R.C.C. m. 1 ②

In History of civil engineering structures reveals that the material used for construction was those which were readily available in nature. Primitive civil engineering structures were built using wooden elements, in the advancement of civil engineering knowledge and manufacturing of tools, structures were built using bricks, stones etc. Stone structures was considered as one of the best materials since, the material was homogeneous. isotropic and had good compressive and tensile strength. The major draw-back of stone structure was it must be quarried in a particular place and then transported to a particular location and then used to placed in required position. These element were very heavy and hence used to pose a big challenge to civil engineers. In the advent of various materials cement was one of the material were mixed with aggregates and mortar behaved as a plastic material which could be moulded to any shape and this materials after hardening resembled a rocky element hence, this element is referred as cement concrete was used to replace the stone elements of a civil engineering structure. Any complex civil engineering structure can be disintegrated into smaller elements and it was found that 99% of those elements were in some state of flexure/Bending. Due to this bending, bending stress develops within the material which gives rise to internal compressive and tensile stress. Concrete, when tested against external forces it was found that it had good compressive stress while the tensile strength was negligible or practically zero. To use concrete as a structural element the material had to resist the internal stresses due to bending. Hence, the strength or c/s material has to be increased. The materials in smaller quantities was introduced into concrete in the tension zone so that the tensile strength is enhanced such a material is referred as reinforced cement concrete.

· Various elements like bamboo, aluminium, copper, steel (material having good Tensile strength) was considered as reinforcing materials. Of the above material steel is considered as a ideal material to reinforce concrete. Hence the word RCC refers to cement concrete element reinforce with steel.

Basic assumptions used in design of RCC

Concrete is weak in tension and there its tensile strength is ignored. In other words, in the tensile zone the entire tension is taken by reinforcement which is in the form of steel rods.

There exists the perfect bond b/n concrete and steel

The assumption that the concrete does not take tension is not true. The fact is that concrete also take a part of tension. However since the amount of tension taken by concrete is very small and uncertain. The tensile strength of concrete is ignored in the interest of saftey.

Note: In the tension zone it is assumed that the entire tension is taken up by steel. this only means that the concrete simply remains as a binding material in tension zone

Concrete is classified into various grades depending upon its characteristic strength which is nothing but its cube crushing strength obtained after 28 days of curing. In order to determine the cube crushing strength, A cube of 150 mm size is prepared for the given concrete. The cube is cured for 28 days and then subjected to a compression test. The axial compressive force required to crush a cube is determine. For example, if the load required to crush a cube is 225 KN then the cube crushing strength of concrete after 28 day of curing

$$= \frac{\text{crushing load}}{\text{c/s area of cube}}$$

$$= \frac{250 \times 10^3}{150 \times 150}$$

$$= 10 \, N/mm^2$$

The 28 days strength of concrete is considered as the characteristic strength of concrete for all design purpose. Depending on the characteristic strength concrete is classified under various grades like M_{10}, M_{20}, M_{25}. For example M_{10} grade concrete is defined as that concrete whose cube crushing strength after curing is $10\ N/mm^2$.

As per IS 456 - 2000 the min grade of concrete to be used in RC elements is M_{20}

Density of concrete is also referred as unit weight of concrete which varies between $24\ KN/m^3$ - $25\ KN/m^3$. For RCC elements the density of concrete is taken as $25\ KN/m^3$. Since, Concrete is a very heavy material self weight of the structure cannot be ignored while designing an RC structure

Steel: The steel reinforcements used in RCC is in the form of steel rods. Steel employed in RCC is mainly of 2 types

1. Plain MS bars referred as Grade 1 steel which has an yield strength of $250\ N/mm^2$ hence it is designated as Fe 250

2. High Yield strength deform bars - usually referred as TOR steel this steel is further classified as HYSD or cold twisted deform bars

- The various grades of steel available in this category:

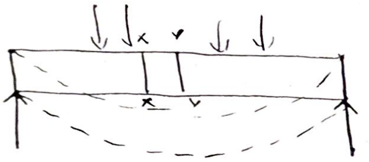
Fe 415, Fe 500, Fe 550

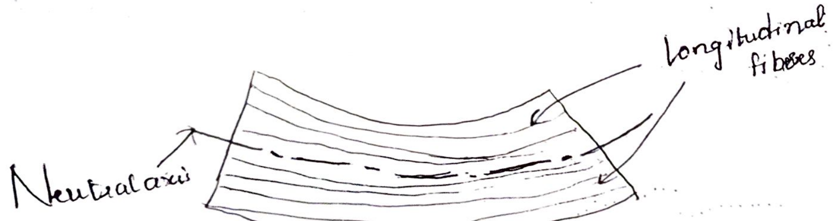
Brief revision of elementary strength of materials

Whenever a body is subjected to external forces, it develops internal resistance, which is referred as the stress. The various stresses which we encounter in practise can be broadly classified

1. Normal or Direct stresses
2. Tangential | shear stresses.

The Normal stresses can be either tensile or compressive in nature. Generally, the stress is defined as $= \frac{\text{Load}}{\text{c/s area}}$. Here we assume that the load is uniform over c/s. and we assume that load is applied gradually. Here the gradually applied load means that the intensity of load is increased from 0 to max. value of P at a constant rate





A beam is a structural element, where the load the loads act perpendicular to the longitudinal axis of the beam. due to this lateral forces the beam | member bends where in its longitudinal fibres are subjected to varying elongation and contraction as shown in fig. The fibres which are situated at the largest distances from the Neutral fibre are subjected to max amount of compression or elongation. The fibres which donot undergo any deformation are reffered as the Neutral fibre. From the

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we have

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Where M is the bending moment at the section

I is the moment of Inertia at c/s

σ intensity of the longitudinal stress in the fibre placed at distance y from the Neutral axis

E = Young modulus

R = Radius of curvature.

At a given section M and I, E and R are constants, it so

$$\frac{\sigma}{y} = \text{constant}$$

$$\sigma = (\text{constant})\, y$$

It shows that intensity of the stress varies linearly wrt to distance of fibre from the N.A. Hence stress is zero at NA and max at the extreme ends

Composite sections

The material is said to be a composite section when the c/s is made of more than one material having different modulus of elasticity. In such type of materials. The deformation in all materials remains same and the load on the section is carried by all the materials. Hence we have $P_1 + P_2 = P$ where P_1 and P_2 are the load carried by individual materials and P is the total load carried. $\Delta l_1 = \Delta L_2 = \Delta L$.

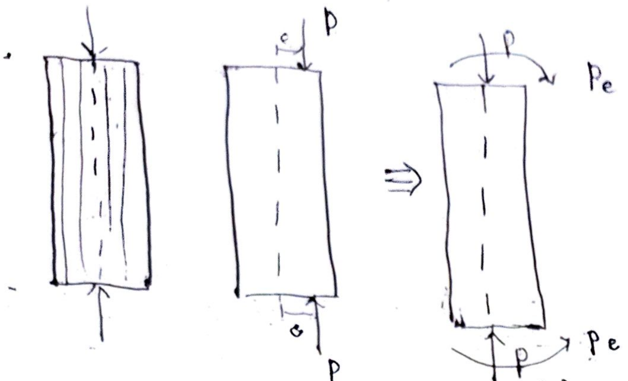
using these two conditions we have

$$\frac{\sigma_1}{\sigma_2} = \frac{E_1}{E_2} = m$$

m = modular ratio.

$$\frac{\frac{P_1}{A_1}}{\frac{P_2}{A_2}} = m \qquad \frac{P_1}{A_1} = \frac{P_2}{A_2}\, m$$

This modular ratio is used to express steel area in terms of concrete so that the location of the neutral fibre in an RCC element can be determined.



axially loaded column

(a)

(b) eccentrically loaded column

- Columns are basically compression members, these compression members are classified as long column, short column or intermediate columns.

i, Short column: An axially loaded compression member is said to be a short column when, the member tends to fail due to crushing of the material. This happens when the slenderness ratio $\left(\frac{l}{r}\right)$ is less than 50

ii, Long column: These elements even though are axially loaded fail due to bending of the element along its longitudinal axis. This condition prevails when $\frac{l}{r} > 120$ such elements are reffered as long columns and they fail due to buckling only.

iii, Intermediate column: When the slenderness ratio of the axially loaded member fall b/n 50 and 120 then the materials or the element fail due combined action of bending and crushing such elements is reffered as intermediate columns.

In case of a compression member, the compressive force may not lime along the axis of the member as shown in fig(b)

Here the compressive force is displaced from axis of the member by a distance e is referred as eccentricity and such columns are referred eccentrically loaded columns. these columns are analyzed as a member subjected to axial force p and together with a bending moment Pe Hence the stress at any fibre in an eccentrically loaded column will be

$$\sigma = \frac{P}{A} + \left(\frac{Pe}{I}\right)y$$

DESIGN OF RC ELEMENTS

Rcc elements can be designed using

1. working stress method of design
2. ultimate load method
3. limit state method

1 Working stress method of design:

- This is the oldest method of design of an element composed of any material. It is based on the assumption that even under the worst combination of the loads, the stress in the material donot exceed, the permissible values. which are usually refered as working stress or simple design stress. further these working stresses are selected in such a manner as to ensure that every material continues to remain well within its elastic limit

*** NOTE: From the above said permissible stresses or working stress we observe that a factor of saftey of a sound 3 is employed in case of concrete where as a factor of saftey of about 2 is employed incase of steel This only indicates that we are confident more on the quality of steel than concrete.

Pg - 80 ANNEX B

Design consideration of working stress method

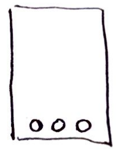
Basic assumption made WSM of design

1. C/s are plane before and after bending
2. Concrete is strong in comp. and weak in tension and entire tension is borne by steel
3. The stress-strain ~~relationship~~ dis of steel and concrete under working.
3. The stress-strain distribution across the depth of c/s is linear.
4. The modular ratio is given by $m = \dfrac{280}{3\sigma_{cbc}}$

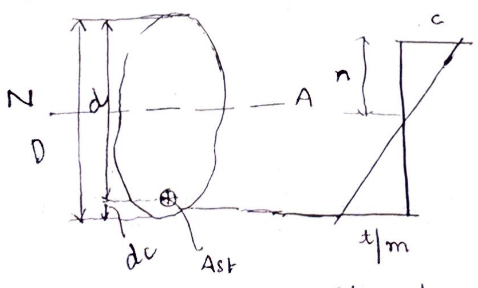
Where σ_{cbc} = permissible stress in concrete in bending compression which is given in table 21 pg-81 of IS 456-2000

- **Singly reinforced crosssection :**

There are member which are reinforced with steel in the tension zone only.



* Analysis of a singly reinforced beam using WSM of design :



Stress distribution

$$\frac{\sigma_s}{\sigma_c} = m = \frac{E_s}{E_c}$$

$$\sigma_c = \frac{\sigma_s}{m}$$

The above fig gives a C/s of a reinforced concrete beam where 'D' refers to the overall depth of the section

d refers to effective depth of the section which is distance b/n a extreme compression edge to centroid of the steel area in the tension zone d_c is effective cover

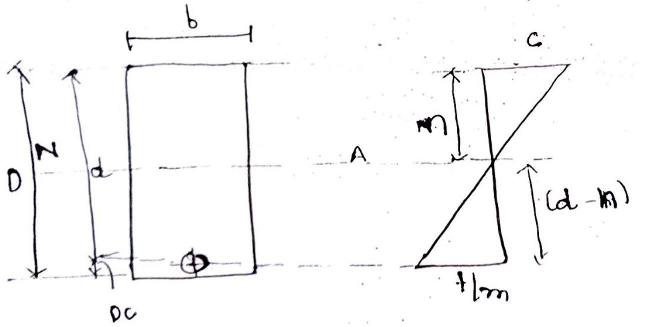
d_c = distance of the centroid of steel to concrete edge

σ_s and σ_c be the

t/m value gives the corresponding stress in the concrete at the level of tension steel.

using this condn the stress - strain diagram can be draw which is linear access the depth of C/s as shown in fig.

Analysis of single reinforced rectangular beam



Consider a rectangular beam such that b = width of the section, D = overall depth of section, d_c is effective cover, d = effective depth = D - d_c

From strain distribution diagram there is one particular point where stress is zero, and this gives us the location of N.A. Let the N.A lie at a distance N measured from the compression edge.

From the property of triangle we have

$$\frac{c}{n} = \frac{t/m}{d-n}$$

$$\boxed{\frac{mc}{t} = \frac{n}{d-n}} \quad \text{——— (1)}$$

Here C and t refer to ~~compressive~~ compressive stress in concrete and tensile stress in steel which depends on amount of steel (A_{st}) provided in the beam. However the max value of the stresses in concrete in bending compression is σ_{cbc} and in steel in tension σ_{st}. Hence $c \not> \sigma_{cbc}$

$t \not> \sigma_{st}$

If $c = \sigma_{cbc}$ and $t = \sigma_{st}$, it means that both concrete and steel attain their permissible stresses simultaneously. Such a section is referred as a balanced | ideal section. The neutral axis of such section is reffered as the critical neutral axis denoted as 'n_c'

$$\boxed{\frac{m\,\sigma_{cbc}}{\sigma_{st}} = \frac{n_c}{d - n_c}} \quad \text{——— } (2)$$

Using eq. (2) we can calculate the depth of critical neutral axis.

Here, incase of a balanced section, the stresses in conc and steel will be equal to their permissible working | design stresses. Reinforcing steel available will be avaible in form of rods of specified diameter, Hence, the c/s can't be provided with an exact value of Reinforcing area ie A_{st}, Which is required for a balanced section. If the steel area provide is less than that required for a balanced section, then such a section is required as under reinforced section. Incase, the area of steel provided is more that required for a balanced section then, the section is reffered as over reinforced section. Due to the above condition actual NA do not coincide with that of a critical neutral axis of the balanced section.

To derive the expression for the actual NA

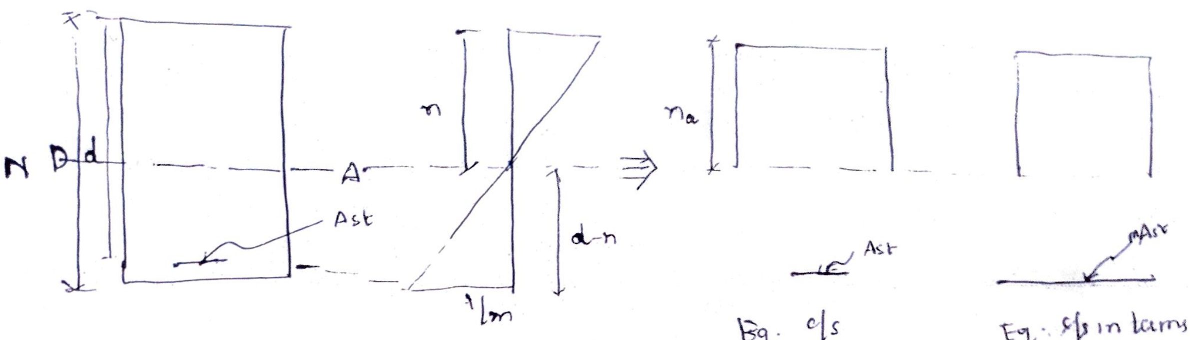


fig (a) Eq. c/s f(b) Eq. c/s in terms of concrete

Let na be the depth of the actual neutral axis considered from the compression edge

The neutral axis of any c/s passes through its centroidal point. The axis through the centroid will divide the section into two parts such that the moment of the area present on one side of the section about the centroidal axis = moment of the area present on the other side of the axis about the centroidal axis

Rcc beam composite section consisting of conc in the compression zone and steel in Tension zone. Hence to locate the centroidal or neutral axis. Steel is replaced by an equivalent area of conc. placed at the level steel only. The entire c/s is expressed in terms of concrete only as shown in fig b. Hence the centroidal location can be located using the property of moments ~~Area~~

Area of concrete in compression = ~~bxnaNA~~ bxna

Equivalent of area of tensile steel in term of concrete = m Ast

Equating the moments of compressive and tensile area about the NA we have

$$b\,na\,\frac{na}{2} = m\,A_{st}\,(d-n_a)$$

$$\boxed{\frac{b\,na^2}{2} = m\,A_{st}\,(d-n_a)} \quad \text{---} (6)$$

using eq. (2) we can locate actual NA of section

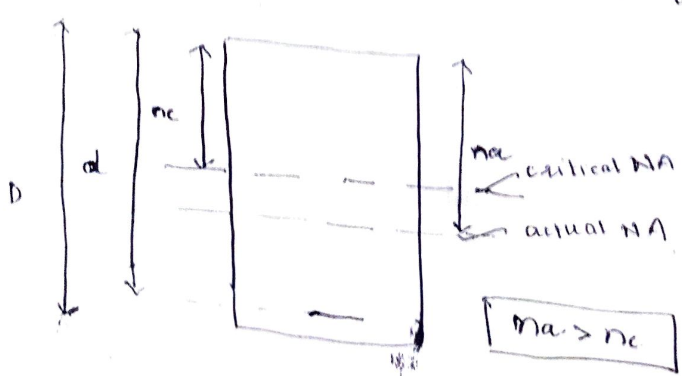
Balanced | Critical sections :

A RCC section is said to be balanced both conc and steel attain their permissible stresses simultaneously in such a section actual neutral axis and critical NA are coincident. However a balanced section is purely theoretical and can't be realized in practice. Further a balanced section and a critical section is also called the most economical section. However it is most economical in the sense that, the strength of conc and steel is fully utilized.

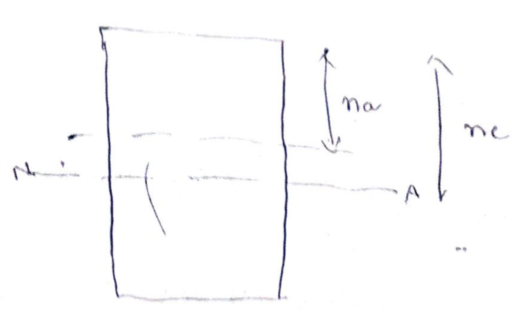
From financial point of view an under reinforced section are usually economical.

Over reinforced section :

A RCC section is said to be over reinforced when, the amount of tension steel provided in tension section is greater than that required in the balanced section. In such case, concrete attains its permissible stresses earlier than steel. The condition that gives in over reinforced section ($n_a > n_c$)



under reinforced section



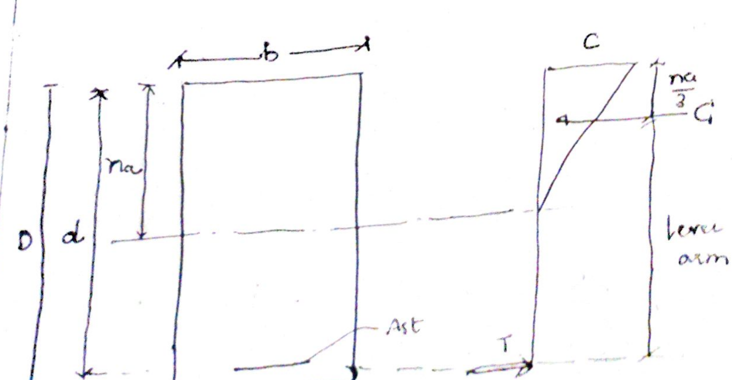
As the name implies a under reinforced section is one in which the amount of tension steel provided is less than that required for a balanced section In such a section tensile steel attain its permissible stresses earlier than concrete, that condition which provides an under reinforced section i.e na cnc · [Lastly any RC beam will either beam under reinforced or over reinforced

Moment of resistance of the section

Whenever a member is subjected to bending a certain bending moment develops across the section. The bending moment so developed can be regarded as an action on the beam. We know that at every section of the beam for every action, there should the equal and opposite reaction, as such every section is require to develop a reacting ~~cop~~ couple or resisting couple. To develop a couple, two parallel forces of equal magnitude but opp in direction is required. The distance between equal unlike parallel forces is called the lever arm. Hence in an RCC section which is subjected to bendings the total compressive force developed in the compressive zone of concrete is equal to, total tensile force located at the tension steel and hence, these two force will constitue a couple

Thus, Moment of Resistance can be defined as the resisting couple that can be developed by a given section. Here the moment of resistance of an RCC section depend on the size of the section, Amount of tension steel reinforcement and compression steel reinforcement contained in a section. The main object of determining the moment resistance of a section is to calculate the load carry capacity of a section. It will be always equal to the max. BM likely to develop across the section.

To derive the expression for moment of resistance



Consider a rectangular section being reinforced with a tensile steel.

Let

c be the compressive stress at the outermost concrete fibre.

Area of concrete under compression $= bna$

Average compressive stress $= \frac{c}{2}$

Total comp. force in conc. = Area of con section in compression

$\times$

Average compressive stress

$$= bna \times \frac{c}{2} = C$$

This compressive force lies at a distance of $\frac{na}{3}$ from compression edge

Total tensile force developed in steel $= T = t \cdot A_{st}$

Further, for equilibrium of the section

$$C = T \quad \text{(Numerically)}$$

Here C and T are two equal unlike parallel forces seperated by a dist called lever arm

Hence, the moment of this couple will give the moment of resistance of the section.

$$\text{lever arm} = d - \frac{na}{3}$$

$$\therefore MR = C(\text{lever arm}) = T(\text{lever arm})$$

$$= \frac{bnac}{2}\left(d - \frac{na}{3}\right) = t\,A_{st}\left(d - \frac{na}{3}\right)$$

15

Further, for a balanced section moment of resistance =

$$MR = \frac{b\,n_c\,\sigma_{cbc}}{2}\left(d - \frac{n_c}{3}\right) = \sigma_{st} A_{st}\left(d - \frac{n_c}{3}\right)$$

For under reinforced section $(n_a < n_c)$

$t = \sigma_{st}$

$c < \sigma_{cbc}$

$$\therefore MR = t A_{st}\left(d - \frac{n_a}{3}\right)$$

For over reinforced section $(n_a > n_c)$

$t < \sigma_{st}$

$c = \sigma_{cbc}$

$$\therefore MR = \frac{b\,n_a\,\sigma_{cbc}}{2}\left(d - \frac{n_a}{3}\right)$$

Types of failure of a RCC section

Failure due to Bending in RCC is mainly ~~cass~~ classified as ~~bit~~ brittle failure and ductile failure

* When an under reinforced beam is load compressive and Tensile stress develops in C and S. The load on beam increases gradually and stage is reached when the depth of the neutral axis (n_a). At this stage the steel would have reached it permissible limit stress σ_{st}

on further increase of the load on the beam, the compressive stress on concrete ↑ while steel being very ductile material elongates with the stress being constant. Due to ~~this~~ condn

further increases of the load these ...
stage is reached when $c = \sigma_{cbc}$. any increase in
...ding beyond the stage will lead to crushing of concrete and hence
beam fails. Such a failure of a RCC member where sufficient
...nings are provided through development of visible cracks before
...ilure is referred as ductile failure.

- When a over Reinf. Beam is loaded gradually a stage is reached when depth of neutral axis is na ($na > n\bar{a}$). at this particular stage concrete has reached its permissible stresses while the stress in tension steel is less than its permissible value. Any increase in loading beyond the stage will lead to crushing of concrete in the compression zone which ultimately results into a dissastrous collapse of the member. Such a failure is a sudden failure without any warning and is termed as Brittle failure.

- In RCC section it is preferred to have an under reinforced section such that users will have sufficient ~~warg~~ warning before failure. Hence, sufficient precaution can be taken.

Limit state method of Design: (LSM)

WSM of design was a very conservative design procedure where in the size of the members and the steel used was quite high. since, high value of FOS was considered. This led to the development ultimate load design. The ultimate load design is based on the assumption that, the structure reaches a collapse condition forming a mechanism when a certain certain load is applied which is referred as ultimate load. The ratio of ultimate load to working load is load factor. This method of design was an improvement over ~~an~~ working state method design. Members designed using this method had adequate strength to resist flexure. However, a member designed by this method was very slender hence the member used to exhibit excessive deflections and cracks which were objectionable. Hence, LSM of design was introduced which a further improvement on ultimate load design procedure. In this LASM, the structure is designed to with stand all loads that are likely to act on it in the duration of its life span and also to satisfy the serveceibility requirements like limitation of deflection and cracking. Hence limit state means, it is an acceptable limit for saftey and servecibility of the structure before failure can occur.

Important limit states considered are

i, limit state of collapse

ii, limit state of serveeibility

i. Limit state of collapse :

The design based on limit state of collapse provides necessary safety of the structure against partial or total collapse of the structure. Hence, the structure should be designed such that the resistance available at any section of the member shall not be less than the value corresponding the most unfavourable loading on the structure / beam

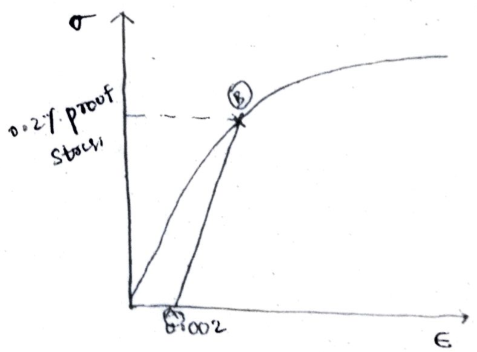
ii,

This limit state has be introduced to prevent objectionable deflection and crack

Mild steel exhibits definite yeild points. Hence the feild stress of mildsteel is considered as the Characteristic strength.

steel other than mild steel donot exhibit perfect yeild location. Hence the concept of proof stress is introduced.

Proof stress is that stress in the material at which a material undergoes a specified amount of permanent strain. Hence 0.2% proof stress means it is that stress in steel at which steel undergo permanent strain of 0.2% (0.002)



Tension test is conducted on the steel specimen and stress strain curve is plotted in the figure. consider point A on ox-ordinate = 0.002 strain From A Draw a straight parallel intial straight line portion of the stress strain curve to meet the curve at B The stress corresponding to B is refered as 0.2% proof stress. These stress is considered as yeild stress of the material

Characteristic load means, it is the value of the which has 95% probability of not being exceeded during the lifespan of the structure. The characteristic load for dead loads or the selfweight of the material is specified in IS 875 part I The char. load for live load (imposed load), wind loads are specified in IS 875 part II, III. Seismic loads as specified in IS 1893-2002.

the loads and the material properties to arrive at the designed values
The partial saftey factor for limit state of collapse is given below.

a Material | strength of material

The design strength of the material is given by the relation

$$f_d = \frac{f}{\gamma_m}$$

Where

f_d = design strength

f = Characteristic strength

γ_m = partial saftey factor for the material

for concrete $f = f_{ck}$, $\gamma_m = 1.5$

for steel $f = f_y$, $\gamma_m = 1.15$

b Loads (γ_L)

Partial saftey factor for loads depends on various load combination considered on the structure

Load combination	Design loads
D.L + L.L	1.5 (D.L + L.L)
DL + LL ± EL	1.5 DL + 1.2 LL ± 1.5 EL
DL ± EL	1.5 (DL ± EL)
	0.9 DL ± 1.5 EL
DL + WL	1.5 (DL + WL)
DL + LL + WL	1.5 DL + 1.2 LL + 1.5 WL

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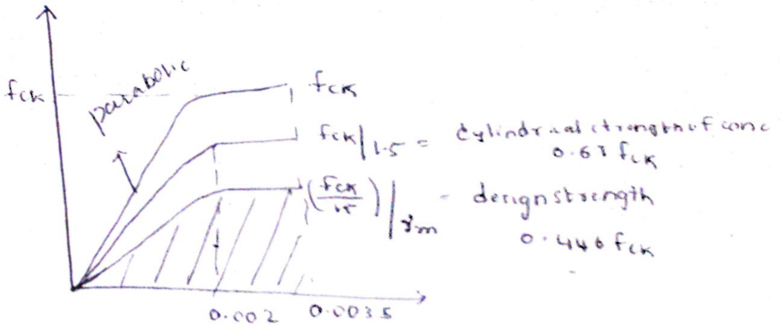
Assumptions made in limit state method of collapse :

1. Plane sections remains plane before and after bending.

2. The maximum compressive strain in concrete at the outermost compression fiber in bending is taken as 0.0035

3. The ultimate strength of concrete is considered as 0.0035 irrespective of grade of concrete

4. The relation ship between compressive stress and strain in concrete may be assumed to rectangle, trapezoidal, parabolic or any other shape such that it results in the prediction of strength with substantial agreement with test results



5. Strain is considered to varying linearly across the section

Tensile stress of concrete is ignored. Hence entire tension is taken by steel

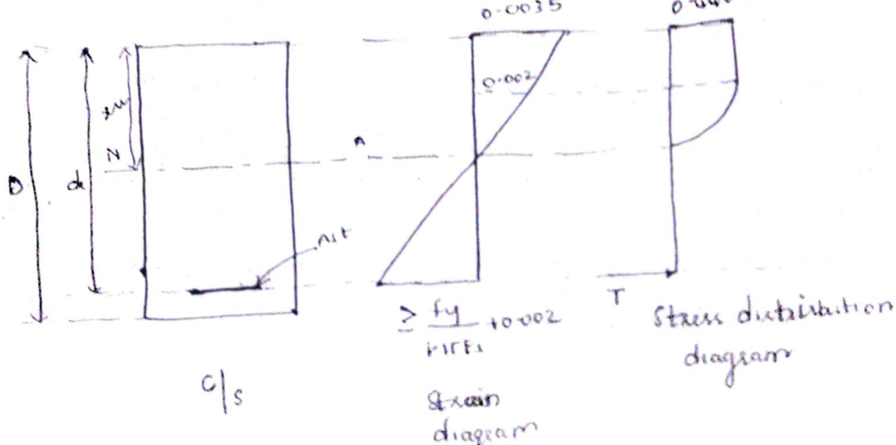
7. The partial saftey factor for conc and steel is considered as 1.5 and 1.15 respectively

8. The maximum strain in tensile reinforcement in the section at failure shall not be less than $\frac{f_y}{1.15 E_s} + 0.002$

By ensuring the strain is more than the specified value, it means that steel reaches yield stress earlier than concrete attains it. permissible stress. Hence the section usually be under reinforced section.

- Evaluate the stress block parameters of a rectangular RC section subjected to flexure.

- Consider a rectangular beam subjected to a bending moment M and shear force 'V'. Having following particulars as shown in fig.



In the above σ - ε diagrams the strain in conc is 0.0035 and strain in steel is $\frac{f_y}{1.15 E_s} + 0.002$. This gives us ideal section for limit state of collapse of design.

Let x be the depth of Neutral axis measured from the extreme compression fibre.

From the strain dia we have

$$\frac{0.0035}{x_u} = \frac{\frac{f_y}{1.15 E_s} + 0.002}{d - x_u}$$

Here Young's modulus remains constant of all grades of steel

$= 2 \times 10^5 \ N/mm^2$

$$(0.0035)(d - x_u) = \left(\frac{f_y}{1.15 E_s} + 0.002\right) x_u$$

$$0.0035d - 0.0035x_u = \left(\frac{f_y}{1.15 \times 2 \times 10^5} + 0.002\right) x_u$$

$$0.0035d = \left(\frac{f_y}{1.15 \times 2 \times 10^5} + 0.0055\right) x_u$$

$$x_u = \left(\frac{0.0035}{\frac{0.87 f_y}{2 \times 10^5} + 0.0055}\right) d \longrightarrow (1)$$

Hence, the value of x_u depends on the strain in steel which is in denominator in eq - (1), which means that with any increase in the strain in the steel beyond $\frac{f_y}{1.15E_s} + 0.002$ will tend to decrease the depth of the neutral axis. Therefore x_u will be max. if the strain in the steel is less than $\frac{f_y}{1.15E_s} + 0.002$ then this x_u is referred as the depth of limiting neutral axis designated as $X_{u\,max}$

From eq: 1

for Fe 250 $f_y = 250\ N/mm^2$

$X_{u\,max} = 0.53\ d$

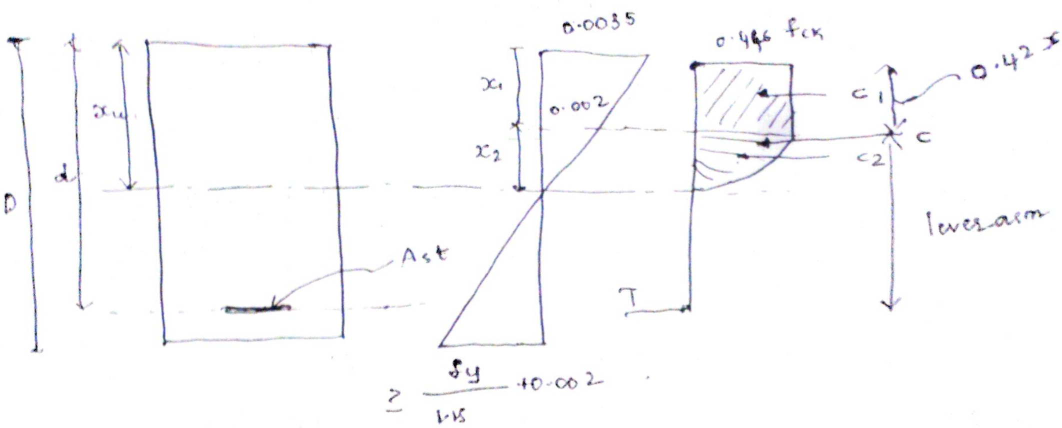
for Fe415 $f_y = 415\ N/mm^2$

$x_{u\,max} = 0.48\ d$

for Fe 500 $f_y = 500\ N/mm^2$

$x_{u\,max} = 0.46\ d$

Derive the stress block parameters



1)

Consider a rectangular RCC beam reinforced as shown in figure

From strain dia. we have

$\frac{1}{E_s} + 0.002$

refore

$- + 0.002$

ral axis

$$\frac{0.0035}{x_u} = \frac{0.002}{x_2}$$

$$x_2 = \frac{0.002}{0.0035} x_u$$

$$x_2 = \frac{4}{7} x_u$$

$$x_1 + x_2 = x_u$$

$$x_1 = \frac{3}{7} x_u$$

Let C_1 and C_2 be compressive forces in the section due to rectangular and parabolic stress block respectively

Let C be total compressive force in the section and T be the total tensile force in tension steel

$$C_1 = (0.446 f_{ck} x_1) b$$

$$= 0.446 f_{ck} \frac{3}{7} b x_u$$

$$C_2 = \frac{2}{3}(0.446 f_{ck} x_2) b$$

$$C_2 = 0.446 \times \frac{2}{3} f_{ck} \times \frac{4}{7} b x_u$$

Further we have the total compression force $= C = C_1 + C_2$

$$= 0.446 f_{ck} \frac{3}{7} b x_u + 0.446 \times \frac{2}{3} \times \frac{4}{7} f_{ck} b x_u$$

$$C = 0.36 f_{ck} b x_u$$

Taking moments of C, C_1, C_2 about the extreme compression edge

$$C(\bar{x}) = C_1\left(\frac{x_1}{2}\right) + C_2\left(x_1 + \frac{3}{8} x_2\right)$$

$$0.36 \cancel{f_{ck}} \cancel{b x_u} (\bar{x}) = 0.446 \cancel{f_{ck}} \frac{3}{7} \cancel{b x_u} \left(\frac{3}{7} \frac{x_u}{2}\right) + 0.446 \cancel{f_{ck}} \frac{2}{3} \times \frac{4}{7} \cancel{b x_u} \left(\frac{3}{7} x_u + \frac{3}{8} \times \frac{4}{7} x_u\right)$$

$$0.36 \bar{x} = 0.446 \times \frac{3}{7} \times \frac{3}{7} \times \frac{x_u}{2} + 0.446 \times \frac{2}{3} \times \frac{4}{7}$$

$$\bar{x} = 0.418 x_u \approx 0.42 x_u$$

For equilibrium of the section total compressive force = total tensile force

$$0.36 f_{ck} b x_u = \frac{f_y}{1.15} (A_{st})$$

$$0.36 f_{ck} b x_u = 0.87 f_y (A_{st})$$

$$\boxed{\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b d}} \quad (1)$$

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Th.. - This is as per IS 456 - 2000 Annexure - G equation A

ge. x_u page 96

: using the above equation we can calculate or determine the depth of actual neutral axis for the given section.

Incase of URS steel attains its permissible limit earlier than concrete hence in the above section C and T ~~and~~ are two equal unlike parallel forces which constitue a couple. The moment of this couple is referred as moment of resistance. and will be equal to externally applied ~~beam~~ BM a/s the section

$$\therefore MR = T(\text{lever arm})$$

$$\cancel{x_u}\left(\frac{3}{7}x_u + \frac{3}{8}\times\frac{4}{7}x_u\right)$$

$$M_u = 0.87 f_y A_{st}(d - 0.42 x_u)$$

$$= 0.87 f_y A_{st}\, d\left[1 - 0.42\frac{x_u}{d}\right]$$

$$\frac{2}{3}\times\frac{4}{7}$$

$$= 0.87 f_y A_{st}\, d\left[1 - 0.42\frac{0.87 f_y A_{st}}{0.36 f_{ck} bd}\right]$$

as per equation

$$= 0.87 f_y A_{st} d\left[1 - \frac{A_{st}}{bd\, f_{ck}}\right] \quad \text{(II)}$$

=

as per LSM of design for collapse when Ast is increased x_u also increases. how ever since over reinforced design is not allowed in this design. The max. amount of Ast allowed such that $x_u = x_{umax}$. Hence $x_u \not> x_{umax}$

If $x_u = x_{umax}$, then the resulting section will be a critical section where in the resultant moment of resistance is referred as limiting BM which means that the max. BM that a beam support / carry as a singly reinforced section.

When $x_u = x_{umax}$ then $M_u = M_{umax}$

$$M_{u\,limit} = C \text{ (lever arm)}$$

$$= 0.36 f_{ck} b x_u (d - 0.42 x_u)$$

$$= 0.36 f_{ck} b x_{umax} (d - 0.42 x_{umax})$$

$$= 0.36 f_{ck} b \frac{x_{umax}}{d} \left(1 - 0.42 \frac{x_{umax}}{d}\right) d^2$$

$$M_{u\,limit} = 0.36 f_{ck} b \left(\frac{x_{umax}}{d}\right) d^2 \left(1 - 0.42 \left(\frac{x_{umax}}{d}\right)\right)$$

This is as per eq. c annexure G pg. 96 of IS 456 : 2000

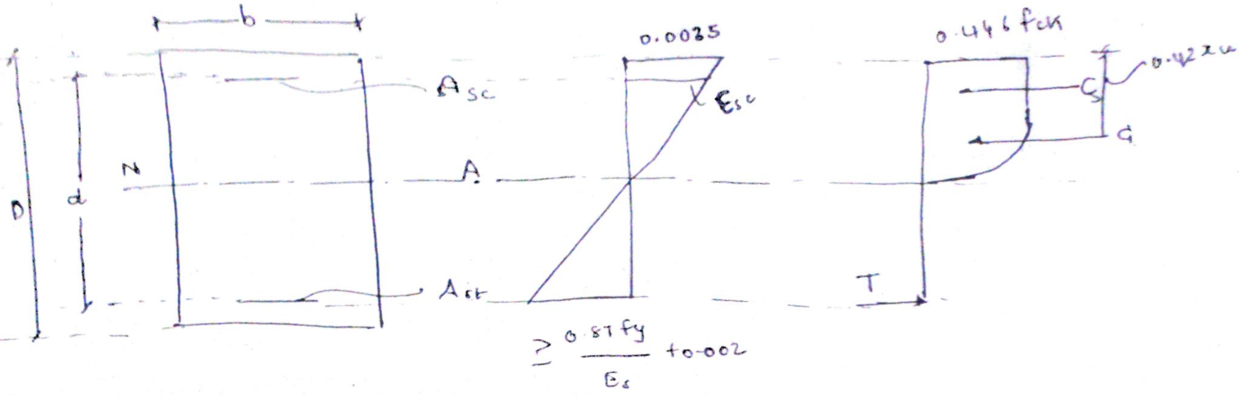
The above equation gives the maximum B.M the beam can carry as singly reinforced beam.

<u>Analysis of Doubly reinforced beam</u>

A doubly reinforced section [illegible] provided when [illegible] limit the size of beam section. The size of the section may be limited to reduce the selfweight of the structure. Further, the size of the section can also be limited due to asthetic point of view or to provide sufficient head room

XX Note : Head room is the minimum vertical distance required from the bottom or soffit of the beam upto the floor, so that space can be utilized effectively

Here, the size of the beam means that it is the depth of the beam under consideration



Consider a DRB with tensile and comp. reinforcement as shown in Fig. Let 'C' be the total compressive force in concrete

C_s = total compression force in compression reinforcement

T = total tensile force in tension reinforcement. Since the section behaves as a composite section and due to the composite action, the strain of comp. reinf. will be equal to the strain of concrete at the level comp. steel ϵ_{sc}.

From the ~~strai~~ strain distribution diagram we have

$$\frac{0.0035}{x_u} = \frac{\epsilon_{sc}}{x_u - d_c}$$

$$\epsilon_{sc} = \left(\frac{0.0035}{x_u}\right)(x_u - d_c)$$

Further the stress in compression steel is $f_{sc} = \epsilon_{sc} E_s$

Where
E_s = Young's modulus / modulus of elasticity of steel

$= 2 \times 10^5 \ N/mm^2$

However $f_{sc} \not> 0.87 f_y$

Further for equilibrium of the section. Total compressive force = Total comp tensile force

$$C + C_s = T$$

$$0.36\, f_{ck}\, b\, x_u + f_{sc}\, A_{sc} = 0.87\, f_y\, A_{st} \quad \text{— (1)}$$

using this equation we can determine the location of actual neutral axis

However $x_u \not> x_{u\,max}$

Here, the moment of resistance of the section will be equal to

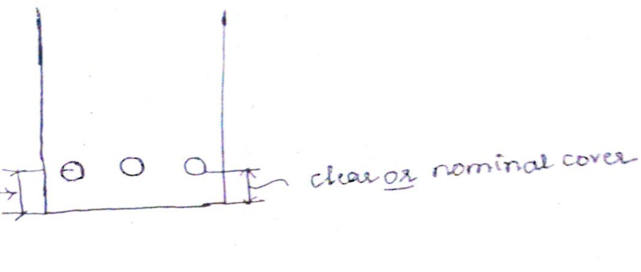
$$MR = C\,(\text{lever arm}) + C_s\,(\text{lever arm})$$

$$M_u = 0.36\, f_{ck}\, b\, x_u\,(d - 0.42\, x_u) + f_{sc}\, A_{sc}\,(d - d_c)$$

• Steel Reinforcements are provided in the form of rods of specified diameters. The diameter available and their characteristics are shown in the following table

dia	Area (mm²)	W/m (kgs)
6	27	
8	50	0.24
10	78.5	0.6
12	113	0.9
16	201	1.6
20	314	2.5
25	490.9	3.87

Effective cover :



- Nominal cover is the clear distance measured from the steel to the outermost fibre of concrete. Nominal cover has to provided so that steel is embedded in concrete so that it develops a perfect bond. Further the nominal cover also helps to protect steel against the possible degradation due to the environmental effects. The nominal cover to be considered depends on the type of exposure of the section. It is given in table 16. pg. 47 of IS 456 : 2000. The nominal cover values varies b/n 20 – 75 mm depending of the exposure

- Effective cover is the distance of the centroid ~~from~~ the of the steel area to the extreme fiber. This value is used to calculate the effective depth of the beams

- Problems on Analysis of RC beams :

- steps to be followed for the analysis of SRB

~~Step~~
Step 1 : Calculate area of tension reinforcement Ast and the effective depth of the beam 'd'

Step 2 : Calculate the depth of limiting neutral axis x_{umax}

Step 3 : Calculate the depth of actual neutral axis using eq. A of IS 456 pg 96

Step 4 : If $x_u < x_{umax}$ it means that the section is under reinforced. Hence calculate the moment of resistance of the section using equation B of pg: 96 IS 456

Step 5 : If $x_u > x_{umax}$ then ...

and calculate the moment of resistance of the section using eq (c) of IS 456 : 2000 pg : 96

Step : 6 : Calculate the working B.M i.e $M = \frac{M_u}{1.5}$

Step 7 : Equate the working B.M to max. BM on the beam to calculate maximum permissible or safe load the beam can carry

- Steps to be followed for DRB

Step 1 : Calculate A_{st}, A_{sc}, d, x_{umax}

Step 2 : Calculate the depth of actual neutral axis and check the stress in compression steel and such that $f_{sc} \ngtr 0.87 f_y$

Step 3 : Calculate the moment of resistance of the section

Step 4 : Calculate the max. permissible load that the beam can carry safely

1. A beam 230 x 450 mm in section is reinforced with 2 bars of 25 mm and 1 bar of 16 mm dia. Which is placed with a clear cover of 25 mm. Calculate the maximum permissible UDL the beam can carry safely over an effective span of 4.8 m. Use M20 concrete and Fe 415 steel

Sol Given data

$b = 230$ mm

$D = 450$ mm

$A_{st} = 2 - 25\phi + 1 - 16\phi$

Clear cover = 25 mm

$l_{eff} = 4.8$ m

M20 concrete Fe 415

** Note : unless and otherwise specified assume the effective cover d_c as 40 mm for both tension and compression reinforcement.

$$d_c = 25 + \frac{25}{2} = 37.5 \text{ mm}$$

$\therefore$ d = effective depth = $D - d_c$

$= 450 - 37.5$

$= 412.5$ mm

$A_{st} = 1182.8 \text{ mm}^2$

For Fe 415 steel

$x_{umax} = 0.48\,d$

$= 0.48 \times 412.5$

$= 198$ mm

To determine depth of actual neutral axis

As per IS 456 : 2000 equation A page 96 we have

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck}\, b \cdot d}$$

$$x_u = \frac{0.87 \times 415 \times 1182.8}{0.36 \times 20 \times 230}$$

$= 257.88$ mm $> x_{u,max}$

Hence the section is over reinforced

$\therefore$ $x_u \not> x_{umax}$

$\boxed{\therefore\ x_u = x_{u,max}}$

As per IS 456 : 2000 pg 96 equation c we have

$$M_{u.limit} = \frac{0.36\ x_{u\,max}}{d}\left[1 - \frac{0.42\,x_{u\,max}}{d}\right] b d^2 f_{ck}$$

$$= \frac{0.36 \times 198}{412.5}\left[1 - \frac{0.42 \times 198}{412.5}\right] 230 \times 412.5^2 \times 20$$

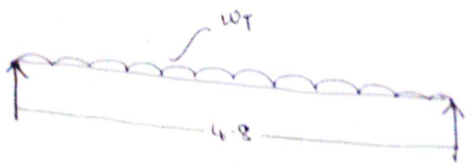
$$= 107.98 \times 10^6 \text{ N.mm}$$

Working bending moment

$$B.M = \frac{M_{u.limit}}{1.5}$$

$$= 71.9 \times 10^6 \text{ N mm}$$

$$= 71.9 \text{ kN.m}$$



$$B.M = \frac{w_r\, l_{eff}^{\,2}}{8}$$

$$71.9 = \frac{w_r\, 4.8^2}{8}$$

$$w_r = 24.96 \text{ kN/m}$$

Self weight of the beam per meter length = volume of the beam per meter length x density of concrete

$W_s = (0.23) \times (0.45) \times (1) \times 25$

$= 2.6 \; KN/m$

$\therefore$ applied UDL $= W_L = W_T - W_s$

$= 24.96 - 2.6$

$W_L = 22.36 \; KN/m$

Hence, the beam can carry safely an imposed udL (live load) of intensity 22.4 KN/m

* Note : Here W_T is reffered as all inclusive UDL

(2) A beam 230 x 750 mm in section is reinforced with 2 bars of 16 mm dia and 1 bar of 12 mm dia calculate the maximum permissible central point load the beam can carry safely over an effective span of 4 m

$b = 230 \, mm$

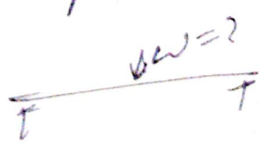
$D = 750 \, mm$

$A_{st} = 2 \times 16\phi + 1 \times 12\phi$

$L_{eff} = 4m$

max BM

$M_{max} = \frac{WL}{4}$



Note ; Unless otherwise specification specified use M_{20} concrete and Fe 415 steel

$f_{ck} = 20 N/mm^2$

$f_y = 415 N/mm^2$

$^2 f_{ck}$

$2.5^2 \times 20$

beam

$$d_c = 20 + \frac{16}{2} = 28\,mm$$

$$d_o = 20 + \frac{16}{2} = 28\ mm$$

Nominal cover as per IS 456 : 2000 pg - 47
Exposure is mild

$$d = 750 - 28 = 722\ mm$$

$$x_{umax} = 0.48\,d$$
$$= 346.56\ mm$$

To determine depth of neutral axis

As per IS : 456 : 2000 pg : 96 equation (a) we have

$$\frac{x_u}{d} = \frac{0.8\, f_y\, A_{st}}{0.36\, f_{ck}\, b \cdot d} \qquad A_{st} = 515\ mm^2$$

$$= \frac{0.8 \times 415 \times 515}{0.36 \times 20 \times 230 \times 722}$$

$$x_u = 112.28 < x_{umax}$$ Hence the beam is under reinforced

To calculate Moment of Resistance

$$M_u = 0.87\, f_y\, A_{st}\, d\left[1 - \frac{A_{st}\, f_y}{b d\, f_{ck}}\right]$$

$$= 0.87 \times 415 \times 515 \times 722\left[1 - \frac{515 \times 415}{230 \times 722 \times 20}\right]$$

$$= 125.6 \times 10^6\ N\text{-}mm$$

$$= 125.6\ KN\text{-}m$$

$$BM = \frac{M_u}{1.5} = 83.73\ KN\text{-}m$$

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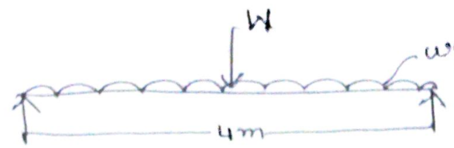


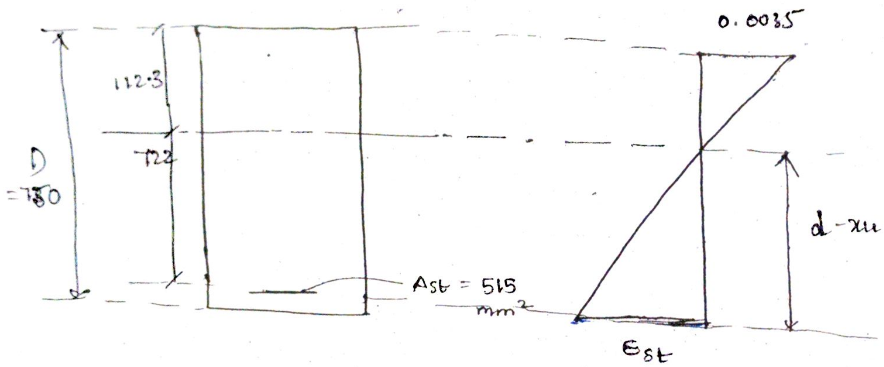
$$BM_{max} = \frac{w_g l^2}{8} + \frac{Wl}{4}$$

$$w_g = (0.23)(0.75)(25)$$
$$= 4.31\ kN/m$$

$$83.73 = \frac{4.31 \times 4^2}{8} + \frac{W(4)}{4}$$

$$\boxed{W = 75.15\ kN}$$

For the above c/s determine the strains in the material.

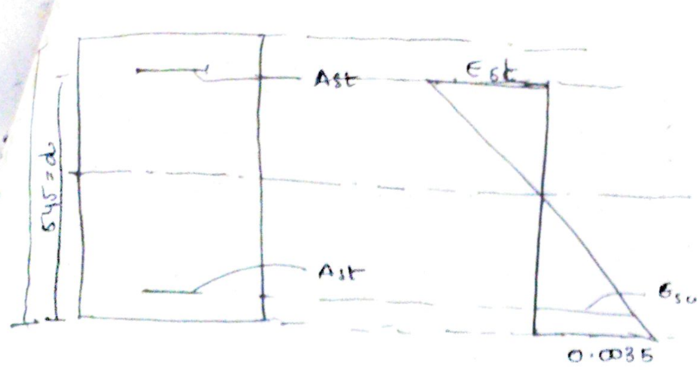


$$\boxed{\frac{0.0035}{x_u} = \frac{\epsilon_{st}}{d - x_u}}$$

$$\frac{0.0035}{112.28} = \frac{\epsilon_{st}}{(722 - 112.28)}$$

$$\epsilon_{st} = 0.019 > \frac{0.87 f_y}{E_s} + 0.002$$

(3) An RCC beam 250 x 800 mm in section is reinforced with two bars of 16 mm ϕ as compression reinforcement and 4 bars of 20 mm ϕ as tensile reinforcement. Calculate the maximum intensity of udl the beam can carry safely over a cantilevered span of 2.2 m. Assume exposure level as severe. Use M20 concrete Fe 500 grade steel



$b = 250$ mm

$D = 600$ mm

$d_c = 45 + \frac{20}{2}$

$= 55$ mm

$\frac{\epsilon_{sc}}{x_u - d_c} = \frac{0.0035}{x_u}$

$\epsilon_{sc} = \frac{0.0035}{x_u}(x_u - d_c)$.

As per IS 456 : 2000 pg 47 for severe exposure Nominal cover = 45

$d = 600 - 55$

$= 545$ mm

$A_{st} = 4 \times 20\phi = 1256\ mm^2$

$A_{sc} = 2 \times 16\phi = 402\ mm^2$

$f_{ck} = 20\ N/mm^2$ $\quad f_y = 500\ N/mm^2$

$x_{umax} = 0.46\ d$

$= 0.46 \times 545$

$= 250.7$ mm

- To locate the neutral axis

total tension = total compression

$0.36 f_{ck} b x_u + f_{sc} A_{sc} = 0.87 f_y A_{st}$

$0.36 f_{ck} b x_u + (\epsilon_{sc} E_s) A_{sc} = 0.87 f_y A_{st}$

$$0.36 f_{ck} b x_u + \frac{0.0035(x_u - 55)}{x_u} A_{sc} = 0.87 f_y A_{st}$$

$$0.36(20) \times 250 \times x_u^2 + 0.0035(x_u - 55)(402) E_s = 0.87 \times 500 \times 1256\, x_u$$

$$1800 x_u^2 - 264960\, x_u - 15.47 \times 10^6 = 0$$

$$x_u = 191.96 \text{ mm}$$

$$x_u < x_{u\,max}$$

c).

Check the value of f_{sc}

cover = 45

$$f_{sc} = \frac{0.0035(x_u - 55)}{x_u} E_s$$

$$f_{sc} = 499.45 \text{ N/mm}^2 > 0.87 f_y$$

Hence $f_{sc} = 0.87 f_y$

Hence we calculate the depth of actual neutral axis by taking $f_{sc} = 0.87 f_y$

$$0.36 f_{ck} b x_u + 0.87 f_y A_{sc} = 0.87 f_y A_{st}$$

$$x_u = 206 \text{ mm} < x_{u\,max}$$

- To calculate the M.R of the section

$$M_u = C(\text{lever arm}) + C_s(\text{lever arm}) \qquad f_{sc} = 0.87 \times f_y$$

$$= 0.36 f_{ck} b x_u (d - 0.42 x_u) + f_{sc} A_{sc}(d - d_c)$$

$$0.36(20) \times (250) \times 206 \,(545 - (0.42 \times 206)) + 0.87(500)(402)(545 -$$

$$= 255.7 \times 10^6 \text{ N mm}$$

$$255.7 \text{ kN m}$$

working B.M. $= \frac{M_u}{1.5}$

$$= \frac{255.7}{1.5} = 170.46 \text{ kN m}$$