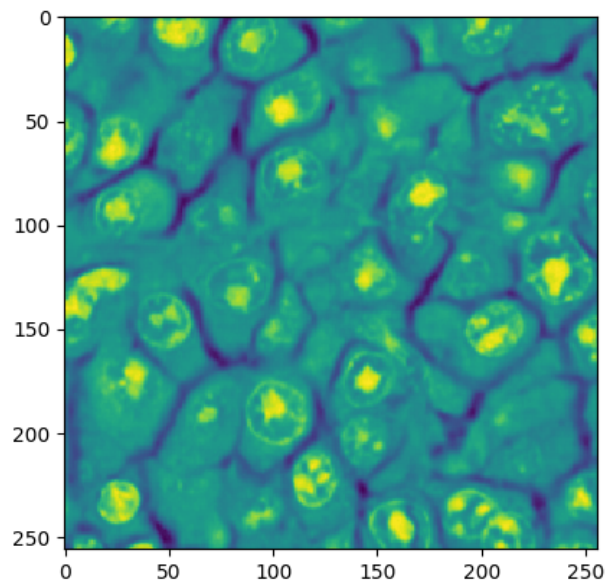


Figure 36: Noiseless S channel

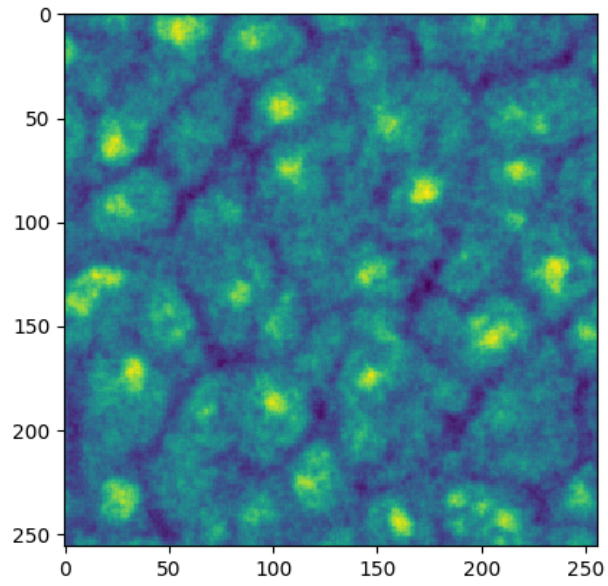


Figure 37: Quadratic Denoised S channel

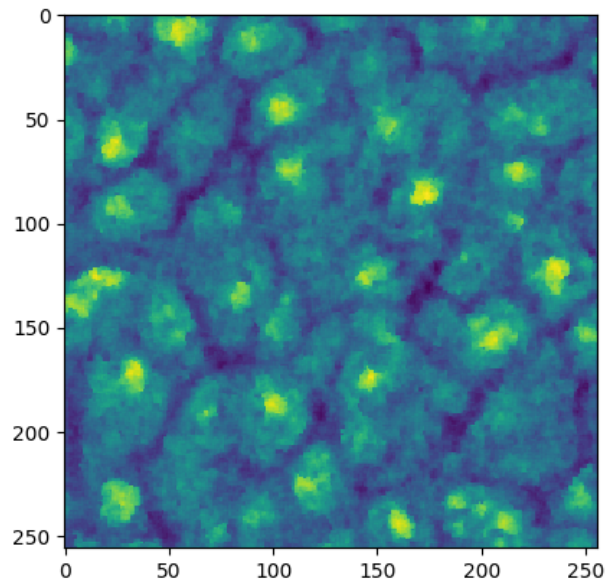


Figure 38: Discontinuity Adaptive Huber Denoised S channel

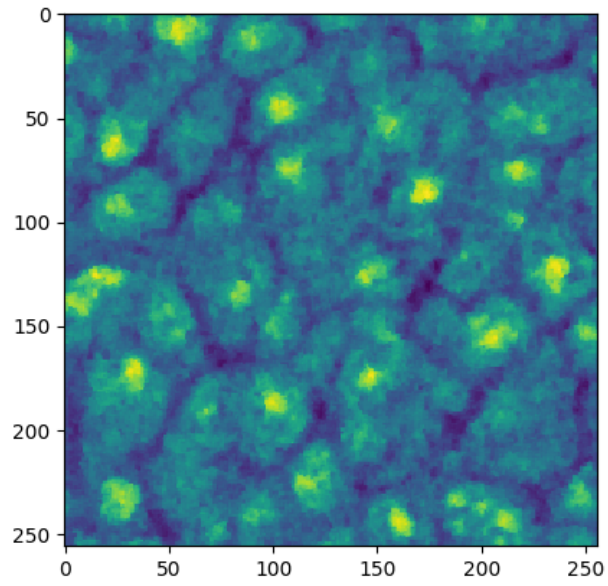


Figure 39: Discontinuity Adaptive S channel

4.4.3 V channel

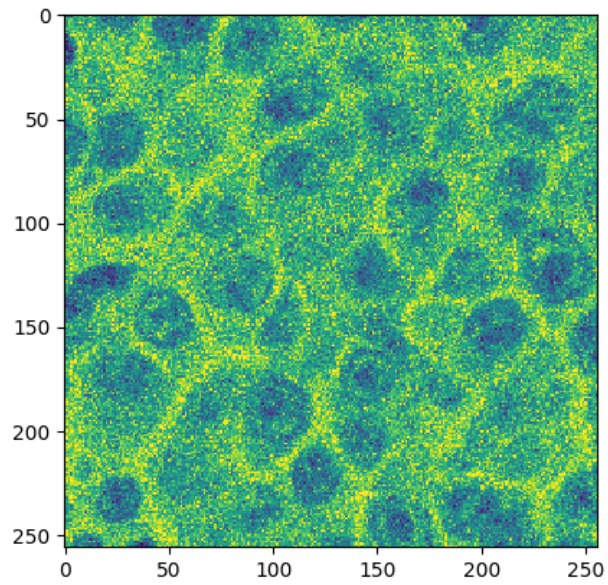


Figure 40: Noisy V channel

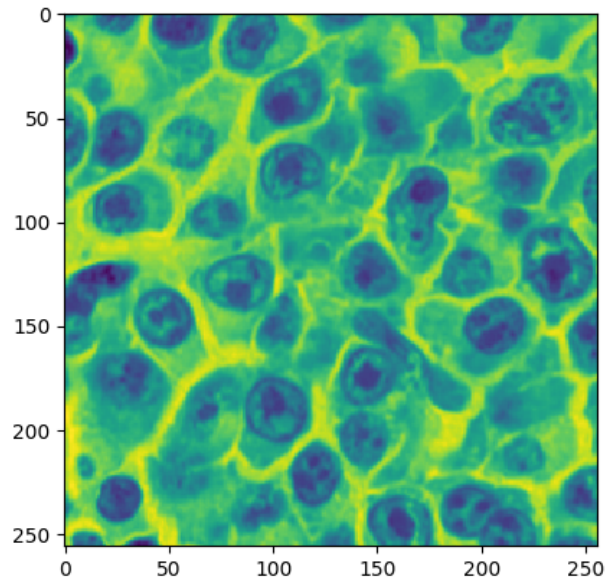


Figure 41: Noiseless V channel

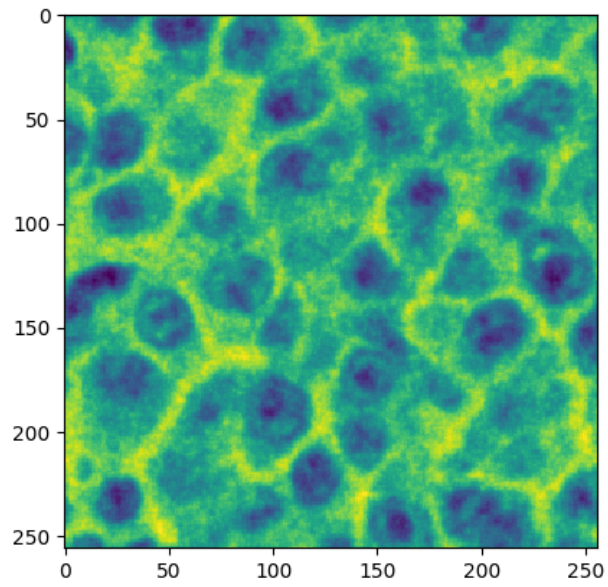


Figure 42: Quadratic Denoised V channel

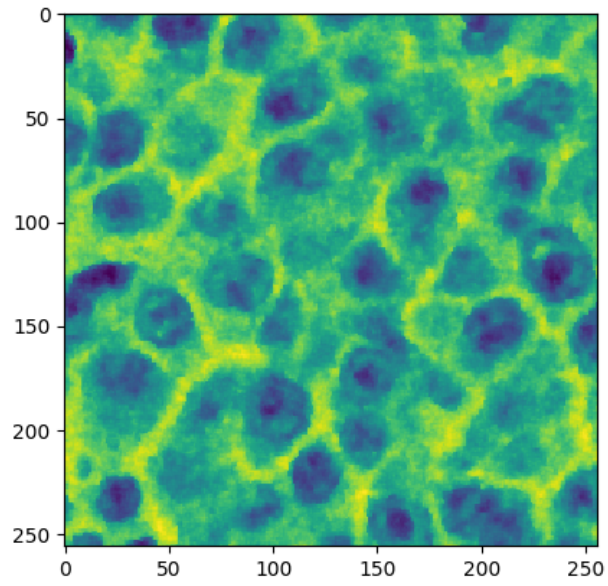


Figure 43: Discontinuity Adaptive Huber Denoised V channel

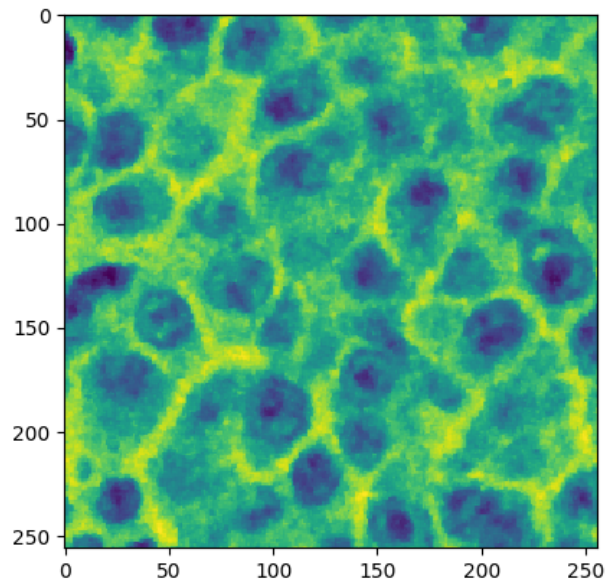


Figure 44: Discontinuity Adaptive V channel

4.5 Objective Function Plots

4.5.1 H channel

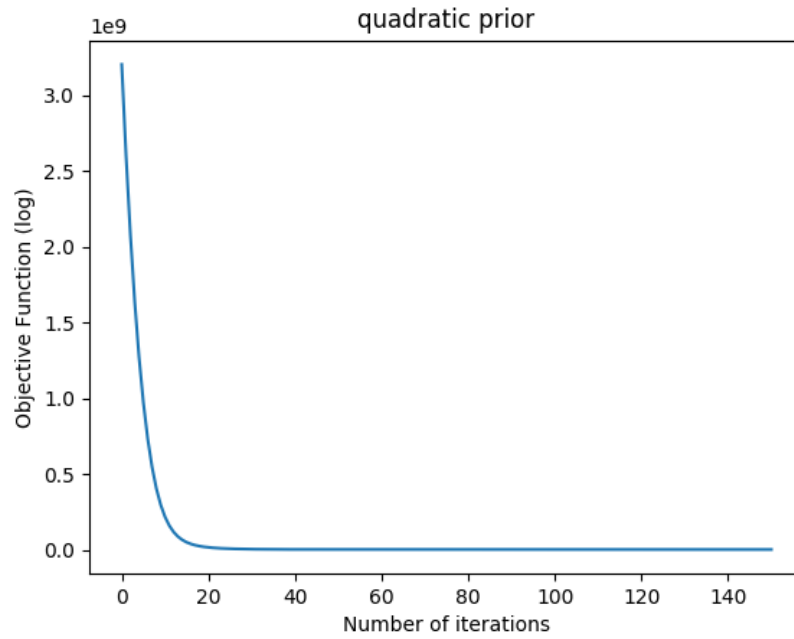


Figure 45: Objective Function Variation: H channel, Quadratic Prior

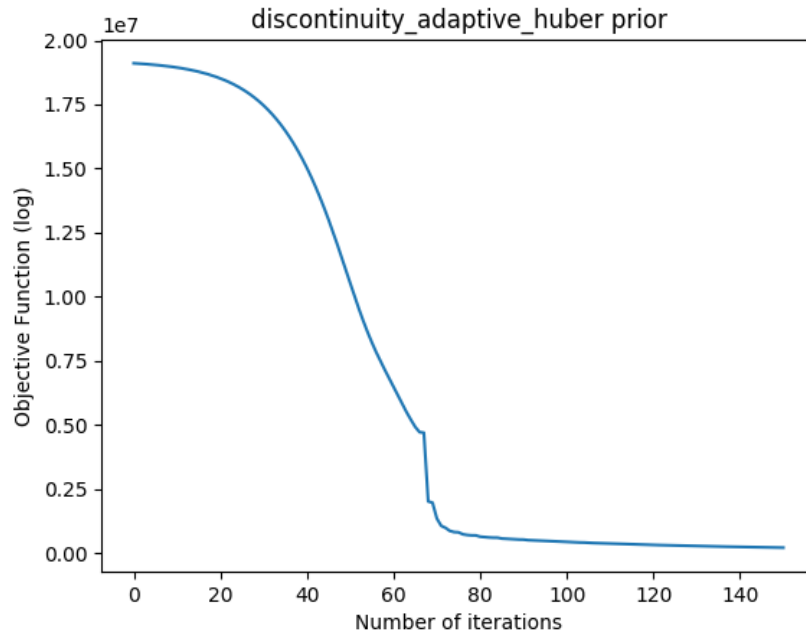


Figure 46: Objective Function Variation: H channel, Discontinuity Adaptive Huber Prior

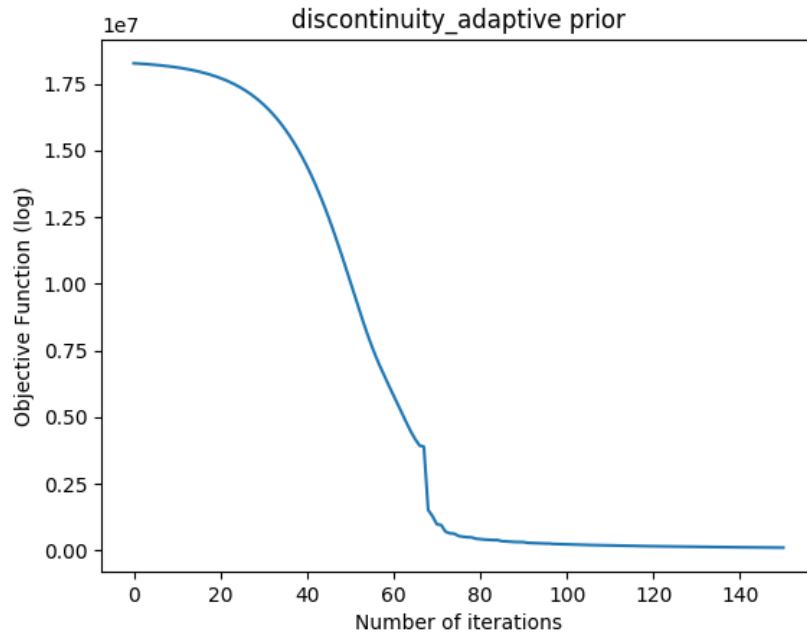


Figure 47: Objective Function Variation: H channel, Discontinuity Adaptive Prior

4.5.2 S channel

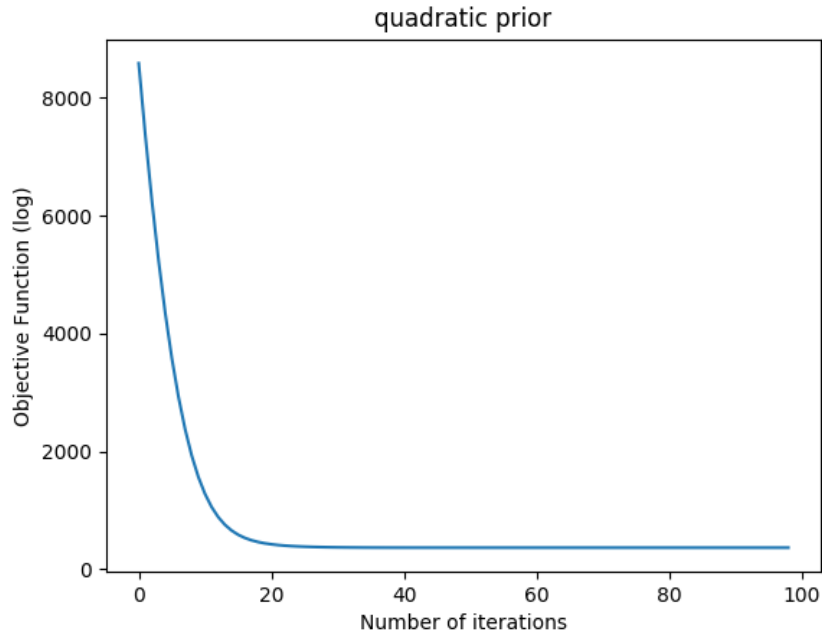


Figure 48: Objective Function Variation: S channel, Quadratic Prior

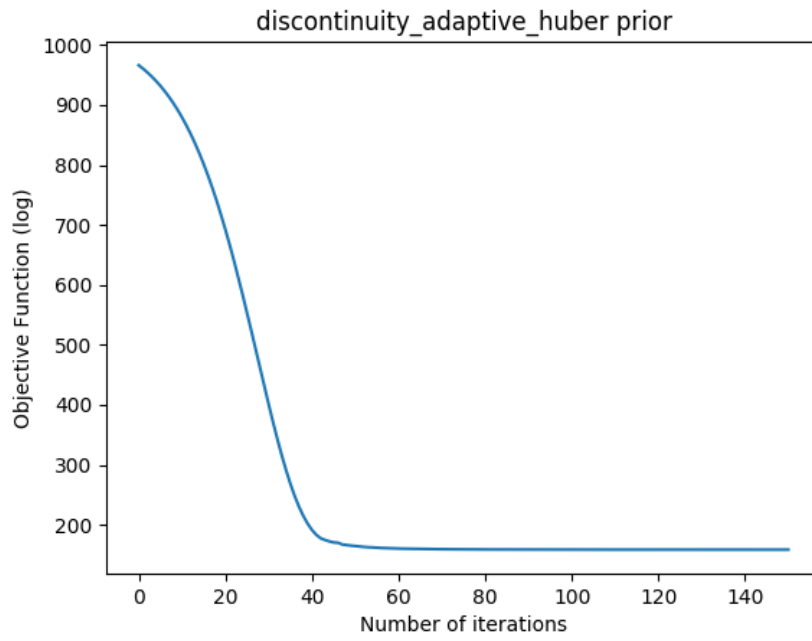


Figure 49: Objective Function Variation: S channel, Discontinuity Adaptive Huber Prior

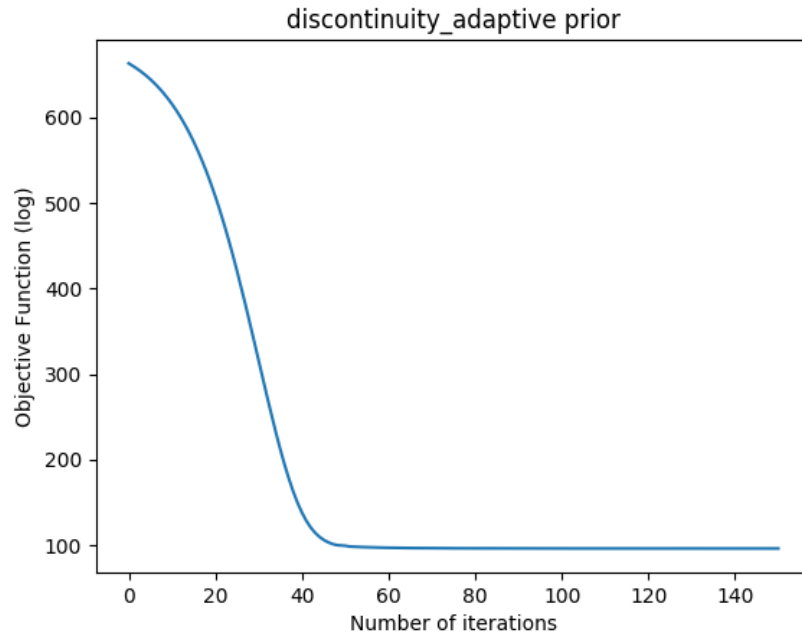


Figure 50: Objective Function Variation: S channel, Discontinuity Adaptive Prior

4.5.3 V channel

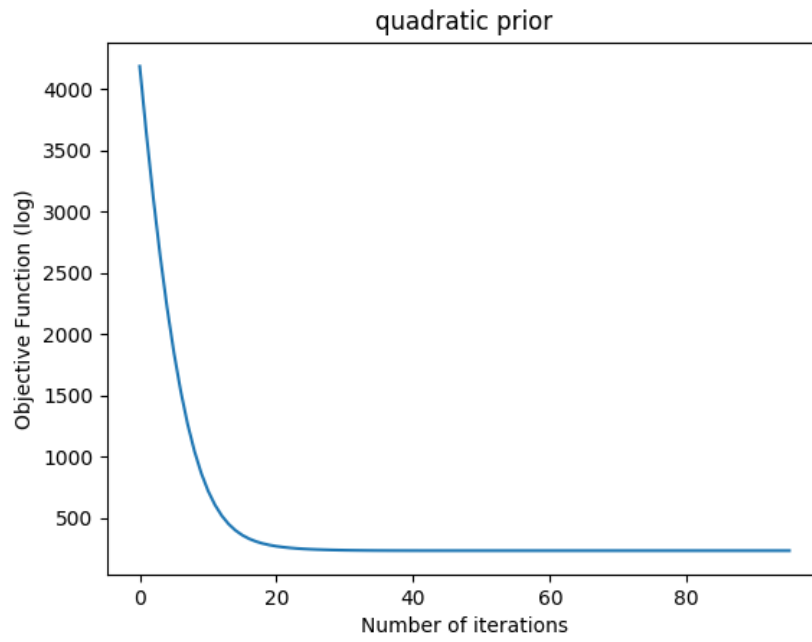


Figure 51: Objective Function Variation: V channel, Quadratic Prior

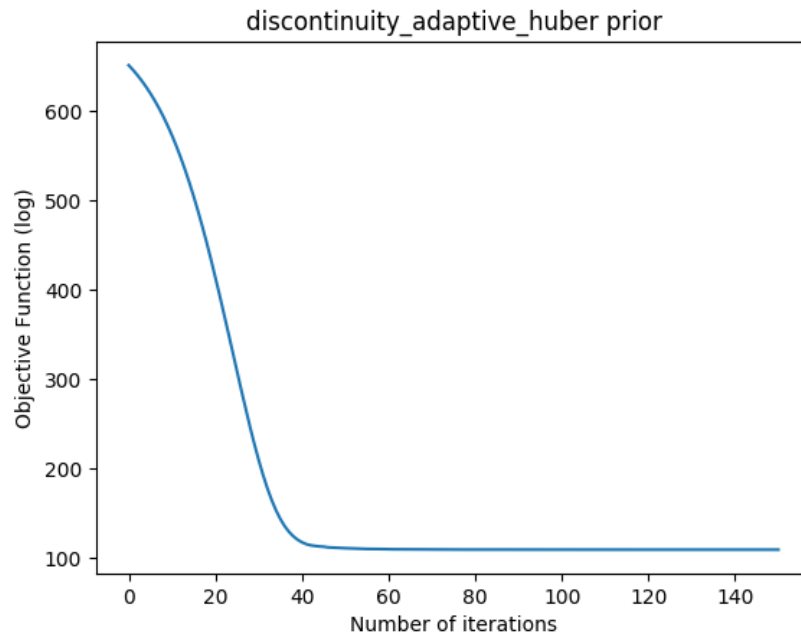


Figure 52: Objective Function Variation: V channel, Discontinuity Adaptive Huber Prior

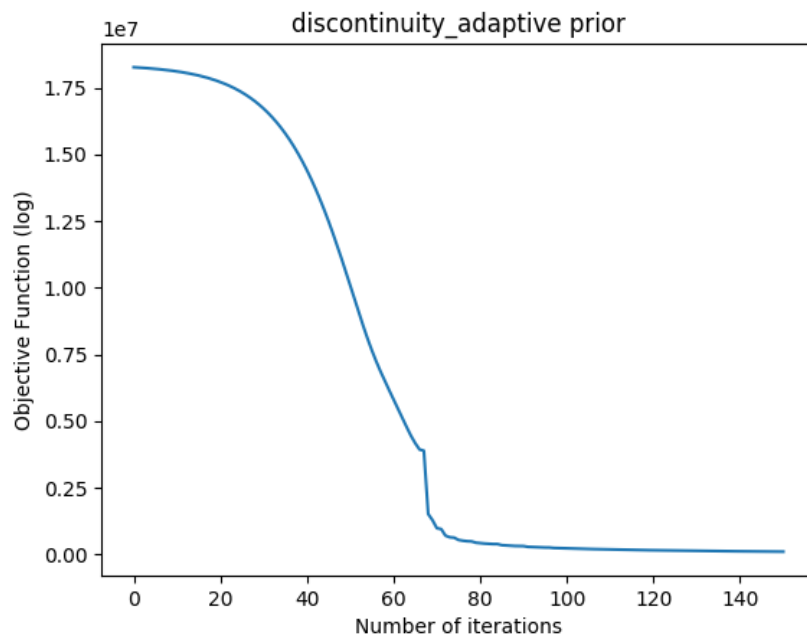


Figure 53: Objective Function Variation: V channel, Discontinuity Adaptive Prior