ME 766- Homework 1

Openmmp analysis of numerical integration methods

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Serial and openmp parallel codes for numerical integration of the function $f(x) = sin(x)$ in the interval $[0,\pi]$ using trapezoid and montecarlo methods are present in the folder that contained this report.

Convergence study

From the convergence study, it can be seen that the integral asnwer converges to the correct answer on increasing the number of sampling points for both trapezoidal and montecarlo method.

We know that $\int_0^{\pi} sin\ x\ dx = 2$

Error (in percentage) has been calculated as: Error $= 100 \times \frac{(Ans - 2)}{2}$

From the tables below, it can be observed that Error reduces on increasing the number of divisions/sampling points.

Trapezoid integration

	Integration answer	Error percentage
N=5	1.9337655981	-3.311720
N=10	1.9835235375	-0.823823
N=50	1.9993419831	-0.032901
N=100	1.9998355039	-0.008225
N=500	1.9999934203	-0.000329
N=1000	1.9999983551	-0.000082
N=5000	1.9999999342	-0.000003
N=10000	1.9999999836	-0.000001
N=50000	1.9999999993	-0.0000000329
N=100000	1.999999999	-0.0000000082
N=1000000	2.0000000000	-0.0000000001
N=10000000	2.0000000000	0.0000000000
N=100000000	2.0000000000	0.0000000000

Montecarlo Integration

	Integration answer	Error percentage
N=10	1.5707963268	-21.4601836603
N=100	1.8221237391	-8.8938130459
N=1000	2.0765927440	3.8296372011
N=10000	2.042977702	2.1488851314
N=100000	2.0068808030	0.3440401520
N=1000000	2.0017442991	0.0872149525
N=10000000	2.0010471796	0.0523589820
N=100000000	2.0002793430	0.0139671490
N=1000000000	2.0000990313	0.0049515635

Time analysis with no. of threads

I made a numbers sheet for analysing execution time taken by the serial and parallel codes (with different number of threads). I have also plotted the graph of variation of execution time vs number of sampling points for different number of threads.

From the graph and analysis, it is evident that the parallel code outperforms serial code for large number of sampling points. For comparatively small number of sampling points, the serial code does better than parallel code which I believe is due to the reason that overhead for making different number of threads is greater than the advantage gained due to parallelization which is supported by the fact that for $N<1000$, execution time increases on increasing the number of threads. But for this range of N, execution time$<<1$ and as one can see from the graph, as the execution time begins significantly more than 0, parallel code outperforms serial code.

I exported the numbers sheet into pdf format and merged the pdf with this pdf.

Trapezoid Integration with serial code

No of divisions	T1	T2	T3	T4	T5	Total time (serial)
5	0.000008000	0.000008000	0.000008000	0.000007000	0.000008000	0.000007600
10	0.000008000	0.000008000	0.000010000	0.000009000	0.000009000	0.000008800
50	0.000009000	0.000010000	0.000010000	0.000010000	0.000010000	0.000009800
100	0.000012000	0.000009000	0.000011000	0.000011000	0.000012000	0.000010800
500	0.000015000	0.000017000	0.000017000	0.000017000	0.000018000	0.000016600
1000	0.000026000	0.000026000	0.000023000	0.000026000	0.000027000	0.000025400
5000	0.000094000	0.000093000	0.000094000	0.000094000	0.000093000	0.000093800
10000	0.000183000	0.000151000	0.000179000	0.000178000	0.000180000	0.000173800
50000	0.000856000	0.000855000	0.000829000	0.000829000	0.000792000	0.000839600
100000	0.001597000	0.001650000	0.001558000	0.001667000	0.001631000	0.001627800
1000000	0.014988000	0.014982000	0.014975000	0.014991000	0.015050000	0.014985400
10000000	0.14892500	0.144910000	0.144702000	0.160700000	0.147820000	0.1519874
100000000	1.266733000	1.268790000	1.249646000	1.227740000	1.250506000	1.248129800

Trapezoid integration with num_threads=2

No of divisions	T1	T2	T3	T4	T5	Total time (2 threads)
5	0.000130000	0.000133000	0.000136000	0.000136000	0.000131000	0.000133200
10	0.000188000	0.000184000	0.000133000	0.000132000	0.000133000	0.000154000
50	0.000139000	0.000136000	0.000136000	0.000132000	0.000131000	0.000134800
100	0.000138000	0.000195000	0.000159000	0.000114000	0.000169000	0.000155000
500	0.000130000	0.000142000	0.000159000	0.000140000	0.000169000	0.000148000
1000	0.000182000	0.000171000	0.000151000	0.000234000	0.000229000	0.000193400
5000	0.000228000	0.000231000	0.000232000	0.000202000	0.000228000	0.000224200
10000	0.000313000	0.000337000	0.000290000	0.000264000	0.000317000	0.000304200
50000	0.000704000	0.000728000	0.000729000	0.00067600	0.000514000	0.0006702
100000	0.001285000	0.001295000	0.001268000	0.001315000	0.001314000	0.001295400
1000000	0.010186000	0.010141000	0.010255000	0.010198000	0.010156000	0.010187200
10000000	0.089338000	0.089760000	0.088805000	0.091914000	0.088611000	0.089685600
100000000	0.836195000	0.833050000	0.820643000	0.826729000	0.821368000	0.827597000

Trapezoid integration with num_threads=4

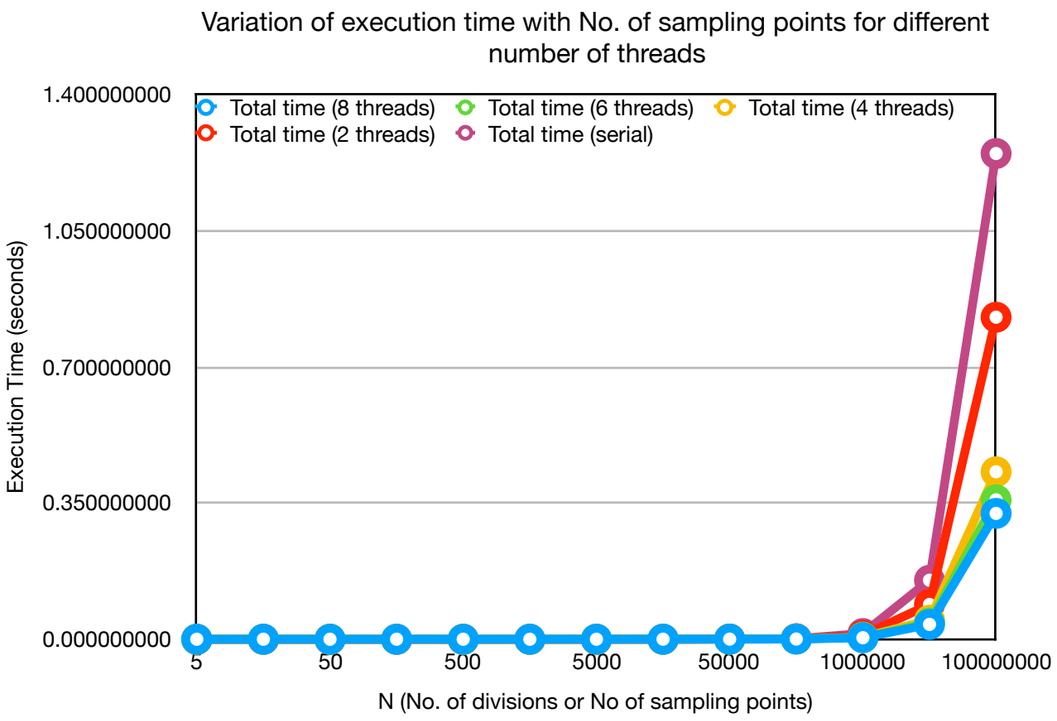
No of divisions	T1	T2	T3	T4	T5	Total time (4 threads)
5	0.000214000	0.000249000	0.000229000	0.000253000	0.000165000	0.000222000
10	0.000181000	0.000221000	0.000230000	0.000199000	0.000224000	0.000211000
50	0.000167000	0.000220000	0.000224000	0.000261000	0.000204000	0.000215200
100	0.000260000	0.000214000	0.000220000	0.000223000	0.000212000	0.000225800
500	0.000187000	0.000241000	0.000199000	0.000208000	0.000285000	0.000224000
1000	0.000211000	0.000227000	0.000248000	0.000257000	0.00022200	0.000233
5000	0.000242000	0.000250000	0.000223000	0.00027900	0.000267000	0.0002522
10000	0.000288000	0.000350000	0.000338000	0.000318000	0.000268000	0.000312400
50000	0.000522000	0.000501000	0.000559000	0.000509000	0.000516000	0.000521400
100000	0.000795000	0.000794000	0.000878000	0.000801000	0.000757000	0.000805000
1000000	0.005263000	0.006235000	0.006047000	0.005341000	0.005115000	0.005600200
10000000	0.049571000	0.055140000	0.044112000	0.048402000	0.049346000	0.049314200
100000000	0.436778000	0.429710000	0.431399000	0.428527000	0.426413000	0.430565400

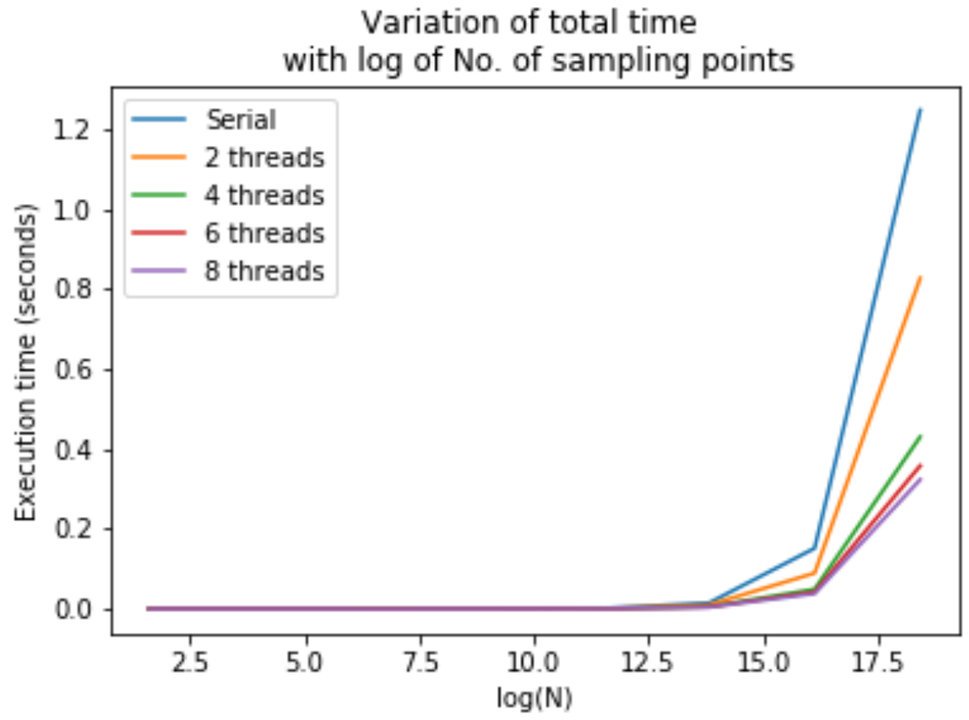
Trapezoid integration with num_threads=6

No of divisions	T1	T2	T3	T4	T5	Total time (6 threads)
5	0.000250000	0.000373000	0.000312000	0.000374000	0.000334000	0.000328600
10	0.000274000	0.000408000	0.000276000	0.000242000	0.000349000	0.000309800
50	0.000337000	0.000326000	0.000310000	0.000334000	0.000322000	0.000325800
100	0.000273000	0.000315000	0.00035900	0.000352000	0.000335000	0.0003268
500	0.000310000	0.000361000	0.000356000	0.000376000	0.000287000	0.000338000
1000	0.000322000	0.000339000	0.000340000	0.000356000	0.000326000	0.000336600
5000	0.000302000	0.000377000	0.000372000	0.00043600	0.000434000	0.0003842
10000	0.000419000	0.000405000	0.000375000	0.000420000	0.000382000	0.000400200
50000	0.000593000	0.000604000	0.000638000	0.000549000	0.000564000	0.000589600
100000	0.000830000	0.000844000	0.000796000	0.000781000	0.000990000	0.000848200
1000000	0.005329000	0.005042000	0.005134000	0.005022000	0.005793000	0.005264000
10000000	0.040685000	0.046396000	0.042768000	0.041590000	0.040471000	0.042382000
100000000	0.355143000	0.358015000	0.365992000	0.354809000	0.357990000	0.358389800

Trapezoid integration with num_threads=8

No of divisions	T1	T2	T3	T4	T5	Total time (8 threads)
5	0.000311000	0.000389000	0.000340000	0.000352000	0.000316000	0.000341600
10	0.000433000	0.000386000	0.000362000	0.000374000	0.000391000	0.000389200
50	0.00034100	0.000368000	0.000307000	0.000321000	0.000348000	0.000337
100	0.000419000	0.000409000	0.000391000	0.000311000	0.000405000	0.000387000
500	0.000376000	0.000426000	0.000311000	0.000291000	0.000396000	0.000360000
1000	0.000317000	0.000392000	0.000333000	0.000320000	0.000302000	0.000332800
5000	0.000418000	0.000454000	0.000368000	0.000350000	0.000350000	0.000388000
10000	0.000371000	0.00047100	0.000426000	0.000497000	0.000461000	0.0004452
50000	0.000581000	0.000614000	0.000652000	0.000701000	0.000591000	0.000627800
100000	0.000890000	0.000791000	0.000871000	0.000943000	0.000827000	0.000864400
1000000	0.003912000	0.004071000	0.004248000	0.00436400	0.004507000	0.0042204
10000000	0.037614000	0.037090000	0.038038000	0.041742000	0.037777000	0.038452200
100000000	0.327913000	0.321709000	0.323197000	0.322337000	0.322872000	0.323605600





Montecarlo Serial

No of divisions	T1	T2	T3	T4	T5	Total time (serial)
10	0.000020	0.000020	0.000019	0.000021	0.000023	0.0000206
100	0.000022	0.000023	0.000023	0.000024	0.000021	0.0000226
1000	0.000069	0.000069	0.000083	0.000068	0.000070	0.0000718
10000	0.000459	0.000520	0.000514	0.000466	0.000460	0.0004838
100000	0.004636	0.004715	0.004512	0.004276	0.004574	0.0045426
1000000	0.043627	0.046067	0.044066	0.044036	0.044612	0.0444816
10000000	0.393987	0.393375	0.383068	0.386500	0.387596	0.3889052
100000000	3.819941	3.764502	3.481265	3.465112	3.527861	3.6117362
1000000000	34.460900	33.949691	34.992481	35.329546	36.28153	35.0028296

Montecarlo integration with 2 threads

No of divisions	T1	T2	T3	T4	T5	Total time (2 threads)
10	0.000199000	0.000208000	0.000192000	0.000196000	0.000212000	0.000201400
100	0.000140000	0.000166000	0.000162000	0.000168000	0.00019400	0.000166
1000	0.000214000	0.000199000	0.000196000	0.000209000	0.000200000	0.000203600
10000	0.000616000	0.000542000	0.000601000	0.000603000	0.000544000	0.000581200
100000	0.003630000	0.003330000	0.003941000	0.003996000	0.004169000	0.003813200
1000000	0.034896000	0.038329000	0.037899000	0.038217000	0.038080000	0.037484200
10000000	0.343648000	0.334229000	0.335073000	0.330751000	0.329925000	0.334725200
100000000	3.113214000	3.257694000	3.179795000	3.136985000	3.174902000	3.172518000
1000000000	30.564994000	30.408618000	31.265543000	31.195445000	30.670171000	30.820954200

Montecarlo integration with 4 threads

No of divisions	T1	T2	T3	T4	T5	Total time (4 threads)
10	0.000224000	0.000259000	0.000344000	0.000174000	0.000261000	0.000252400
100	0.000312000	0.000216000	0.000223000	0.000218000	0.000274000	0.000248600
1000	0.000219000	0.000255000	0.000292000	0.000287000	0.00039900	0.0002904
10000	0.000517000	0.000575000	0.000576000	0.000547000	0.000543000	0.000551600
100000	0.002968000	0.003017000	0.003106000	0.002925000	0.003086000	0.003020400
1000000	0.032867000	0.031383000	0.033577000	0.032400000	0.03169500	0.0323844
10000000	0.251253000	0.239202000	0.272946000	0.238773000	0.243867000	0.249208200
100000000	2.510907000	2.505397000	2.481992000	2.149006000	2.149274000	2.359315200
1000000000	25.035145000	21.332298000	24.992016000	20.944748000	24.881240000	23.437089400

Montecarlo integration with 6 threads

No of divisions	T1	T2	T3	T4	T5	Total time (6 threads)
10	0.000346000	0.000391000	0.000427000	0.000487000	0.000397000	0.000409600
100	0.000473000	0.000451000	0.000338000	0.000453000	0.000444000	0.000431800
1000	0.000444000	0.000349000	0.000439000	0.000349000	0.000461000	0.000408400
10000	0.000645000	0.000596000	0.000655000	0.000540000	0.000653000	0.000617800
100000	0.002470000	0.002702000	0.002932000	0.002882000	0.003025000	0.002802200
1000000	0.028794000	0.027113000	0.027519000	0.027480000	0.028784000	0.027938000
10000000	0.209541000	0.211528000	0.208958000	0.215693000	0.204718000	0.210087600
100000000	1.779249000	1.961993000	1.794548000	1.942048000	1.922966000	1.880160800
1000000000	17.857557000	17.806907000	19.169396000	19.220044000	17.641390000	18.339058800

Montecarlo integration with 8 threads

No of divisions	T1	T2	T3	T4	T5	Total time (8 Threads)
10	0.000337000	0.000349000	0.000424000	0.000285000	0.000496000	0.000378200
100	0.000310000	0.000372000	0.000364000	0.000464000	0.000427000	0.000387400
1000	0.000419000	0.000390000	0.00045400	0.000404000	0.000332000	0.0003998
10000	0.000573000	0.000722000	0.000691000	0.000633000	0.000667000	0.000657200
100000	0.002918000	0.002909000	0.002955000	0.002902000	0.002816000	0.002900000
1000000	0.027253000	0.027993000	0.028339000	0.027961000	0.027004000	0.027710000
10000000	0.184652000	0.183312000	0.187988000	0.181005000	0.187797000	0.184950800
100000000	1.621574000	1.594254000	1.534135000	1.536196000	1.637058000	1.584643400
1000000000	15.599301000	15.122645000	15.387323000	15.542041000	15.328572000	15.395976400

Time is in seconds everywhere