MODULE - 1

Introduction to limit state method [LSM] and Serviceability

RCC :- when reinforcing bars are introduced in the Concrete it makes the concrete and excellent building structure which can bear or take Significant amount of loads or Stress and strains.

Design codes :-

Different countries have formulated their own codes which provides guidelines for the construction and design of structures. They Serve for the following objectives or purposes like (1) They ensure structural stability and Safety which are required for designing.

(2) They make the task of engineers very Simple by providing the values in the form of charts or tables.

(3) They provide consistency in procedures which is adopted by various designers in the country.

(4) They protect the designers against the

Structural failure that are caused due to improper sight construction practise.

Necessity of Designing RC Structures

The principle aim of structural design is that the structure should perform it's intended function safely at ultimate loads within their life time and also servicable during their service period.

The term safety includes the following

(1) Strength of the material

(2) Durability of the structure

(3) Thus safety implies the possibility of the failure of the structure is very low at ultimate loads

Serviceability :-

It implies that the structure should perform its intended function very well at working loads. Thus the basic object is to fulfill the serviceability criteria can be summarised as

(1) The designed structure should perform

its function at Service loads Satisfactorily.

* The Structure should bear all the loads within its permeasible limits

* The Structure should be durable enough.

* The Structure must adequately resist the hazardors effect of Structural ~~vee~~ misuse and fire.

Objectives of Structural design :

① Stability - The Structure must be Stable enough to resist the failure in terms of over turning, Sliding, buckling of the Structure under Severe action of loads

② Strength :- The structure must be able to carry Safely, the Stress induced by the combination of loads acting on the Structure.

③ Serveceability :- The Structure must be well Serveceable enough to perform its function. This implies that the deflection, vibration, crack - widths, permeability, to water etc.

④ Asthetics appearance :- The structure must be in harmony with the Surroundings and Should look pleasing. It is purely based on architectural consideration.

(5) Economy :- At last economy place the most important role in the structural design

Design policy for RC Structure

working Stress method :- Diff b/w working Stress method and limit state method

Assumptions:-

(1) The material behaves in linear elastic manner

(2) Adequate safety can be ensured by restricting the stress in the material that are induced due to application of working or Service load.

(3) The 2nd assumption introduced the concept of factor of safety where is expressed as

$$FOS = \frac{(SOM)\ \text{Strength of material}}{\text{Allowable Stress in material}}$$

It is an indirect relationship between steel and concrete which is expressed in terms of modular ratio [m] is equal to St

$$m = \frac{\text{Stress in Steel}}{\text{Stress in Concrete}}$$

$$m = \frac{\text{Young's modulus of Steel}}{\text{Young's modulus of concrete}}$$

Limitations of working stress method of designing

(1) It fails to give relative importance to different types of loads.

(2) It is suitable only for large sections of designed RC Structure.

Ultimate load method :-

This method is the improvised version of traditional WSM design and it takes the short-commings of earlier method. It is also called as load factor method. or ultimate strength method.

The probleme faced due to modular ratio is entirely avoided in this method.

→ The safety factor is taken care by the concept of load factor which is expressed as

$$\text{Load factor} = \frac{\text{ultimate or design load}}{\text{working or service load}}$$

Limitations of ultimate load method :-

The major drawbacks of this method is that we cant say with 100% assurance that if a structure performs well at ultimate load the same structure will perform its function satisfactorily at service loads also.

(2) Another major drawback of this method is that the assumed non linear behaviour of concrete and steel is correct only if non linear is designed on the structure

Limit state method of design :-

unlike the other two methods that is WSM and ULM. This LSM takes into account both safety at ultimate load and serveaciability at service load.

Limit state :- It is the state of about to collapse beyond which the structure is not of any practical use. i.e, the structure will collapse will or it will become unservicable.

There are two types of limits

(1) Limit state of collapse :-

This limit state deals with the strength of the structure in terms of over turning, sliding and buckling.

Various state of collapse are

(i) Flexure

(ii) Compression

(iii) Shear

(iv) Torsion

Limit State of Serviceability :-

This is the limit state that deals with the deformation of the structure to such an extent that the structure becomes unserviceable due to excessive deflection, cracks, vibration, leakage etc.

The various limit state of serviceability are

(1) Deflection

(2) Excessive Vibration

(3) Corrossion

(4) cracking

Assumptions made in limit state method

(1) The plane section remains plane even after bending

(2) The material is homogeneous and isotrophic

(3) Maximum strain in the concrete at outer most compression fibre is taken as 0.0035 in bending.

(4) The tensile strength of concrete is ignored.

(5) The compressive strength of concrete is assumed to be 0.67 times the characteristic strength

(6) The maximum strain in the tension reinforcement in the section at failure shall not be less than

$$\frac{f_y}{1.15\,E_s} + 0.002$$

(7) The partial factor of safety for concrete is 1.5 and for steel 1.15

Characteristic Strength :-

It is defined as the value of strength of material below which not more than 5% of the result are expected to fail.

[Same defination is used for both steel and concrete]

Characteristic load :-

It is the value of load which has 95% probability of not being exceeded during the life of the structure.

Design strength :-

The factor is used to allow for the

possible difference between the material Strength obtained in the actual Structure and the characteristic Strength. It allows for Variation in workmanship or quality control in the manufacture of material and reduces the characteristic Strength to the lower value which is known as design Strength. Therefore

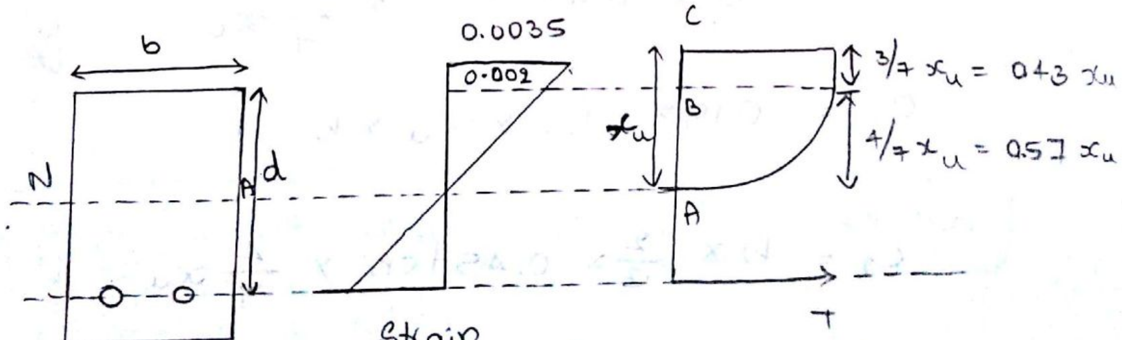
$$\text{design strength} = \frac{\text{characteristic strength}}{\text{factor of safety}}.$$

Design load :-

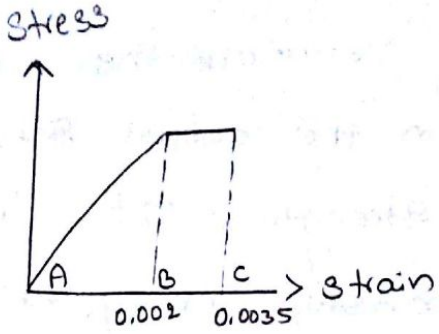
Factors are used to allow for the possible difference in the loads that may actually come on the Structure as compared to their characteristic value. Therefore d

$$\text{Design load} = \text{Characteristic load} \times \text{factor of Safety}$$

STRESS - BLOCK PARAMETERS



c/s of Beam   Strain diagram   stress diagram



$$\frac{0.67 f_{ck}}{\gamma_m}$$

Ratio of $\frac{AB}{AC} = \frac{0.002}{0.0035} = \frac{4}{7}$

Ratio of $\frac{BC}{AC} = \frac{0.0035 - 0.002}{0.0035} = \frac{3}{7}$

depth of neutral axis is represented as x_u

$\therefore$ $\boxed{AB = 0.57\ x_u}$

$\boxed{BC = 0.43\ x_u}$

Step (2) :- compressive force

C_u = width x area of Stress diagram.

$C_1 = b \times a$

$C_1 = b \times 0.45 f_{ck} \times \frac{3}{7} x_u$

$C_1 = 0.193\ f_{ck} \times x_u \times b$

$C_2 = b \times \frac{2}{3} \times 0.45 f_{ck} \times \frac{4}{7} x_u$

$= 0.171\ f_{ck} \times x_u \times b$

Total compressive force =

$$C_1 + C_2$$

$$= b \times 0.193 f_{ck} x_u + 0.171 f_{ck} x_u b$$

$$= 0.364 f_{ck} \times x_u \times b$$

Step ③ :- This total compressive force C_u acts from the distance $\bar{y}$ from top of the compression.

i.e, $$\bar{y} = \frac{C_1 y_1 + C_2 y_2}{C_1 + C_2}$$

$$y_1 = \frac{1}{2} \times \frac{3}{7} x_u \qquad y_2 = \frac{3}{7} x_u + \frac{3}{8} \times \frac{4}{7} x_u$$

$$y_1 = \frac{3}{14} x_u \qquad = \frac{3}{7} x_u + \frac{3}{14} x_u$$

$$y_2 = \frac{9}{14} x_u$$

$$\bar{y} = \frac{(0.193 f_{ck} x_u b)\left(\frac{3}{14} x_u\right) + (0.171 f_{ck} x_u b)\left(\frac{9}{14} x_u\right)}{(0.193 f_{ck} x_u b) + (0.171 f_{ck} x_u b)}$$

$$\bar{y} = \frac{0.0413 f_{ck} x_u^2 b + 0.1099 f_{ck} x_u^2 b}{0.364 f_{ck} x_u . b}$$

$$\boxed{\bar{y} = 0.415 x_u} \approx \boxed{\bar{y} = 0.42 x_u}$$

Step ④ :- Tensile force

$$T_u = f_{st} . A_{st}$$

where $f_{st} = 0.87 f_y$

$\therefore \quad T_u = f_{st} \cdot A_{st}$

$= 0.87 f_y \cdot A_{st}$

Step: 5 Actual depth of neutral axis

For any given section the location of neutral axis will be such that it satisfies the condition of equilibrium.

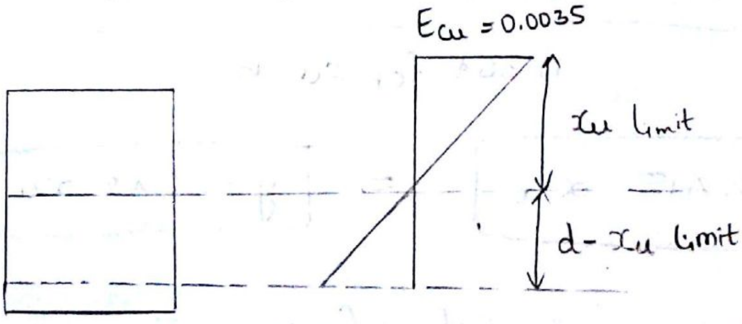
i.e, $C_u = T_u$

$$0.36 f_{ck} \cdot b \cdot x_u = 0.87 f_y \cdot A_{st}$$

$$x_u = \frac{0.87 f_y \cdot A_{st}}{0.36 f_{ck} \cdot b}$$

$$\boxed{x_u = 2.41 \frac{f_y \cdot A_{st}}{f_{ck} \cdot b}}$$

Limiting depth of neutral axis



$$E_{st} = \frac{f_y}{1.15 E_s} + 0.002$$

step ① :- $\dfrac{0.0035}{x_u\ limit} = \dfrac{\dfrac{f_y}{1.15\,E_s} + 0.002}{d - x_u\ limits}$

$$\frac{d - x_{u\,lim}}{x_u\ limit} = \frac{\dfrac{f_y}{1.15E_s} + 0.002}{0.0035}$$

$$x_{u\,lim} = \left[\frac{700}{0.87 f_y + 1100}\right] \cdot d$$

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Concept of balanced, under and over reinforced Section

Balanced Section

In balanced RC Section the percentage of Steel is Such that the Stress in the Steel becomes equal to the Stress in the concrete when beam is Subjected to load. In this kind of Sections the actual neutral axis is always will coincide with balanced neutral axis.

i.e, $\boxed{x_u = x_{u\,max}}$

Under reinforced Section

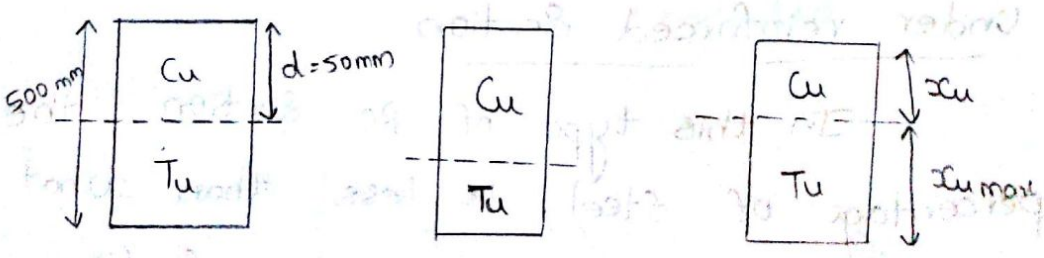
In this type of RC Section, the percentage of Steel is less than what it is required for balancing the Section.

Hence Stress in the Steel reaches first before the concrete. There fore the Structure fails by excess yielding of Steel. But before the Structure fails it will give Sufficient warning.

i.e, $\boxed{x_u < x_{u\,max}}$

Over reinforced Section

In this type of RC Section the percentage of Steel is greater than what is required for balanced Section. Hence Stress in the Concrete reaches first than Steel. Therefore, beam fails by crushing of concrete in compression zone. Hence this type of failure is Sudden and it won't give any warning before it fails. Therefore IS 456 : 2000 does not permit oven reinforced design.



Balanced   under Reinforced   Tu > Cu

This theory holds good for horizontally load transfering members.

Deflection :-

The deflection of the structure or its parts adversly effect the appearance or efficiency of the structure or finishing or partition etc, There are two types of deflection (1) Short time deflection :-

The factors affecting short time deflection are (i)

(i) magnitude and distribution of live load
(ii) Span and end conditions
(iii) Cross section properties including steel percentage.
(iv) Grade of concrete and steel

(2) Long time deflection :-

(i) The factors affecting long time deflection are :-

(i) Humidity and temperature at the time of curing.
(ii) Age of concrete at the time of loading
(iii) Shrinkage, creep, type and size of aggregate water - cement ratio, presence of composite steel etc.

According to code the total deflection due to all the loadings which includes the effect of temperature, Shrinkage and Creep [should not be greater than] $\ngtr \frac{span}{250}$.

Limiting deflection :-

$D_{LL} \leq L/360$ [LL - Live load]

$D_{TL} \leq L/240$ [TL - Total load]

✱ Cracks in structural concrete member

In RC members, cracks occurs mainly due to Shrinkage and loadings. Shrinkage cracking can be minimised by proper design of concrete mix.

→ cracking is a complex phenomena and is influenced by number of factors such as

(i) Stress in Steel
(ii) Surface characteristics of steel
(iii) Diameter and Spacing of steel
(iv) Cover to bars
(v) quality of concrete
(vi) Load distribution and rate of application
(vii) Shear Stirrups.

The code IS: 456 try to control cracks and deflection by

(i) Restricting the maximum spacing of bars

(ii) Restricting the span by depth ratio

(iii) Splendour

Splendour limits of beams for stability

The ratio of overall depth 'D' to width 'b' of rectangular beam section should generally lie within the range of 1.5 to 2. This ratio can extend upto 3 for beam carrying heavy load.

→ In designs the beam depth is often taken as $1/10^{th}$ to $1/15^{th}$ of the span for simply supported and continous beams. For cantilever beams, the ratio adopted is very less.

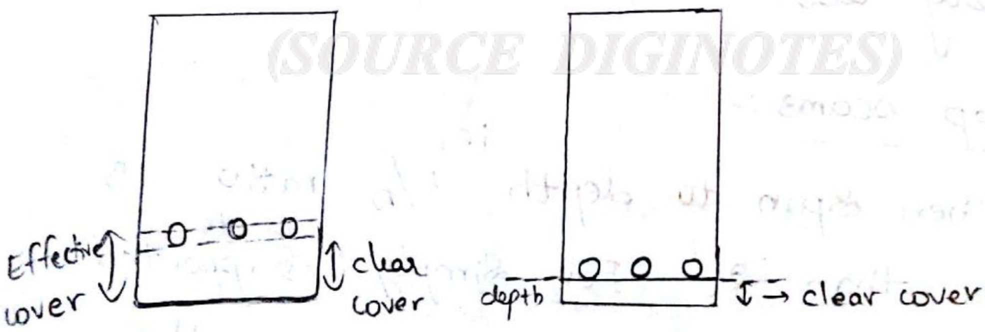
Deep beams :-

when span to depth i.e. L/D ratio is less than 2. For simply supported and 2.5 for continous beams then such beams are called as Deep beams.

Slender beams :-

when the Span of beam is very large as compared to cross section dimension of the beam then such beams are called as slender beams. There is always possibility of instability in this kind of slender beams and hence IS 456 : 2000 specifies slenderness limit to ensure lateral stability. The cleared distance between lateral restrains must be minimum of 60 b or 250 b^2/d, for simply supported and continous beams and minimum of 25 b or 100 b^2/d for cantilever beams.

Side face reinforcement



For slabs and staircase, clear cover will be 20 mm and for beam it is 30 mm, for coloumn it is 40 mm, For footing 50 mm.

Effective cover - clear cover + 1/2 of diameter of bars

Effective depth = overall depth – effective cover

Diameter of bars available in market

8 mm, 10 mm, 12 mm, 16 mm, 18 mm, 20 mm,

22 mm, 25 mm, 28 mm, 30 mm, 32 mm, 36 mm

40 mm, 45 mm, 50 mm. In this

problems :-

Calculation of short term deflection

Clause C_2 of annexure C of IS : 456 - 2000 Specifies the steps of calculating the short term deflection. [page no - 88 IS 456 : 2000]

Long term deflection

(1) Deflection due to Shrinkage - clause C_3 IS 456 : 2000 [page no - 88]

Calculation of dueflection due to creep – clause C_4 in IS 456: 2000 [page no - 89]

creep deflection due to permanent loads

Check on total deflection :-

$$\Delta_{DL+LL} + \Delta_{\text{long term}} \leq \frac{\lambda}{250}$$

$$\Delta_{LL} + \Delta_{long\ term} \leq \frac{l}{350} \text{ or } 20 \text{ mm}$$

{whichever is less}

where $\Delta_{long\ term} = \Delta_{shrinkage} + \Delta_{creep} + \Delta_{tempera}$

If these limits exceeds then the member needs to be redesigned by increasing the depth of the member.

Limits on crack width :-

Annexure - F of IS 456 : 2000 [page no 95] describes the expression for calculating the crack width.

① For a cantilever beam of span 6000 mm and cross section 200 mm x 500 mm. The details are as given below. calculate the total deflection $\Delta_{DL} = 0.025$ mm, $\Delta_{LL} = 0.0175$ mm

$\alpha_I = 0.01$ mm, $A_{st} = 125 \text{ mm}^2$, $A_{sc} = 100 \text{ mm}^2$

$\epsilon_{cs} = 0.002$, $\alpha_{i,cc} = 0.105$ mm, $\Delta_{temp} = 0.195$ mm

Sol^n $\Delta_{DL} + \Delta_{LL} = 0.025 + 0.0175$

$= 0.0425$

Δ shrinkage

$$a_{cs} = K_3 \, \psi_{cs} \, l^2$$

$$\psi_{cs} = K_4 \frac{E_{cs}}{D} \qquad P_t = \frac{100 \, A_{st}}{bd}$$

$$P_t = \frac{100 \times 125}{200 \times 500} = 0.125$$

$$P_c = \frac{100 \, A_{sc}}{bd}$$

$$P_c = 100 \times \frac{100}{200 \times 500} = 0.1$$

$$Pt - Pc = 0.125 - 0.1 = 0.025 < 1$$

$$\therefore \; K_4 = 0.72 \times \frac{Pt - P_c}{\sqrt{Pt}}$$

$$= 0.72 \times \frac{0.025}{\sqrt{0.125}}$$

$$K_4 = 0.0509$$

$$\psi_{cs} = 0.0509 \times \frac{0.002}{500}$$

$$\psi_{cs} = 2.036 \times 10^{-7}$$

$$\alpha_{cs} = K_3 \, \psi_{cs} \, l^2$$

$$= 0.5 \times 2.036 \times 10^{-7} \times (6000)^2$$

$$= 3.66 \text{ mm}$$

Δ creep :-

$\Delta_{creep} = \alpha_{cs} = \alpha_{icc} - \alpha_i$

$= 0.105 - 0.01$

$\alpha_{cs} = 0.095$

$\Delta_{long\ term} = \Delta_{shrinkage} + \Delta_{creep} + \Delta_{temp}$

$= 3.66 + 0.095 + 0.195$

$= 3.95$

$\Delta_{DL} + LL + \Delta_{long\ term}$

$0.0425 + 3.95$

$3.99 < 24$

② For a Simply Support beam of Span 8.5 m and cross Section 250 $\cancel{x}$ 600 mm. Taluculate the total deflection and other details that are asked below.

① $\Delta_{LL} = 0.15$ mm, $\Delta_{DL} = 0.30$ mm, $\Delta_{temp} = 0.025$ mm

α_i or $\Delta_i = 0.1$ m, $\alpha_{cc} = 0.05$ mm,

$A_{st} = 130\ mm^2$ $, $A_{sc} = 95\ mm^2$

Solⁿ

$\Delta_{DL} + \Delta_{LL} = 0.30 + 0.15$

$= 0.45$ mm

Δ shrinkage $a_{cs} = k_3 \psi_{cs} l^2$

$$\psi_{cs} = K_4 \frac{E_{cs}}{D}$$

$$P_t = \frac{100\ A_{st}}{bd}$$

$$= \frac{100 \times 130}{250 \times 600}$$

$$\boxed{P_t = 0.086}$$

$$P_c = \frac{100\ A_{sc}}{bd} = \frac{100 \times 95}{250 \times 600} = 0.063$$

$$\boxed{P_s = 0.064}$$

$$P_t - P_c = 0.087 - 0.064$$

$$= 0.023$$

$$\therefore \quad K_4 = 0.72 \times \frac{P_t - P_c}{\sqrt{P_t}}$$

$$= 0.72 \times \frac{0.086 - 0.064}{\sqrt{0.086}}$$

$$= 0.72 \times \frac{0.022}{0.2932}$$

$$\boxed{K_4 = 0.054}$$

$$\psi_{cs} = K_4 \times \frac{E_{cs}}{D}$$

$$= 0.054 \times \frac{0.003}{600}$$

$$\boxed{\psi_{cs} = 2.7 \times 10^{-7}}$$

$$\alpha_{cs} = K_3 \psi_{cs} l^2$$

$$= 0.125 \times 2.7\times10^{-7} \times (8500)^2$$

$$\alpha_{cs} = 2.438 \text{ mm}$$

$$\Delta_{creep} = \alpha_{cs} = \alpha_{icc} - \alpha_i$$

$$= 0.\phi5 - 0.1$$

$$= 0.05$$

$$\Delta_{long\ term} = \Delta_{Shrinkage} + \Delta_{creep} + \Delta_{temp}$$

$$= 2.438 + 0.05 + 0.025$$

$$= 2.513 \quad 2.689$$

$$\Delta_{DL} + \Delta_{LL} + \Delta_{long\ term} \leq l/250$$

$$0.45 + 2.513 \leq 8500/250$$

$$2.963 \leq 34 \quad 3.1395$$

Calculation of deflection due to ultimate Creep co-efficient

$$= \frac{\text{Creep strain}}{\text{Initial elastic strain}}$$

If creep strain and Initial elastic creep is not given then the value 'θ' will be

$\theta = 2.2 \longrightarrow 7$ days

$1.6 \longrightarrow 28$ days

$1.1 \longrightarrow 365$ days [1 year]

③ For a cantilever beam of span 5 m and cross section 200 x 450 mm. calculate the total deflection of a beam which is casted before '1' year and further details are given below

$\Delta_{LL} = \cancel{0.15 mm}$, $\Delta_{DL} = \cancel{0.30 mm}$, $\Delta_{temp} = \cancel{0.025 mm}$

α_i or $\Delta_i = \cancel{0.1 m}$, $\alpha_{cc} = \cancel{0.05 mm}$, $A_{st} = \cancel{130 mm^2}$

$A_{sc} = \cancel{95 mm^3}$

$\Delta_{DL} = 0.025$ mm, $\Delta_{LL} = 0.0175$ mm, $\alpha_i = 0.01$ mm,

$A_{st} = 125\ mm^2$, $A_{sc} = 100 mm^2$, $E_{cs} = 0.002$,

$\alpha_{icc} = 0.105$ mm, $\Delta_{temp} = 0.195$ mm

Sol^n $\Delta_{DL} + \Delta_{LL} = 0.025 + 0.0175$

$= 0.0425$

$\Delta_{shrinkage}$

$\alpha_{cs} = K_3 \psi_{cs} l^2$

$= \cancel{0.5} \times$

$\Delta_{shrinkage}$ $a_{cs} = K_3 \psi_{cs} l^2$

$$\psi_{cs} = K_4 \frac{E_{cs}}{D}$$

$$K_4 = 0.72 \times \frac{P_t - P_c}{\sqrt{P_t}}$$

$$P_t = \frac{100 A_{st}}{bd} \qquad P_c = \frac{100 A_{sc}}{bd}$$

$$= \frac{100 \times 125}{200 \times 450} \qquad = \frac{100 \times 100}{200 \times 450}$$

$$P_t = 0.138 \qquad P_c = 0.111$$

$$K_4 = 0.72 \times \frac{0.138 - 0.111}{\sqrt{0.138}}$$

$$= 0.72 \times \frac{0.027}{0.371}$$

$$K_4 = 0.0523$$

$$P_t - P_c = 0.138 - 0.111 = 0.027$$

$$\psi_{cs} = K_4 \frac{E_{cs}}{D}$$

$$= 0.0523 \times \frac{0.002}{450}$$

$$\psi_{cs} = 2.324 \times 10^{-7}$$

$$\alpha_{cs} = K_3 \psi_{cs} l^2$$

$$= 0.5 \times 2.324 \times 10^{-7} (5000)^2$$

$$\alpha_{cs} = 2.905 \text{ mm}$$

Δ_{creep} = 1.1 for 1 year

Δ_{creep} = 1.1 = θ

Δ_{temp} = 0.195

$\Delta_{long\ term} = \Delta_{shrinkage} + \Delta_{creep} + \Delta_{temp}$

$= 2.905 + 1.1 + 0.195$

$\Delta_{long\ term} = 4.2$

Total deflection $= \Delta_{LL+DL} + \Delta_{long\ term}$

$= 0.0425 + 4.2$

$= 4.2425$

<u>Limits on crackwidth</u>

Annexure - F : IS 456 : 2000 describes the expressions for calculating the crack width

① For a simply supported beam of span 6m and cross section 250 x 500. The average strain is 0.04 mm and the depth of neutral axis is exactly at the centre. The distance of the surface from the nearest longitudinal bar is 45 mm. calculate the crack width.

<u>Regarding deflection</u>

② For a simply supported beam of span 7m and cross section 300 x 600 mm. calculate the total deflection and details are as given below

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Partial factor of safety or partial safety factor

The value of partial safety factor for material and load is taken for the following reasons.

(i) possibility of deviation of strength of materials.

(ii) possibility of deviation of sectional dimension

(iii) Accuracy of calculation procedure

(iv) Unusual increase in the load

(v) Inaccurate assessment of the effects of loads

According to code partial safety factor for

Concrete - 1.5

Steel - 1.15

Dead load and live load - 1.5

(vi) The assumptions in limit state of collapse - Flexure. Refer IS 456 page No: 67

①

For a simply supported beam of span 6m and cross section 250 × 500. The average strain is 0.04 mm and the depth of neutral axis is exactly at the centre. The distance of the surface from the nearest longitudinal bar is 45 mm. calculate the crack width.

Sol^n:

Given:

$$w_{cr} = \frac{3 a_{cr} * E_m}{1 + \frac{2 (a_{cr} - C_{min})}{h - x}}$$

a_{cr} = 45 mm

E_m = 0.04

C_{min} = 30 mm for beams

h = 500 mm

x = 250 mm (Neutral axis exactly at centre)

$$w_{cr} = \frac{3 a_{cr} \cdot E_m}{1 + \frac{2 a_{cr} . E_m}{\cancel{1+} h - x}}$$

$$= \frac{3 \times 45 \times 0.04}{1 + \frac{2 (45 - 30)}{500 - 25}}$$

$$= \frac{5.4}{1.12}$$

$$\boxed{w_{cr} = 4.821 \text{ mm}}$$