AE 240 - Assignment

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APRIL, 2018

Mission Information

Launch Vehicle: Delta 3910
Payload: IRAS

Space Mission Objective

All-sky survey of astronomical IR bodies. Spacecraft engaged in research and exploration of the upper atmosphere or outer space (US Cat B).

Mission Specifications

Launch Site: Vandenberg (USA) - 34.7420° N, 120.5724° W
Launch Complex: Vandenberg SLC2W
LV Family: Thor
Launch Vehicle: Delta 3910

General Information and Orbital and Trajectory Parameters of the Launch vehicle

Delta 3910, an American orbital launch vehicle, is part of Thor Family. It is a 3-stage

Figure 1: Launch of Delta 3910 in 1983

vehicle consisting of 9 x Castor 4 + 1 x ELT Thor/RS-27 + 1 x Delta P /TR-201.

The Launch vehicle was retired in 1988. The first launch of Delta 3910 was on 1980-02-14 and its last launch was on 1988-02-08.

Specifications of Delta 3910 are as follows:

Thrust: 3188.00 kN
Gross Mass: 190,000 kg
Height: 35.00 m
Diameter: 2.44 m
Apogee: 1000 km
No. of Stages: 3
Stages: 1 $\times$ Delta P, 1 $\times$ Delta Thor RS27,9 $\times$ Castor 4

Stage information

Delta P stage

Propellant: N2O4/Aerozine-50 *Thrust*: 41.92 kN
Gross Mass: 5434 kg
Unfuelled Mass: 820 kg
Specific Impulse: 301 s
Burn Time: 322 s
Height: 5.97 m
Diameter: 1.38 m

Delta Thor RS27

Propellant: Lox/kerosene *Thrust*: 1030.22 kN
Gross Mass: 84,368 kg
Unfuelled Mass: 4360 kg
Specific Impulse: 296 s
Burn Time: 223 s
Height: 22.37 m
Diameter: 2.44 m

Castor 4

Propellant: Solid *Thrust*: 407.20 kN
Gross Mass: 10,534 kg
Unfuelled Mass: 1269 kg
Specific Impulse: 261 s
Burn Time: 54 s
Height: 9.07 m
Diameter: 2.44 m

General Information and Orbital and Trajectory Parameters of the spacecraft

IRAS was a Dutch-American-UK infrared Astronomy Satellite which was built for the purpose of all-sky survey of astronomical IR bodies.

Following information about the spacecraft orbit was gathered:

Class: Astronomy
Mass: 1073 kg
Apogee: 903 km
Perigee: 885 km
Inclination: 99.0000°
Time Period: 102.90 min

Overall Mission

IRAS was the first observatory to perform an all-sky survey at infrared wavelengths. It mapped 96 percent of the sky four times, at 12, 25, 60 and 100 micrometers, with resolutions ranging from 30 arc seconds at 12 micrometers to 2 arc minutes at 100 micrometers. It discovered about 350,000 sources, many of which are still awaiting identification. About 75,000 of those are believed to be star burst galaxies, still enduring their star-formation stage. Many other sources are normal stars with disks of dust around them, possibly the early stage of planetary system formation. New discoveries included a dust disk around Vega and the first images of the Milky Way's core.

IRAS was designed to catalog fixed sources, so it scanned the same region of sky several times. This led to the discovery of three asteroids, including 3200 Phaethon (an Apollo asteroid and the parent body of the Geminid meteor shower), six comets, and a

Figure 2: View from above satellite

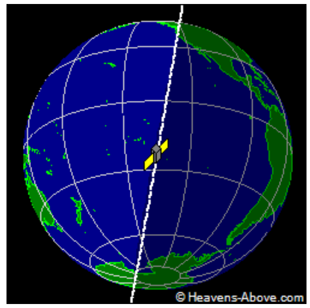
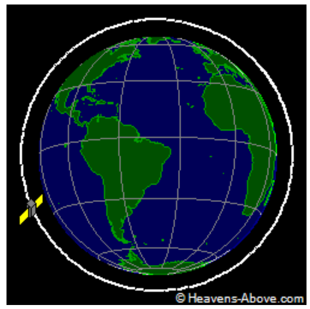


Figure 3: View from the orbital plane



huge dust trail associated with comet 10P/Tempel. The comets included 126P/IRAS, 161P/Hartley–IRAS, and comet IRAS–Araki–Alcock (C/1983 H1), which made a close approach to the Earth in 1983. Out of the six comets IRAS found, four were long period and two were short period comets.

Mission Status

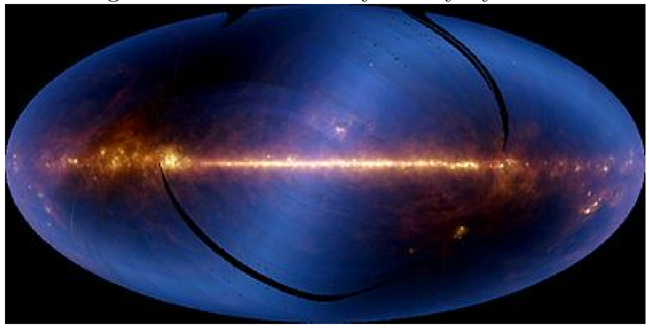
The mission was decomposed and deactivated on 21 November 1983. The spacecraft continues to orbit the Earth.

Success

The orbiting telescope discovered wealth of new phenomena in the universe during its l0-month lifetime. Engineering tests will be conducted for about one week before the satellite ceases operations, according to mission officials. The observatory made headlines briefly with the announcement on some "unknown objects" which after further analysis revealed that, nine were distant galaxies and the tenth was "intergalactic cirrus".

During its mission, IRAS (and later the Spitzer Space Telescope) detected odd infrared signatures around several stars which led to the systems being targeted by the Hubble Space Telescope's NICMOS instrument between 1999 and 2006, but nothing was detected. In 2014, using new image processing techniques on the Hubble data, researchers discovered planetary disks around these stars. The spacecraft also discovered asteroids such as 3200 Phaethon,3728 IRA, (10714) 1983 QG and (100004) 1983 VA

Figure 4: Infrared all-sky survey by IRAS



<u>*Limitations*</u>

IRAS's life was limited by its cooling system. To effectively work in the infrared domain, a telescope must be cooled to cryogenic temperatures. The on-board supply of liquid helium was depleted after 10 months on 21 November 1983, causing the telescope temperature to rise, preventing further observations. At launch, IRAS was expected to operate for only seven months. After launch, based on flight data which measures the rate at which the helium was being used, mission engineers estimated that the 75 kilograms of refrigerant would last through early January, l984. But there were uncertainties in the calibration estimates of the flow rate of helium, which accounts for the difference.

Trajectory Parameters of the spacecraft

Since the satellite revolves around the earth, its periapsis and apoapsis are same as its perigee and apogee.

Given apogee $r_a = 903 + 6371 = 7274\ km$ and perigee $r_p = 885 + 6371 = 7256\ km$

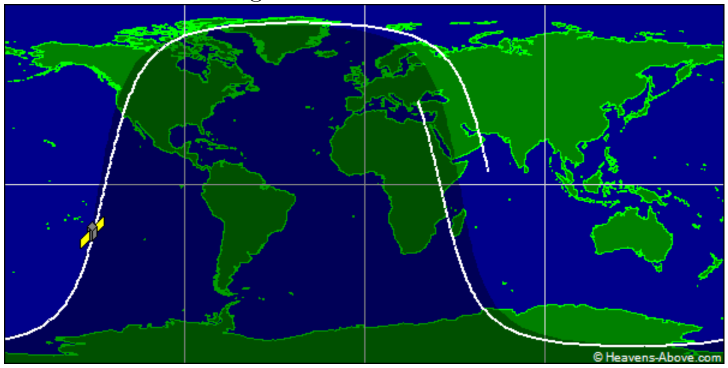
$\implies a(1+e) = 7281\ km$ and $a(1-e) = 7256\ km$

$\implies e = 0.00124$

Hence, the spacecraft's trajectory was elliptical

So, $a = 7264.99\ km$

Figure 5: Ground Track



Hence Total time period of the trajectory $T = 2\pi\sqrt{\frac{a^3}{\mu}}$

$\implies T = 102.71\ min$

This gives an error of 0.186% with respect to the given time, which is 102.9 minutes.

Other Orbital parameters of the spacecraft trajectory (the following data has been taken on 15 April 2018, 10:45 UTC):

Epoch: 15 April 2018 10:45:44 UTC
Right ascension of ascending node: 294.8057°
Argument of perigee: 304.4631°
Revolutions per day: 14.00372682
Mean anomaly at epoch: 125.2331
Orbit number at epoch: 46857

The above stated orbit data is extracted from the following two-line orbital elements,

1 13777U 83004A 18105.44842885 -.00000051 +00000-0 -48346-5 0 9996
2 13777 098.9447 294.8057 0019510 304.4631 125.2331 14.00372682468570

Nature of the orbit

The orbit of IRAS was connected with the mission in the following ways:

1. The IRAS orbit was chosen with several factors in mind. First, the heat-sensitive telescope must always point more than 60 degrees away from the Sun and more than

88 degrees from the brightness of the Earth's limb, or edge. The power-producing solar panels must also receive sunlight at least part of the time. The satellite therefore follows a nearly polar orbit closely aligned with the Earth's terminator, or sunrise-sunset line. The orbit is Sun-synchronous, it shifts approximately one degree each day to keep the same attitude relative to the Sun as the Earth travels its seasonal journey. Horizon sensors on the satellite warn it away from bright objects and keep it looking at dark space. They also help in the computer-guided, gyro-assisted attitude control system. Star sensors are used to achieve a telescope pointing accuracy down to only a few seconds of arc.

2. The purpose of all this caution and engineering is to maximize the amount of radiation striking the IRAS telescope's sensitive infrared detectors on each survey scan. There are 62 detectors in all, sensitive in four wavelength bands. Rectangular in shape, they average about the size of a medium-length printed word on this page. They work on the principle that exposure to infrared radiation reduces the electrical resistance of their crystals by a known amount, so that the amount of radiation reaching the telescope's focal plane can be read directly as an increase in current. The Band 1 detectors, observing at shorter wavelengths, typically reveal the emissions of hotter point sources such as stars. Band 4, on the other hand, observes cooler or more extended objects, such as dust clouds, with its longer wavelength sensitivity.

Design of the Launch Vehicle trajectory

I have made the following assumptions in designing the trajectory of the Launch vehicle:

1. In the 1st stage rocket undergoes vertical lift-off under the influence of gravity for the first 10 seconds and for the rest of the trajectory, the rocket undergoes constant pitch rate motion.
2. The rocket is given a pitch kick of $10°$ at the beginning of the constant pitch rate motion followed by further pitch kicks of $5°$ at the start of 2nd stage and $10°$ at the start of the third stage.
3. The stage-wise information is stated above. Note that 9 Castor 4 engines constitute the first stage, followed by Delta Thor RS27 and then Delta P stages.
4. The burn rate of the propellant is assumed to be constant.
5. Terminal condition provided for this manoeuvre is that the rocket is injected at an altitude of 1000 km. This point is assumed to be at a temporary orbit, which is assumed to be circular. As the orbit is assumed circular, the exact location of the point does not matter as velocity in a circular orbit is constant. The apogee of the final orbit is less than this altitude, hence to transfer the spacecraft at its

final orbit, lowering of perigee and apogee is required.

6. Velocity at the terminal point of the launch vehicle trajectory is calculated as per the assumed trajectory as stated above.
7. The launch vehicle trajectory has been designed as a combination of vertical takeoff, constant pitch rate solution and upper atmosphere orbital manoeuvre.
8. Finally, angle will be 90° at the injection point.
9. In the entire calculations, the effect of gravity has been accounted in calculation of angle θ and altitude, even in case of vertical lift-off.

NOTE: All the calculations have been done in a ipython notebook, which has been submitted with the report. The final answers and formulas used for calculation have been reported below.

Below is the information about each stage:

Stage	T_B	I_sp	m_p	m_{gross}	β
1	54	261	83385	94806	5967.21; 538.93
2	223	296	80008	84368	358
3	322	301	4614	5434	14

Also note that $\sum_i m_{gross,i} + m^* = 186,754kg$ ($m^* = 2 \times 1073kg$ as the launch vehicle carried another payload PIX 2 of the same mass as IRAS)

However given that $M_0 = 190,000kg$. So it is further assumed that the difference in mass = 3246 kg is the propellant mass which is used for upper atmosphere manoeuvres and spacecraft orbit manoeuvres.

β for stage 1 is 5967.21 during vertical launch and 538.93 during constant pitch rate trajectory.

Launch Azimuth Angle

It is assumed that the Launch vehicle is launched at an azimuth to ensure that the inclination of the orbit is the desired 99.0000°. This is efficient as it will save the fuel that would have been required for plane change orbital manoeuvre
So, we have the relation: $cos(i) = cos(\phi)sin(\beta)$
where
i: inclination of the orbit
ϕ: Latitude of the launch site

β: Launch azimuth

For Vandenberg, $\phi = 34.7420°$.
Substituting the values of i and ϕ, we get:
$\beta = 190.9745°$ due east or $\beta = 10.9745°$ due west

For this value of azimuth, the final orbit will have the required inclination of 99.0000°.

Formulas used

For vertical lift-off, the altitude and velocity have been calculated using the formula:

$$\Delta V = g_0 I_{sp} log(\frac{m_0}{m_t}) - gt$$
$$\Delta H = \frac{m_0 g_0 I_{sp}}{\beta}(log(1-\Lambda)+\Lambda) \text{ where } \Lambda = \frac{m_p}{m_0}$$

For constant pitch trajctory, the altitude, velocity, final θ has been calculated using the following formula:

$$\theta_f = \theta_0 + qt$$
$$V = \frac{g sin\theta_f}{q}$$
$$H = \frac{g}{4q^2}(cos2\theta_0 - cos2\theta_f) + H_0$$

First Stage

Burnout time of 1st stage: 54s

The Launch vehicle is designed to undergo vertical liftoff for for the 1st 10 seconds with a burn rate of 5967.21 kg/s. The high burn rate is distributed among the 9 Castor 4 engines. So, the burn rate in each engine is 663.02 kg/s.

After 10 seconds or the vertical lift-off:

Altitude $H_0 = 3165.26m$
Velocity $V_0 = 867.15m/s$

After the vertical lift-off, a pitch kick of $10°$ is given to the rocket and the pitch rate is then constant for the entire trajectory.

So, pitch rate $q = \dfrac{gsin\theta}{V_0}$
$\implies q = 0.001921$

After the burnout of 1st stage:

Theta $\theta_1 = 15.72°$
Velocity $V_1 = 697.73 km/s$
Altitude $H_1 = 29.88 km$
Burn rate $\beta_1 = 538.93 kg/s$

Second Stage

Burnout time of 1st stage: 223s

A further pitch kick of $5°$ is given at the beginning of this stage.

After the burnout of 2nd stage:

Theta $\theta_2 = 45.26°$
Velocity $V_2 = 3627.62 m/s$
Altitude $H_2 = 386.08 km$
Burn rate $\beta_2 = 358 kg/s$

Third Stage

Burnout time of 1st stage: 322s

A pitch kick of $10°$ is given at the beginning of this stage.

After the burnout of 3nd stage:

Theta $\theta_3 = 90.71°$
Velocity $V_3 = 5106.32 m/s$
Altitude $H_3 = 947.49 km$
Burn rate $\beta_3 = 14 kg/s$

Upper Atmosphere Orbital Manoeuvres

After this, a velocity impulse is given.

$\Delta V = V_4 - V_3$
Here V_4 is the velocity of the spacecraft in a circular orbit of radius 7371 km (1000 km altitude).
$\implies V_4\sqrt{\frac{\mu}{r}}$
$\implies V_4 = 7353.69 m/s$

Therefore $\Delta V = 2247.37 m/s$

The spacecraft is now on a circular orbit of radius 7371 km. The 3rd stage is separated and spacecraft is injected at this point.

The final orbital parameters obtained are as follows:

Theta $\theta_3 = 90.71°$
Velocity $V_4 = 7353.69 m/s$
Altitude $H_3 = 947.49 km$

As per the terminal conditions, following errors in θ, altitude and velocity are observed:

Error in $\theta = 0.79$ %
Error in velocity = 0 % (After the manoeuvre, exact velocity required at that altitude is imparted)
Error in altitude = 5.25 %

Orbital Manoeuvres of the spacecraft

In the design of the Launch Vehicle trajectory, we have placed the spacecraft in a circular orbit of radius 7371 km. Now, we need to transfer the satellite to an elliptical orbit with perigee 7256 km and apogee 7274 km. To achieve the desired orbit, two velocity impulses will be imparted to the spacecraft. The first one will be imparted at the apogee to lower the perigee, while the second impulse is given once, the satellite reaches the perigee of the orbit, to lower the apogee.

First Velocity Impulse

This impulse will put the spacecraft in a transfer orbit which will be an ellipse with apogee 7371 km and perigee 7256 km.

Velocity of the spacecraft in the circular orbit $V_0 = 7353.69m/s$
Velocity of the spacecraft in the transfer orbit $V_1 = \sqrt{\mu\Big(\frac{2}{r} - \frac{1}{a}\Big)}$

For the transfer orbit: $r = 7371km$ and $a = \dfrac{7371 + 7256}{2} = 7313.5km$
$\implies V_5 = 7324.73m/s$

$\Delta V_1 = V_1 - V_0$
$\implies \Delta V_1 = -28.96m/s$

Second Velocity Impulse

This impulse will put the spacecraft in the final orbit which will be an ellipse with apogee 7274 km and perigee 7256 km. After the 1st velocity impulse, the spacecraft travels from the apogee to the perigee where the second velocity impulse is provided.

At the perigee, $V_2 = \sqrt{\mu\Big(\frac{2}{r} - \frac{1}{a}\Big)}$ where a = 7313.5 km and r = 7256 km
$\implies V_2 = 7440.82m/s$

Similarly for $V_3, r = 7256km$ and $a = \dfrac{7256 + 7274}{2} = 7265km$
$\implies V_3 = 7416.33m/s$

$\Delta V_2 = V_3 - V_2$
$\implies \Delta V_2 = -24.49m/s$

Total Velocity Impulse

$\Delta V = \Delta V_1 + \Delta V_2$
$\implies \Delta V = -53.45m/s$

Now the spacecraft is finally in its desired orbit and our launch vehicle and orbit manoeuvre design is complete.

Time of perigee passage

Under the assumptions made and furthre assuming that the velocity impulses are imparted instantaneously, Time of perigee passage will be the half of the total time period of the transfer orbit.

For the transfer orbit, a = 7313.5 km

$\implies$ Time of perigee passage $T = \frac{1}{2}\left(2\pi\sqrt{\frac{a^3}{\mu}}\right)$

$\implies T = \pi\sqrt{\frac{a^3}{\mu}}$

$\implies T = 51.87$ minutes

Key events in the spacecraft trajectory

The Key events taken by the spacecraft are as follows:

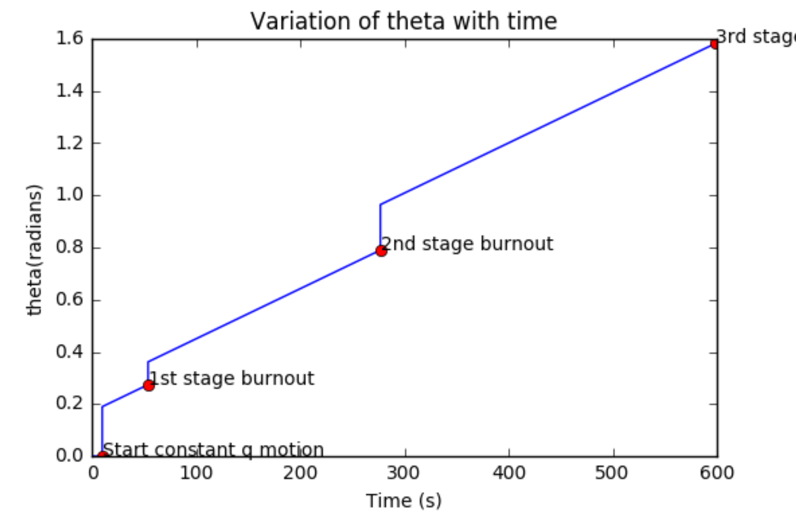
Table III.C.1 Mission Chronology

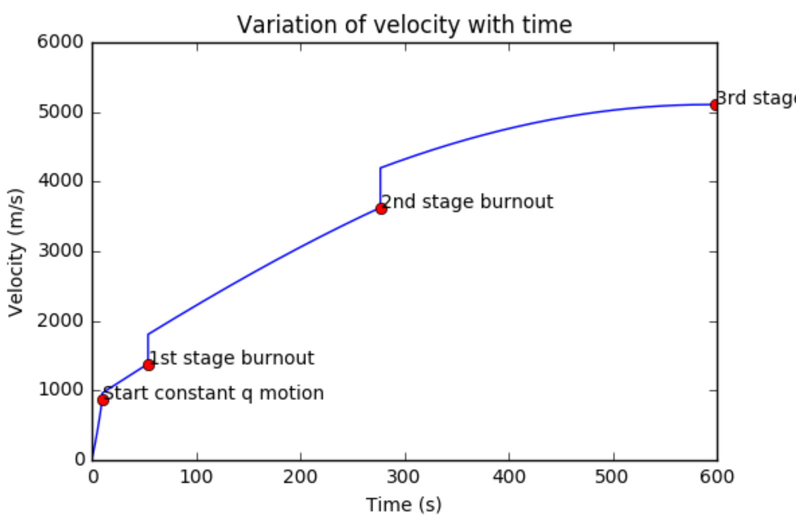
(All dates are 1983 and given in GMT)

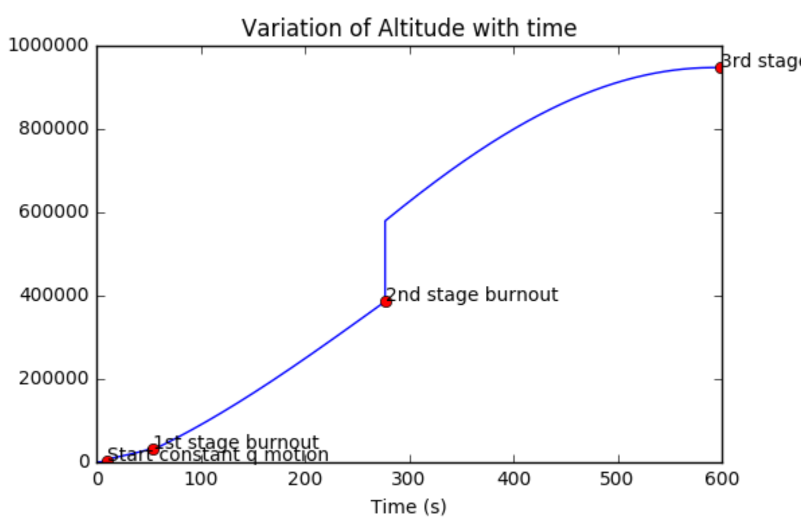
Date	SOP	Event
26 Jan	1	Launch 02h 17m
26-31 Jan	1-12	In Orbit Checkout (Cover on) Outgassing of satellite
31 Jan	12	Cover ejection 19h 37m
31 Jan-8 Feb	12-28	In Orbit Checkout (Cover off)
9 Feb	29	SAA contour A (Fig. III.B.6) usage begins
9-10 Feb	29-30	Minisurvey layer 1 Hand made scans
10 Feb	31	Start first two hours-confirming coverages using half circles.
11-12 Feb	33-34	Minisurvey layer 2 Hand made scans
13-14 Feb	37-38	Minisurvey layer 3 Hand made scans
15 Feb	41	Minisurvey layer 4a Hand made scans
16 Feb	43	Minisurvey layer 4b Hand made scans
23 Feb	57	Half circle method ended
23 Feb	58	Lune method started
3 Apr	135	Moon avoidance radius lowered from 25 to 20°
9 May	207	SAA contour B (Fig. III.B.6) usage begins
26 Aug	425	End of first two hours-confirming coverages
26 Aug	426	Start third hours-confirming coverage
26 Aug	426	Moon avoidance radius lowered from 20 to 13°
9 Sep	454	Moon avoidance radius raised from 13 to 20°
18 Nov	593	First eclipse. Fallback to safety mode
21 Nov	600	Survey operations resumed 19h 40m
22 Nov	600	Liquid helium ran out 00h 16m
22 Nov	600	Last survey scan started 03h 34m
23 Nov	603	12 μm detector baselines saturated 09h 30m

Plots and simulations

The following plots show the variation of parameters with time in the launch vehicle trajectory.

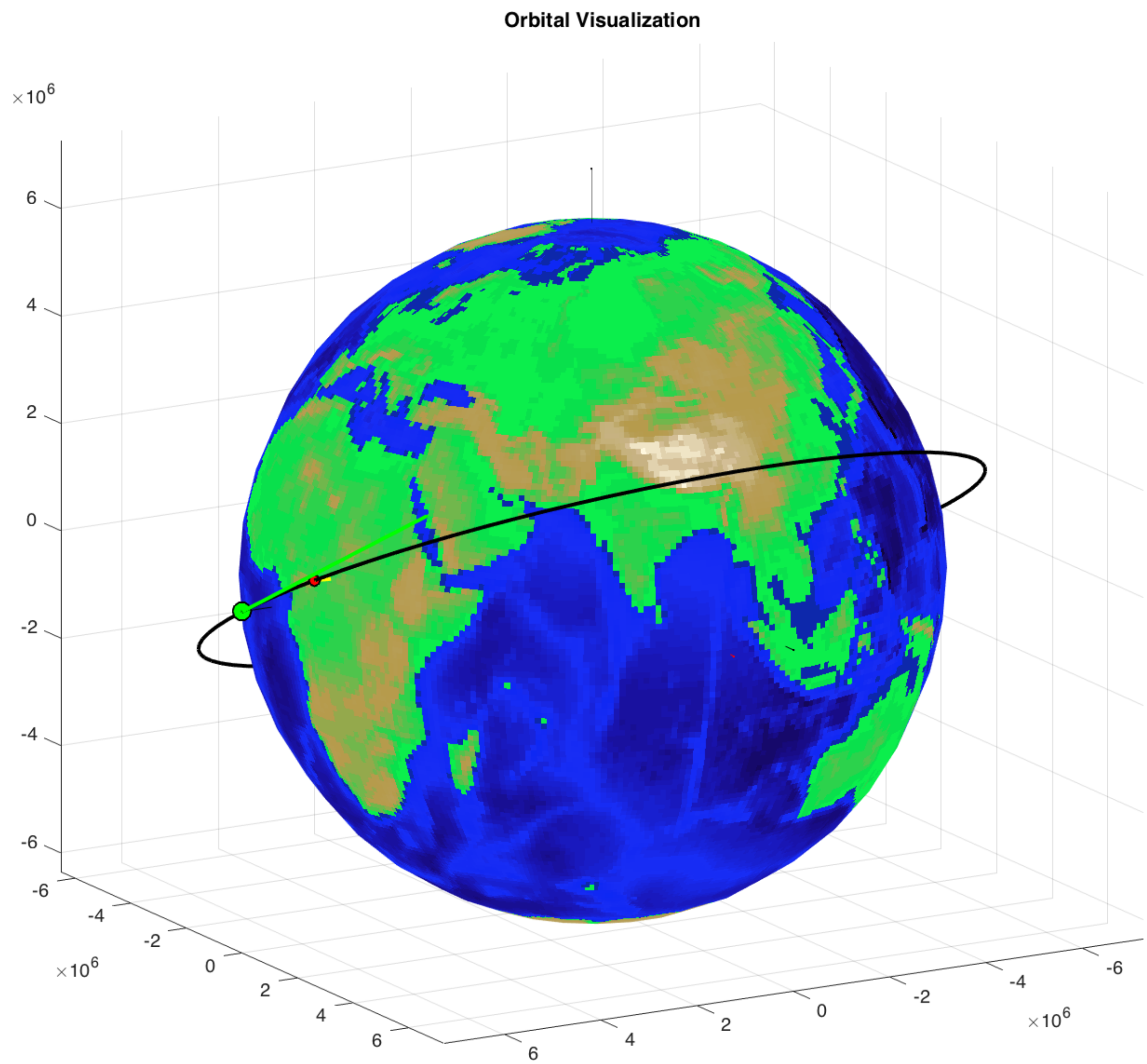






For the simulation of the spacecraft orbit, I have used a satellite orbit simulation repository by deedy. The simulation has been made in Matlab and is very realistic and sophisticated. I have modified and restructured the code to model the simulation for the IRAS satellite. After the simulation has been run, the ground track of the satellite is also traced. Below is a snapshot of the simulation:

Click on the image to play the video of the simulation



The code for this simulation has been submitted along with this report.

References

http://www.astronautix.com/d/delta3910.html
http://www.astronautix.com/i/iras.html
https://en.wikipedia.org/wiki/Delta_(rocket_family)#Delta_3000-Series
https://www.jpl.nasa.gov/news/news.php?feature=5953
http://heavens-above.com/orbit.aspx?satid=13777
https://er.jsc.nasa.gov/seh/infra.html
https://www.youtube.com/watch?v=44vzI2udcRU&list=PLrDsNBDqvNLGHAt-F4xAMwV
index=1&t=0s
Infrared Astronomical Satellite (IRAS) - Catalog and Atlases Vol.1 Explanatory Supplement

Feedback

For any feedback or suggestion regarding the report or the codes involved, you can file an issue in my github repository for this assignment. The repository can be found here.

AE240

April 17, 2018

1 AE 240 Assignement- Launch Vehicle Trajectory Code

```
In [115]: from __future__ import print_function
          from numpy import *
          import pandas as pd

In [116]: import matplotlib.pyplot as plt
          %matplotlib inline

In [117]: q=0.001921
          g0=9.81


          def g_h(h): # value of g at an altitude h
              factor=( 6371.0/(h+6371.0) )**2
              return g0*factor

          def beta(m_i, m_f, t): #burn rate for the vertical launch (Assumed Constant)
              return (m_i-m_f)/t

In [118]: #Functions for parameters in a constant pitch rate trajectory

          def TOF(theta_a, theta_b): #Time of flight to go from angle A to B in constant pitch t
              return (theta_a-theta_b)/q

          def theta_final(theta_initial, TOF ): #Final angle for a given initial angle and TOF
              return q*TOF+theta_initial

          def velocity1(theta, g):
              return g*sin(theta)/q

          def altitude_constant_q(theta_i, theta_f, g, h0):
              return ( g*(cos(2*theta_i)-cos(2*theta_f))/(4*q*q) +h0 )

In [119]: #Functions for parameters in a vertical launch

          def vel_ideal(m_i,m_f,Isp): #Ideal Velocity without the impact of gravity
              return g0*Isp*log(m_i/m_f)
```

```
def vel_gravity(m_i, m_f, Isp, g, t): #Velocity taking into account gravity
    return (vel_ideal(m_i,m_f,Isp) - g*t)

def altitude_vertial_launch(m_i, m_f, Isp, g, t): #Altitude attained by the vertical l
    L = (m_f-m_i)/m_i
    b=beta(m_i,m_f,t)
    H = ( (m_i*g0*Isp)/b)*((1-L)*log(1-L) + L) -0.5*g*t*t
    return H
```

1.1 Stage 1

```
In [120]: #These data have been mentioned in the report and have been obtained from http://www.a

          m01 = 190000
          m11 = 106615
          Tb1 = 54
          Isp1= 261

In [121]: t1 = 10
          t2 = Tb1-t1
```

For a certain time t1, the Launch vehicle undergoes vertical motion. After that, it receives a pitch kick of theta0, and executes a constant pitch rate gravity turn till angle theta1 in the 1st stage

```
In [133]: #Calculations for the vertical launch
          m = 130327.8497

          V0 = vel_gravity(m01, m, Isp1, g0, t1)
          H0 = altitude_vertial_launch(m01, m, Isp1, g0, t1)
          g01 = g_h(H0/1000.0)
          g=(g01+g0)/2.0
          V0 = vel_gravity(m01, m, Isp1, g, t1)
          H0 = altitude_vertial_launch(m01, m, Isp1, g, t1)
          beta0 = beta(m01, m, t1)
          print("H0: ", H0)
          print("V0: ", V0)
          print("Burn rate for the vertical launch: ",beta0, "kg/s")

H0:  3165.26640262
V0:  867.148698105
Burn rate for the vertical launch:  5967.21503 kg/s


In [134]: #Calculations from time t1 to time Tb

          theta0=arcsin(V0*q/g0)
          theta1=theta_final(theta0, Tb1)
          altitude1=altitude_constant_q(theta0, theta1, g0, H0)
```

```
g01 = g_h(altitude1)
g1=(g0+g01)/2.0
altitude1=altitude_constant_q(theta0, theta1, g1, 0)

V1=velocity1(theta1,g1)

beta1 =beta(m, m11, t2)

print("Pitch kick at the start: ", theta0*180/pi)
print("Altitude at stage 1 separation: ", altitude1/1000, "km")
print("Theta at stage 1 separation: ", theta1*180/pi, "degrees")
print("Velocity at stage 1 separation: ", V1/1000, "km/s")
print("Burn rate after vertical launch to stage 1 separation: ",beta1, "kg/s")
```

```
Pitch kick at the start:  9.77651472565
Altitude at stage 1 separation:  29.8769284886 km
Theta at stage 1 separation:  15.7200351177 degrees
Velocity at stage 1 separation:  0.697734586421 km/s
Burn rate after vertical launch to stage 1 separation:  538.928402273 kg/s
```

1.2 Stage 2

```
In [135]: #These data have been mentioned in the report and have been obtained from http://www.a
          Tb2 = 223
          Isp2 = 296
          m12 = 95194
          m22 = 15186
```

```
In [136]: #Calulculations from stage 1 separation to stage 2 separation

          #pitch kick of 5 degrees at the start of this stage
          theta1+=0.0872664
          theta2=theta_final(theta1, Tb2)

          altitude2=altitude_constant_q(theta1, theta2, g1, altitude1)
          g12=g_h((altitude1+altitude2)/1000.0)
          g2=(g12+g1)/2.0
          altitude2=altitude_constant_q(theta1, theta2, g2, altitude1)

          V2=velocity1(theta2,g0)

          beta2 = beta(m12, m22, Tb2)

          print("Altitude at stage 2 separation: ", altitude2/1000, "km")
          print("Theta at stage 2 separation: ", theta2*180/pi, "degrees")
          print("Velocity at stage 2 separation: ", V2/1000, "km/s")
          print("Burn rate from stage 1 separation to stage 2 separation: ",beta2, "kg/s")
```

```
Altitude at stage 2 separation:  386.079479528 km
Theta at stage 2 separation:  45.2645694461 degrees
Velocity at stage 2 separation:  3.62762857794 km/s
Burn rate from stage 1 separation to stage 2 separation:  358 kg/s
```

1.3 Stage 3

In [137]:
```
#These data have been mentioned in the report and have been obtained from http://www.a

Tb3 = 322
Isp3 = 301
m13 = 10826
m23 = 6306
```

In [138]:
```
#Calculations from stage 2 separation to stage 3 separation

theta3 = theta_final(theta2, Tb3)
theta3+=0.17453

altitude3=altitude_constant_q(theta2, theta3, g2, altitude2)
g13=g_h((altitude3+altitude2)/1000.0)
g3=(g+g2)/2.0
altitude3=altitude_constant_q(theta2, theta3, g3, altitude2)

V3 = velocity1(theta3, g0)

beta3 = beta(m13, m23, Tb3)

print("Altitude at stage 3 separation: ", altitude3/1000, "km")
print("Theta at stage 3 separation: ", theta3*180/pi, "degrees")
print("Velocity at stage 3 separation: ", V3/1000, "km/s")
print("Burn rate from stage 2 separation to stage 3 separation: ",beta3, "kg/s")
```

```
Altitude at stage 3 separation:  947.491994488 km
Theta at stage 3 separation:  90.7053938117 degrees
Velocity at stage 3 separation:  5.10632824069 km/s
Burn rate from stage 2 separation to stage 3 separation:  14 kg/s
```

1.4 Error in the final parameters obtained

In [139]:
```
theta = pi/2 #Since final theta of the trajectory is 90 degrees
altitude = 1000000 # Launch vehicle trajectory is assumed circular with the given alti
```

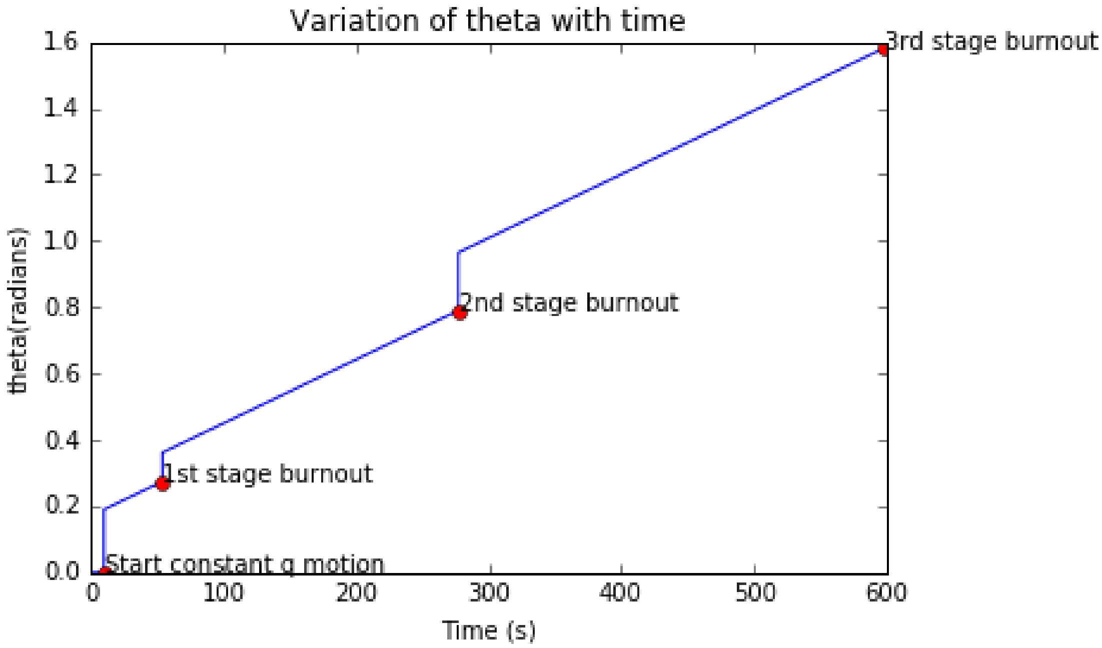
In [140]:
```
error_theta = 100*(theta-theta3)/theta
error_altitude = 100*(altitude-altitude3)/altitude
```

```
print("Error in theta: ", error_theta, "%")
print("Error in altitude: ", error_altitude, "%")
```

```
Error in theta:  -0.783770901896 %
Error in altitude:  5.25080055115 %
```

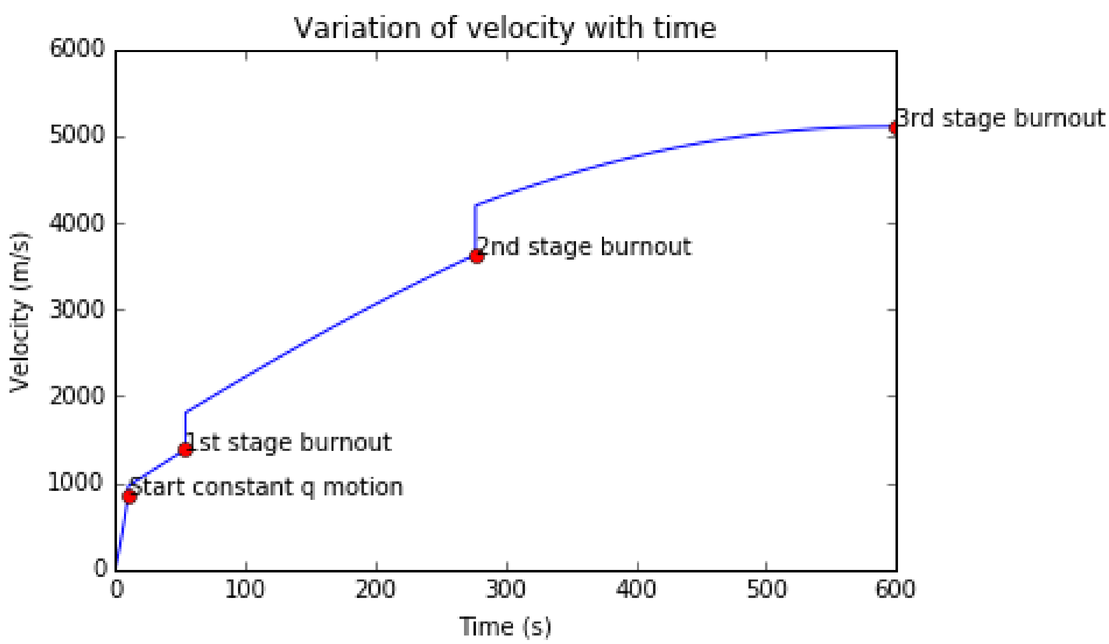
1.5 Graphs and Plots

```
In [141]: #Plotting theta as a function of time
theta_values=[]
time_values=[]

#stage 1
time1=linspace(0,10,10)
theta_list_1=repeat(0,10)
time2=linspace(10, 54, 44)
theta_list_2=time2*q+0.1705

#stage 2
time3=linspace(54, 277, 223)
theta_list_3=(time3-54)*q+theta_list_2[-1]+0.0872664

#stage 3
time4=linspace(277, 599, 322)
theta_list_4=(time4-277)*q+theta_list_3[-1]+0.17453

time_values = concatenate((time1,time2,time3, time4), axis=0)
theta_values = concatenate((theta_list_1,theta_list_2,theta_list_3, theta_list_4), axi

plt.plot(time_values, theta_values)
plt.title("Variation of theta with time")
plt.xlabel("Time (s)")
plt.ylabel("theta(radians)")
plt.annotate("Start constant q motion", xy=(time1[-1], theta_list_1[-1]))
plt.annotate("1st stage burnout", xy=(time2[-1], theta_list_2[-1]))
plt.annotate("2nd stage burnout", xy=(time3[-1], theta_list_3[-1]))
plt.annotate("3rd stage burnout", xy=(time4[-1], theta_list_4[-1]))
plt.plot(time1[-1], theta_list_1[-1], 'ro')
plt.plot(time2[-1], theta_list_2[-1], 'ro')
plt.plot(time3[-1], theta_list_3[-1], 'ro')
plt.plot(time4[-1], theta_list_4[-1], 'ro')
plt.savefig("theta.png")
```




```
In [144]: #Plotting velocity as a function of time
          velocity_values=[]
          time_values=[]

          #stage 1
          time1=linspace(0,10,10)
          velocity_list_1=vel_gravity(m01, m01-beta0*time1, Isp1, g, time1)
          time2=linspace(10, 54, 44)
          velocity_list_2=velocity1(theta_list_2,g0)

          #stage 2
          time3=linspace(54, 277, 223)
          velocity_list_3=velocity1(theta_list_3,g0)

          #stage 3
          time4=linspace(277, 599, 322)
          append(time4, array([599]))
          velocity_list_4=velocity1(theta_list_4,g0)
          append(velocity_list_4, array([7273.6]))

          time_values = concatenate((time1,time2,time3, time4) , axis=0)
          velocity_values = concatenate((velocity_list_1,velocity_list_2,velocity_list_3, veloci

          plt.plot(time_values, velocity_values)
          plt.title("Variation of velocity with time")
          plt.xlabel("Time (s)")
```

```
plt.ylabel("Velocity (m/s)")

plt.annotate("Start constant q motion", xy=(time1[-1], velocity_list_1[-1]))
plt.annotate("1st stage burnout", xy=(time2[-1], velocity_list_2[-1]))
plt.annotate("2nd stage burnout", xy=(time3[-1], velocity_list_3[-1]))
plt.annotate("3rd stage burnout", xy=(time4[-1], velocity_list_4[-1]))
#plt.annotate("Final velocity after manoeuvre", xy=(time4[-1], 7273.6))
plt.plot(time1[-1], velocity_list_1[-1], 'ro')
plt.plot(time2[-1], velocity_list_2[-1], 'ro')
plt.plot(time3[-1], velocity_list_3[-1], 'ro')
plt.plot(time4[-1], velocity_list_4[-1], 'ro')
plt.savefig("Velocity.png")
```



In [145]:
```
#Plotting height as a function of time
alt_values=[]
time_values=[]

#stage 1
time1=linspace(1e-1,10,10)
alt_list_1=altitude_vertial_launch(m01, m, Isp1, g0, time1)
time2=linspace(10, 54, 44)
alt_list_2=altitude_constant_q(theta0, theta_list_2, g1, H0)

#stage 2
time3=linspace(54, 277, 223)
alt_list_3=altitude_constant_q(theta1, theta_list_3, g2, altitude1)
```

```
#stage 3
time4=linspace(277, 599, 322)
alt_list_4=altitude_constant_q(theta2, theta_list_4, g3, altitude2)

time_values = concatenate((time1,time2,time3, time4) , axis=0)
alt_values = concatenate((alt_list_1,alt_list_2,alt_list_3, alt_list_4), axis=0)

plt.plot(time_values, alt_values)
plt.title("Variation of Altitude with time")
plt.xlabel("Time (s)")
plt.ylabel("Altitude (m)")

plt.annotate("Start constant q motion", xy=(time1[-1], alt_list_1[-1]))
plt.annotate("1st stage burnout", xy=(time2[-1], alt_list_2[-1]))
plt.annotate("2nd stage burnout", xy=(time3[-1], alt_list_3[-1]))
plt.annotate("3rd stage burnout", xy=(time4[-1], alt_list_4[-1]))
plt.plot(time1[-1], alt_list_1[-1], 'ro')
plt.plot(time2[-1], alt_list_2[-1], 'ro')
plt.plot(time3[-1], alt_list_3[-1], 'ro')
plt.plot(time4[-1], alt_list_4[-1], 'ro')
plt.savefig("altitude.png")
```