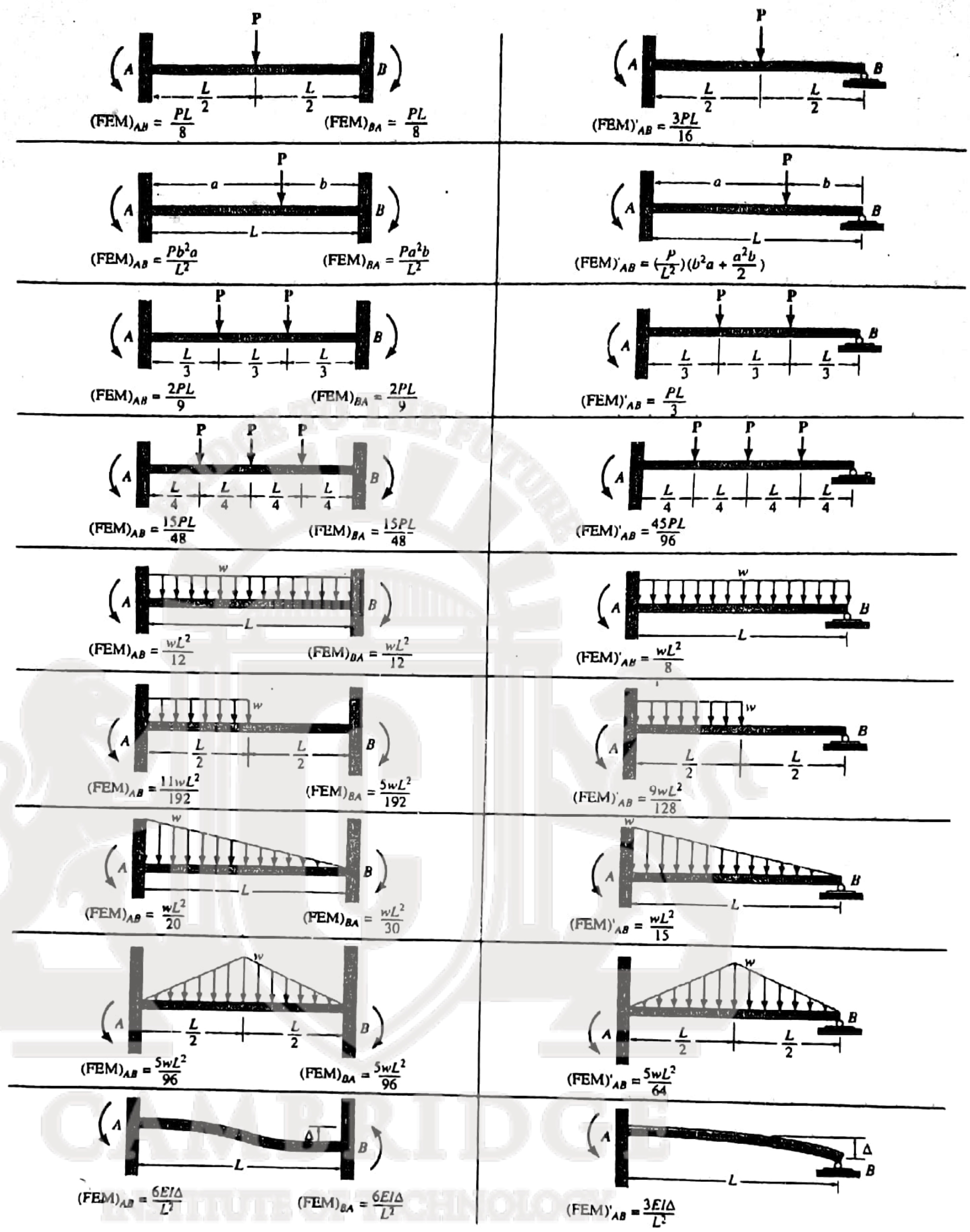
Fixed End Moments



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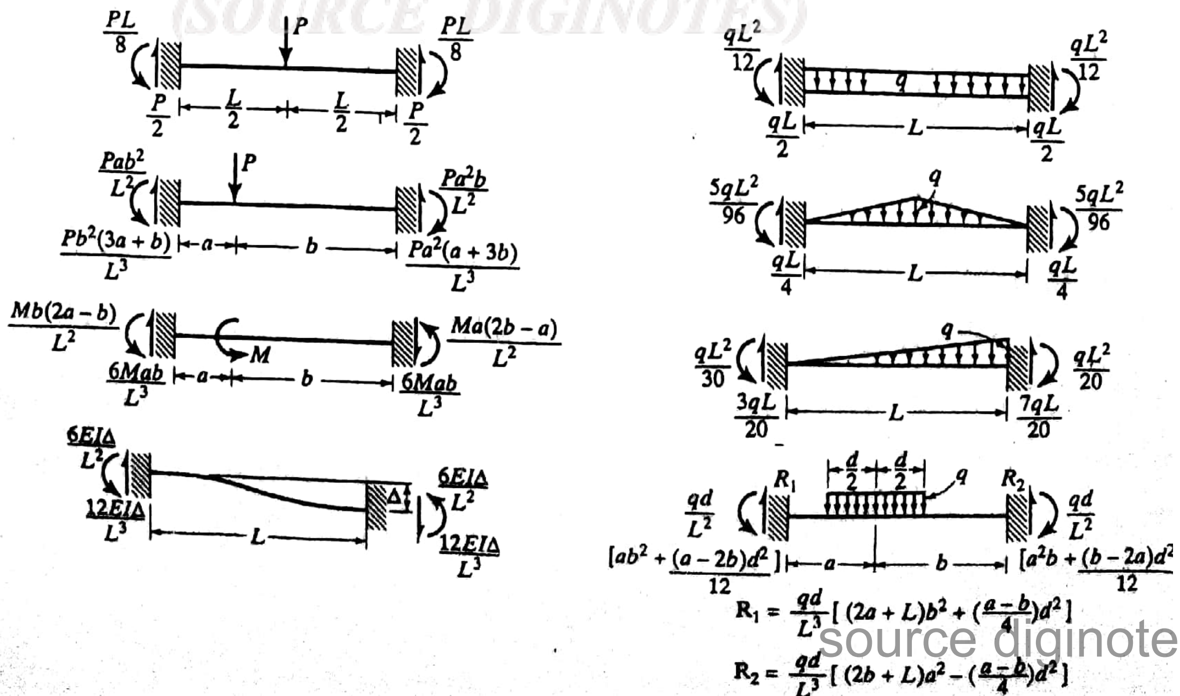

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Determinate structure :-

The structure which can be analysed i.e the support reaction, bending moment & shear force can be determined with the equations of static equillibrium only are known as Determinate structure.

Ex :-

*  ⇒ Simply supported beam.

*  ⇒ Cantilever beam.

Indeterminate structure :-

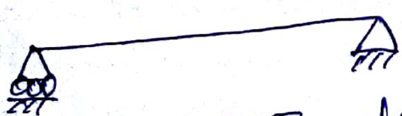
The structure which cannot be analysed i.e if the support reactions, bending moment & shear forces can't be determined using the equations of static equillibrium are called Indeterminate structures.

Ex :-  ⇒ fixed beam.

Degree of static Indeterminacy :-

The number of eq^ns required over and above the equations of static equillibrium for the analysis of structure is known as Degree of indeterminacy.

No. of reactions = 3



DOI = No. of reactions − eq^ns of equilibrium

⇒ 3−3 = 0 //.

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Degree of kinematic Indeterminacy :-

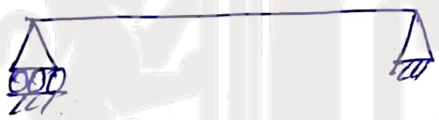
Degree of kinematic indeterminacy is equal to the degree of freedom. The number of independent displacement & rotations at the supports is known as degree of freedom.

Ex :-

*

DOF = 4

*



DOF = 3

*

DOF = 0

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Module-1

SLOPE DEFLECTION METHOD

Introduction :-

* In this method all the joints are considered to be rigid.
* Because they are rigid, angles b/w members at the joints are considered not to change.
* This method uses a classical approch based on equillibrium.
* A serious of simultaneous equations, each expressing the relation b/w moments acting at the ends of the members are written in terms of slope and deflection.
* The rotations at the joints are considered as unknowns.
* The solution of the slope deflection equation along with the equilibrium equations gives the values of the unknowns.
* Knowing these rotations, the end moments are calculated using the slope-deflection equations.
* Here fixed end moments are to be known.

Sign Conventions :-

* Clockwise moments and rotations at the ends of the member are considered positive.

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* Anticlockwise moments & rotations at the ends of the members are considered negative.
* Upward deflection or displacement of right end relative to the left end producing clockwise fixed end moments are considered positive.

Developement of Slope deflection equations :-

Consider a beam AB hinged at ends A & B and subjected to external loading as shown in the figure. Due to the external loading ~~as shown~~ the beam will be bent and the ends will rotate.

Let us now apply end moments M_{FAB} & M_{FBA} at the ends A & B of such magnitude that the slopes θ_A & θ_B due to the external loading are reduced to zero. Such moments which keeps the slope at zero at the ends is called fixed end moments.

The fixed end moments can be calculated from the standard fixed beam formulae for a given system of loading on the beam.

Let 'B' deflect by an amount δ w.r.to 'A'

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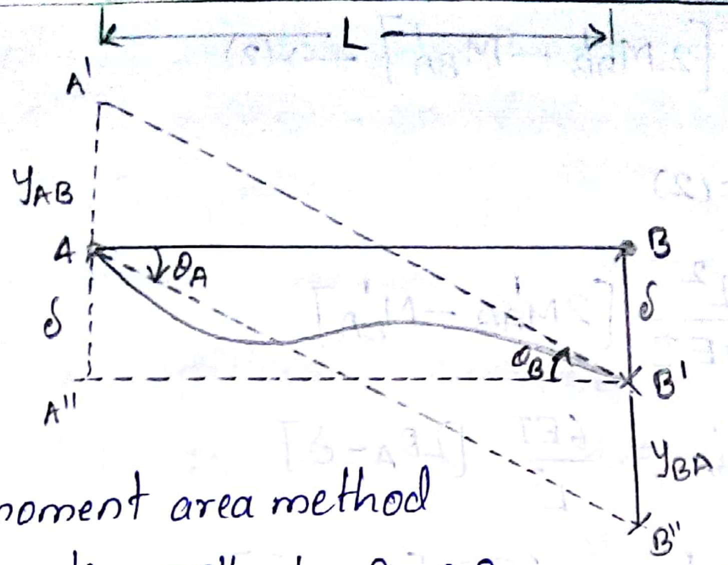


From moment area method

W.K.T Δ^{le} ABB'', $\tan\theta_A \simeq \theta_A$

$$\theta_A = \frac{BB''}{AB}$$

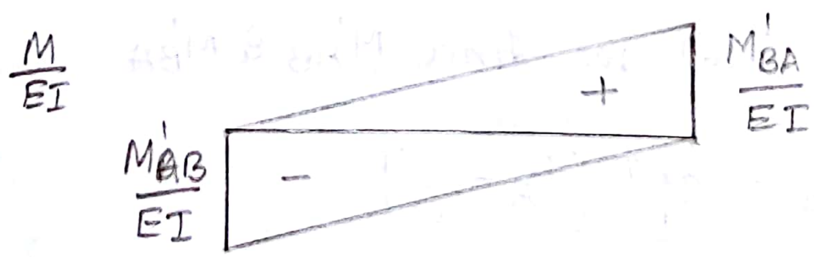
$$\theta_A L = y_{BA} + \delta$$

$$y_{BA} = L\theta_A - \delta \rightarrow (1)$$

According to 2nd moment area theorem.

$$y_{BA} = \bar{x}\int_B^A \frac{M}{EI}\,dx$$

Here an additional moment M_{AB}' & M_{BA}' are introduced at A & B respectively. Hence the $\frac{M}{EI}$ diagram for the addition moments can be written as.



$$y_{BA} = -\left[\frac{1}{2} \times L \times \frac{M_{BA}'}{EI}\right]\left(\frac{L}{3}\right) + \left[\frac{1}{2} \times L \times \frac{M_{AB}'}{EI}\right]\left(\frac{2L}{3}\right)$$

$$y_{BA} = \frac{L^2 M_{AB}'}{3EI} - \frac{L^2 M_{BA}'}{6EI}$$

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$$y_{BA} = \frac{L^2}{6EI}\left[2M'_{AB} - M'_{BA}\right] \longrightarrow (2)$$

W.K.T eq^n (1) = (2)

$$L\theta_A - \delta = \frac{L^2}{6EI}\left[2M'_{AB} - M'_{BA}\right]$$

$$2M'_{AB} - M'_{BA} = \frac{6EI}{L^2}\left[L\theta_A - \delta\right]$$

$$2M'_{AB} - M'_{BA} = \frac{6EI}{L}\left[\theta_A - \frac{\delta}{L}\right] \longrightarrow (3)$$

Similarly to find y_{AB}, W.K.T $y_{AB} = \bar{x}\int_A^B \frac{M}{EI}\,dx$

$$y_{AB} = L\theta_B - \delta \quad [\text{from } \Delta^{le}\ BA'A''] \longrightarrow (4)$$

$$y_{AB} = \left[\frac{1}{2} \times L \times \frac{M'_{AB}}{EI}\right]\left(\frac{L}{3}\right) + \left[\frac{1}{2} \times L \times \frac{M'_{BA}}{EI}\right]\left(\frac{2L}{3}\right)$$

$$y_{AB} = \frac{L^2}{6EI}\left[2M'_{BA} - M'_{AB}\right] \longrightarrow (5)$$

eq^n (4) = (5)

$$L\theta_B - \delta = \left[2M'_{BA} - M'_{AB}\right]\frac{L^2}{6EI}$$

$$2M'_{BA} - M'_{AB} = \frac{6EI}{L}\left[\theta_B - \frac{\delta}{L}\right] \longrightarrow (6)$$

Using eq^{ns} (3) & (6) we find M'_{AB} & M'_{BA}

$$2M'_{AB} - M'_{BA} = \frac{6EI}{L}\left[\theta_A - \frac{\delta}{L}\right]$$

$$2M'_{BA} - M'_{AB} = \frac{6EI}{L}\left[\theta_B - \frac{\delta}{L}\right] \div \text{by } 2$$

$$\frac{3}{2}M'_{AB} = \frac{6EI}{L}\left[\theta_A - \frac{\delta}{L}\right] + \frac{3EI}{L}\left[\theta_B - \frac{\delta}{L}\right]$$

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$$M'_{AB} = \frac{2EI}{L}\left[2\theta_A + \theta_B - \frac{3\delta}{L}\right]$$

$$M'_{BA} = \frac{2EI}{L}\left[2\theta_B + \theta_A - \frac{3\delta}{L}\right]$$ //.

Thus the additional moments M'_{AB} & M'_{BA} are known

Therefore the final moments $M_{AB} = M_{FAB} + M'_{AB}$

$M_{BA} = M_{FBA} + M'_{BA}$ //.

Analysis of contineous beams with settlement :-

A contineous beam is essentially a statically indeterminate structure which must satisfy both the conditions of geometry as well as statical equillibrium.

Steps in solving the problems :-

* Find the kinematic indeterminacy and the degrees of freedom.
* Divide the beam into parts based on the number of joints.
* Consider each part to be a fixed beam and find the fixed end moments.
* Using the slope deflection equations find M'_{AB} & M'_{BA} etc by considering equilibrium condition at the joints [becoz joints are considered to be riged]
* find the final moments by knowing θ_A & θ_B from step-4.

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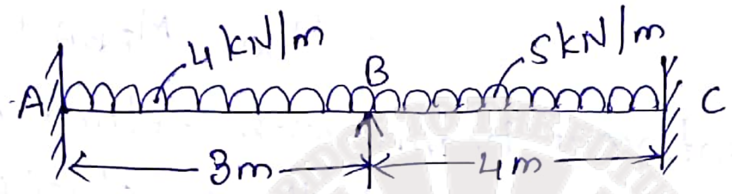
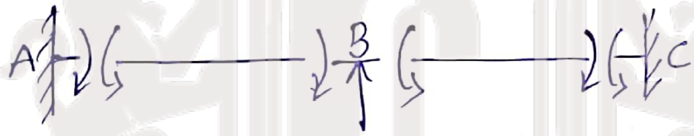
1. A contineous beam ABC consists of spans AB=3m and BC=4m the ends A & C being fixed. AB & BC carry UDL of intensity 4kN/m and 5kN/m respectively. Find the support moments & draw the bending moment diagram for the beam. The beam is of uniform cross section througout



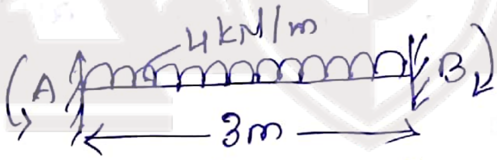
Step-1:- DKI = DOF = 2

Step-2:- Divide the beam in to parts.



Step-3:- ~~S~~ fixed end moments

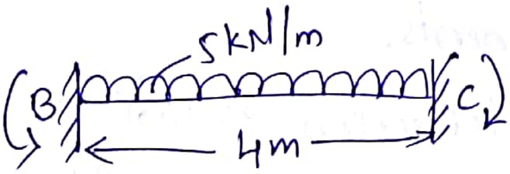
* AB



$$M_{FAB} = -\frac{WL^2}{12} = -\frac{4\times 3^2}{12} = -3\ kN\text{-}m$$

$$M_{FBA} = \frac{WL^2}{12} = \frac{4\times 3^2}{12} = 3\ kN\text{-}m$$

* BC



$$M_{FBC} = -\frac{WL^2}{12} = -\frac{5\times 4^2}{12} = -6.67\ kN\text{-}m$$

$$M_{FCB} = \frac{WL^2}{12} = \frac{5\times 4^2}{12} = 6.67\ kN\text{-}m$$

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Step-4:- Slope deflection equations.

Since joints A & C are fixed $\theta_A = \theta_C = 0$, $\delta = 0$

* $M'_{AB} = \frac{2EI}{L}\left[2\cancel{\theta_A}^{0} + \theta_B - \frac{3\cancel{\delta}^{0}}{L}\right]$

$$\boxed{M'_{AB} = \frac{2}{3}EI\theta_B}$$

* $M'_{BA} = \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_A}^{0} - \frac{3\cancel{\delta}^{0}}{L}\right]$

$$\boxed{M'_{BA} = \frac{4}{3}EI\theta_B}$$

* $M'_{BC} = \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_C}^{0} - \frac{3\cancel{\delta}^{0}}{L}\right]$

$$\boxed{M'_{BC} = EI\theta_B}$$

* $M'_{CB} = \frac{2EI}{L}\left[2\cancel{\theta_C}^{0} + \theta_B - \frac{3\cancel{\delta}^{0}}{L}\right]$

$$\boxed{M'_{CB} = \frac{EI\theta_B}{2}}$$

Step-5:- $M_{AB} = M_{FAB} + M'_{AB}$

$$= -3 + \frac{2}{3}EI\theta_B$$

$$M_{BA} = M_{FBA} + M'_{BA}$$

$$= 3 + \frac{4}{3}EI\theta_B$$

$$M_{BC} = M_{FBC} + M'_{BC}$$

$$= -6.67 + EI\theta_B$$

$$M_{CB} = M_{FCB} + M'_{CB}$$

$$= 6.67 + \frac{EI\theta_B}{2}$$

Considering the joints 'B' to be in equilibrium

$$M_{BA} + M_{BC} = 0$$

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$$3 + \frac{4}{3} EI\theta_B - 6.67 + EI\theta_B = 0$$

$$\boxed{EI\theta_B = 1.575}$$

Step-6:- Final moments.

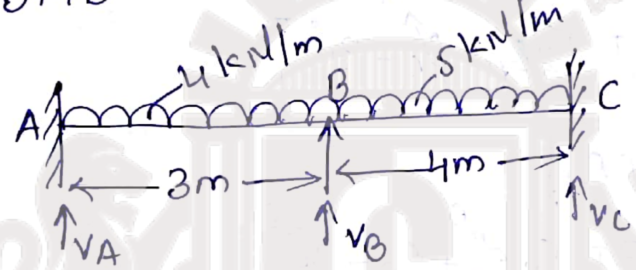
$$M_{AB} = -3 + \frac{2}{3} EI\theta_B = -1.95 \text{ kN-m}$$

$$M_{BA} = 3 + \frac{4}{3} EI\theta_B = 5.1 \text{ kN-m}$$

$$M_{BC} = -6.67 + EI\theta_B = -5.1 \text{ kN-m}$$

$$M_{CB} = 6.67 + \frac{EI\theta_B}{2} = 7.45 \text{ kN-m.}$$

Step-7:- BMD



$\Sigma V = 0,$ $\quad V_A + V_B + V_C = 32 \longrightarrow (1)$

$M_B = 5.1$ kN-m

$\Rightarrow -5.1 = V_A \times 3 - 4 \times 1.5 \times 3 - 1.95$

$$\boxed{V_A = 4.95 \text{ kN}}$$

$\Rightarrow 5.1 = -V_C \times 4 + 5 \times 4 \times 2 + 7.45$

$$\boxed{V_C = 10.59 \text{ kN}}$$

$$\boxed{V_B = 16.46 \text{ kN}}$$

* $BM @ AB = 4.95 \times 1.5 - 4 \times 1.5 \times \frac{1.5}{2} - 1.95$

$= 0.972$ kN-m

$BM @ BC = -10.59 \times 2 + 5 \times 2 \times 1 + 7.45$

$= 3.73$ kN-m

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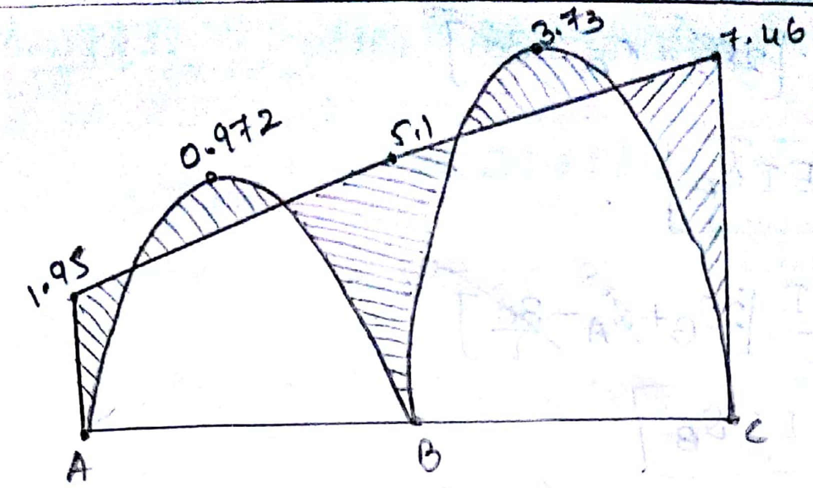
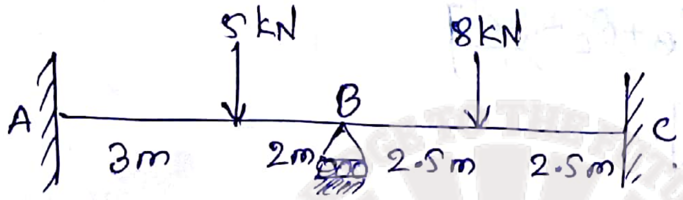
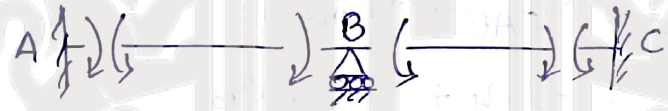


2.



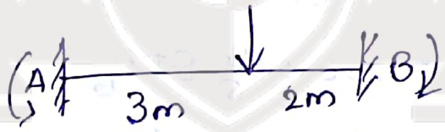
Step-1 :- DKI = DOF = 2

Step-2 :- Divide the beam in to parts



Step-3 :- FEM's

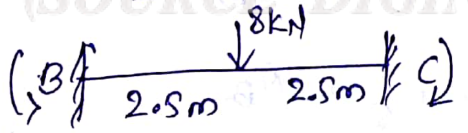
* Span AB



$$M_{FAB} = -\frac{Pb^2a}{L^2} = \frac{5 \times 2^2 \times 3}{5^2} = -2.4 \text{ kN-m}$$

$$M_{FBA} = \frac{Pa^2b}{L^2} = \frac{5 \times 3^2 \times 2}{5^2} = 3.6 \text{ kN-m}$$

* BC



$$M_{FBC} = -\frac{PL}{8} = -\frac{8 \times 5}{8} = -5 \text{ kN-m}$$

$$M_{FCB} = \frac{PL}{8} = \frac{8 \times 5}{8} = 5 \text{ kN-m}$$

Step-4 :- Slope deflection equations

Since joints A & C are fixed θ_A & $\theta_C = 0$

$\delta = 0$

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* $M'_{AB} = \frac{2EI}{L}\left[2\cancel{\theta_A}^{\to 0} + \theta_B - \cancel{\frac{3\delta}{L}}^{\to 0}\right]$

$$\boxed{M'_{AB} = \frac{2}{5} EI\theta_B}$$

* $M'_{BA} = \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_A}^{\to 0} - \cancel{\frac{3\delta}{L}}^{\to 0}\right]$

$$\boxed{M'_{BA} = \frac{4}{5} EI\theta_B}$$

* $M'_{BC} = \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_C}^{\to 0} - \cancel{\frac{3\delta}{L}}^{\to 0}\right]$

$$\boxed{M'_{BC} = \frac{4}{5} EI\theta_B}$$

* $M'_{CB} = \frac{2EI}{L}\left[2\cancel{\theta_C}^{\to 0} + \theta_B - \cancel{\frac{3\delta}{L}}^{\to 0}\right]$

$$\boxed{M'_{CB} = \frac{2}{5} EI\theta_B}$$

Step-5:- $M_{AB} = M_{FAB} + M'_{AB}$

$= -2.4 + \frac{2}{5} EI\theta_B$

$M_{BA} = M_{FBA} + M'_{BA}$

$= 3.6 + \frac{4}{5} EI\theta_B$

$M_{BC} = M_{FBC} + M'_{BC}$

$= -5 + \frac{4}{5} EI\theta_B$

$M_{CB} = M_{FCB} + M'_{CB}$

$= 5 + \frac{2}{5} EI\theta_B$

Considering the joint B to be in equilibrium

$M_{BA} + M_{BC} = 0$

$3.6 + \frac{4}{5} EI\theta_B - 5 + \frac{4}{5} EI\theta_B = 0$

$$\boxed{EI\theta_B = 0.875}$$

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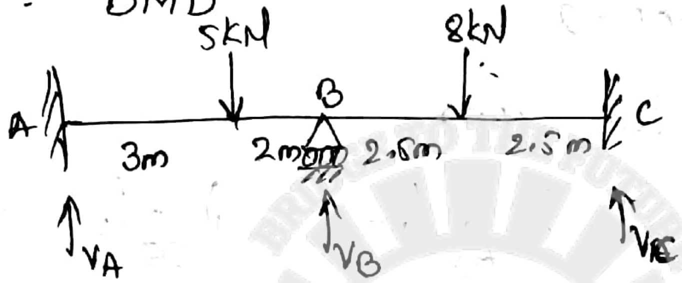
Step-6:- $M_{AB} = -2.4 + \frac{2}{5}(0.875) = -2.05\ kN\text{-}m$

$$M_{BA} = 3.6 + \frac{4}{5}(0.875) = 4.3\ kN\text{-}m$$

$$M_{BC} = -5 + \frac{4}{5}(0.875) = -4.3\ kN\text{-}m$$

$$M_{CB} = 5 + \frac{2}{5}(0.875) = 5.35\ kN\text{-}m.$$

Step-7:- BMD



$$V_A + V_B + V_C = 13\ kN \longrightarrow (1)$$

$M_B = -4.3\ kN$

$\Rightarrow V_A \times 5 - 5 \times 2 - 2.05 + 4.3 = 0$

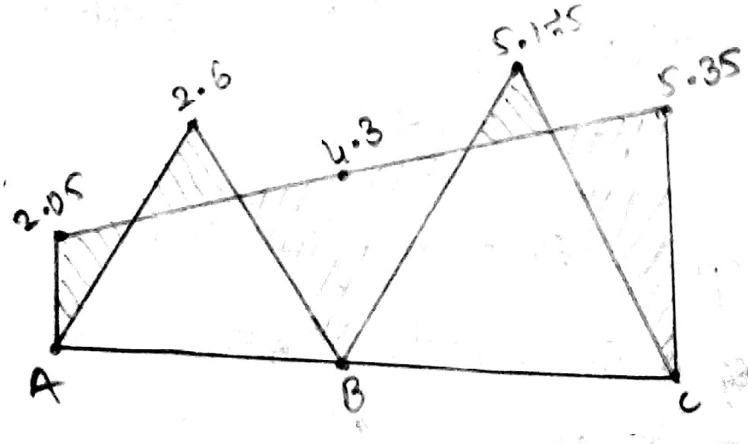
$\boxed{V_A = 1.55\ kN}$

$\Rightarrow -V_C \times 5 + 8 \times 2.5 - 4.3 + 5.35 = 0$

$\boxed{V_C = 4.21\ kN}$

$\boxed{V_B = 7.24\ kN}$

* BM @ AB = $1.55 \times 3 - 2.05 = 2.6\ kN\text{-}m$
* BM @ BC = $4.21 \times 2.5 - 5.35 = 5.175\ kN\text{-}m$



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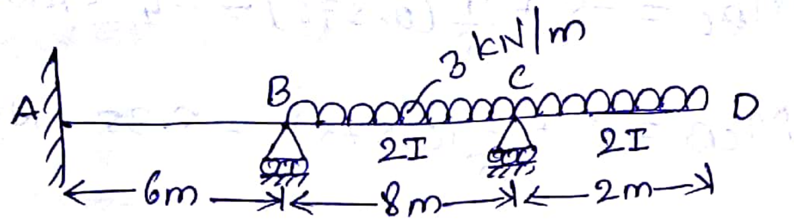
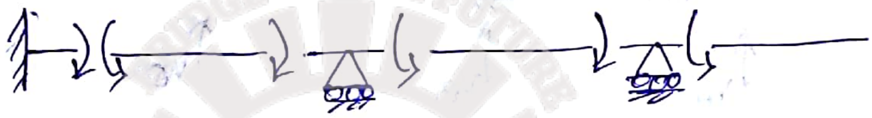
3. For the beam shown in the fig. determine support reactions for all the members using slope-deflection method also draw the BM diagram.



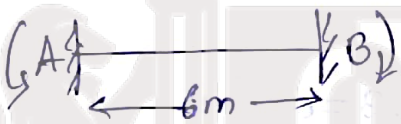
Step-1 :- DKI = DOF = 2

Step-2 :- Divide the beam



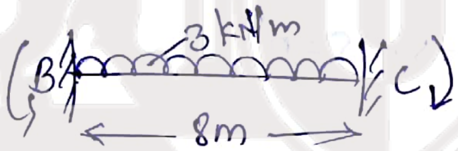
Step-3 :- FEM's

* AB



$M_{FAB} = M_{FBA} = 0$

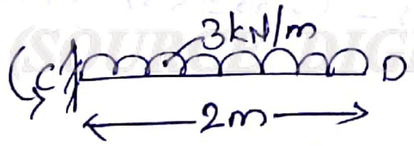
* BC



$$M_{FBC} = -\frac{WL^2}{12} = -\frac{3\times 8^2}{12} = -16 \text{ kN-m}$$

$$M_{FCB} = \frac{WL^2}{12} = \frac{3\times 8^2}{12} = 16 \text{ kN-m}$$

* CD



$$M_{FCD} = M_{CD} = -\frac{WL^2}{2} = -\frac{3\times 2^2}{2} = -6 \text{ kN-m}$$

$$M_{FDC} = M_{DC} = 0 \text{ kN-m}$$

Step-4 :- Slope deflection equations

Since A is fixed, $\theta_A = 0$, $\delta = 0$

$$* \; M'_{AB} = \frac{2EI}{L}\left[2\cancel{\theta_A}^{0} + \theta_B - \cancel{\frac{3\delta}{L}}^{0}\right]$$

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$$\boxed{M'_{AB} = \frac{1}{3} EI\theta_B}$$

* $M'_{BA} = \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_A}^{\,0} - \cancel{\frac{3\delta}{L}}^{\,0}\right]$

$$\boxed{M'_{BA} = \frac{2}{3} EI\theta_B}$$

* $M'_{BC} = \frac{2EI}{L}\left[2\theta_B + \theta_C - \cancel{\frac{3\delta}{L}}^{\,0}\right]$

$$\boxed{M'_{BC} = \frac{EI\theta_B}{2} + \frac{EI\theta_C}{4}}$$

* $M'_{CB} = \frac{2EI}{L}\left[2\theta_C + \theta_B - \frac{3\delta}{L}\right]$

$$\boxed{M'_{CB} = \frac{EI\theta_C}{2} + \frac{EI\theta_B}{4}}$$

Step-5 :-

$M_{AB} = \frac{1}{3} EI\theta_B$

$M_{BA} = \frac{2}{3} EI\theta_B$

$M_{BC} = -16 + \frac{EI\theta_B}{2} + \frac{EI\theta_C}{4}$

$M_{CB} = 16 + \frac{EI\theta_B}{4} + \frac{EI\theta_C}{2}$

$M_{CD} = -6$ ~~kN~~

$M_{DC} = 0$

Considering the joint B ~~&C~~ to be in equillibrium.

$\therefore M_{BA} + M_{BC} = 0$

$$\frac{2}{3} EI\theta_B + \frac{E(2I)\theta_B}{2} + \frac{E(2I)\theta_C}{4} - 16 = 0$$

$$\frac{5}{3} EI\theta_B + \frac{1}{2} EI\theta_C = 16 \longrightarrow (1)$$

$\therefore M_{CB} + M_{CD} = 0$

$$16 + \frac{E(2I)\theta_C}{2} + \frac{E(2I)\theta_B}{4} = 6$$

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$$EI\theta_C + \frac{EI\theta_B}{2} = -10 \rightarrow (2)$$

$\boxed{EI\theta_B = 14.89}$ $\boxed{EI\theta_C = -17.44}$

Step-6:- $M_{AB} = 4.96$ kN-m

$M_{BA} = 9.92$ kN-m

$M_{BC} = -9.92$ kN-m

$M_{CB} = +6$ kN-m

$M_{CD} = -6$ kN-m

$M_{DC} = 0$ kN-m

Step-7:- BMD

$V_A + V_B + V_C = 30$

$M_B = 9.92$ kN-m

$\Rightarrow V_A \times 6 + 4.96 + 9.9 = 0$

$\boxed{V_A = -2.46 kN}$

$M_C = 6$ kN-m

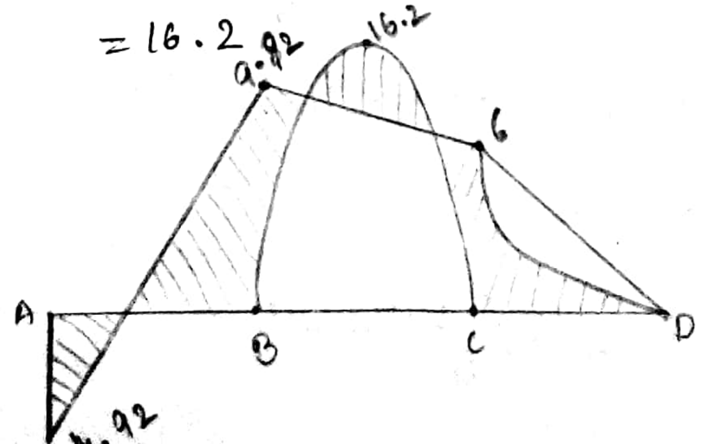
$\Rightarrow -2.46 \times 14 + V_B \times 8 + 4.96 - 3 \times 8 \times 4 + 6 = 0$

$\boxed{V_B = 14.95 kN}$

$\boxed{V_C = 17.52 kN}$

* BM @ AB = 0
* BM @ BC = $-2.46 \times 10 + 4.96 + 14.96 \times 4 - 3 \times 4 \times 2$ $\cancel{=0}$

$= 16.2$



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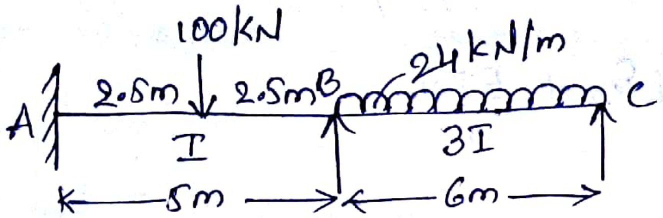
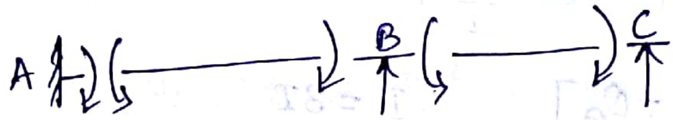
4. Analyse the contineous beam as shown in the figure.



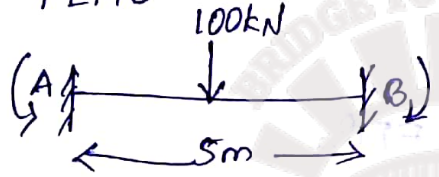
Step-1 :- DKI = 2 = DOF

Step-2 :- divide the beam



Step-3 :- FEM's

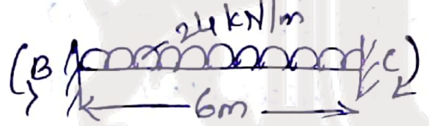
* AB



$$M_{FAB} = \frac{-PL}{8} = \frac{-100\times5}{8} = -62.5\,kN\text{-}m$$

$$M_{FBA} = \frac{PL}{8} = \frac{100\times5}{8} = 62.5\,kN\text{-}m$$

* BC



$$M_{FBC} = \frac{-WL^2}{12} = \frac{-24\times6^2}{12} = -72\,kN\text{-}m$$

$$M_{FCB} = \frac{WL^2}{12} = \frac{24\times6^2}{12} = 72\,kN\text{-}m$$

Step-4 :- Slope deflection equations

Since 'A' is fixed, $\theta_A = 0$, $\delta = 0$

* $M'_{AB} = \frac{2EI}{L}\left[2\cancel{\theta_A}^{0} + \theta_B - \frac{3\cancel{\delta}^{0}}{L}\right]$

$$\boxed{M'_{AB} = \frac{2}{5}EI\theta_B}$$

* $M'_{BA} = \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_A}^{0} - \frac{3\cancel{\delta}^{0}}{L}\right]$

$$\boxed{M'_{BA} = \frac{4}{5}EI\theta_B}$$

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* $M'_{BC} = \frac{2EI}{L}\left[2\theta_B + \theta_C - \cancel{\frac{3\delta}{L}}^{0}\right]$

$M'_{BC} = \frac{2}{3}EI\theta_B + \frac{EI\theta_C}{3}$

$I = 3I$

$$\boxed{M'_{BC} = 2EI\theta_B + EI\theta_C}$$

* $M'_{CB} = \frac{2EI}{L}\left[2\theta_C + \theta_B - \cancel{\frac{3\delta}{L}}^{0}\right]$

$= \frac{\cancel{6}EI}{3}\left[2\theta_C + \theta_B\right] \quad \therefore I = 3I$

$$\boxed{M'_{CB} = 2EI\theta_C + EI\theta_B}$$

Step-5 :- $M_{AB} = -62.5 + \frac{2}{5}EI\theta_B$

$M_{BA} = 62.5 + \frac{4}{5}EI\theta_B$

$M_{BC} = -72 + 2EI\theta_B + EI\theta_C$

$M_{CB} = 72 + 2EI\theta_C + EI\theta_B = 0 \longrightarrow (1)$

Considering 'B' is to be in equillibrium.

$M_{BA} + M_{BC} = 0$

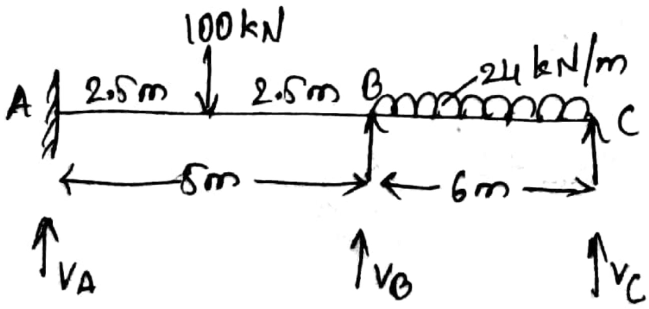
$62.5 + \frac{4}{5}EI\theta_B - 72 + 2EI\theta_B + EI\theta_C = 0$

$\frac{14}{5}EI\theta_B + EI\theta_C - 9.5 = 0 \longrightarrow (2)$

from eqn (1) & (2)

$\boxed{EI\theta_B = 19.78}$ $\quad$ $\boxed{EI\theta_C = -45.89}$

Step-6 :- BMD



$V_A + V_B + V_C = 244 \longrightarrow (3)$

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$M_{AB} = -54.58$ kN-m

$M_{BA} = 78.32$ kN-m

$M_{BC} = -78.33$ kN-m

$M_{CB} = 0$ kN-m

$\Rightarrow M_B = 78.32$ kN-m

$\Rightarrow V_A \times 5 - 100 \times 2.5 - 54.58 + 78.32 = 0$

$$\boxed{V_A = 45.25 \text{ kN}}$$

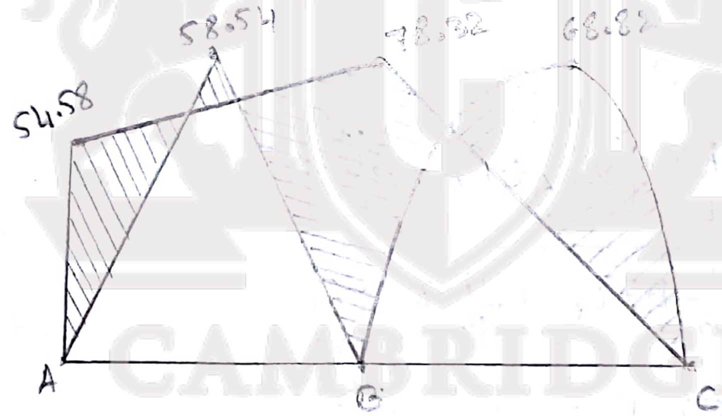
$\rightarrow -V_C \times 6 + 24 \times 6 \times 3 - 78.33$

$$\boxed{V_C = 58.94 \text{ kN}}$$

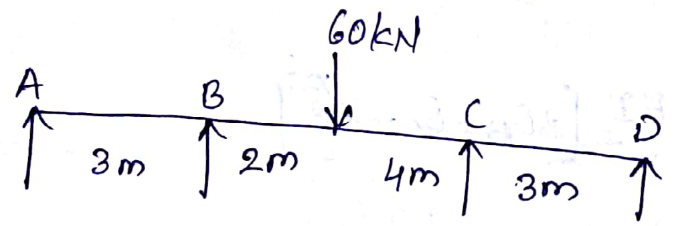
$$\boxed{V_B = 139.81 \text{ kN}}$$

* BM @ AB $= 45.25 \times 2.5 - 54.58 = 58.54$ kN-m

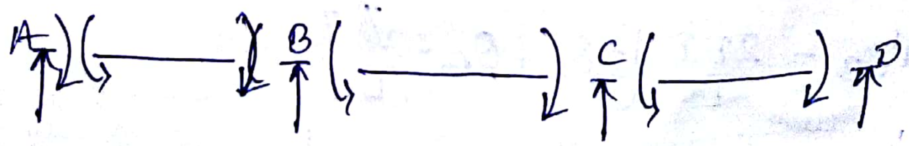
* BM @ BC $= 58.94 \times 3 - 24 \times 3 \times 1.5 = 68.82$ kN-m



5. Analyse the contineous beam and find the support moments, also draw BMD.



Step-1:- Divide the beam



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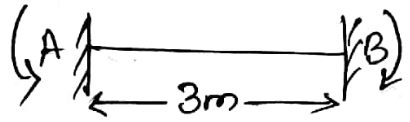
Step-2:- FEM's

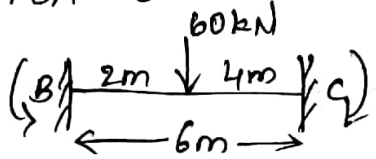
* AB



$M_{FAB} = 0$

$M_{FBA} = 0$

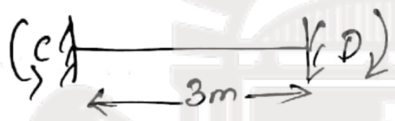
* BC



$$M_{FBC} = \frac{-Pb^2a}{L^2} = -\frac{60 \times 4^2 \times 2}{6^2} = -53.3\,kN$$

$$M_{FCB} = \frac{Pa^2b}{L^2} = \frac{60 \times 2^2 \times 4}{6^2} = 26.66\,kN.$$

* CD



$M_{FCD} = M_{FDC} = 0$

Step-3:-

$\because \delta = 0$

* $M'_{AB} = \frac{2EI}{L}\left[2\theta_A + \theta_B - \cancel{\frac{3\delta}{L}}^{\,0}\right]$

$$\boxed{M'_{AB} = \frac{2}{3}EI\theta_B + \frac{4}{3}EI\theta_A}$$

* $M'_{BA} = \frac{2EI}{L}\left[2\theta_B + \theta_A - \cancel{\frac{3\delta}{L}}^{\,0}\right]$

$$\boxed{M'_{BA} = \frac{4}{3}EI\theta_B + \frac{2}{3}EI\theta_A}$$

* $M'_{BC} = \frac{2EI}{L}\left[2\theta_B + \theta_C - \cancel{\frac{3\delta}{L}}^{\,0}\right]$

$$\boxed{M'_{BC} = \frac{2}{3}EI\theta_B + \frac{EI\theta_C}{3}}$$

* $M'_{CB} = \frac{2EI}{L}\left[2\theta_C + \theta_B - \cancel{\frac{3\delta}{L}}^{\,0}\right]$

$$\boxed{M'_{CB} = \frac{2}{3}EI\theta_C + \frac{EI\theta_B}{3}}$$

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* $M'_{CD} = \frac{2EI}{L}\left[2\theta_C + \theta_D - \frac{3\delta}{L}\right]$

$$\boxed{M'_{CD} = \frac{4}{3}EI\theta_C + \frac{2}{3}EI\theta_D}$$

* $M'_{DC} = \frac{2EI}{L}\left[2\theta_D + \theta_C - \frac{3\delta}{L}\right]$

$$\boxed{M'_{DC} = \frac{4}{3}EI\theta_D + \frac{2}{3}EI\theta_C}$$

Step-4:- $M_{AB} = \frac{2}{3}EI\theta_B + \frac{4}{3}EI\theta_A = 0 \rightarrow (1)$

$M_{BA} = \frac{4}{3}EI\theta_B + \frac{2}{3}EI\theta_A$

$M_{BC} = -53.3 + \frac{2}{3}EI\theta_B + \frac{1}{3}EI\theta_C$

$M_{CB} = 26.6 + \frac{2}{3}EI\theta_C + \frac{1}{3}EI\theta_B$

$M_{CD} = \frac{4}{3}EI\theta_C + \frac{2}{3}EI\theta_D$

$M_{DC} = \frac{4}{3}EI\theta_D + \frac{2}{3}EI\theta_C = 0 \rightarrow (2)$

Considering B & C are in equillibrium

$\therefore M_{BA} + M_{BC} = 0$

$\frac{4}{3}EI\theta_B + \frac{2}{3}EI\theta_A + \frac{2}{3}EI\theta_B + \frac{1}{3}EI\theta_C - 53.3 = 0$

$\frac{2}{3}EI\theta_A + 2EI\theta_B + \frac{1}{3}EI\theta_C = 53.5 \rightarrow (3)$

$\therefore M_{CB} + M_{CD} = 0$

$26.6 + \frac{2}{3}EI\theta_C + \frac{1}{3}EI\theta_B + \frac{4}{3}EI\theta_C + \frac{2}{3}EI\theta_D = 0$

$\frac{1}{3}EI\theta_B + 2EI\theta_C + \frac{2}{3}EI\theta_D = -26.6 \rightarrow (4)$

$eq^n(1) \Rightarrow \frac{4}{3}EI\theta_A + \frac{2}{3}EI\theta_B = 0$

$$\boxed{EI\theta_A = -\frac{1}{2}EI\theta_B}$$

$eq^n(2) \Rightarrow \frac{4}{3}EI\theta_D + \frac{2}{3}EI\theta_C = 0$

$$\boxed{EI\theta_D = -\frac{1}{2}EI\theta_C}$$

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$eq^n(3) \Rightarrow \frac{5}{3} EI\theta_B + \frac{1}{3} EI\theta_C = 53.5 \rightarrow (5)$

$eq^n(4) \Rightarrow \frac{5}{3} EI\theta_C + \frac{1}{3} EI\theta_B = -26.6 \rightarrow (6)$

$\boxed{EI\theta_B = 36.75}$ $\boxed{EI\theta_A = -18.37}$

$\boxed{EI\theta_C = -23.25}$ $\boxed{EI\theta_D = 11.62}$

Step-5:- $M_{AB} = 0$ kN-m

$M_{BA} = 36.64$ kN-m

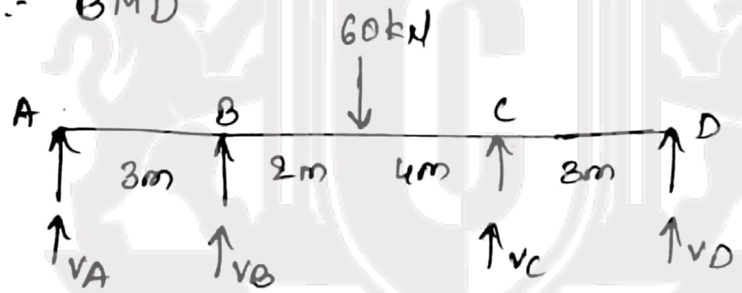
$M_{BC} = -36.64$ kN-m

$M_{CB} = 23.28$ kN-m

$M_{CD} = -23.28$ kN-m

$M_{DC} = 0$ kN-m

Step-6:- BMD



$V_A + V_B + V_C + V_D = 60 \rightarrow (7)$

$M_B = 36.75$ kN-m

$\rightarrow V_A \times 3 + 36.75 = 0$

$\boxed{V_A = -12.25 \text{ kN}}$

$M_C = -23.25$ kN-m

$\Rightarrow V_D \times 3 + 23.38 = 0$

$\boxed{V_D = -7.79 \text{ kN}}$

$\rightarrow V_A \times 9 + V_B \times 6 - 60 \times 4 + 23.25 = 0$

$\boxed{V_B = 54.5 \text{ kN}}$

$\boxed{V_C = 25.54 \text{ kN}}$

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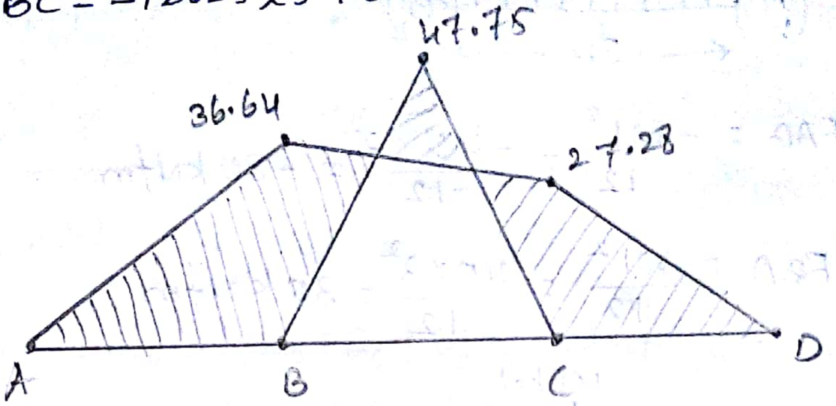
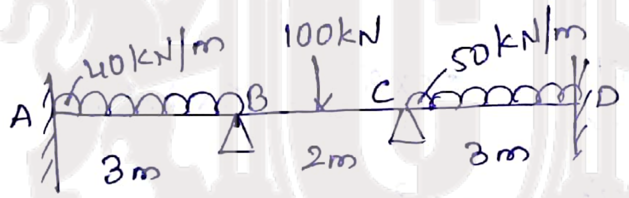
* BM @ AB & CD = 0
* BM @ BC = $-12.25 \times 5 + 54.4 \times 2 = 47.55$ kN-m



Problems on contineous beam with settlements :-

1. Determine the support moments for the contineous gurder as shown in the figure. If the support 'B' sinks 6mm x 2.5mm. for all the members $I = 3.5 \times 10^7 \, mm^4$ & $E = 200 \, kN/mm^2$



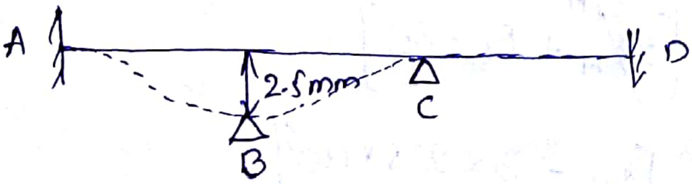
given :- $\delta_B = 2.5 mm = 2.5 \times 10^{-3} m$

$I = 3.5 \times 10^7 \, mm^4$ ~~= 8.~~

$E = 200 \, kN/mm^2$

$$EI = \frac{3.5 \times 10^7 \times 10^{-12} \times \cancel{200}}{10^{-6}}$$

$$\boxed{EI = 7000 \, kN\text{-}m^2}$$



Step-1 :- DKI = 2 = DOF

Step-2 :- divide the beam

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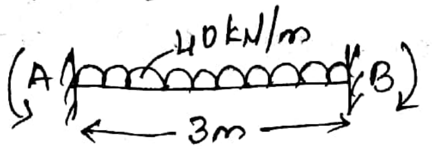
Step-3 :- FEM's

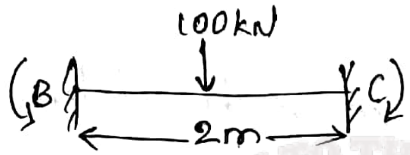
* AB



$$M_{FAB} = -\frac{WL^2}{12} = -\frac{40\times 3^2}{12} = -30\ kN\text{-}m$$

$$M_{FBA} = \frac{WL^2}{12} = \frac{40\times 3^2}{12} = 30\ kN\text{-}m$$

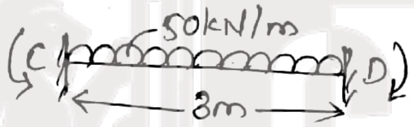
* BC



$$M_{FBC} = -\frac{PL}{8} = -\frac{100\times 2}{8} = -25\ kN\text{-}m$$

$$M_{FCB} = \frac{PL}{8} = \frac{100\times 2}{8} = 25\ kN\text{-}m$$

* CD



$$M_{FCD} = -\frac{WL^2}{12} = -\frac{50\times 3^2}{12} = -37.5\ kN\text{-}m$$

$$M_{FDC} = \frac{WL^2}{12} = \frac{50\times 3^2}{12} = 37.5\ kN\text{-}m$$

Step-4 :- Slope deflection eq^ns

Since A & D are fixed, $\theta_A = \theta_D = 0$

for portion AB, $\delta = 2.5\times 10^{-3}\ m$

for portion BC, $\delta = -2.5\times 10^{-3}\ m$

for portion CD, $\delta = 0$

* $M'_{AB} = \frac{2EI}{L}\left[2\cancel{\theta_A}^{0} + \theta_B - \frac{3\delta}{L}\right]$

$$= \frac{2\times 7000}{3}\left[\theta_B - \frac{3\times 2.5\times 10^{-3}}{3}\right]$$

$$\boxed{M'_{AB} = 4666.67\theta_B - 11.67}$$

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* $M'_{BA} = \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_A}^{0} - \frac{3\delta}{L}\right]$

$= \frac{2\times7000}{3}\left[2\theta_B - \frac{3\times2.5\times10^{-3}}{3}\right]$

$\boxed{M'_{BA} = 9333.3\theta_B - 11.67}$

* $M'_{BC} = \frac{2EI}{L}\left[2\theta_B + \theta_C - \frac{3\delta}{L}\right]$

$= \frac{2\times7000}{2}\left[2\theta_B + \theta_C + \frac{3\times2.5\times10^{-3}}{2}\right]$

$\boxed{M'_{BC} = 14000\theta_B + 7000\theta_C + 26.25}$

* $M'_{CB} = \frac{2EI}{L}\left[2\theta_C + \theta_B - \frac{3\delta}{L}\right]$

$= \frac{2\times7000}{2}\left[2\theta_C + \theta_B + \frac{3\times2.5\times10^{-3}}{2}\right]$

$\boxed{M'_{CB} = 14000\theta_C + 7000\theta_B + 26.25}$

* $M'_{CD} = \frac{2EI}{L}\left[2\theta_C + \cancel{\theta_D}^{0} - \cancel{\frac{3\delta}{L}}^{0}\right]$

$= \frac{2\times7000}{3}[2\theta_C]$

$\boxed{M'_{CD} = 9333.33\theta_C}$

* $M'_{DC} = \frac{2EI}{L}\left[2\cancel{\theta_D}^{0} + \theta_C - \cancel{\frac{3\delta}{L}}^{0}\right]$

$= \frac{2\times7000}{3}[\theta_C]$

$\boxed{M'_{DC} = 4666.67\theta_C}$

Step-5:-

$M_{AB} = -30 + 4666.67\theta_B - 11.67$

$M_{BA} = 30 + 9333.33\theta_B - 11.67$

$M_{BC} = -25 + 14000\theta_B + 7000\theta_C + 26.25$

$M_{CB} = 25 + 14000\theta_C + 7000\theta_B + 26.25$

$M_{CD} = -37.5 + 9333.33\theta_C$

$M_{DC} = 37.5 + 4666.67\theta_C$

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Considering 'B' & 'C' are in equilibrium

$\therefore M_{BA}+M_{BC}=0$

$30+9333.33\,\theta_B-11.67-25+14000\,\theta_B+7000\,\theta_C+26.25=0$

$23333.33\,\theta_B+7000\,\theta_C=-19.58 \rightarrow (1)$

$\therefore M_{CB}+M_{CD}=0$

$25+7000\,\theta_B+14000\,\theta_C+26.25-37.5+9333.33\,\theta_C=0$

$7000\,\theta_B+23333.33\,\theta_C=-13.75 \rightarrow (2)$

from eq^n (1) & (2)

$\boxed{\theta_B=-7.278\times10^{-4}}$ $\boxed{\theta_C=-3.709\times10^{-4}}$

Step-6 :- $M_{AB}=-45.06$ kN-m

$M_{BA}=11.537$ kN-m

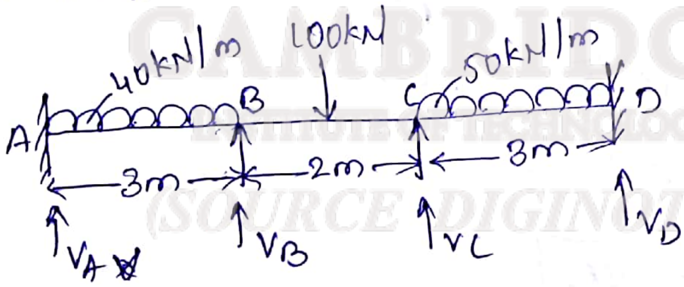
$M_{BC}=-11.535$ kN-m

$M_{CB}=40.963$ kN-m

$M_{CD}=-40.962$ kN-m

$M_{DC}=35.769$ kN-m

Step-7 :- BMD



$V_A+V_B+V_C+V_D=120+100+150=370$ kN $\rightarrow (3)$

$M_B=11.537$ kN-m

$\Rightarrow V_A\times3-45.06-40\times3\times\frac{3}{2}+11.537=0$

$\boxed{V_A=+71.17\text{kN}}$

$\Rightarrow M_C=40.962$ kN-m

$\Rightarrow +71.17\times5-45.06-40\times3\left(\frac{3}{2}+2\right)-100\times1+V_B\times2+40.962=0$

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$\boxed{V_B = 84.124\,kN}$

$\Rightarrow -V_D \times 3 + 50 \times 3 \times \frac{3}{2} + 35.769 - 40.962 = 0$

$\boxed{V_D = 73.269\,kN}$

$\boxed{V_C = 141.437\,kN}$

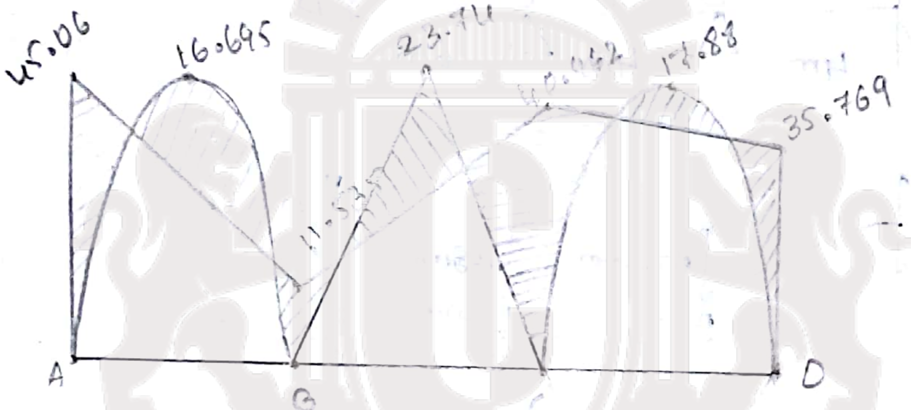
* $BM\ @\ AB = 71.17 \times 1.5 - 45.06 - 40 \times 1.5 \times \frac{1.5}{2} = 16.695\,kN\text{-}m$

* $BM\ @\ BC = 71.17 \times 4 - 45.06 - 40 \times 3 \times (1.5+1) + 84.124 \times 1$

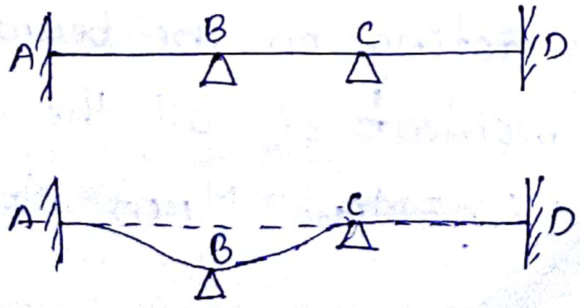
$= 23.744\,kN\text{-}m$

* $BM\ @\ CD = +73.269 \times 1.5 + 35.769 + 50 \times 1.5 \times \frac{1.5}{2}$

$= +17.884\,kN\text{-}m$



Note :- There are different sign conventions followed by different authors. The sign convention that downward deflection is negative throughout the span is considered by same authors but different sign convention as shown in the figure below is being followed by most of the authors.



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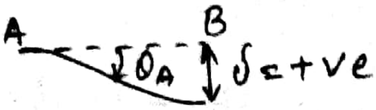
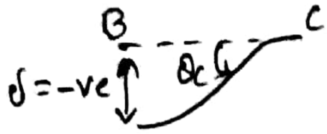
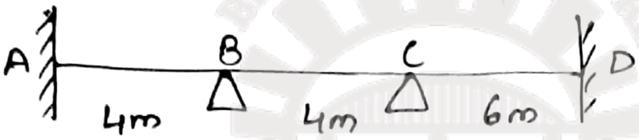
Span AB :-

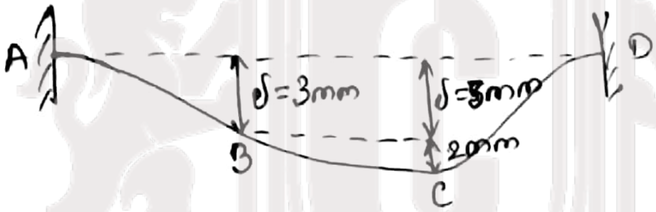


Span BC :-



2. Analyse the continuous beam as shown in the fig. If the supports B & C sink by 3mm & 5mm respectively. For the beam, take $I = 4 \times 10^7 mm^4$ & $E = 200\ kN/mm^2$



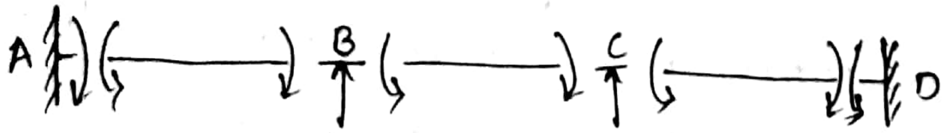


$$EI = \frac{4 \times 10^7 \times 10^{-12} \times 200}{10^{-6}}$$

$$\boxed{EI = 8000\ kN\text{-}m^2}$$

Step-1 :- DKI = DOF = 2

Step-2 :- Divide the beam



Step-3 :- FEM'S

No loads is acting on the beam, the fixed end moments of all the sections = 0

$$M_{FAB} = M_{FBA} = M_{FCB} = M_{FBC} = M_{FCD} = M_{FDC} = 0$$

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Step-4:- Slope deflection eqns

* $M'_{AB} = \frac{2EI}{L}\left[2\cancel{\theta_A}^{0} + \theta_B - \frac{3\delta}{L}\right]$

Since A & D are fixed, $\theta_A = \theta_D = 0$

portion AB, $\delta = 3mm = 3\times10^{-3}m$

portion BC, $\delta = 2\times10^{-3}m$

portion CD $= \delta = -5\times10^{-3}m$

$$M'_{AB} = \frac{2\times8000}{4}\left[\theta_B - \frac{3\times3\times10^{-3}}{4}\right]$$

$$\boxed{M'_{AB} = 4000\,\theta_B - 9}$$

* $M'_{BA} = \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_A}^{0} - \frac{3\delta}{L}\right]$

$$= \frac{2\times8000}{4}\left[2\theta_B - \frac{3\times3\times10^{-3}}{4}\right]$$

$$\boxed{M'_{BA} = 8000\,\theta_B - 9}$$

* $M'_{BC} = \frac{2EI}{L}\left[2\theta_B + \theta_C - \frac{3\delta}{L}\right]$

$$= \frac{2\times8000}{4}\left[2\theta_B + \theta_C - \frac{3\times2\times10^{-3}}{4}\right]$$

$$\boxed{M'_{BC} = 8000\,\theta_B + 4000\,\theta_C - 6}$$

* $M'_{CB} = \frac{2EI}{L}\left[2\theta_C + \theta_B - \frac{3\delta}{L}\right]$

$$= \frac{2\times8000}{4}\left[2\theta_C + \theta_B - \frac{3\times2\times10^{-3}}{4}\right]$$

$$\boxed{M'_{CB} = 8000\,\theta_C + 4000\,\theta_B - 6}$$

* $M'_{CD} = \frac{2EI}{L}\left[2\theta_C + \theta_D - \frac{3\delta}{L}\right]$

$$= \frac{2\times8000}{6}\left[2\theta_C + \theta_D + \frac{3\times5\times10^{-3}}{6}\right]$$

$$= 5333.3\,\theta_C + 2666.67\,\cancel{\theta_D}^{0} + 6.67$$

$$\boxed{M'_{CD} = 5333.3\,\theta_C + 6.67}$$

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* $M'_{DC} = \frac{2EI}{L}\left[2\cancel{\theta_D}^{0} + \theta_C - \frac{3\delta}{L}\right]$

$= \frac{2\times 8000}{6}\left[\theta_C + \frac{3\times 5\times 10^{-3}}{6}\right]$

$\boxed{M'_{DC} = 2666.67\theta_C + 6.67}$

Step-5:-

$M_{AB} = 4000\theta_B - 9$

$M_{BA} = 8000\theta_B - 9$

$M_{BC} = 8000\theta_B + 4000\theta_C - 6$

$M_{CB} = 8000\theta_C + 4000\theta_B - 6$

$M_{CD} = 5333.3\theta_C + 6.67$

$M_{DC} = 2666.67\theta_C + 6.67$

Considering 'B' & 'C' is in equilibrium

$\therefore M_{BA} + M_{BC} = 0$

$8000\theta_B - 9 + 8000\theta_B + 4000\theta_C - 6 = 0$

$16000\theta_B + 4000\theta_C = 15 \rightarrow (1)$

$\therefore M_{CB} + M_{CD} = 0$

$8000\theta_C + 4000\theta_B - 6 + 5333.3\theta_C + 6.67 = 0$

$4000\theta_B + 13333.3\theta_C = -0.67 \rightarrow (2)$

from eq^n (1) & (2)

$\boxed{\theta_B = 1.027\times 10^{-3}}$ $\boxed{\theta_C = -3.58\times 10^{-4}}$

Step 6:- $M_{AB} = -4.892$ kN-m

$M_{BA} = -0.78$ kN-m

$M_{BC} = 0.78$ kN-m

$M_{CB} = -4.75$ kN-m

$M_{CD} = 4.75$ kN-m

$M_{DC} = 5.71$ kN-m

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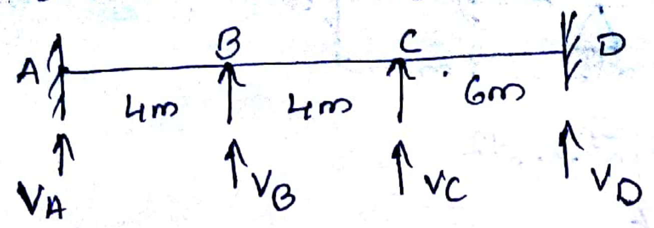
Step-7:- BMD



$$V_A + V_B + V_C + V_D = 0$$

$M_B = +0.78$ kN-m

$\Rightarrow V_A \times 4 - 4.892 + 0.78 = 0$

$$\boxed{V_A = 1.03 \text{ kN}}$$

$M_C = 4.75$ kN-m

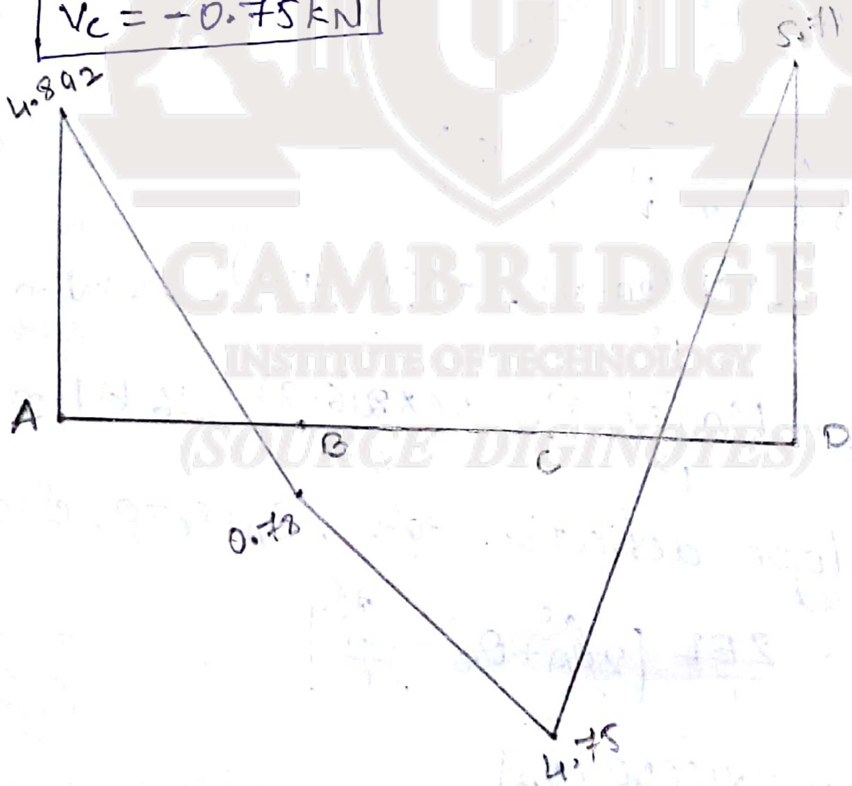
$\Rightarrow 1.03 \times 8 + V_B \times 4 - 4.892 + 4.75 = 0$

$$\boxed{V_B = -2.02 \text{ kN}}$$

$\Rightarrow -V_D \times 6 + 5.71 + 4.75 = 0$

$$\boxed{V_D = 1.74 \text{ kN}}$$

$$\boxed{V_C = -0.75 \text{ kN}}$$



3. Analyse the contineous beam as shown in the figure consider modulus of elasticity to be 200 kN/mm^2 & moment of inertia 5×10^7 mm^4.

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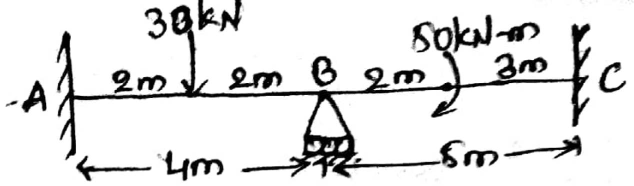
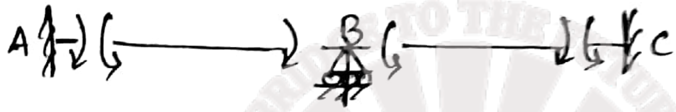


$$EI = \frac{200 \times 5 \times 10^{7} \times 10^{-12}}{10^{-6}}$$

$$\boxed{EI = 10000\ kN\text{-}m^2}$$

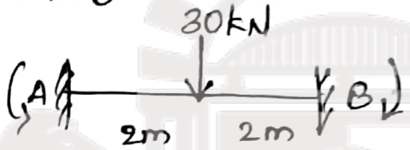
Step-1 :- DKI = DOF = 2

Step-2 :- divide the beam



Step-3 :- FEM's

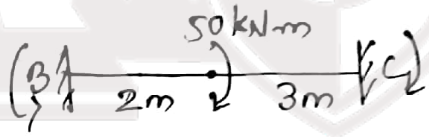
* AB



$$M_{FAB} = -\frac{PL}{8} = -\frac{30 \times 4}{8} = -15\ kN\text{-}m$$

$$M_{FBA} = \frac{PL}{8} = \frac{30 \times 4}{8} = 15\ kN\text{-}m$$

* BC



$$M_{FBC} = -\frac{Mb(2a-b)}{L^2} = -\frac{50 \times 3(4-3)}{5^2} = -6\ kN\text{-}m$$

$$M_{FCB} = \frac{Ma(2b-a)}{L^2} = \frac{50 \times 2(6-2)}{5^2} = 16\ kN\text{-}m$$

Step-4 :- Slope deflection eq^{n}s , $\theta_A = \theta_C = 0$, $\delta = 0$

* $M'_{AB} = \frac{2EI}{L}\left[2\cancel{\theta_A}^{0} + \theta_B - \cancel{\frac{3\delta}{L}}^{0}\right]$

$$= \frac{2 \times 10000}{4}[\theta_B]$$

$$\boxed{M'_{AB} = 5000\theta_B}$$

* $M'_{BA} = \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_A}^{0} - \cancel{\frac{3\delta}{L}}^{0}\right]$

$$= \frac{2 \times 10000}{4}[2\theta_B]$$

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$$\boxed{M'_{BA} = 10000\,\theta_B}$$

$$* \; M'_{BC} = \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_C}^{\,0} - \frac{3\cancel{\delta}^{\,0}}{L}\right]$$

$$= \frac{2 \times 10000}{5}\left[2\theta_B\right]$$

$$\boxed{M'_{BC} = 8000\,\theta_B}$$

$$* \; M'_{CB} = \frac{2EI}{L}\left[2\cancel{\theta_C}^{\,0} + \theta_B - \frac{3\cancel{\delta}^{\,0}}{L}\right]$$

$$= \frac{2 \times 10000}{5}\left[\theta_B\right]$$

$$\boxed{M'_{CB} = 4000\,\theta_B}$$

Step-5 :- $M_{AB} = -15 + 5000\,\theta_B \Rightarrow -17.5$ kN-m

$M_{BA} = 15 + 10000\,\theta_B \Rightarrow 10$ kN-m

$M_{BC} = -6 + 8000\,\theta_B \Rightarrow -10$ kN-m

$M_{CB} = 16 + 4000\,\theta_B \Rightarrow 14$ kN-m

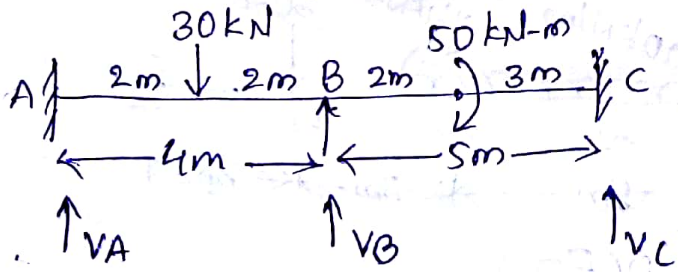
Considering 'B' is in equilibrium.

$\therefore M_{BA} + M_{BC} = 0$

$15 + 10000\,\theta_B - 6 + 8000\,\theta_B = 0$

$$\boxed{\theta_B = -5 \times 10^{-4}}$$

Step-6 :- BMD



$V_A + V_B + V_C = 30 \longrightarrow (1)$

$M_B = 10$ kN-m

$\Rightarrow V_A \times 4 - 30 \times 2 - 17.5 + 10 = 0$

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$\boxed{V_A = 16.875 \text{ kN-m}}$

$\Rightarrow -V_C \times 5 + 60 - 10 + 14 = 0$

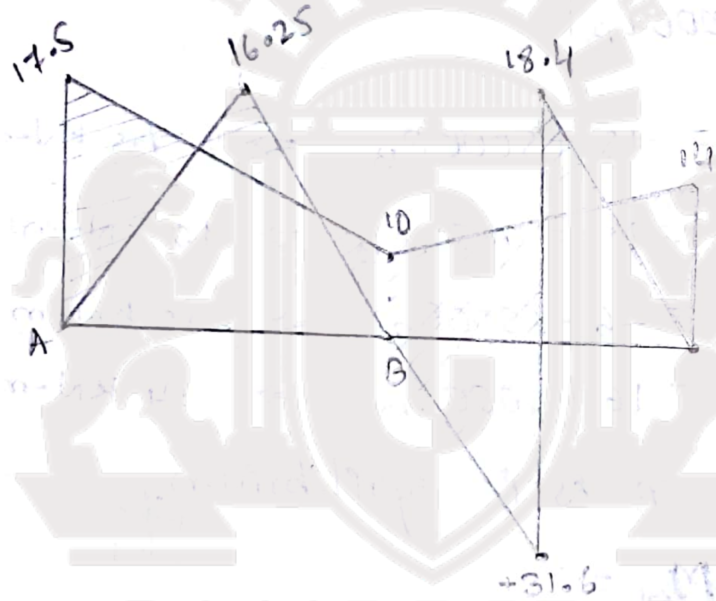
$\boxed{V_C = 10.8 \text{ kN}}$

$\boxed{V_B = 2.325 \text{ kN}}$

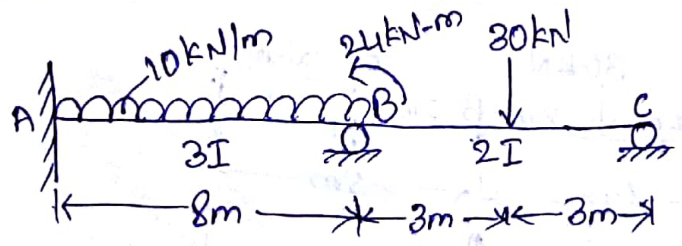
* BM @ AB $= 16.875 \times 2 - 17.5 = 16.25$ kN-m

* BM @ $BC_L = 16.875 \times 6 - 30 \times 4 - 17.5 + 2.325 \times 2 = -31.6$ kN-m

* BM @ $BC_R = +2.325 \times 3 - 14 = 18.4$ kN-m

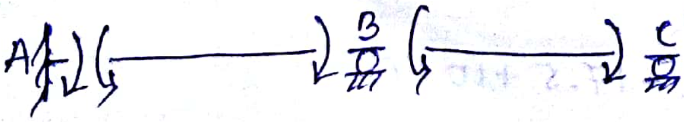


4. Analyse the contineous beam shown in the figure. Using slope deflection method then draw the SFD & BMD.



Step-1:- DKI = DOF = 3

Step-2:- divide the beam



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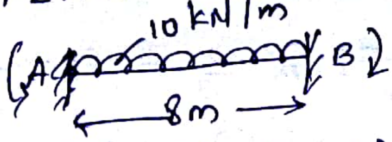
Step-3 :- FEM's

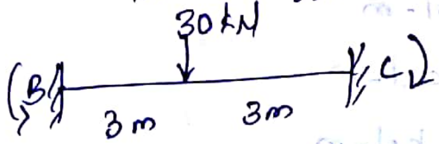
* AB



$$M_{FAB} = \frac{-WL^2}{12} = \frac{-10\times 8^2}{12} = -53.33 \text{ kN-m}$$

$$M_{FBA} = \frac{WL^2}{12} = \frac{10\times 8^2}{12} = 53.33 \text{ kN-m}$$

* BC



$$M_{FBC} = \frac{-PL}{8} = \frac{-30\times 6}{8} = -22.5 \text{ kN-m}$$

$$M_{FCB} = \frac{PL}{8} = \frac{30\times 6}{8} = 22.5 \text{ kN-m}$$

Step-4 :- Slope deflection eq's

Since A is fixed, $\theta_A = 0$, $\delta = 0$

$I = 3I$ in portion AB

$I = 2I$ in portion BC

* $M'_{AB} = \frac{2EI}{L}\left[2\theta_A + \theta_B - \frac{3\delta}{L}\right]$

$$\boxed{M'_{AB} = \frac{3}{4} EI\theta_B}$$

* $M'_{BA} = \frac{2EI}{L}\left[2\theta_B + \theta_A - \frac{3\delta}{L}\right]$

$$\boxed{M'_{BA} = \frac{3}{2} EI\theta_B}$$

* $M'_{BC} = \frac{2EI}{L}\left[2\theta_B + \theta_C - \frac{3\delta}{L}\right]$

$$\boxed{M'_{BC} = \frac{4}{3} EI\theta_B + \frac{2}{3} EI\theta_C}$$

* $M'_{CB} = \frac{2EI}{L}\left[2\theta_C + \theta_B - \frac{3\delta}{L}\right]$

$$\boxed{M'_{CB} = \frac{2}{3} EI\theta_B + \frac{4}{3} EI\theta_C}$$

Step-5 :- $M_{AB} = -53.33 + \frac{3}{4} EI\theta_B$

$M_{BA} = 53.33 + \frac{3}{2} EI\theta_B$

$M_{BC} = -22.5 + \frac{4}{3} EI\theta_B + \frac{2}{3} EI\theta_C$ → (1)

$M_{CB} = 22.5 + \frac{2}{3} EI\theta_B + \frac{4}{3} EI\theta_C$

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$\therefore M_{BA} + M_{BC} = -24$

$$53.33 + \frac{3}{2} EI\theta_B - 22.5 + \frac{4}{3} EI\theta_B + \frac{2}{3} EI\theta_C = -24$$

$$\frac{17}{6} EI\theta_B + \frac{2}{3} EI\theta_C = -54.83 \rightarrow (2)$$

$\boxed{EI\theta_B = -17.432}$ $\boxed{EI\theta_C = -8.159}$

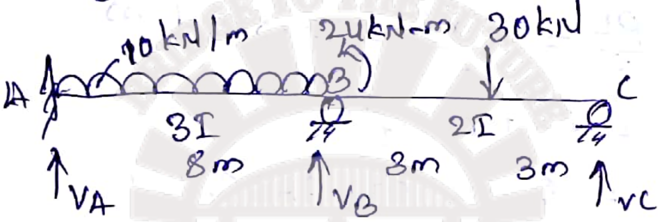
Step-6 :-
$M_{AB} = -66.4 \text{ kN-m}$

$M_{BA} = +27.182 \text{ kN-m}$

$M_{BC} = -51.182 \text{ kN-m}$

$M_{CB} = 0 \text{ kN-m}$

Step-7 :-



$SF A_R = 40.26 \text{ kN}$

$SF B_L = 40.26 - 10 \times 8$
$= -39.74 \text{ kN}$

$SF B_R = 40 - 10 \times 8 + 50.2$
$= 10.47 \text{ kN}$

$SF C_L = -19.53 \text{ kN}$

$$V_A + V_B + V_C = 10 \times 8 + 30 = 110 \text{ kN}$$

$M_B = -24 \text{ kN-m}$

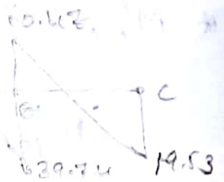
$\Rightarrow V_A \times 8 - 10 \times 8 \times 4 - 66.4 + 40.256 = -24$

$\boxed{V_A = 40.26 \text{ kN}}$

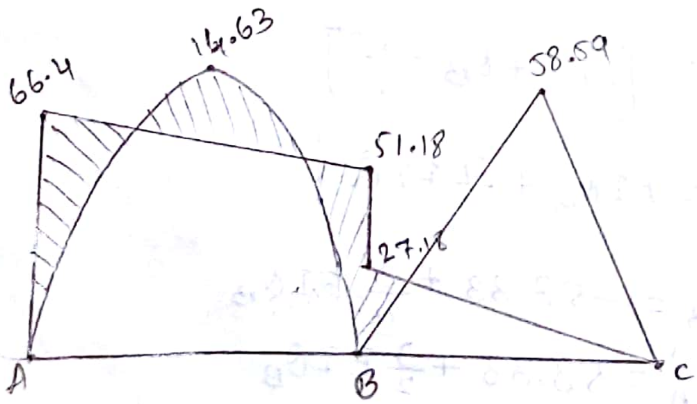
$\Rightarrow V_C \times 6 - 30 \times 3 - 51.182 = -24$

$\boxed{V_C = 19.53 \text{ kN}}$

$\boxed{V_B = 50.21 \text{ kN}}$



* BM @ AB $= 40.26 \times 4 - 10 \times 4 \times 2 - 66.404 = 14.636 \text{ kN-m}$
* BM @ BC $= 19.53 \times 3 = 58.59 \text{ kN-m}$



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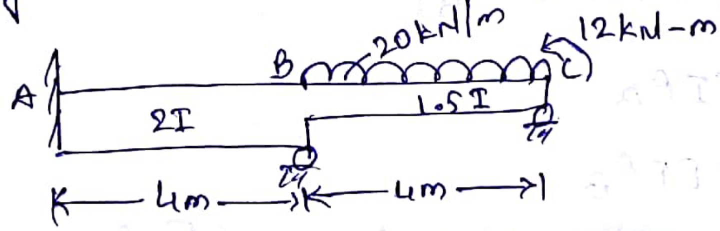
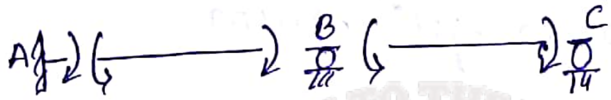
5. Analyse the contineous beam shown in the fig. using SDM & draw the SFD & BMD



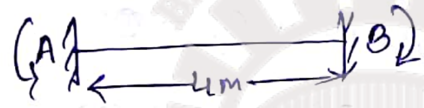
Step-1:- DKI = DOF = 3

Step-2:- Divide the beam.



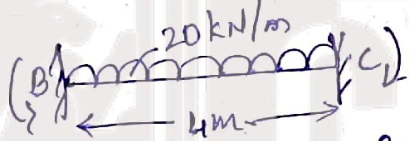
Step-3:- FEM's

* AB



$M_{FAB} = M_{FBA} = 0$

* BC



$M_{FBC} = \frac{-WL^2}{12} = \frac{-20\times4^2}{12} = -26.67$ kN-m

$M_{FCB} = \frac{WL^2}{12} = \frac{20\times4^2}{12} = 26.67$ kN-m

Step-4:- Slope deflection eq^{n}s.

Since 'A' is fixed, $\theta_A = 0$, $\delta = 0$

$I = 2I$ in portion AB

$I = 1.5I$ in portion BC

* $M'_{AB} = \frac{2EI}{L}\left[2\cancel{\theta_A}^{0} + \theta_B - \frac{3\cancel{\delta}^{0}}{L}\right]$

$\boxed{M'_{AB} = EI\,\theta_B}$

* $M'_{BA} = \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_A}^{0} - \frac{3\cancel{\delta}^{0}}{L}\right]$

$\boxed{M'_{BA} = 2EI\,\theta_B}$

* $M'_{BC} = \frac{2EI}{L}\left[2\theta_B + \theta_C - \frac{3\cancel{\delta}^{0}}{L}\right]$

$\boxed{M'_{BC} = 1.5EI\theta_B + 0.75\,\theta_C}$

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$* \; M'_{CB} = \frac{2EI}{L}\left[2\theta_C + \theta_B - 3\frac{\Delta}{L}\right]$

$$\boxed{M'_{CB} = 1.5EI\theta_C + 0.75EI\theta_B}$$

Step-5 :- $M_{AB} = EI\theta_B$

$M_{BA} = 2EI\theta_B$

$M_{BC} = -26.67 + 1.5EI\theta_B + 0.75EI\theta_C$

$M_{CB} = 26.67 + 1.5EI\theta_C + 0.75EI\theta_B = 12 \rightarrow (1)$

Considering eqn 'B' in equilibrium

$\therefore M_{BA} + M_{BC} = 0$

$2EI\theta_B - 26.67 + 1.5EI\theta_B + 0.75\theta_C = 0$

$3.5EI\theta_B + 0.75\theta_C = 26.67 \rightarrow (2)$

eqn (1) $\Rightarrow 0.75EI\theta_B + 1.5EI\theta_C = -14.67 \rightarrow (3)$

$\boxed{EI\theta_B = 10.88}$ $\boxed{EI\theta_C = -15.22}$

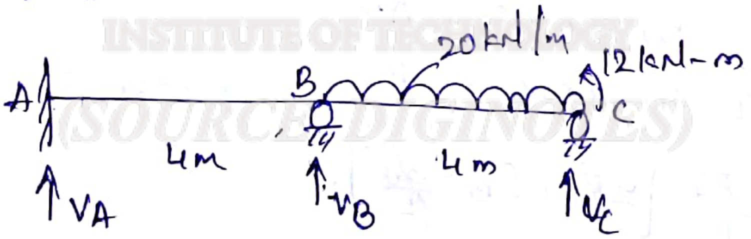
$M_{AB} = 10.88$ kN-m

$M_{BA} = 21.76$ kN-m

$M_{BC} = -21.765$ kN-m

$M_{CB} = 12$ kN-m

Step-6 :- BMD



$V_A + V_B + V_C = 80$ kN $\rightarrow (4)$

$\Rightarrow M_B = 21.76$ kN-m

$\Rightarrow V_A \times 4 + 10.88 + 21.76 = 0$

$$\boxed{V_A = -8.16 \text{ kN}}$$

$\Rightarrow -V_C \times 4 + 20 \times 4 \times 2 - 12 + 12 - 21.76 = 0$

$$\boxed{V_C = 34.56 \text{ kN}}$$

$$\boxed{V_B = 53.6 \text{ kN}}$$

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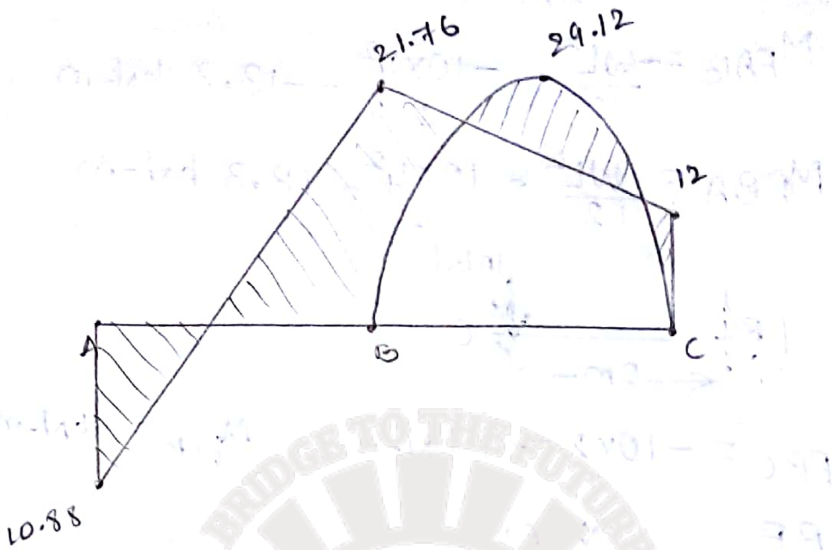
* BM @ AB = 0

* BM @ BC = $-8.16\times6 + 10.88 + 53.6\times2 - 20\times2\times1$

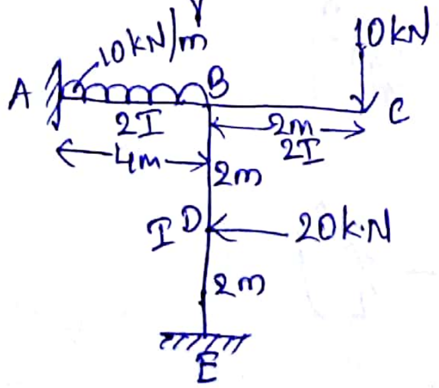
$= 29.12$ kN-m



* SF @ $A_R = -8.16$ kN
* SF @ $B_L = -8.16$ kN
* SF @ $B_R = -8.16 + 53.6 = 45.44$ kN
* SF @ $C_L = -8.16 + 53.6 - 20\times4 = -34.56$ kN

Non-Sway Frames :- [Analysis]

1. Analyse the frame by slope deflection method as shown in figure draw the BMD & SFD for the beam



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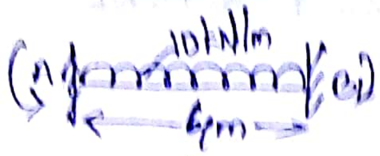
Step-1:- FEM's

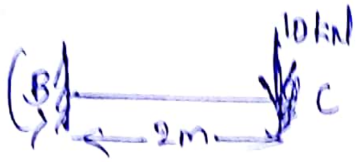
* AB



$M_{FAB} = -\frac{WL^2}{12} = \frac{-10\times4^2}{12} = -13.3$ kN-m

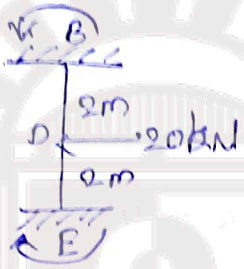
$M_{FBA} = \frac{WL^2}{12} = \frac{10\times4^2}{12} = 13.3$ kN-m

* BC



$M_{BC} = M_{FBC} = -10\times2 = -20$ kN-m $M_{FCB} = 0$ kN-m $= M_{CB}$

* Span BE



$M_{FBE} = -\frac{WL}{8} = \frac{-20\times4}{8} = -10$ kN-m

$M_{FEB} = \frac{WL}{8} = \frac{20\times4}{8} = 10$ kN-m.

Step-2:- Slope deflection eq^{n}s.

Since A & E are fixed, $\theta_A = \theta_E = 0$, $\delta = 0$

portion AB = ~~2I~~ = 2I

portion BC, I = 2I

portion BE, I = I

* $M'_{AB} = \frac{2EI}{L}\left[2\cancel{\theta_A}^{0} + \theta_B - \cancel{\frac{3\delta}{L}}^{0}\right]$

$\boxed{M'_{AB} = EI\theta_B}$

* $M'_{BA} = \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_A}^{0} - \cancel{\frac{3\delta}{L}}^{0}\right]$

$\boxed{M'_{BA} = 2EI\theta_B}$

* $M'_{BC} = \frac{2EI}{L}\left[2\theta_B + \theta_C - \cancel{\frac{3\delta}{L}}^{0}\right]$

M'_B.

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Step-1 :- FEM's

* AB

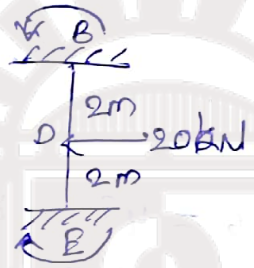
$$M_{FAB} = -\frac{WL^2}{12} = -\frac{10\times4^2}{12} = -13.3 \text{ kN-m}$$

$$M_{FBA} = \frac{WL^2}{12} = \frac{10\times4^2}{12} = 13.3 \text{ kN-m}$$

* BC

$M_{BC} = M_{FBC} = -10\times2 = -20 \text{ kN-m}$ $\quad M_{FCB} = 0 \text{ kN-m} = M_{CB}$

* Span BE



$$M_{FBE} = -\frac{WL}{8} = -\frac{20\times4}{8} = -10 \text{ kN-m}$$

$$M_{FEB} = \frac{WL}{8} = \frac{20\times4}{8} = 10 \text{ kN-m.}$$

Step-2 :- Slope deflection eq^ns

Since A & E are fixed, $\theta_A = \theta_E = 0$, $\delta = 0$

portion AB $= \cancel{2}I = 2I$

portion BC, $I = 2I$

portion BE, $I = I$

* $M'_{AB} = \frac{2EI}{L}\left[2\cancel{\theta_A}^{0} + \theta_B - \cancel{\frac{3\delta}{L}}^{0}\right]$

$\boxed{M'_{AB} = EI\theta_B}$

* $M'_{BA} = \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_A}^{0} - \cancel{\frac{3\delta}{L}}^{0}\right]$

$\boxed{M'_{BA} = 2EI\theta_B}$

* $M'_{BC} = \frac{2EI}{L}\left[2\theta_B + \theta_C - \cancel{\frac{3\delta}{L}}^{0}\right]$

M'_B

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$$M'_{BE} = \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_E}^{\,0} - \cancel{\frac{3\delta}{L}}^{\,0}\right]$$

$$\boxed{M'_{BE} = EI\theta_B}$$

$$M'_{EB} = \frac{2EI}{L}\left[2\cancel{\theta_E}^{\,0} + \theta_B - \cancel{\frac{3\delta}{L}}^{\,0}\right]$$

$$\boxed{M'_{EB} = \frac{EI\theta_B}{2}}$$

Step-3 :-

$M_{AB} = -13.3 + EI\theta_B$

$M_{BA} = 13.3 + 2EI\theta_B$

$M_{BC} = -20$ kN-m

$M_{CB} = 0$ kN-m

$M_{BE} = -10 + EI\theta_B$

$M_{EB} = 10 + 0.5\, EI\theta_B$

Considering 'B' is in equilibrium

$\therefore M_{BA} + M_{BC} + M_{BE} = 0.$

$13.3 + 2EI\theta_B - 20 - 10 + EI\theta_B = 0$

$3EI\theta_B = +16.7$

$\boxed{EI\theta_B = +5.56}$

Step-4 :-

$M_{AB} = -7.74$ kN-m

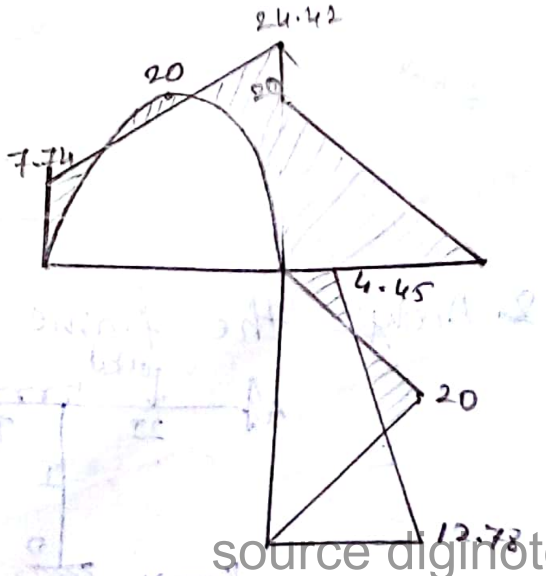
$M_{BA} = 24.42$ kN-m

$M_{BC} = -20$ kN-m

$M_{CB} = 0$ kN-m

$M_{BE} = -4.44$ kN-m

$M_{EB} = 12.78$ kN-m



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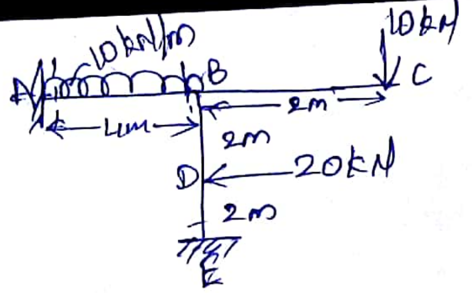
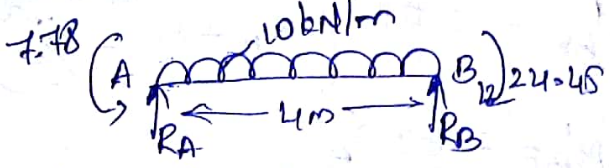




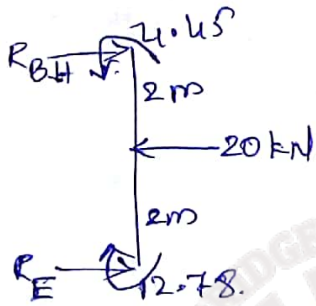
$\Sigma M_A = 0$ ↓+ve

$-R_B \times 4 + 10 \times 4 \times \frac{4}{2} - 7.78 + 24.45 = 0$

$\boxed{R_B = 24.16 kN}$

$R_A = 40 - R_B$

$\boxed{R_A = 15.84 kN}$



$R_{BH} + R_E = 20 kN$

$R_{BH} = 0$ ↓+ve

$-R_E \times 4 + 20 \times 2 - 4.45 + 12.78 = 0$

$\boxed{R_E = 12.08 kN}$

$\boxed{R_{BH} = 7.91 kN}$

SF @ A_R = 15.84 kN

SF @ B_L = 15.84 − 40 = −24.16 kN.

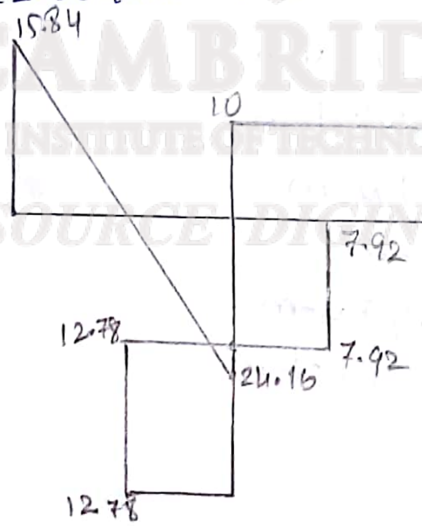
SF @ B_R = ~~15.84 − 40 + 24.16~~ = 10 kN SF @ C_L = +10 kN

SF @ E_D = −12.08 kN

SF @ D_D = −12.08 kN

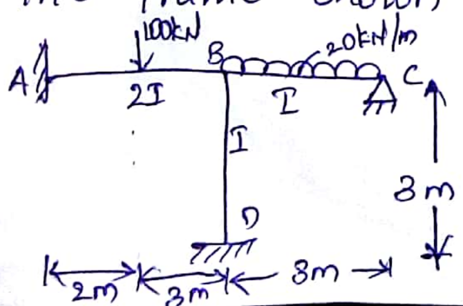
SF @ D_U = −12.08 + 20 = 7.92 kN

SF @ B_D = −12.08 + 20 = 7.92 kN



−8.644

2. Analyse the frame shown in a figure by SDM



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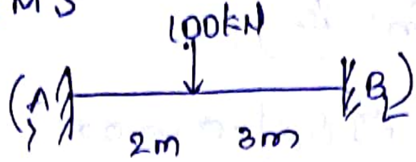
Step-1 : FEM's

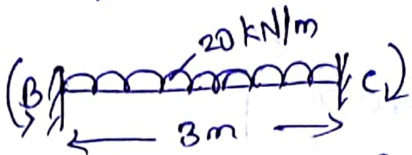
* AB ⇒



$$M_{FAB} = -\frac{Pb^2a}{l^2} = -\frac{100\times3^2\times2}{5^2} = -72 \text{ kN-m}$$

$$M_{FBA} = \frac{Pa^2b}{l^2} = \frac{100\times2^2\times3}{5^2} = 48 \text{ kN-m}$$

* BC ⇒



$$M_{FBC} = -\frac{WL^2}{12} = -\frac{20\times3^2}{12} = -15 \text{ kN-m}$$

$$M_{FCB} = \frac{WL^2}{12} = \frac{20\times3^2}{12} = 15 \text{ kN-m}$$

* BD

$M_{FBD} = M_{FDB} = 0.$

Step-2 : SD eq^ns

Since A & D are fixed $\theta_A = \theta_D = 0,\ \delta = 0$

* AB, $I = 2I$ * BC, $I = I$, * BD, $I = I$

$$*\ M_{AB} = -72 + \frac{2EI}{L}\left[2\cancel{\theta_A}^{0} + \theta_B - \frac{3\cancel{\delta}^{0}}{L}\right]$$

$$\boxed{M_{AB} = -72 + \frac{4}{5}EI\theta_B}$$

$$*\ M_{BA} = 48 + \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_A}^{0} - \frac{3\cancel{\delta}^{0}}{L}\right]$$

$$\boxed{M_{BA} = 48 + \frac{8}{5}EI\theta_B}$$

$$*\ M_{BC} = -15 + \frac{2EI}{L}\left[2\theta_B + \theta_C - \frac{3\delta}{L}\right]$$

$$\boxed{M_{BC} = -15 + \frac{4}{3}EI\theta_B + \frac{2}{3}EI\theta_C}$$

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* $M_{CB} = 15 + \frac{2EI}{L}\left[2\theta_C + \theta_B - \cancel{\frac{3\delta}{L}}^{\to 0}\right]$

$$\boxed{M_{CB} = 15 + \frac{2}{3}EI\theta_B + \frac{4}{3}EI\theta_C} = 0 \to (2)$$

* $M_{BD} = \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_D}^{\to 0} - \cancel{\frac{3\delta}{L}}^{\to 0}\right]$

$$\boxed{M_{BD} = \frac{4}{3}EI\theta_B}$$

* $M_{DB} = \frac{2EI}{L}\left[2\cancel{\theta_D}^{\to 0} + \theta_B - \cancel{\frac{3\delta}{L}}^{\to 0}\right]$

$$\boxed{M_{DB} = \frac{2}{3}EI\theta_B}$$

Considering B is in equilibrium

$\therefore M_{BA} + M_{BC} + M_{BD} = 0$

$$48 + \frac{8}{5}EI\theta_B - 15 + \frac{4}{3}EI\theta_B + \frac{2}{3}EI\theta_C + \frac{4}{3}EI\theta_B = 0$$

$$\frac{64}{15}EI\theta_B + \frac{2}{3}EI\theta_C = -33 \to (1)$$

$$\boxed{EI\theta_B = -6.483} \quad \boxed{EI\theta_C = -8.008}$$

Step-3:- $M_{AB} = -77.186$ kN-m

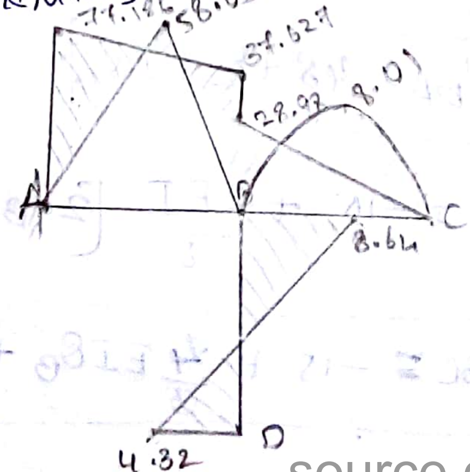
$M_{BA} = 37.627$ kN-m

$M_{BC} = -28.98$ kN-m

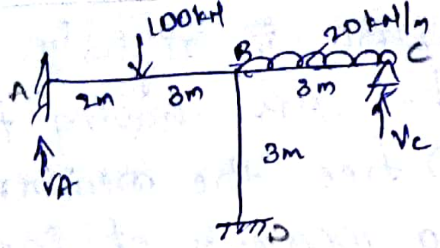
$M_{CB} = \cancel{6.67\times10^{-4}\ \text{kN-m}} = 0$ kN-m

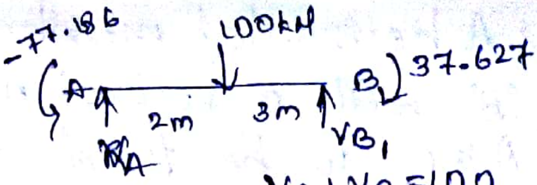
$M_{BD} = -8.644$ kN-m

$M_{DB} = -4.322$ kN-m



Step-4:-





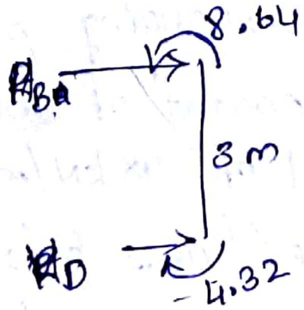
$V_A + V_B = 100$

$\Sigma M_A = 0$

$-V_{B_1} \times 5 + 100 \times 2 - 77.186 + 37.627 = 0$

$\boxed{V_{B_1} = 32.08\ kN}$

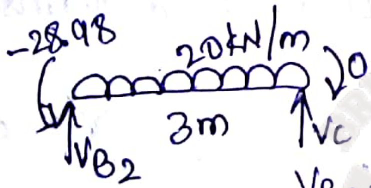
$\boxed{V_A = 67.91\ kN}$



$-H_D \times 3 - 8.644 - 4.32 = 0$

$\boxed{H_D = -4.32\ kN}$ $\quad H_B + H_D = 0$

$\boxed{H_B = 4.32\ kN}$



$V_{B_2} + V_C = 20 \times 3 = 60\ kN$

$-V_C \times 3 + 20 \times 3 \times 1.5 - 28.98 = 0$

$\boxed{V_C = 20.34\ kN}$ $\quad \boxed{V_{B_2} = 39.66\ kN}$

$V_B = 32.08 + 39.66$

$\boxed{V_B = 71.74\ kN}$

⇒ BM @ AB $= 67.91 \times 2 - 77.186 = 58.63\ kN\text{-}m$

BM @ BC $= 20.34 \times 1.5 - 20 \times 1.5 \times \frac{1.5}{2} = 8.01\ kN\text{-}m$

BM @ BD $= 0\ kN\text{-}m$

→ SF @ $A_R = 67.91\ kN$

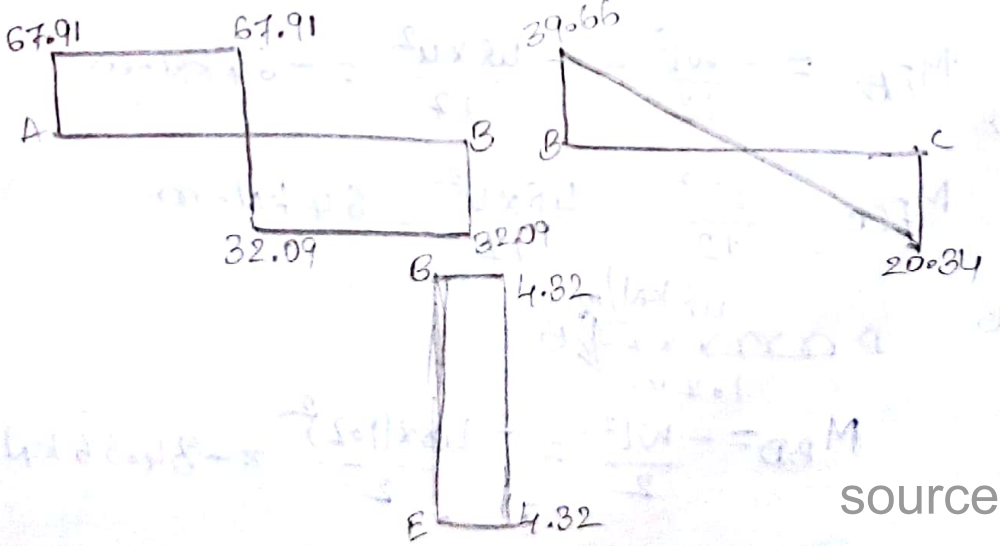
SF @ $B_L = 67.91 - 100 = -32.09\ kN$

SF @ $B_R = -20.34 + 20 \times 3 = 39.66\ kN$

SF @ $D_U = -4.32\ kN$

SF @ $B_D = -4.32\ kN$

SF @ $C_L = -20.34\ kN$



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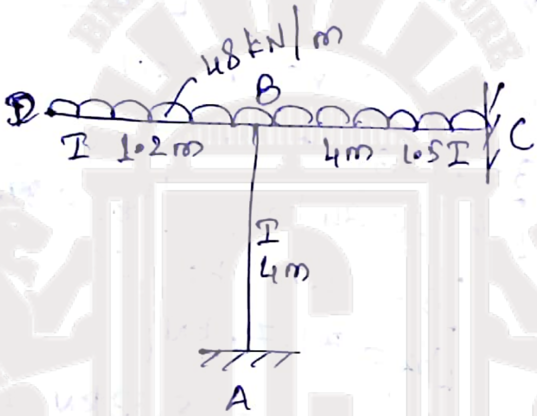
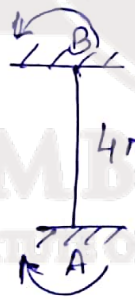
3. A horizontal member DBC is rigidly joint at 'B' with a vertical member AB. having the support A & C to be fixed & 'D' free the members AB & BC are 4m in length having moments of inertia, I an 1.5 I respectively, the overhanging portion BD is 1.2m long with the moment of inertia I. The member DBC carries a udl of 48 kN/m. Draw the BM-diagram of the frame & Calculate the vertical defl^n of the free end 'D' in terms of 'EI'.



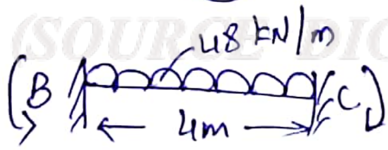
Step-1:- FEM's

* AB



$M_{FAB} = M_{FBA} = 0$

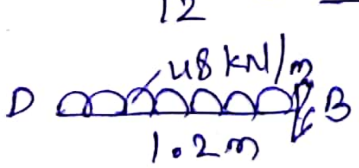
* BC



$$M_{FBC} = -\frac{WL^2}{12} = -\frac{48 \times 4^2}{12} = -64 \text{ kN-m}$$

$$M_{FCB} = \frac{WL^2}{12} = \frac{48 \times 4^2}{12} = 64 \text{ kN-m}$$

* DB



$$M_{BD} = -\frac{WL^2}{2} = -\frac{48 \times (1.2)^2}{2} = -34.56 \text{ kN-m}$$

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Step-2 :- Slope deflection eqns

$$* \; M_{AB} = \frac{2EI}{L}\left[2\cancel{\theta_A}^{0} + \theta_B - \cancel{\frac{3\delta}{L}}^{0}\right]$$

$$\boxed{M_{AB} = \frac{EI\theta_B}{2}}$$

$$* \; M_{BA} = \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_A}^{0} - \cancel{\frac{3\delta}{L}}^{0}\right]$$

$$\boxed{M_{BA} = EI\theta_B}$$

$$* \; M_{BC} = -64 + \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_C}^{0} - \cancel{\frac{3\delta}{L}}^{0}\right]$$

$$\boxed{M_{BC} = -64 + \frac{3}{2}EI\theta_B}$$

$$* \; M_{CB} = 64 + \frac{2EI}{L}\left[2\cancel{\theta_C}^{0} + \theta_B - \cancel{\frac{3\delta}{L}}^{0}\right]$$

$$\boxed{M_{CB} = 64 + \frac{3}{4}EI\theta_B}$$

$$* \; M_{BD} = -34.56$$

$$* \; M_{DB} = 0.$$

Consider 'B' is in equilibrium

$$\therefore M_{BA} + M_{BC} + M_{BD} = 0$$

$$EI\theta_B - 64 + \frac{3}{2}EI\theta_B - 34.56 = 0$$

$$\boxed{EI\theta_B = 39.424}$$

Step-3 :-

$M_{AB} = 19.712$ kN-m

$M_{BA} = 39.424$ kN-m

$M_{BC} = -4.864$ kN-m

$M_{CB} = 93.568$ kN-m

$M_{BD} = -34.56$ kN-m

$M_{DB} = 0$ kN-m

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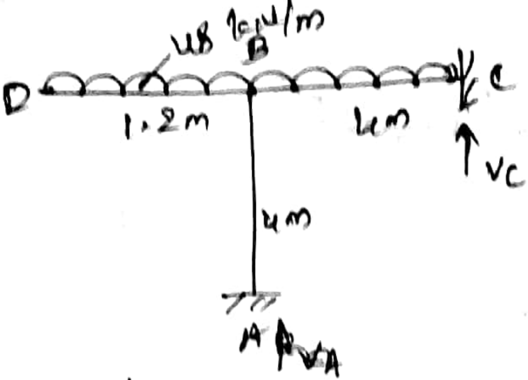
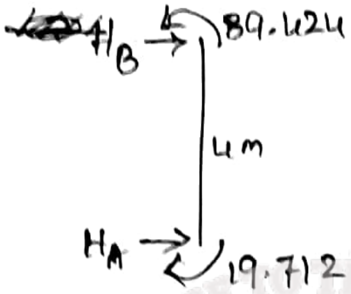
Step-4:-



* AB



$H_A + H_B = 0$

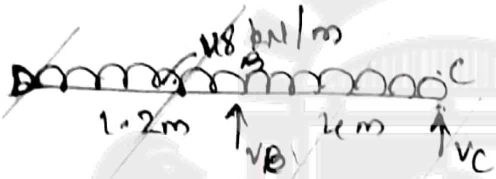
$\Sigma M_B = 0$

$-H_A \times 4 + 19.712 + 89.424 = 0$

$\boxed{H_A = 14.784\ kN}$

$\boxed{H_B = -14.784\ kN}$

*



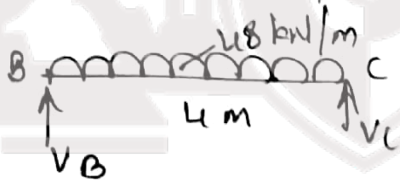
$V_C + V_B = 249.6\ kN$

$\Sigma M_C = 0$

$V_B \times 4 - 48 \times 5.2 \times \frac{5.2}{2} - 4.864 + 93.586 - 346 = 0$

$\boxed{V_B = 17.659\ kN}$

* BC



$V_B + V_C = 192\ kN$

$-V_C \times 4 + 48 \times 4 \times \frac{4}{2} - 4.864 + 93.586 = 0$

$\boxed{V_C = 118.18\ kN}$

$\boxed{V_B = 73.81\ kN}$

Step-5:

* BM @ AB = 0

* BM @ BC = $118.18 \times 2 - 48 \times 2 \times 1 = 140.36\ kN\text{-}m$

* BM @ BD =

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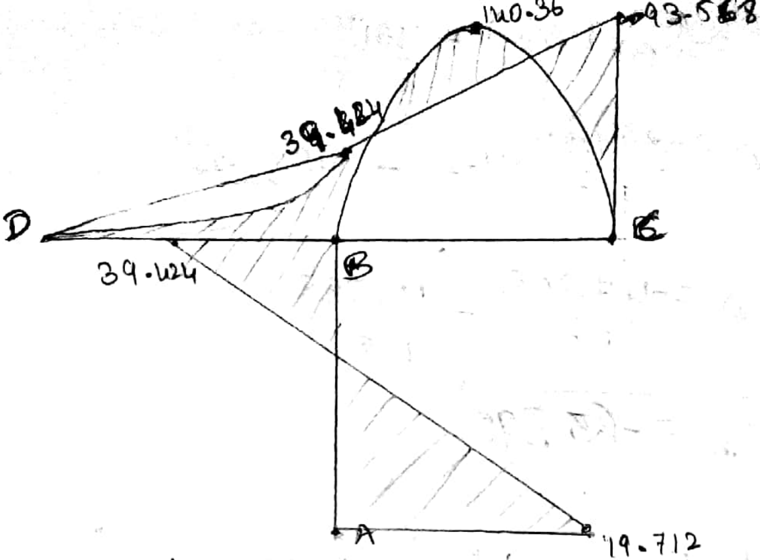


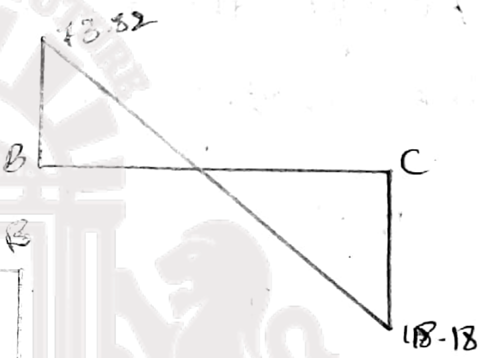
SF@ C_L = -118.18 kN

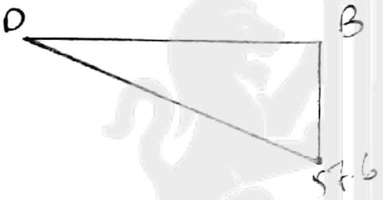
SF@ B_R = 118.18 - 48x4 = +73.82 kN

SF@ B_L = 48x1.2 = -57.6 kN

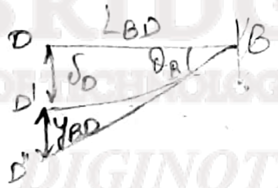
SF@ A_U = 14.784 kN

SF@ B_D = 14.784 kN.





Step-6: Vertical deflection at 'D'.



$$\tan\theta_B = \frac{DD''}{L_{BD}}$$

$$\theta_B \times L_{BD} = DD' + D'D''$$ = A

$$\theta_B \times L_{BD} = \delta_D + y_{BD} \rightarrow (1)$$

$$y_{BD} = \bar{x}\int_B^D \frac{M}{EI}dx$$

$$y_{BD} = \left[\frac{1}{2}\times L\times\frac{WL^2}{2EI}\right]\left[\frac{2}{3}\times L\right] = \frac{WL^4}{6EI}$$

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$$\delta_D = -(\theta_B \times L_{BD}) - \frac{wL^4}{6EI}$$

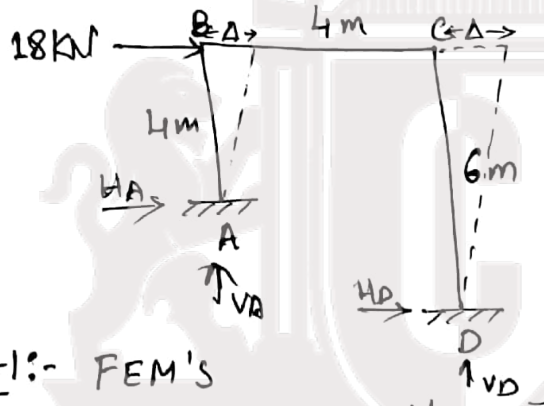
$$\delta_D = -\frac{89.424}{EI} \times 1.2 - \frac{48 \times (1.2)^4}{6EI}$$

$$\delta_D = -\frac{47.308}{EI} - \frac{16.588}{EI}$$

$$\boxed{\delta_D = -\frac{63.898}{EI}}$$

Analysis of Frames With Sway :-

1. Analyse the frame shown in the figure & draw the bending moment diagram.



Step-1 :- FEM's

$$M_{FAB} = M_{FBA} = M_{FBC} = M_{FCB} = M_{FCD} = M_{FDC} = 0$$

Step-2 :- SDE's

$$M_{AB} = \frac{2EI}{L}\left[2\cancel{\theta_A}^{0} + \theta_B - \frac{3\Delta}{L}\right]$$

$$\boxed{M_{AB} = 0.5EI\theta_B - 0.375EI\Delta}$$

$$M_{BA} = \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_A}^{0} - \frac{3\Delta}{L}\right]$$

$$\boxed{M_{BA} = EI\theta_B - 0.375EI\Delta}$$

$$M_{BC} = \frac{2EI}{L}\left[2\theta_B + \theta_C - \frac{3\Delta}{L}\right]$$

$$\boxed{M_{BC} = EI\theta_B + 0.5EI\theta_C - 0.375EI\Delta}$$

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* $M_{CB} = \frac{2EI}{L}\left[2\theta_C + \theta_B - \frac{3\Delta}{L}\right]$

$$\boxed{M_{CB} = 0.5EI\theta_B + EI\theta_C - 0.375EI\Delta}$$

* $M_{CD} = \frac{2EI}{L}\left[2\theta_C + \cancel{\theta_D} - \frac{3\Delta}{L}\right]$

$$\boxed{M_{CD} = 0.67EI\theta_C - 0.167EI\Delta}$$

* $M_{DC} = \frac{2EI}{L}\left[2\cancel{\theta_D} + \theta_C - \frac{3\Delta}{L}\right]$

$$\boxed{M_{DC} = 0.33EI\theta_C - 0.167EI\Delta}$$

∴ $M_{BA} + M_{BC} = 0$

$EI\theta_B - 0.375EI\Delta + EI\theta_B + 0.5EI\theta_C = 0$

$2EI\theta_B + 0.5EI\theta_C - 0.375EI\Delta = 0 \rightarrow (1)$

∴ $M_{CB} + M_{CD} = 0$

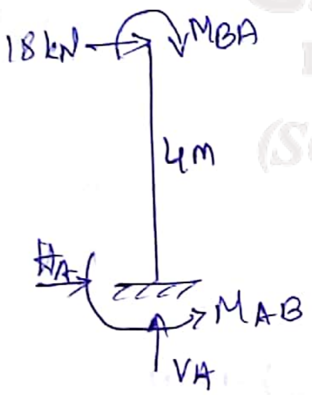
$0.5EI\theta_B + EI\theta_C + 0.67EI\theta_C - 0.167EI\Delta = 0$

$0.5EI\theta_B + 1.67EI\theta_C - 0.167EI\Delta = 0 \rightarrow (2)$

$\Sigma H = 0$

$H_A + H_D + 18 = 0 \rightarrow (3)$

$\Sigma M_B = 0$



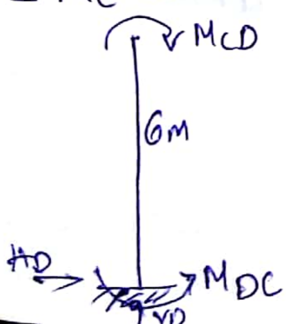
$-H_A \times 4 + M_{BA} + M_{AB} = 0$

$H_A = \frac{M_{BA} + M_{AB}}{4}$

$H_A = \frac{EI\theta_B - 0.375EI\Delta + 0.5EI\theta_B - 0.375EI\Delta}{4}$

$$\boxed{H_A = \frac{1.5EI\theta_B - 0.75EI\Delta}{4}}$$

$\Sigma M_C = 0$



$-H_D \times 6 + M_{DC} + M_{CD} = 0$

$H_D = \frac{0.33EI\theta_C - 0.167EI\Delta + 0.67EI\theta_C - 0.167EI\Delta}{6}$

$$\boxed{H_D = \frac{EI\theta_C - 0.334EI\Delta}{6}}$$

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$$eq^n(3) \Rightarrow \frac{1.5\,EI\theta_B - 0.75\,EI\Delta}{4} + \frac{EI\theta_C - 0.334\,EI\Delta}{6} + 18 = 0.$$

$$0.375\,EI\theta_B + 0.167\,EI\theta_C - 0.243\,EI\Delta = -18 \rightarrow (4)$$

from eqn (1), (2) & (4)

$\boxed{EI\theta_B = -18.68}$ $\boxed{EI\theta_C = 8.04}$ $\boxed{EI\Delta = 106.37}$

$M_{AB} = -30.548$ kN-m

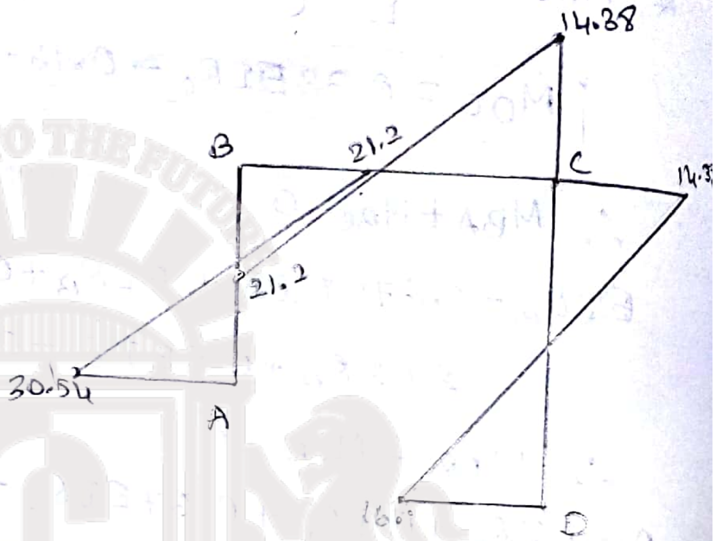
$M_{BA} = -21.208$ kN-m

$M_{BC} = 21.2$ kN-m

$M_{CB} = 14.38$ kN-m

$M_{CD} = -14.38$ kN-m

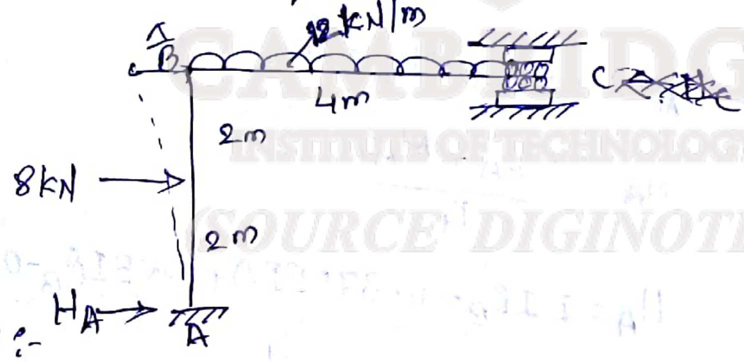
$M_{DC} = -16.1$ kN-m



$\boxed{H_A = -12.939\text{ kN}}$

$\boxed{H_D = -5.08\text{ kN}}$

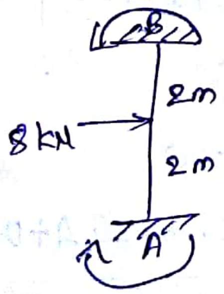
2. Analyse the frame shown in the figure & draw the BM diagram.



Step-1:-

FEM's

* AB



$$M_{FAB} = -\frac{PL}{8} = -\frac{8\times4}{8} = -4\text{ kN-m}$$

$$M_{FBA} = \frac{PL}{8} = \frac{8\times4}{8} = 4\text{ kN-m}$$

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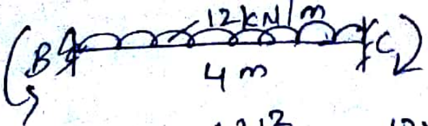
* BC 

$$M_{FBC} = -\frac{wL^2}{12} = -\frac{12\times4^2}{12} = -16 \text{ kN-m}$$

$$M_{FCB} = \frac{wL^2}{12} = \frac{12\times4^2}{12} = 16 \text{ kN-m.}$$

Step-2:- Slope deflection eq^ns

* $M_{AB} = -4 + \frac{2EI}{L}\left[2\cancel{\theta_A}^{\,0} + \theta_B - \frac{3\Delta}{L}\right]$

$$\boxed{M_{AB} = -4 + 0.5EI\theta_B - 0.375EI\Delta}$$

* $M_{BA} = 4 + \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_A}^{\,0} - \frac{3\Delta}{L}\right]$

$$\boxed{M_{BA} = 4 + EI\theta_B - 0.375EI\Delta}$$

* $M_{BC} = -16 + \frac{2EI}{L}\left[2\theta_B + \theta_C - \cancel{\frac{3\Delta}{L}}^{\,0}\right]$

$$\boxed{M_{BC} = -16 + EI\theta_B + 0.5EI\theta_C}$$

* $M_{CB} = 16 + \frac{2EI}{L}\left[2\theta_C + \theta_B - \cancel{\frac{3\Delta}{L}}^{\,0}\right]$

$$\boxed{M_{CB} = 16 + 0.5EI\theta_B + EI\theta_C} = 0 \rightarrow (1)$$

$\therefore M_{BA} + M_{BC} = 0$

$4 + EI\theta_B - 0.375EI\Delta - 16 + EI\theta_B + 0.5EI\theta_C = 0$

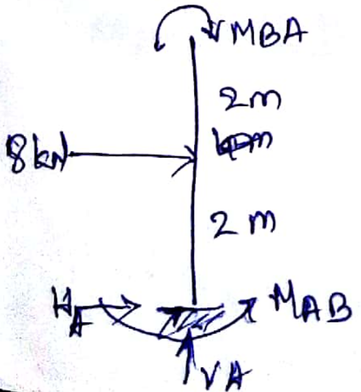
$2EI\theta_B + 0.5EI\theta_C - 0.375EI\Delta = 12 \rightarrow (2)$

$\Sigma H = 0.$

$H_A + 8 = 0$

$H_A = -8 \text{ kN} \rightarrow (3)$

$\Sigma M_B = 0$



$-H_A\times4 - 8\times2 + M_{AB} + M_{BA} = 0$

$$\boxed{H_A = \frac{M_{AB} + M_{BA}}{4} - 4}$$

$$H_A = \frac{-\cancel{4} + 0.5EI\theta_B - 0.375EI\Delta + \cancel{4} + EI\theta_B - 0.375EI\Delta}{4} - 4$$

$$H_A = \frac{1.5EI\theta_B - 0.75EI\Delta}{4} - 4$$

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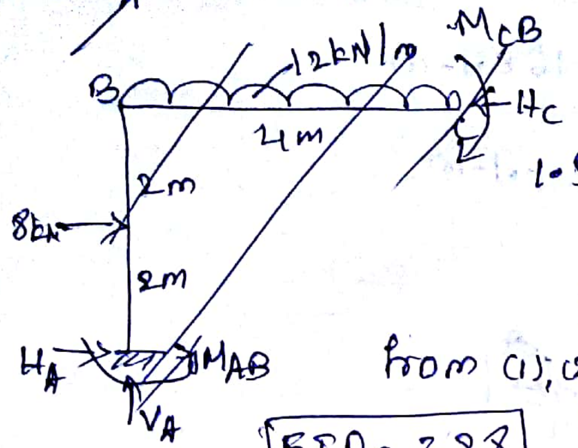
$\Sigma M_A = 0$



$$\frac{1.5EI\theta_B - 0.75EI\Delta}{4} = -8+4$$

$$1.5EI\theta_B - 0.75EI\Delta = -16 \rightarrow (4)$$

from (1), (2) & (4)

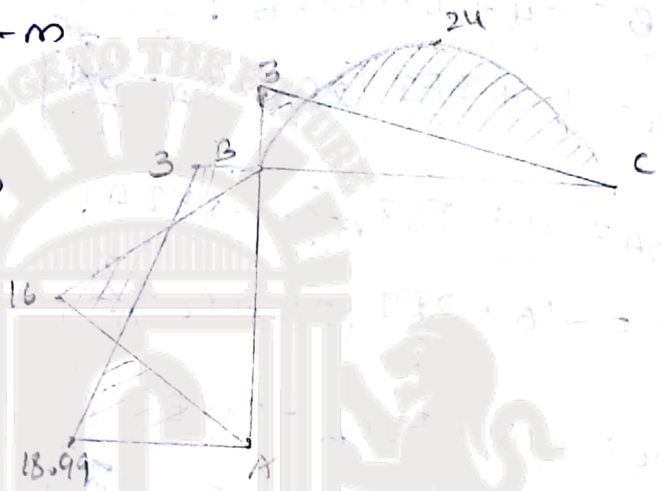
$\boxed{EI\theta_B = 28}$ $\boxed{EI\theta_C = -30}$ $\boxed{EI\Delta = 77.33}$

$M_{AB} = -18.99$ kN-m

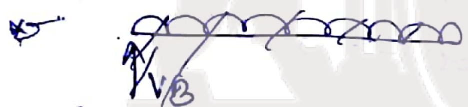
$M_{BA} = 3$ kN-m

$M_{BC} = -3$ kN-m

$M_{CB} = 0$ kN-m

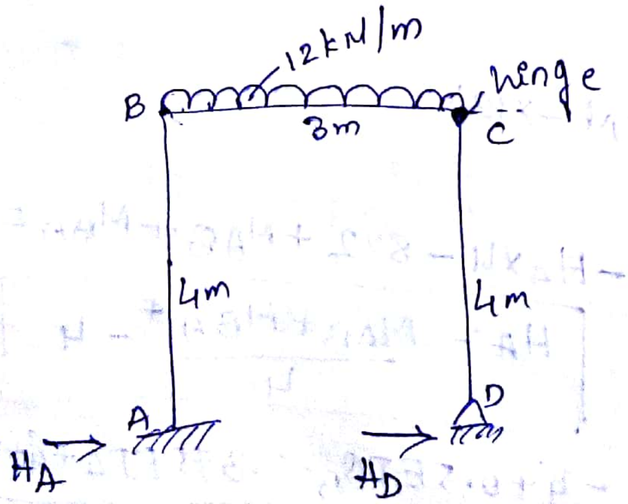


$\boxed{H_A = 8.005 \text{ kN}}$



* BM @ BC = 12×2 = 24 kN-m
* BM @ AB = 8×2 = −16 kN-m

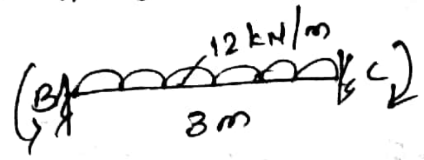
3. Analyse the portal frame as shown in the figure also draw the BMD.



FEM's

* AB $M_{FAB} = M_{FBA} = 0$

* BC



$$M_{FBC} = -\frac{WL^2}{12} = -\frac{12\times3^2}{12} = -9 \text{ kN-m}$$

$$M_{FCB} = \frac{WL^2}{12} = \frac{12\times3^2}{12} = 9 \text{ kN-m}$$

* CD $M_{FCD} = M_{FDC} = 0$

slope deflection eq^ns

* $M_{AB} = \frac{2EI}{L}\left[2\cancel{\theta_A}^{0} + \theta_B - \frac{3\Delta}{L}\right]$

$$\boxed{M_{AB} = 0.5EI\theta_B - 0.375EI\Delta}$$

* $M_{BA} = \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_A}^{0} - \frac{3\Delta}{L}\right]$

$$\boxed{M_{BA} = EI\theta_B - 0.375EI\Delta}$$

* $M_{BC} = \frac{2EI}{L}\left[2\theta_B + \theta_C - \cancel{\frac{3\Delta}{L}}^{0}\right]$

$$\boxed{M_{BC} = 1.33EI\theta_B + 0.67EI\theta_C - 9}$$

* $M_{CB} = \frac{2EI}{L}\left[2\theta_C + \theta_B - \cancel{\frac{3\Delta}{L}}^{0}\right]$

$$\boxed{M_{CB} = 0.67EI\theta_B + 1.33EI\theta_C + 9}$$

* $M_{CD} = \frac{2EI}{L}\left[2\theta_C + \theta_D - \frac{3\Delta}{L}\right]$

$$\boxed{M_{CD} = EI\theta_C + 0.5\cancel{EI\theta_D}^{0} - 0.375EI\Delta} = 0$$

* $M_{DC} = \frac{2EI}{L}\left[2\theta_D + \theta_C - \frac{3\Delta}{L}\right]$

$$\boxed{M_{DC} = 0.5EI\theta_C + \cancel{EI\theta_D}^{0} - 0.375EI\Delta} = 0 \rightarrow (1)$$

$\therefore$ $M_{BA} + M_{BC} = 0$

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$EI\theta_B - 0.375EI\Delta + 1.33EI\theta_B + 0.67EI\theta_C - 9 = 0$

$2.33EI\theta_B + 0.67EI\theta_C - 0.375EI\Delta = 9 \rightarrow (2)$

$\therefore M_{CB} + M_{CD} = 0$

$0.67EI\theta_B + 1.33EI\theta_C + 9 + EI\theta_C + 0.5EI\cancel{\theta_D}^{\to 0} - 0.375EI\Delta = 0$

$0.67EI\theta_B + 2.33EI\theta_C + 0.5EI\cancel{\theta_D}^{\to 0} - 0.375EI\Delta = -9 \rightarrow (3)$

from eq^n (1)

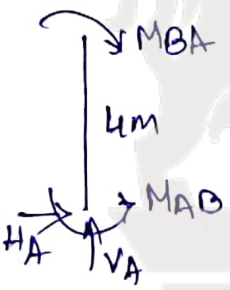
$EI\cancel{\theta_D}^{\to 0} = 0.375EI\Delta - 0.5EI\theta_C$

eq^n(3) $0.67EI\theta_B + 2.33EI\theta_C + 0.1875EI\Delta - 0.25EI\theta_C - 0.375EI\Delta = -9$

$0.67EI\theta_B + 2.08EI\theta_C - 0.1875EI\Delta = -9 \rightarrow (4)$

$\Sigma H = 0$

$H_A + H_D = 0$

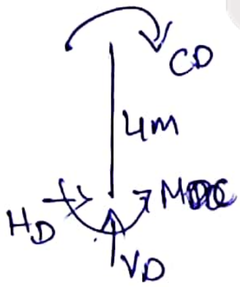


$\Sigma M_B = 0$

$-H_A \times 4 + M_{AB} + M_{BA} = 0$

$H_A = \dfrac{0.5EI\theta_B + 0.375EI\Delta + EI\theta_B - 0.375EI\Delta}{4}$

$\boxed{H_A = \dfrac{1.5EI\theta_B - 0.75EI\Delta}{4}} \rightarrow (5)$



$-H_D \times 4 + M_{DC} + M_{CD} = 0$

$H_D = \dfrac{0.5EI\theta_C - 0.375EI\Delta + EI\theta_C - 0.375EI\Delta}{4}$

$\boxed{H_D = \dfrac{1.5EI\theta_C - 0.75EI\Delta}{4}} \leq 0.$

$0.375EI\theta_B + 0.375EI\cancel{\theta_C}^{\to 0} - 0.375EI\Delta = 0 \rightarrow (\cancel{5})$

from eq^n (2), (3) & (5)

$\boxed{EI\theta_B = 10.89}$ $\boxed{EI\theta_C = -12.254}$ $\boxed{EI\Delta = 21.786}$

$M_{AB} = -2.724$ kN-m

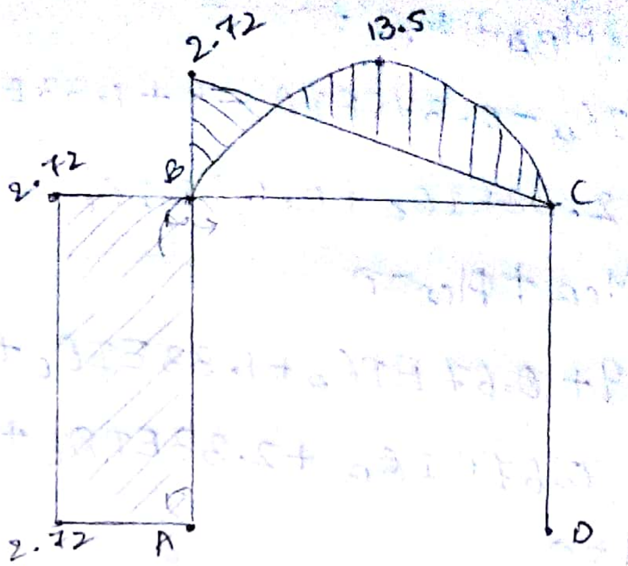
$M_{BA} = 2.72$ kN-m

$M_{BC} = -2.72$ kN-m

$M_{CB} = 0$ kN-m

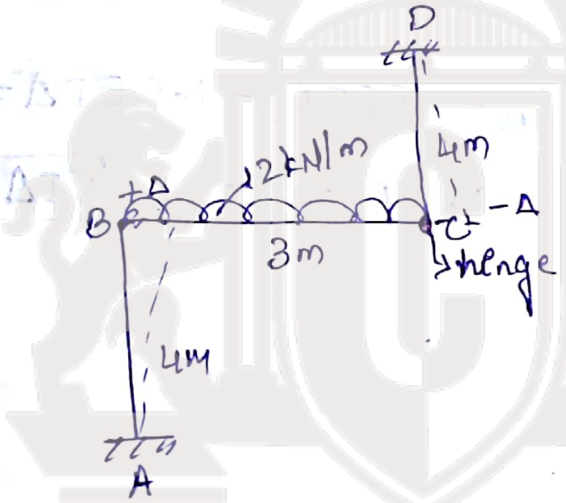
$M_{CD} = 0$ kN-m

$M_{DC} = 0$ kN-m



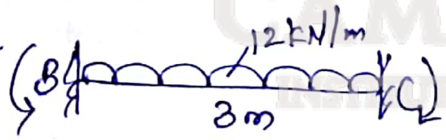
$BM@BC = \frac{12 \times 1.5 \times 1.5}{2} = 13.5$ kN-m

4. Analyse the portal frame as shown in the figure also draw the BMD.



FEM's

* BC



$M_{FBC} = -\frac{WL^2}{12} = -9$ kN-m

$M_{FCB} = 9$ kN-m

Slope deflection eq^{ns}.

* $M_{AB} = 0.5EI\theta_B - 0.375EI\Delta$
* $M_{BA} = EI\theta_B - 0.375EI\Delta$
* $M_{BC} = -9 + 1.33EI\theta_B + 0.67EI\theta_C$
* $M_{CB} = 9 + 0.67EI\theta_B + 1.33EI\theta_C = 0$
* $M_{CD} = EI\theta_C + 0.375EI\Delta = 0$
* $M_{DC} = 0.5EI\theta_C + 0.375EI\Delta$

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$\therefore M_{BA} + M_{BC} = 0$

$EI\theta_B - 0.375EI\Delta - 9 + 1.33EI\theta_B + 0.67EI\theta_C = 0$

$2.33EI\theta_B + 0.67EI\theta_C - 0.375EI\Delta = 9 \rightarrow (1)$

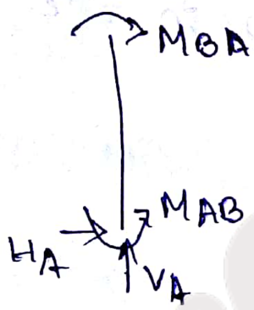
$\therefore M_{CB} + M_{CD} = 0$

$9 + 0.67EI\theta_B + 1.33EI\theta_C + EI\theta_C + 0.375EI\Delta = 0$

$0.67EI\theta_B + 2.33EI\theta_C + 0.375EI\Delta = -9 \rightarrow (2)$

$\Sigma H = 0$

$H_A + H_D = 0$



$H_A = 1.5EI\theta_B - 0.75EI\Delta$

$H_D = 1.5EI\theta_C - 0.75EI\Delta$

$1.5EI\theta_B - 1.5EI\theta_C - 1.5EI\Delta = 0 \rightarrow (3)$

$\boxed{EI\theta_B = 9.89}$ $\boxed{EI\theta_C = -9.89}$ $\boxed{EI\Delta = 19.78}$

$M_{AB} = -2.4725$ kN-m

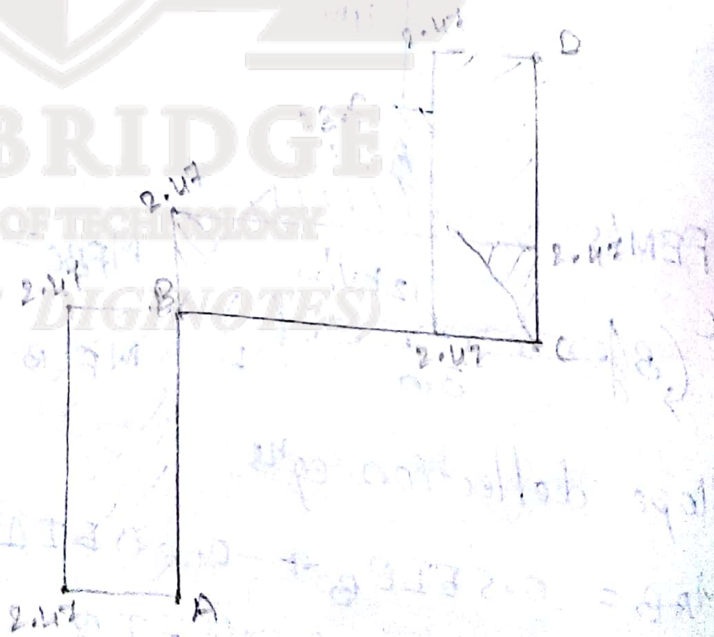
$M_{BA} = 2.4725$ kN-m

$M_{BC} = -2.47$ kN-m

$M_{CB} = 2.4725$ kN-m

$M_{CD} = -2.4725$ kN-m

$M_{DC} = 2.4725$ kN-m



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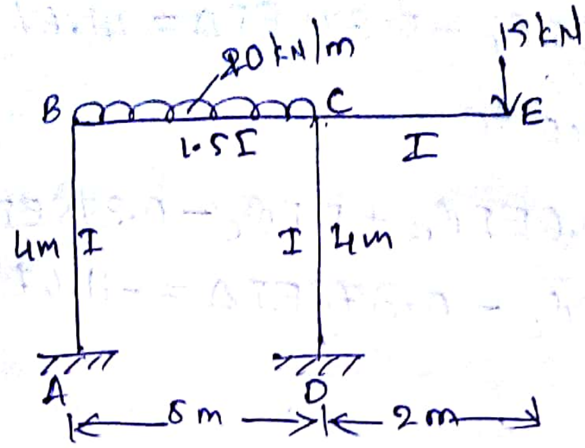
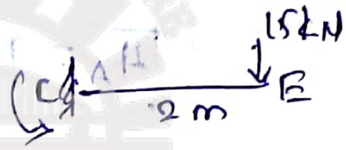
5. Analyse the portal frame as shown in the figure & draw the BMD.



$M_{AB} = 6.5$
$M_{BA} = 18.5$
$M_{BC} = -18.5$
$M_{CB} = 44.83$
$M_{DC} = -10.16$
$M_{CE} = -30$
$M_{CD} = -14.84$

FEM's

* $M_{FBC} = -41.67$ kN-m
* $M_{FCB} = 41.67$ kN-m
* $M_{CE} = -30$ kN-m



Slope deflection eqns.

* $M_{AB} = \frac{2EI}{L}\left[2\cancel{\theta_A}^{0} + \theta_B - \frac{3\Delta}{L}\right]$

$$\boxed{M_{AB} = 0.5EI\theta_B - 0.375EI\Delta}$$

* $M_{BA} = \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_A}^{0} - \frac{3\Delta}{L}\right]$

$$\boxed{M_{BA} = EI\theta_B - 0.375EI\Delta}$$

* $M_{BC} = -41.67 + \frac{2EI}{L}\left[2\theta_B + \theta_C - \cancel{\frac{3\Delta}{L}}^{0}\right]$

$$\boxed{M_{BC} = -41.67 + 1.2EI\theta_B + 0.6EI\theta_C}$$

* $M_{CB} = 41.67 + \frac{2EI}{L}\left[2\theta_C + \theta_B - \cancel{\frac{3\Delta}{L}}^{0}\right]$

$$\boxed{M_{CB} = 41.67 + 0.6EI\theta_B + 1.2EI\theta_C}$$

* $M_{CD} = \frac{2EI}{L}\left[2\theta_C + \cancel{\theta_D}^{0} - \frac{3\Delta}{L}\right]$

$$\boxed{M_{CD} = EI\theta_C - 0.375EI\Delta}$$

* $M_{DC} = \frac{2EI}{L}\left[2\cancel{\theta_D}^{0} + \theta_C - \frac{3\Delta}{L}\right]$

$$\boxed{M_{DC} = 0.5EI\theta_C - 0.375EI\Delta}$$

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$\therefore M_{BA} + M_{BC} = 0$

$EI\theta_B - 0.375EI\Delta - 41.67 + 1.2EI\theta_B + 0.6EI\theta_C = 0$

$2.2EI\theta_B + 0.6EI\theta_C - 0.375EI\Delta = 41.67 \rightarrow (1)$

$\therefore M_{CB} + M_{CD} + M_{CE} = 0$

$41.67 + 1.2EI\theta_C + 0.6EI\theta_B + EI\theta_C - 0.375EI\Delta - 30 = 0$

$0.6EI\theta_B + 2.2EI\theta_C - 0.375EI\Delta = -11.67 \rightarrow (2)$

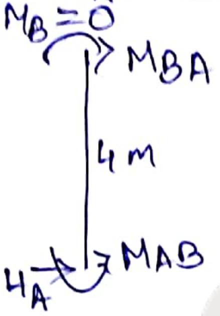
$\Sigma H = 0$

$H_A + H_D = 0$

$\Sigma M_B = 0$

$-H_A \times 4 + 0.5EI\theta_B - 0.375EI\Delta + EI\theta_B - 0.375EI\Delta = 0$

$\boxed{H_A = 1.5EI\theta_B - 0.75EI\Delta}$



$\Sigma M_C = 0$

$-H_D \times 4 + EI\theta_C - 0.375EI\Delta + 0.5EI\theta_C - 0.375EI\Delta = 0$

$\boxed{H_D = 1.5EI\theta_C - 0.75EI\Delta}$

$\therefore 1.5EI\theta_B + 1.5EI\theta_C - 1.5EI\Delta = 0 \rightarrow (3)$

from eqn (1), (2) & (3)

$\boxed{EI\theta_B = 23.98}$ $\boxed{EI\theta_C = -9.35}$ $\boxed{EI\Delta = 14.63}$

$M_{AB} = 6.5$ kN-m

$M_{BA} = 18.5$ kN-m

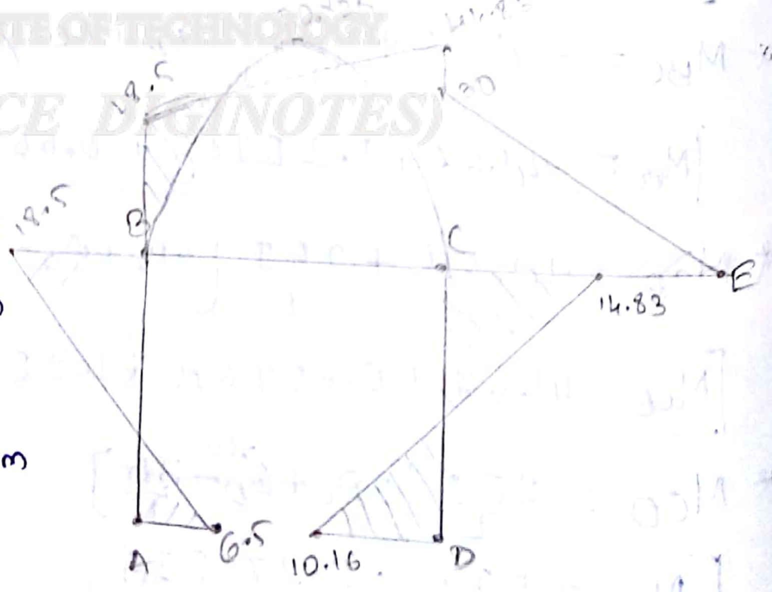
$M_{BC} = -18.5$ kN-m

$M_{CB} = 44.83$ kN-m

$M_{DC} = -10.16$ kN-m

$M_{CD} = -14.83$ kN-m

$M_{CE} = -30$ kN-m



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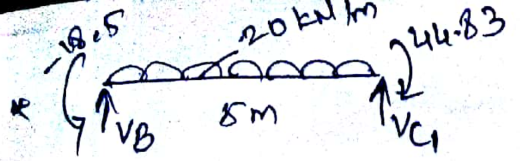
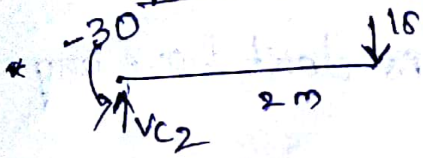


$V_B + V_{C_1} = 100\,kN$

$-V_{C_1} \times 5 + 20 \times 5 \times \frac{5}{2} - 18.5 + 44.83 = 0$

$\boxed{V_{C_1} = 55.26\,kN}$

$\boxed{V_B = 44.73\,kN}$



$V_{C_2} \times 2 - 30 = 0$

$\boxed{V_{C_2} = 15\,kN}$ $\boxed{V_C = 70.26\,kN}$

* BM @ BC $= 44.73 \times 2.5 - 20 \times 2.5 \times \frac{2.5}{2} - 18.5 = 30.825\,kN\text{-}m$

8/9/18 Summary :- Slope deflection method

Step-1 :- degree of kinematic indeterminacy.

Step-2 :- divide the beam.

Step-3 :- fixed end moments.

Step-4 :- Slope deflection equations.

Step-5 :- finding unknowns. or angles

Step-6 :- reactions & bending moments

Step-7 :- bending moment diagram.

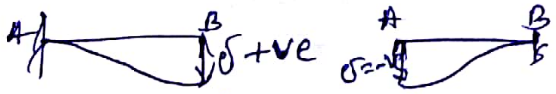
\# Continuous beam :-

=> without settlement

* $\delta = 0$.
* The beam is fixed. $\therefore \theta = 0$ at that point
* for cantilever beam,  $M_{AB} = -WL$

=> with settlement

* δ - exists.



* $\delta = 0$ at fixed ends

* δ = relative settlement.

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Frames:-

⇒ frames without sway

* $\Delta = 0$
* $\theta = 0$ at the fixed ends
* The frames without sway is don't have any horizontal force.

⇒ frames with sway

* Δ value depends on the horizontal force acting on the frame.
* $\Delta = 0$ in horizontal member of a frame
* Δ will changes in vertical members of the frame.
* If hinge is connected in frame is considered as frame with sway & the moments at that point is equal to zero.
*