Date 24/08/08

Structural Analysis - II

(a) Sign Convention

(1) Reaction

$\Sigma V = 0$, ↑ +ve ↓ -ve

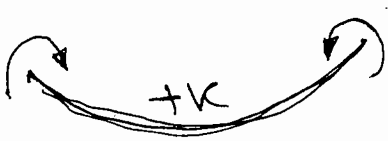
$\Sigma H = 0$, → +ve ← -ve

$\Sigma M = 0$, ↻ +ve ↺ -ve

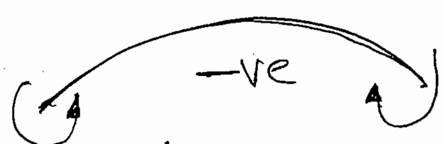
(2) Shear Force

From "Left" to "Right" ↑ +ve ↓ -ve

(3) Bending Moment



Sagging

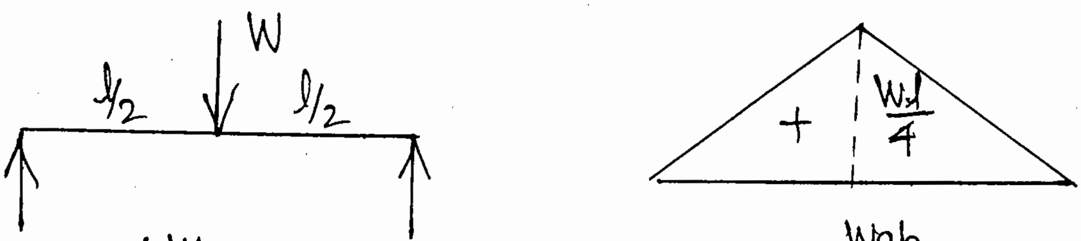


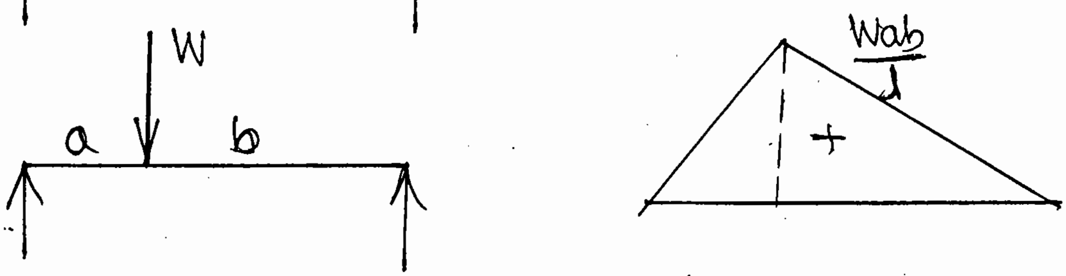
Hogging

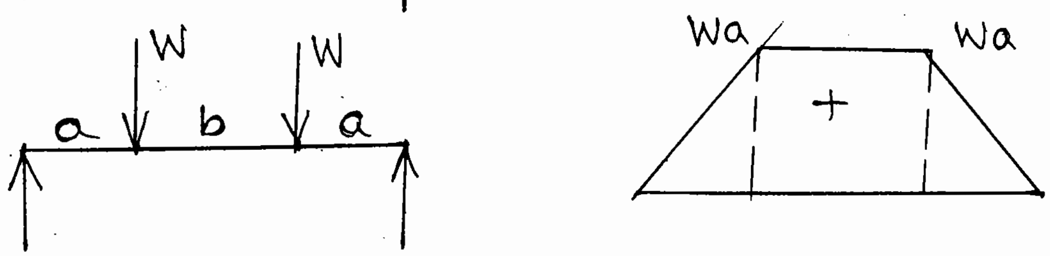
Or Clockwise Moment → +ve

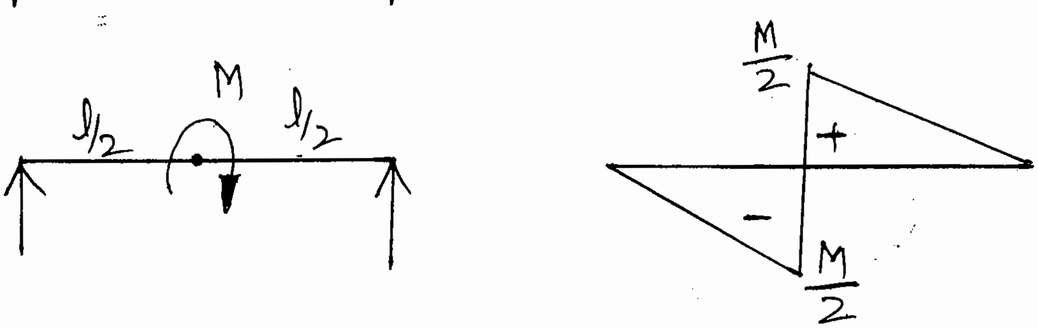
Anti-clockwise " → -ve

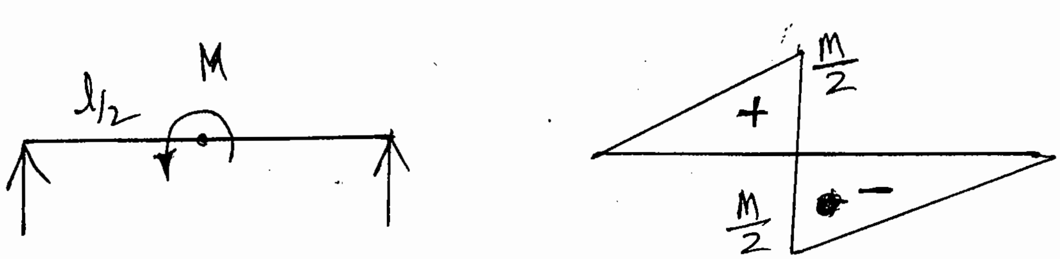
(b) BMD: (Free BMD)

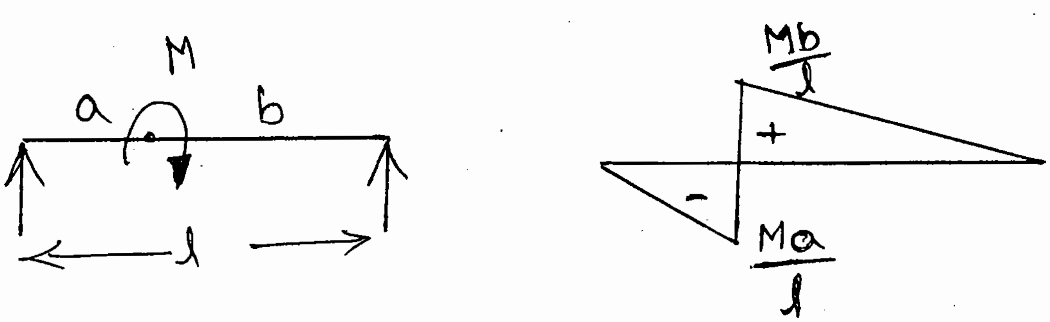




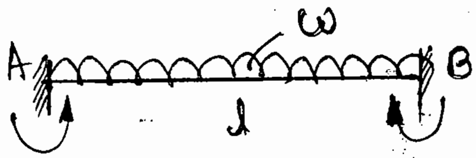




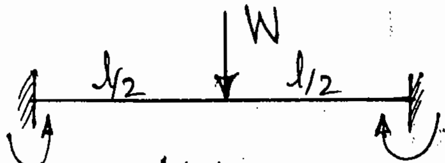




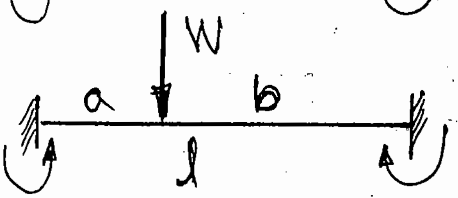
(c) Fixed End Moments :-



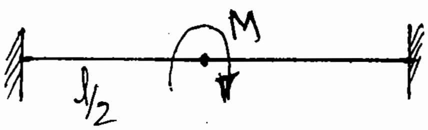
$$M_{FAB} = -\frac{\omega l^2}{12}, \quad M_{FBA} = +\frac{\omega l^2}{12}$$



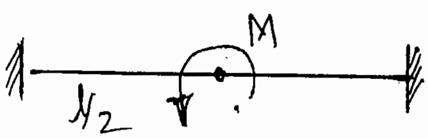
$$M_{FAB} = -\frac{Wl}{8}, \quad M_{FBA} = +\frac{Wl}{8}$$



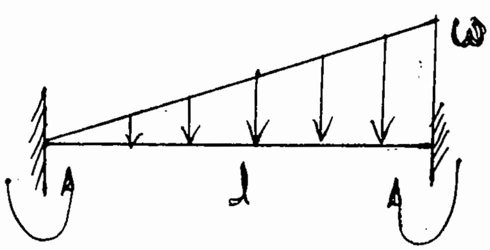
$$M_{FAB} = -\frac{Wab^2}{l^2}, \quad M_{FBA} = +\frac{Wa^2b}{l^2}$$



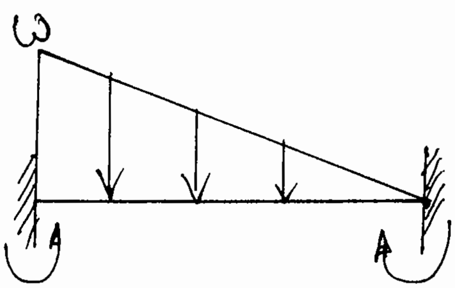
$$M_{FAB} = M_{FBA} = +\frac{M}{4}$$



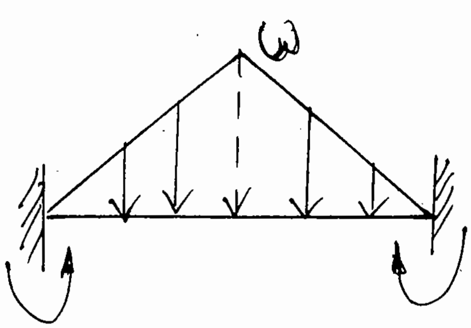
$$M_{FAB} = M_{FBA} = -\frac{M}{4}.$$



$$M_{FAB} = -\frac{\omega l^2}{30}, \quad M_{FBA} = +\frac{\omega l^2}{20}$$

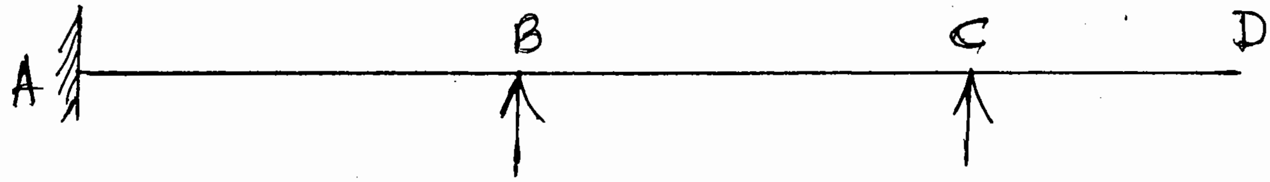


$$M_{FAB} = -\frac{\omega l^2}{20}, \quad M_{FBA} = +\frac{\omega l^2}{30}$$



$$M_{FAB} = -\frac{5\omega l^2}{96} \qquad M_{FBA} = +\frac{5\omega l^2}{96}$$

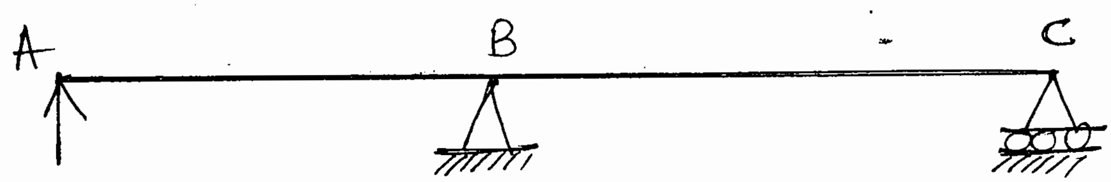
(d) (1) Intermediate Support :



If there is a span on both sides of supports then it is called "intermediate" support

∴ 'B' & 'C' → Intermediate

(2) Last Support :



If span is on one side only, then it is called last simple or hinge or Roller support.

∴ At last support $\boxed{\text{Moment} = 0}$

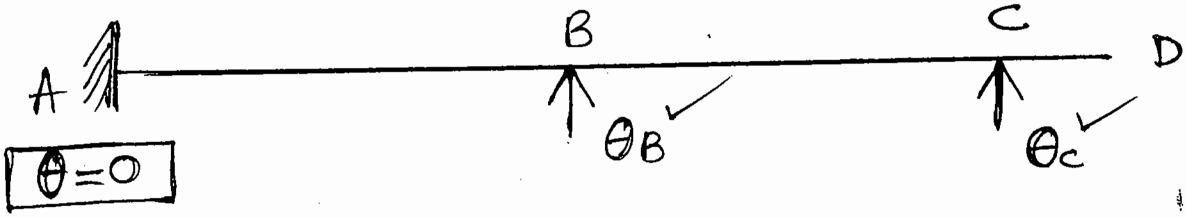
(3) Equillibrium Condition :-

At intermediate support, $\boxed{\text{Sum of Moment} = 0}$

At "B" $\boxed{M_{BA} + M_{BC} = 0}$

At "C" $\boxed{M_{CB} + M_{CD} = 0}$

(I) Slope Deflection Method :



Basic Equation

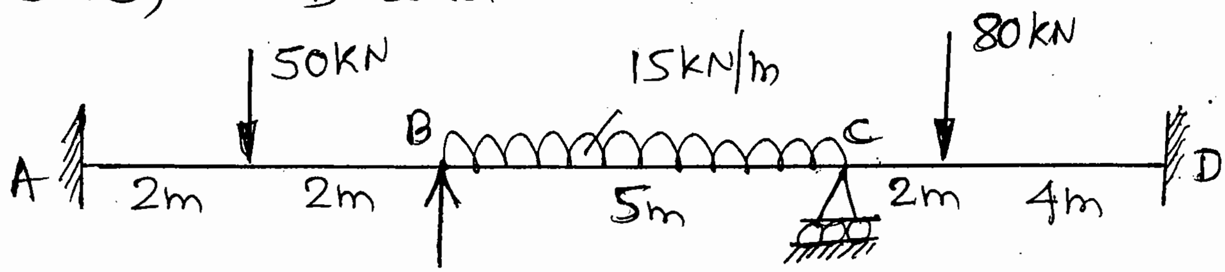
$$\boxed{\begin{aligned} M_{AB} &= \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right] + M_{FAB} \\ M_{BA} &= \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3\delta}{l}\right] + M_{FBA} \end{aligned}}$$

$\theta \rightarrow$ Slope or Rotation

$\delta \rightarrow$ Sinking or Settlement

$EI \rightarrow$ Flexural Rigidity.

Eg:-1] Analyse the continuous beam shown by S.D. method and draw BMD; SFD and EC.



(EI) → Constant

Sol^n

(a) FEM

$$M_{FAB} = -\frac{Wl}{8} = \frac{-50\times4}{8} = -25 \text{ kN-m}$$

$$M_{FBA} = +\frac{Wl}{8} = +25 \text{ kN-m}$$

$$M_{FBC} = -\frac{wl^2}{12} = -\frac{15\times5^2}{12} = -31.25$$

$$M_{FCB} = +\frac{wl^2}{12} = +31.25$$

$$M_{FCD} = -\frac{Wab^2}{l^2} = -\frac{80\times2\times4^2}{6^2} = -71.11 \text{ kN-m}$$

$$M_{FDC} = +\frac{Wa^2b}{l^2} = \frac{80\times2^2\times4}{6^2} = +35.56 \text{ kN-m}$$

(b) S.D. Equation:

$\theta_A = \theta_D = 0$ ($\because$ Fixed Support)

$\delta = 0$ ($\because$ No sinking)

$$\boxed{M_{AB} = \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right] + M_{FAB}}$$

$$M_{AB} = \frac{2EI}{4}\left[0 + \theta_B - 0\right] - 25 = 0.5EI\theta_B - 25 \quad \text{—(i)}$$

$$M_{BA} = \frac{2EI}{4}\left[2\theta_B + 0 - 0\right] + 25 = EI\theta_B + 25 \quad \text{—(ii)}$$

$$M_{BC} = \frac{2EI}{5}\left[2\theta_B + \theta_C - 0\right] - 31.25$$

$$= 0.8EI\theta_B + 0.4EI\theta_C - 31.25 \quad \text{—(iii)}$$

$$M_{CB} = \frac{2EI}{5}\left[2\theta_C + \theta_B - 0\right] + 31.25$$

$$= 0.8EI\theta_C + 0.4EI\theta_B + 31.25 \quad \text{—(iv)}$$

$$M_{CD} = \frac{2EI}{6}\left[2\theta_C + 0 - 0\right] - 71.11 = 0.667EI\theta_C - 71.11 \quad \text{—(v)}$$

$$M_{DC} = \frac{2EI}{6}\left[0 + \theta_C - 0\right] + 35.56 = 0.333EI\theta_C + 35.56 \quad \text{—(vi)}$$

(C) Equillibrium Condition :-

(1) At "B" $\boxed{M_{BA} + M_{BC} = 0}$ → Intermediate support

$$\left[EI\theta_B + 25\right] + \left[0.8EI\theta_B + 0.4EI\theta_C - 31.25\right] = 0$$

$$1.8EI\theta_B + 0.4EI\theta_C = 6.25 \rightarrow \text{(I)}$$

(2) At "C" $\boxed{M_{CB} + M_{CD} = 0}$

$$\left[0.8EI\theta_C + 0.4EI\theta_B + 31.25\right] + \left[0.667EI\theta_C - 71.11\right] = 0$$

$$0.4EI\theta_B + 1.467EI\theta_C = 39.86 \rightarrow \text{(II)}$$

Solving

$$\theta_B = -2.73/EI$$

$$\theta_C = +27.91/EI$$

(d) <u>Final Moments</u> [Substitute 'θ' values in eq^n (i) to (vi)]

$M_{AB} = -26.36$ kN-m

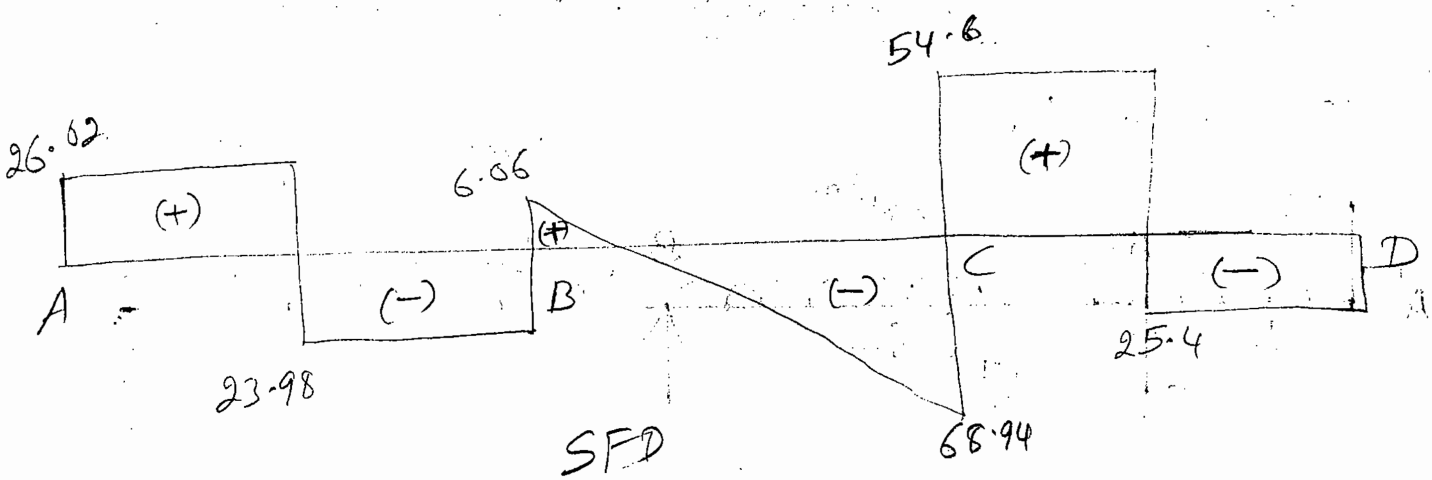
$M_{CB} = 52.48$ kN-m

$M_{BA} = 22.27$ kN-m

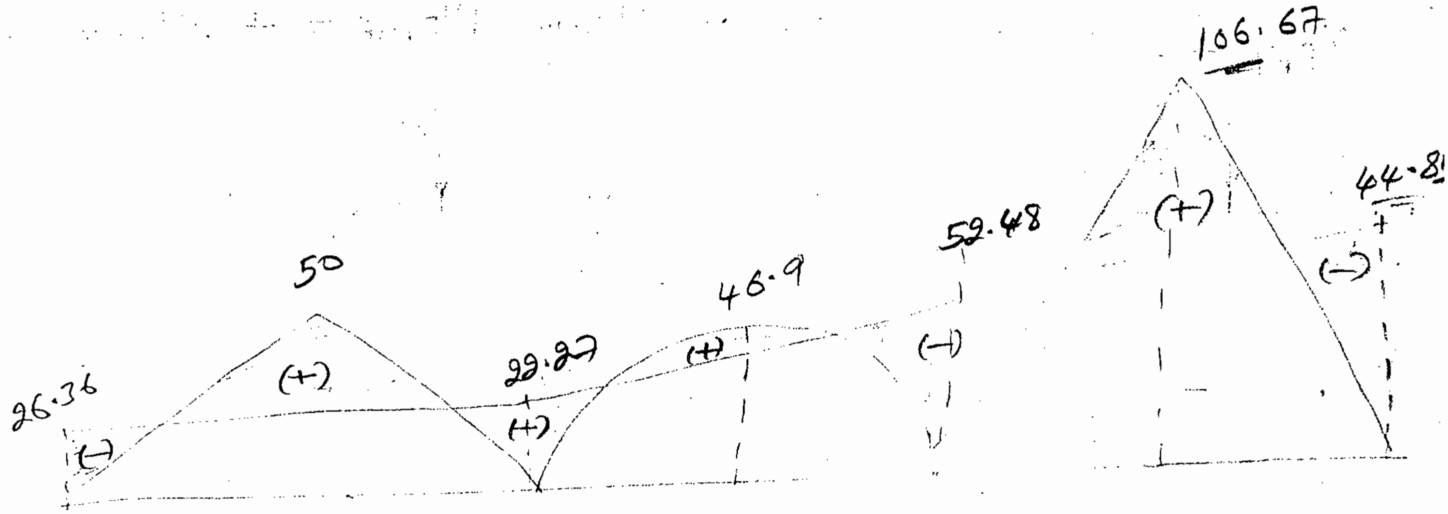
$M_{CD} = -52.49$ kN-m

$M_{BC} = -22.27$ kN-m

$M_{DC} = 44.85$ kN-m

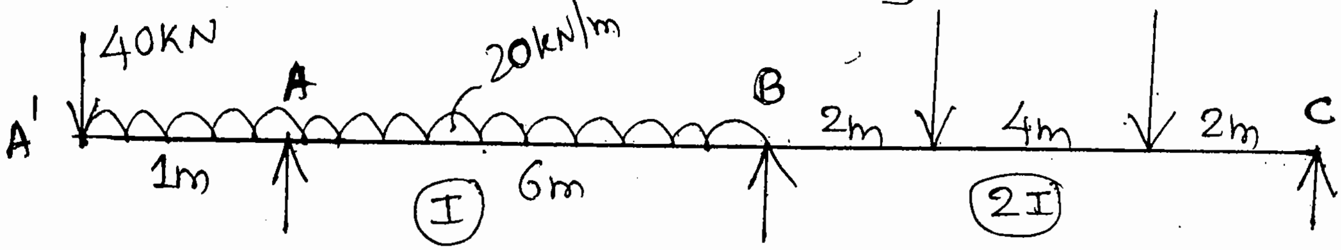

26.62
(+)
6.06
(+)
54.6
(+)
A
(−)
B
(−)
C
D
(−)
23.98
25.4
68.94

SFD


106.67
44.81
(+)
52.48
50
46.9
(−)
(−)
(+)
22.27
(+)
26.36
(−)
(+)
(+)

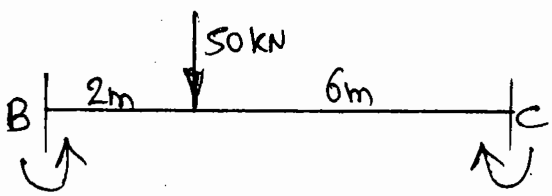
BMD

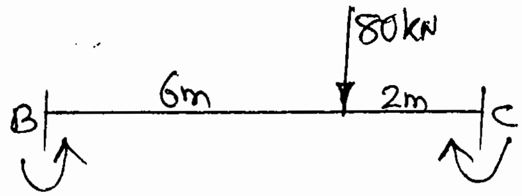
Eg:-2] Analyse the continuous beam shown by <u>S.D. method</u>. And draw BMD, SFD and E.C.



Sol^n (a) <u>FEM</u>

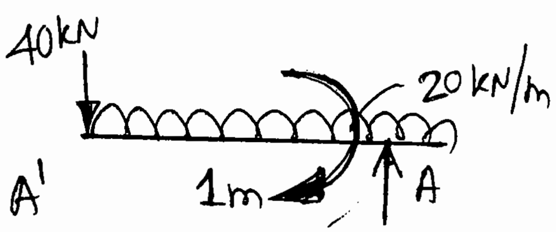
$$M_{FAB} = -\frac{wl^2}{12} = -\frac{20\times6^2}{12} = -60\,kN\text{-}m, \quad M_{FBA} = +60\,kN\text{-}m$$





$$M_{FBC} = -\frac{Wab^2}{l^2} = -\left[\frac{50\times2\times6^2}{8^2} + \frac{80\times6\times2^2}{8^2}\right] = -86.25\,kN\text{-}m$$

$$M_{FCB} = +\frac{Wa^2b}{l^2} = +\left[\frac{50\times2^2\times6}{8^2} + \frac{80\times6^2\times2}{8^2}\right] = +108.75\,kN\text{-}m$$



$$M_{AA'} = +\left[40\times1 + 20\times1\times\tfrac{1}{2}\right]$$

$$= +50\,kN\text{-}m$$

+ve sign for <u>clockwise</u> <u>resisting</u> <u>moment</u>.

(b) S.D. Equation

$\delta = 0$ (No sinking)

$$\boxed{M_{AB} = \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right] + M_{FAB}}$$

$$M_{AB} = \frac{2(1\times EI)}{6}\left[2\theta_A + \theta_B - 0\right] - 60$$

$$= (0.667EI)\theta_A + (0.333EI)\theta_B - 60 \quad \text{—(i)}$$

$$M_{BA} = \frac{2(1\times EI)}{6}\left[2\theta_B + \theta_A - 0\right] + 60$$

$$= (0.667EI)\theta_B + (0.333EI)\theta_A + 60 \quad \text{—(ii)}$$

$$M_{BC} = \frac{2(2EI)}{8}\left[2\theta_B + \theta_C - 0\right] - 86.25$$

$$= EI\theta_B + 0.5EI\theta_C - 86.25 \quad \text{—(iii)}$$

$$M_{CB} = \frac{2(2EI)}{8}\left[2\theta_C + \theta_B - 0\right] + 108.75$$

$$= EI\theta_C + 0.5EI\theta_B + 108.75 \quad \text{—(iv)}$$

★ **Note:-** There is **NO** S.D. Equation for "overhang"

(c) Equillibrium condition :-

(i) At "A" $\boxed{M_{AA'} + M_{AB} = 0}$

$$[50] + [0.667EI\theta_A + 0.333EI\theta_B - 60] = 0$$

$$(0.667EI)\theta_A + (0.333EI)\theta_B = 10 \rightarrow \text{(I)}$$

(ii) At "B" $\boxed{M_{BA} + M_{BC} = 0}$

$$(0.333\,EI)\theta_A + (1.667\,EI)\theta_B + (0.5\,EI)\theta_C = 26.25 \rightarrow \text{(II)}$$

(iii) At "C" $\boxed{M_{CB} = 0}$ ($\because$ last Simple or Hinge or Roller support)

$$(0.5\,EI)\theta_B + (1\,EI)\theta_C = -108.75 \longrightarrow \text{(III)}$$

Solving

$$\theta_A = \frac{-15.197}{EI} \qquad \theta_C = \frac{-138.98}{EI}$$

$$\theta_B = \frac{60.47}{EI}$$

(d) Final Moment:

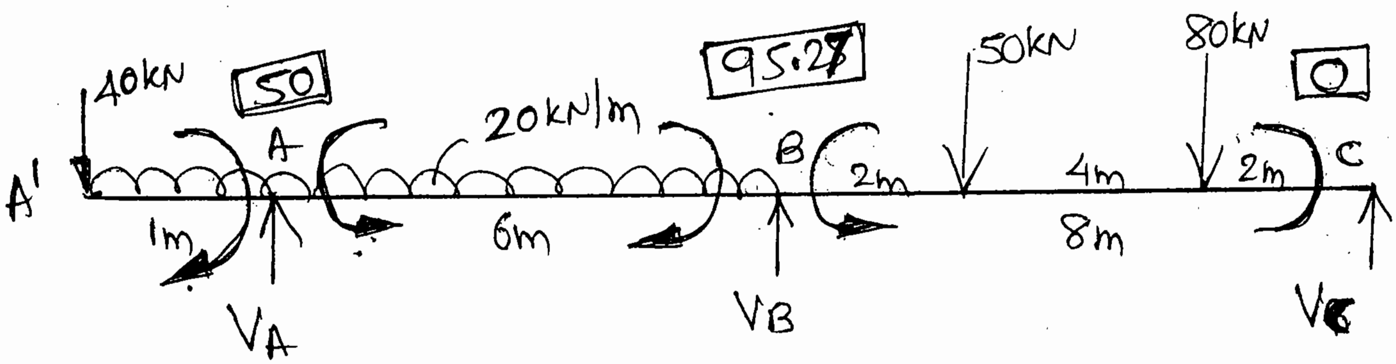
$M_{AB} = -50$ kN-m

$M_{BA} = 95.27$ kN-m

$M_{BC} = -95.27$ "

$M_{CB} = 0$

$M_{AA'} = +50$ kN-m



$\sum V = 0,\quad V_A + V_B + V_C = 40 + 50 + 80 + 20 \times 7 = 310 \; -\text{(i)}$

$\sum M_B = 0 \text{ (RHS)} \quad -V_C \times 8 + 50 \times 2 + 80 \times 6 - 95.27 = 0$

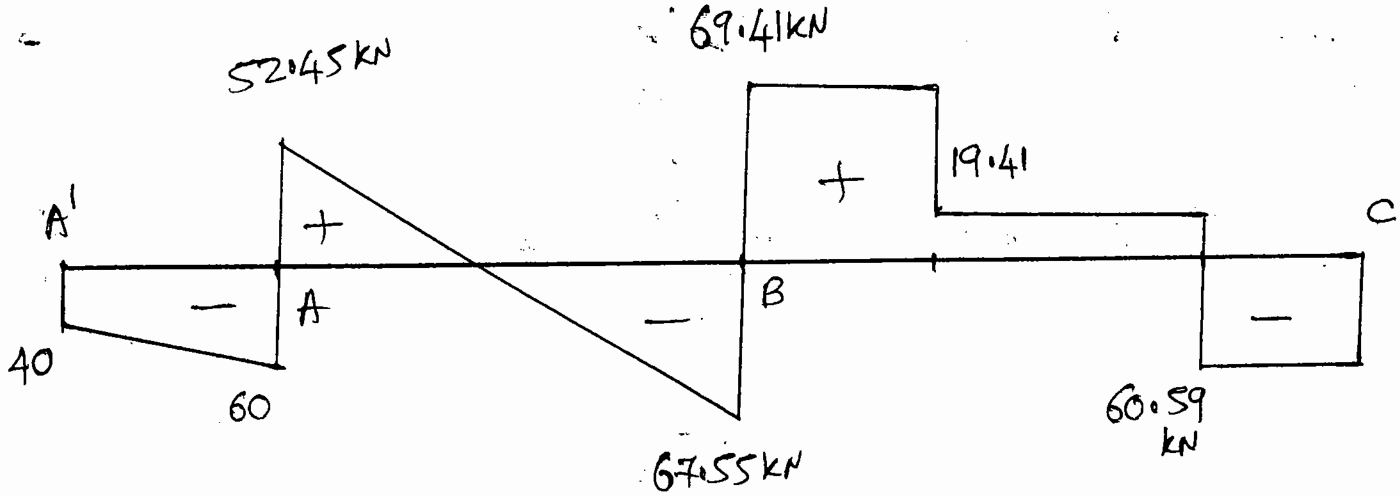
$V_C = 60.59$ kN

$$\Sigma M_B = 0 \ (LHS)$$

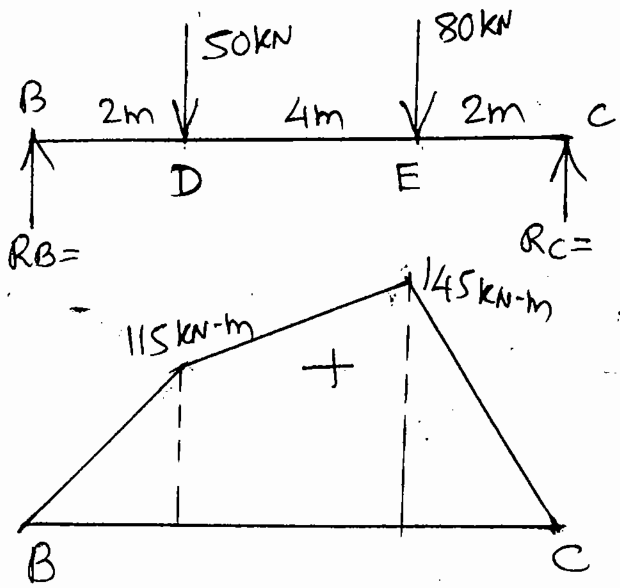
$$V_A \times 6 - 40 \times 7 - 20 \times 7 \times 7/2 + 50 - 50 + 95.27 = 0$$

$$V_A = 112.45 \text{ kN}$$

From (i) $112.45 + V_B + 60.59 = 310$ $\quad \therefore V_B = 136.96$



Free BMD for "BC"



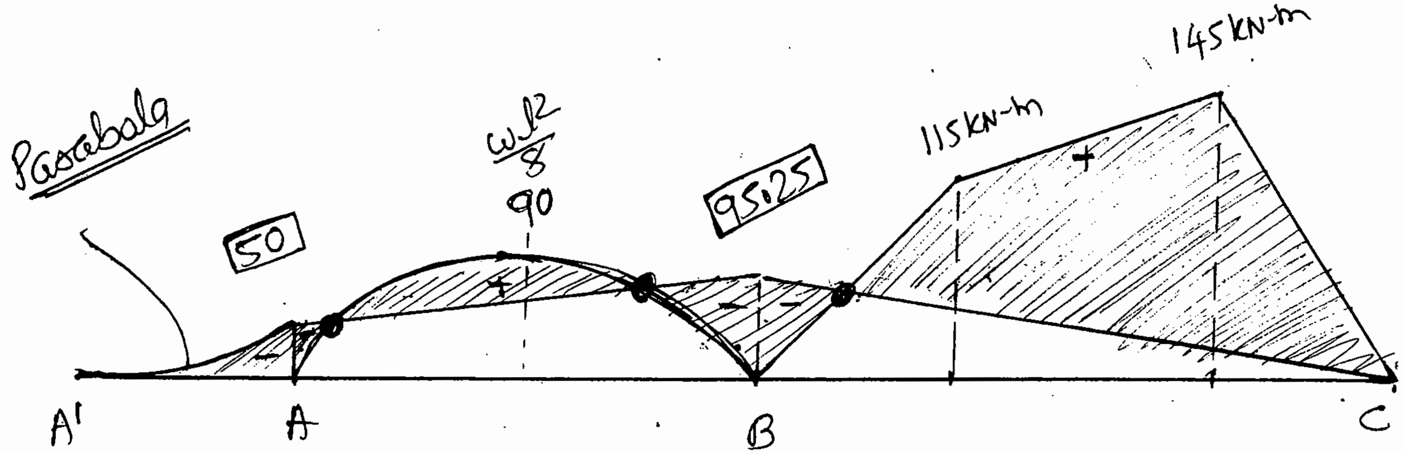
Reaction

$R_C = 72.5$ kN

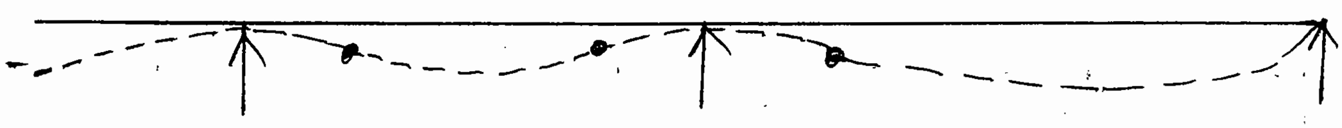
$R_B = 57.5$ kN

$\therefore M_D = R_B \times 2 = 115$ kN-m

$M_E = R_C \times 2 = 145$ kN-m



EC



EC

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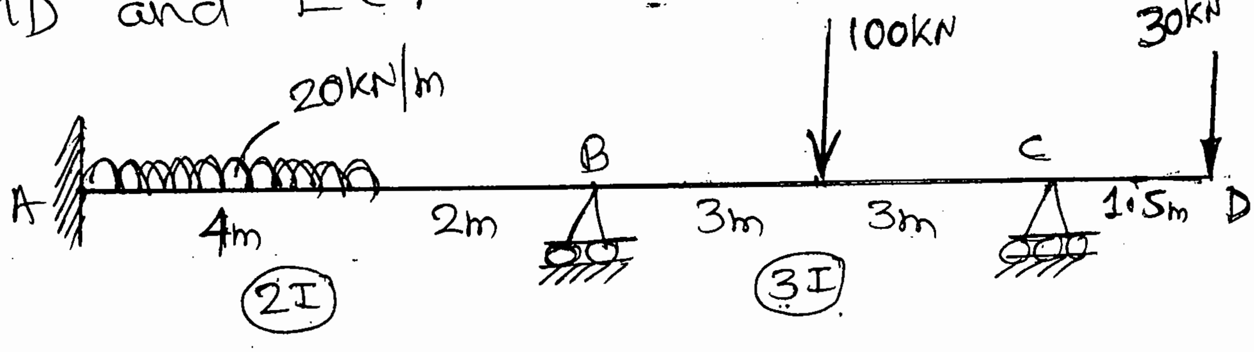
Eg:- 3 Analyse the continuous beam shown by S.D. method Draw SFD, BMD and EC.

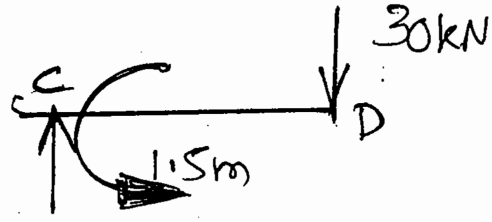


Sol^n

(a) FEM

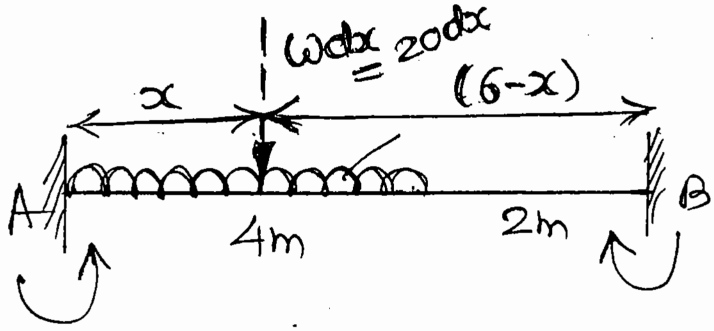
$$M_{FBC} = -\frac{Wl}{8} = -\frac{100 \times 6}{8} = -75 \text{ kN-m}$$

$$M_{FCB} = +\frac{Wl}{8} = +75 \text{ kN-m}$$



$$M_{CD} = -30 \times 1.5 = -45 \text{ kN-m}$$

-ve sign for "Anticlockwise" Resisting Moment



$W = \omega \cdot dx = 20dx$

$a = x$

$b = (6-x)$

$l = 6m$

$$M_{FAB} = -\frac{Wab^2}{l^2} = -\int_0^4 \frac{(20dx)(x)(6-x)^2}{6^2} = -53.33 \text{ kN-m}$$

$$M_{FBA} = +\frac{Wa^2b}{l^2} = +\int_0^4 \frac{(20dx)(x)^2(6-x)}{6^2} = +35.56 \text{ kN-m}$$

(b) S.D. Equation :

$\theta_A = 0$ ($\because$ fixed)

$\delta = 0$ ($\because$ No sinking)

For overhang there is NO SD equation

$$M_{AB} = \frac{2(2EI)}{6}\left[0 + \theta_B + 0\right] - 53.33$$

$$= 0.667\,EI\,\theta_B - 53.33 \quad \text{—(i)}$$

$$M_{BA} = \frac{2(2EI)}{6}\left[2\theta_B + 0 - 0\right] + 35.56$$

$$= 1.333\,EI\,\theta_B + 35.56 \quad \text{—(ii)}$$

$$M_{BC} = \frac{2(3EI)}{6}\left[2\theta_B + \theta_C - 0\right] - 75$$

$$= 2EI\theta_B + EI\theta_C - 75 \quad \text{—— (III)}$$

$$M_{CB} = \frac{2(3EI)}{6}\left[2\theta_C + \theta_B - 0\right] + 75$$

$$= 2EI\theta_C + EI\theta_B + 75 \longrightarrow \text{(IV)}$$

(c) Equillibrium Condition

(i) At "B" $\boxed{M_{BA} + M_{BC} = 0}$

$$3.333EI\theta_B + EI\theta_C = 39.44 \longrightarrow \text{(I)}$$

(ii) At 'C' $\boxed{M_{CB} + M_{CD} = 0}$

$$EI\theta_B + 2EI\theta_C = -30 \longrightarrow \text{(II)}$$

Solving $\theta_B = 19.21/EI$

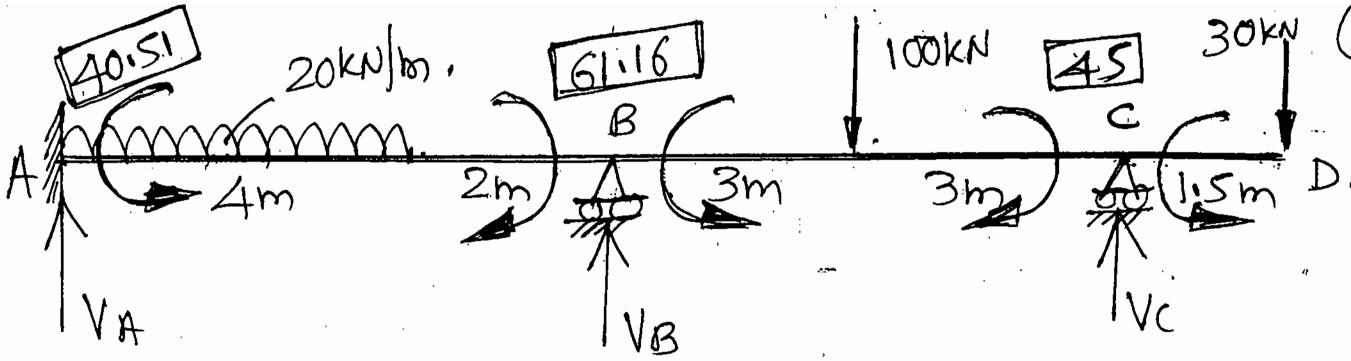
$\theta_C = -24.61/EI$

(d) Final Moment

$M_{AB} = -40.51$ kN-m ↻ $\quad M_{BC} = -61.18$ kN-m ↻

$M_{BA} = 61.16$ kN-m ↺ $\quad M_{CB} = 45$ kN-m ↺

$M_{CD} = -45$ kN-m ↻



$\Sigma V = 0, \quad 20 \times 4 + 100 + 30 = V_A + V_B + V_C$

$\therefore \quad V_A + V_B + V_C = 210 \longrightarrow (i)$

$\Sigma M_B = 0$, (LHS)

$V_A \times 6 - 20 \times 4 \times \left(\frac{4}{2} + 2\right) - 40.51 + 61.16 = 0$

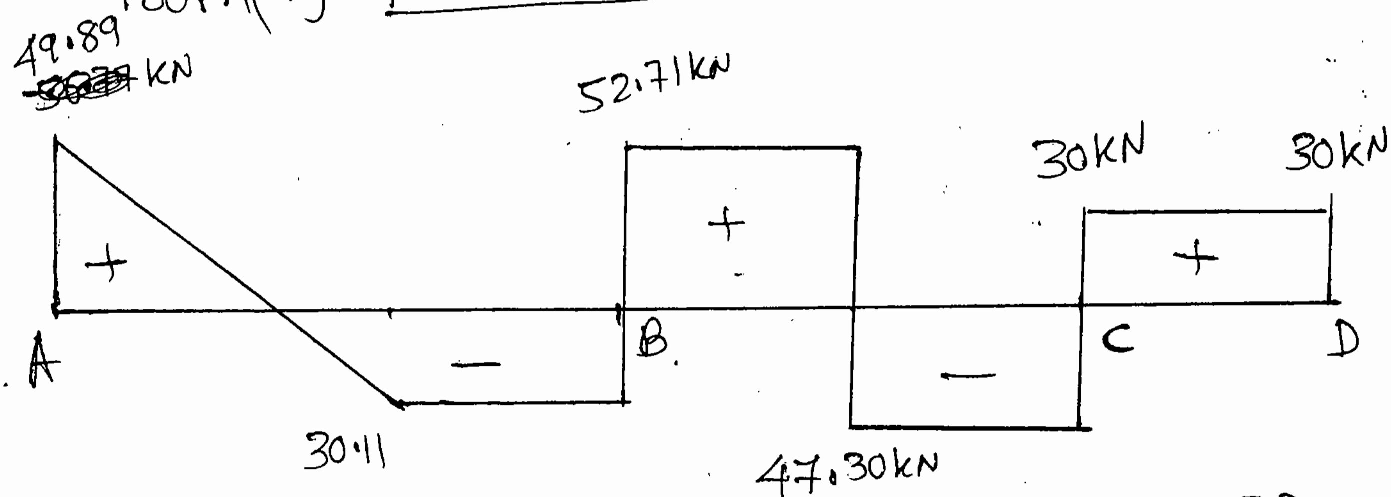
$\boxed{V_A = \cancel{56.77}} \; \boxed{49.89}$ kN

$\Sigma M_B = 0$ (RHS)

$-V_C \times 6 + 100 \times 3 + 30 \times 7.5 - 61.16 + 45 - 45 = 0$

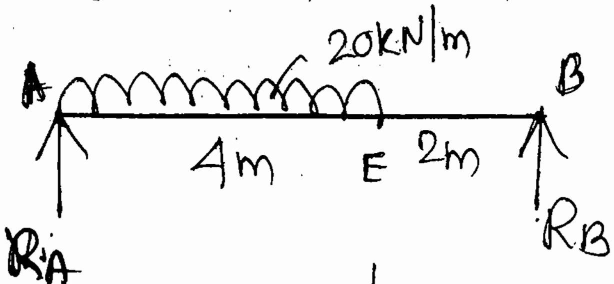
$\boxed{V_C = 77.30 \text{ kN}}$

from (i) $\boxed{V_B = \cancel{78.93}}$ 82.81 kN



SFD

Free BMD for AB

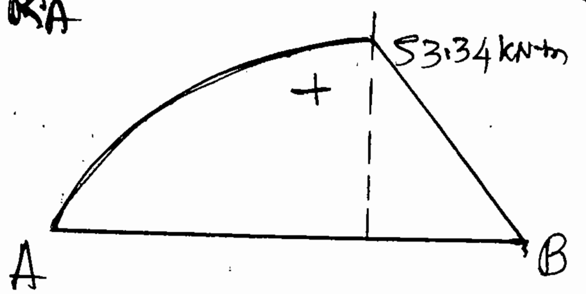


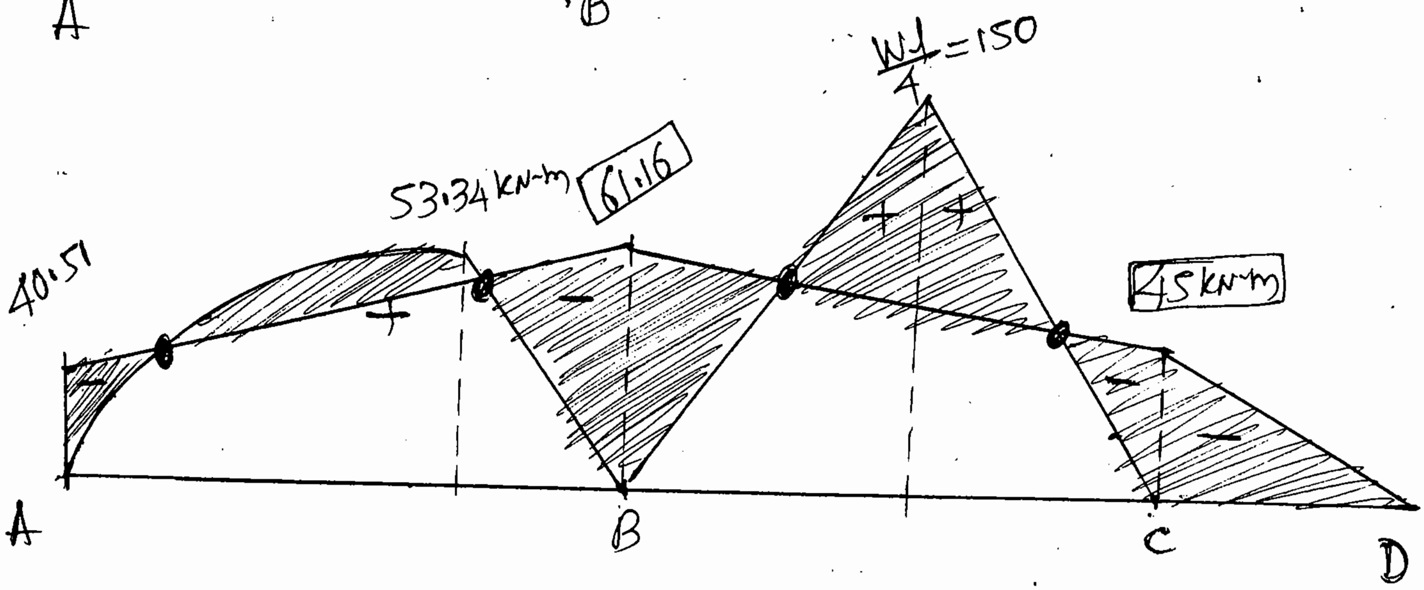
$R_A = 53.33\,kN$

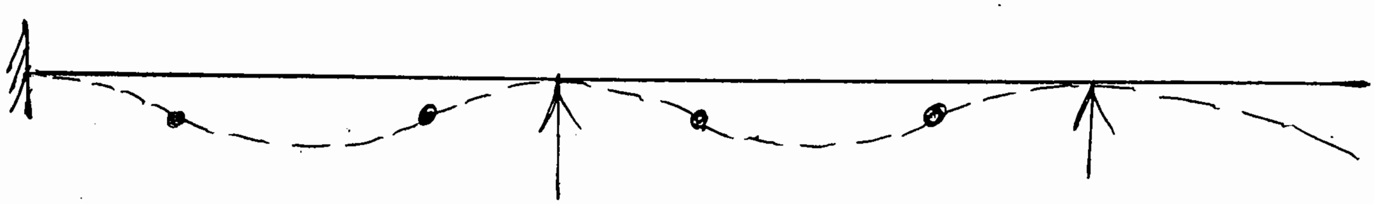
$R_B = 26.67\,kN$

$\therefore M_E = R_B \times 2$

$= 53.34\,kN\text{-}m$





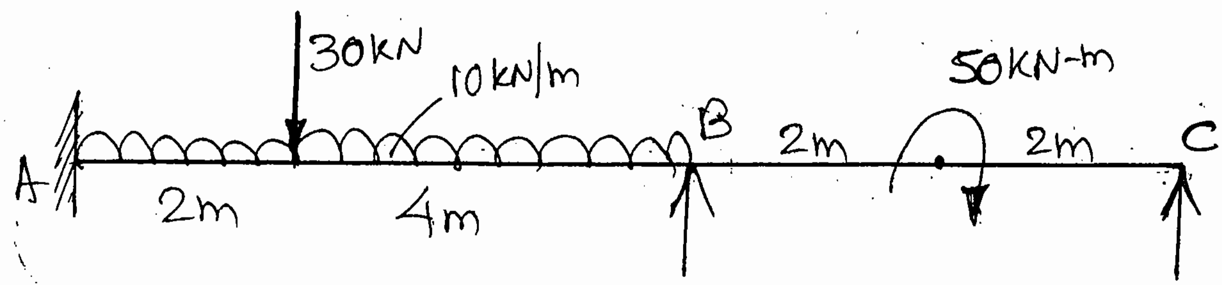


—— × ——

Sinking of Support

Eg:- 4] Analyse the beam shown by SD method and draw BMD, SFD & EC. The support 'B' sinks by 5mm

Take $E = 210 GPa$, $I = 0.1 G mm^4$



Sol/2

$$E = 210 GPa = 210 \times 10^9 \times 10^{-6} = 210 \times 10^3 \, N/mm^2$$

$$I = 0.1 G mm^4 = 0.1 \times 10^9 mm^4$$

$$EI = \underset{\frac{N}{mm^2}}{(210 \times 10^3)} \underset{mm^4}{(0.1 \times 10^9)} = 2.1 \times 10^{13} \, N\text{-}mm^2$$

$$EI = \frac{2.1 \times 10^{13}}{(1000)(1000)^2} = 2.1 \times 10^4 \, kN\text{-}m^2$$

(a) FEM

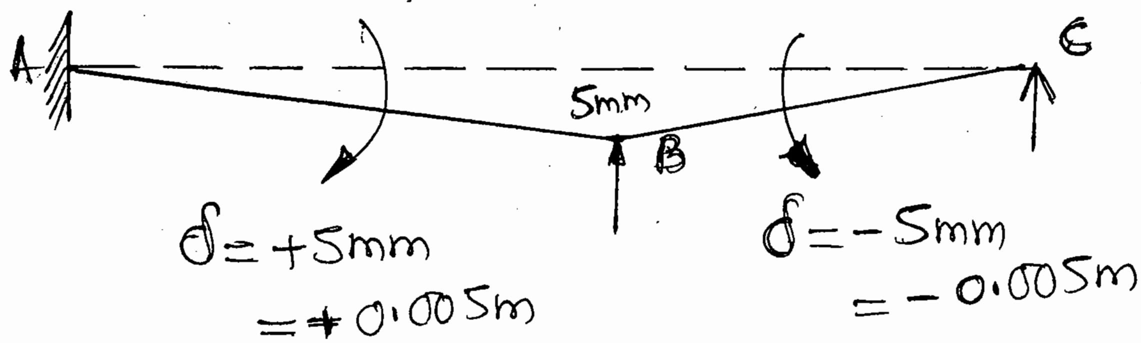
$$M_{FAB} = -\frac{wl^2}{12} - \frac{Wab^2}{l^2} = \frac{-10 \times 6^2}{12} - \frac{30 \times 2 \times 4^2}{6^2} = -56.67$$

$$M_{FBA} = +\frac{wl^2}{12} + \frac{Wa^2b}{l^2} = +\frac{10 \times 6^2}{12} + \frac{30 \times 2^2 \times 4}{6^2} = +43.33$$

$$M_{FBC} = M_{FCB} = +\frac{M}{4} = +12.5 \, kN\text{-}m$$

(b) SD Equation :

$\theta_A = 0$,



$$M_{AB} = \frac{2EI}{6}\left[0 + \theta_B - \frac{3 \times 0.005}{6}\right] - 56.67$$

$$= \frac{2(2.1 \times 10^4)}{6}\left[\theta_B - 2.5 \times 10^{-3}\right] - 56.67$$

$$= 7000\,\theta_B - 74.17 \longrightarrow (i)$$

$$M_{BA} = \frac{2(2.1 \times 10^4)}{6}\left[2\theta_B + 0 - 2.5 \times 10^{-3}\right] + 43.33$$

$$= 14000\,\theta_B + 25.83 \longrightarrow (ii)$$

$$M_{BC} = \frac{2(2.1 \times 10^4)}{4}\left[2\theta_B + \theta_C - \frac{3(-0.005)}{4}\right] + 12.5$$

$$= 21000\,\theta_B + 10500\,\theta_C + 51.87 \longrightarrow (iii)$$

$$M_{CB} = \frac{2(2.1 \times 10^4)}{4}\left[2\theta_C + \theta_B - \frac{3(-0.005)}{4}\right] + 12.5$$

$$= 21000\,\theta_C + 10500\,\theta_B + 51.87 \longrightarrow (iv)$$

(C) Equillibrium Condition

① At "B" $\boxed{M_{BA} + M_{BC} = 0}$

$$35000\,\theta_B + 10500\,\theta_C = -77.7 \rightarrow \text{(I)}$$

② At 'C' $\boxed{M_{CB} = 0}$

$$10500\,\theta_B + 21000\,\theta_C = -51.87 \rightarrow \text{(II)}$$

Solving

$$\theta_B = -1.74 \times 10^{-3}$$

$$\theta_C = -1.6 \times 10^{-3}$$

(d) Final Moment :

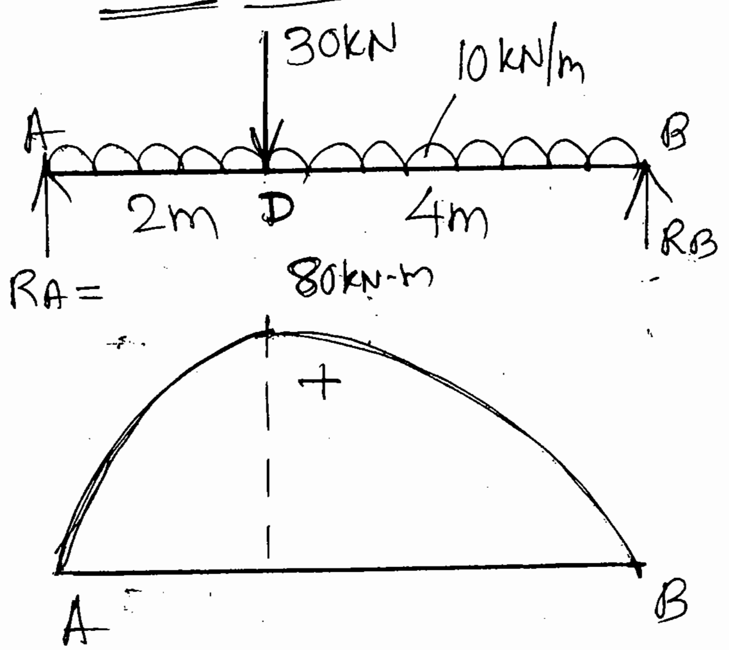
$M_{AB} = -86.35$ kN-m ↻

$M_{BA} = 1.47$ kN-m ↺

$M_{BC} = -1.47$ kN-m ↻

$M_{CB} = 0$

Free BMD for "AB"

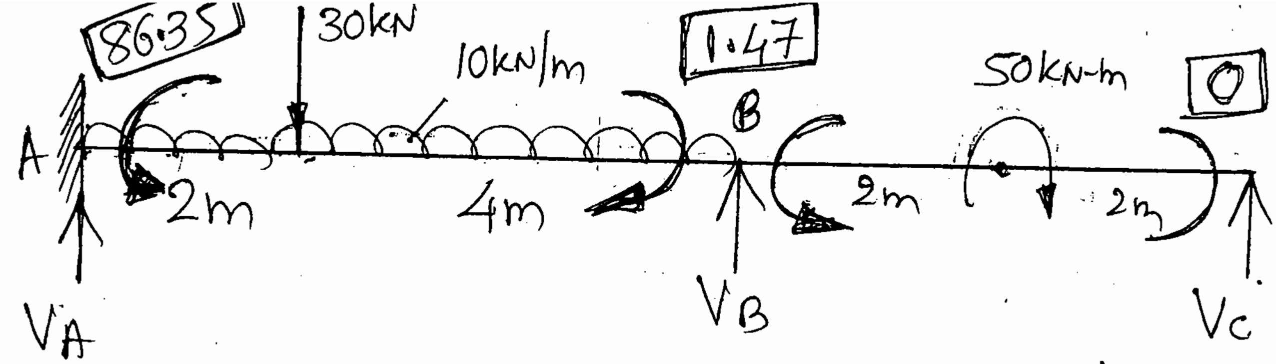


$R_A = 50$ kN

$R_B = 40$ kN

$$\therefore M_D = R_B \times 4 - \frac{10 \times 4 \times 4}{2}$$

$$= 80 \text{ kN-m}$$



$$V_A + V_B + V_C = 30 + 10 \times 6 = 90 \rightarrow (i)$$

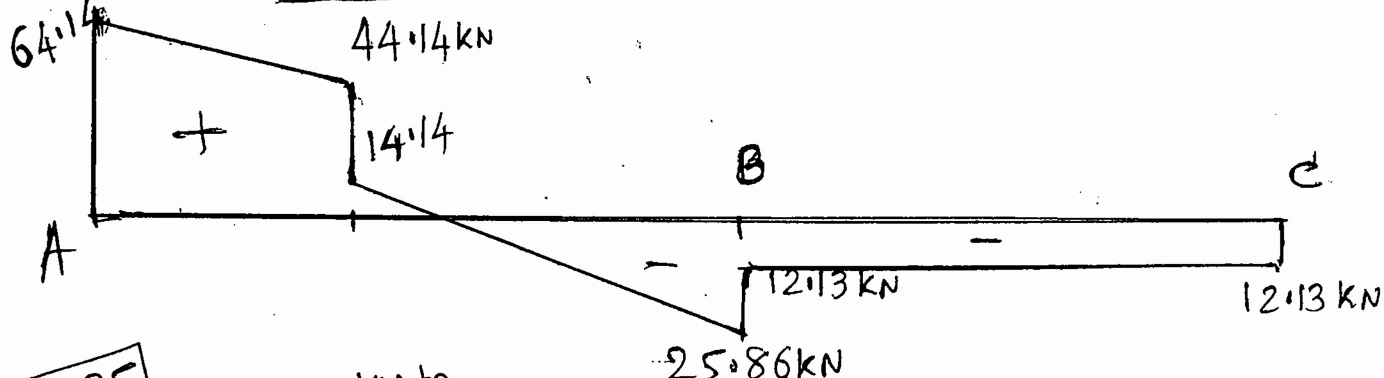
$\Sigma M_B = 0$, (LHS)

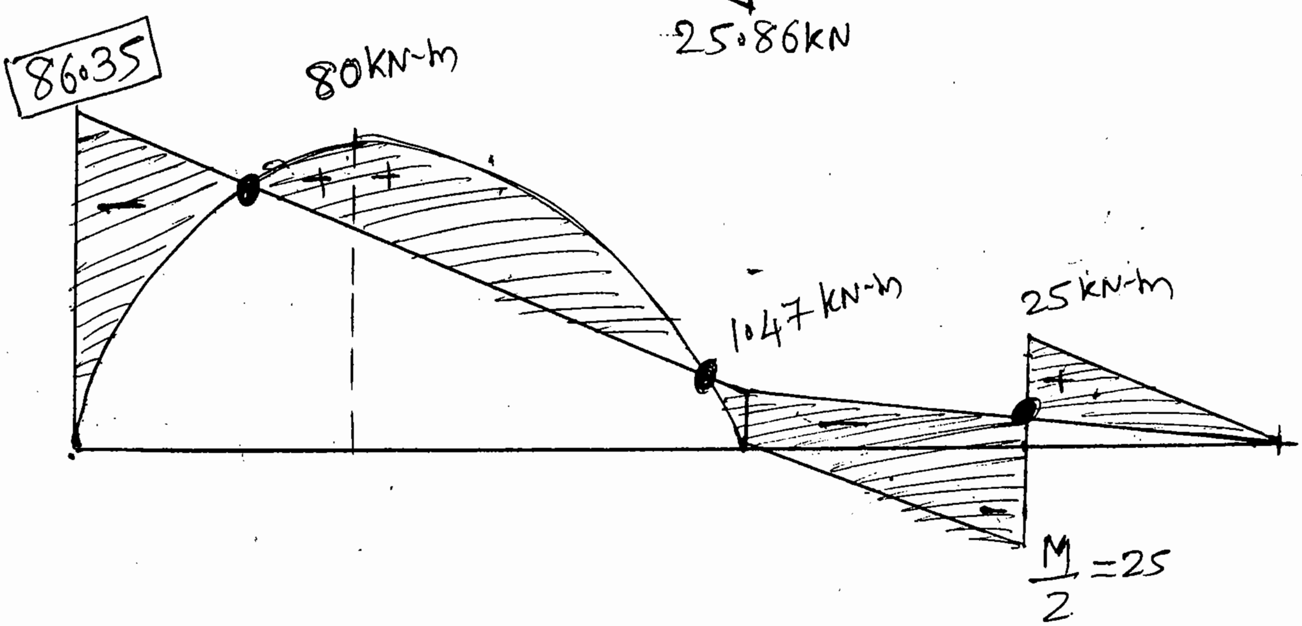
$$V_A \times 6 - 30 \times 4 - 10 \times 6 \times 6/2 - 86.35 + 1.47 = 0$$

$$\boxed{V_A = 64.14}$$

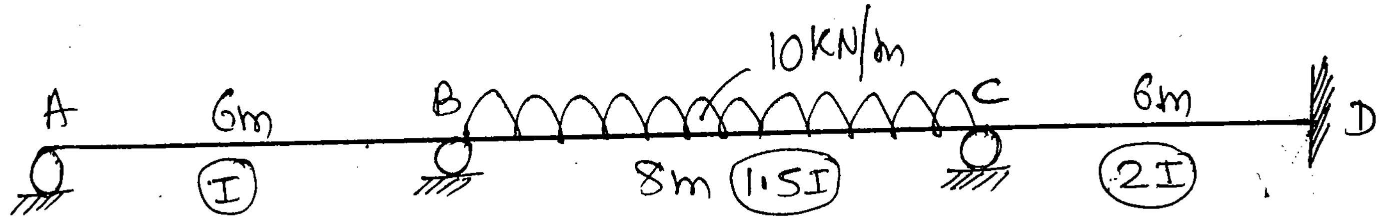
$\Sigma M_B = 0$ (RHS) $-V_C \times 4 + 50 - 1.47 = 0$ $\boxed{V_C = 12.13}$

From (i) $\boxed{V_B = 13.72}$



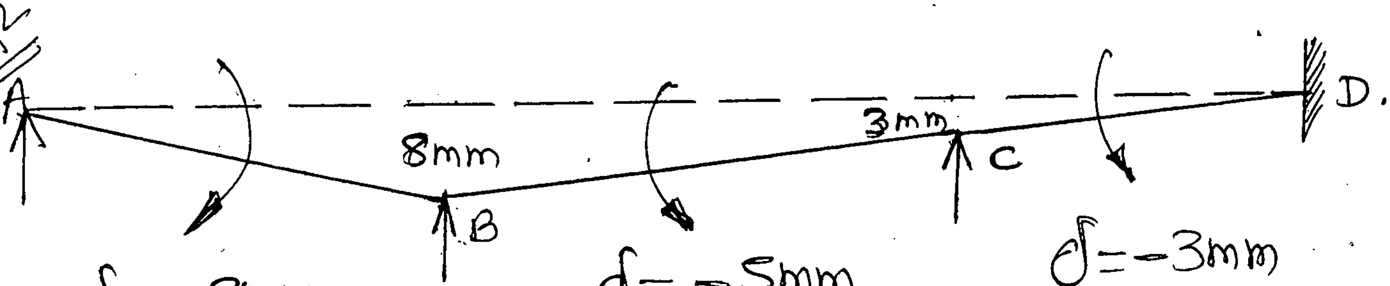


Eg:- 5] Analysis the beam shown by SD method. Draw BMD, SFD & EC.



Support B & C settles by 8mm and 3mm respt. Take $EI = 2\times10^4$ kN-m^2.

Sol^n



$\delta = +8mm$
$0.008m$.

$\delta = -5mm$
$= -0.005m$.

$\delta = -3mm$
$= -0.003m$

(a) FEM

$$M_{FBC} = \frac{-wl^2}{12} = -53.33, \quad M_{FCB} = +53.33$$

(b) SD Equation :

$$\theta_D = 0$$

$$M_{AB} = \frac{2(2\times10^4)}{6}\left[2\theta_A + \theta_B - \frac{3(0.008)}{6}\right] + 0$$

$$= 13333.33\,\theta_A + 6666.67\,\theta_B - 26.67 \rightarrow (i)$$

$$M_{BA} = \frac{2(2\times10^4)}{6}\left[2\theta_B + \theta_A - \frac{3(0.008)}{6}\right] + 0$$

$$= 13333.33\,\theta_B + 6666.67\,\theta_A - 26.67 \quad \text{—(II)}$$

$$M_{BC} = \frac{2(1.5\times2\times10^4)}{8}\left[2\theta_B + \theta_C - \frac{3(-0.005)}{8}\right] - 53.33$$

$$= 15000\,\theta_B + 7500\,\theta_C - 39.26 \quad \text{—(III)}$$

$$M_{CB} = 7500\left[2\theta_C + \theta_B - \frac{3(-0.005)}{8}\right] + 53.33$$

$$= 15000\,\theta_C + 7500\,\theta_B + 67.39 \quad \text{—(IV)}$$

$$M_{CD} = \frac{2(2\times2\times10^4)}{6}\left[2\theta_C + 0 - \frac{3(-0.003)}{6}\right]$$

$$= 26666.67\,\theta_C + 20 \rightarrow \text{(V)}$$

$$M_{DC} = 13333.33\left(0 + \theta_C - \frac{3(-0.003)}{6}\right)$$

$$= 13333.33\,\theta_C + 20 \rightarrow \text{(VI)}$$

(c) Equillibrium Condition

(i) $\boxed{M_{AB} = 0}$

$$13333.33\,\theta_A + 6666.67\,\theta_B = 26.67 \rightarrow \text{(I)}$$

(ii) $\boxed{M_{BA} + M_{BC} = 0}$

$$28333.33\,\theta_B + 6666.67\,\theta_A + 7500\,\theta_C = 65.93 \rightarrow \text{(II)}$$

(iii) $\boxed{M_{CB} + M_{CD} = 0}$

$$7500\,\theta_B + 41666.67\,\theta_C = -87.39 \rightarrow \text{(III)}$$

Solving, $\theta_A = +5.56 \times 10^{-4}$ $\theta_C = -2.617 \times 10^{-3}$

$\theta_B = 2.889 \times 10^{-3}$

(d) Final Values

$M_{AB} \approx 0$

$M_{BA} = 15.55$ kN-m

$M_{BC} = -15.55$ kN-m

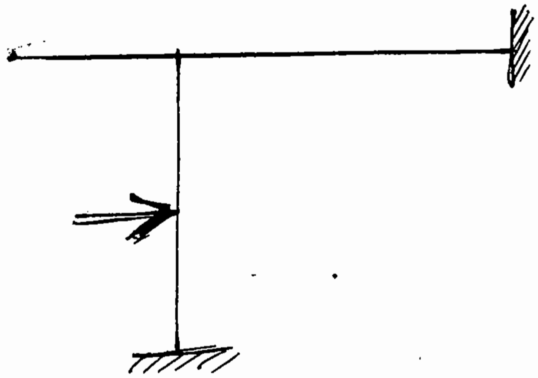
$M_{CB} = 49.80$ kN-m

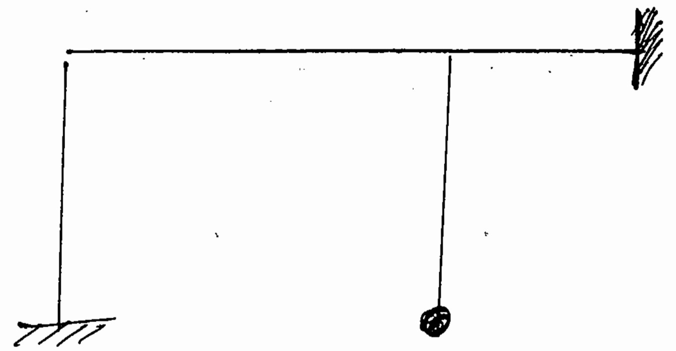
$M_{CD} = -49.80$ kN-m

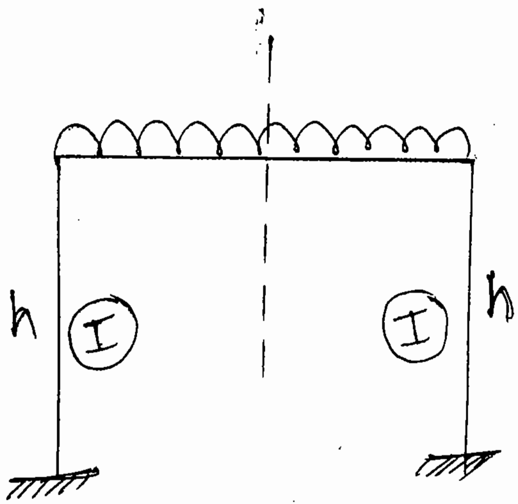
$M_{DC} = -14.89$ kN-m

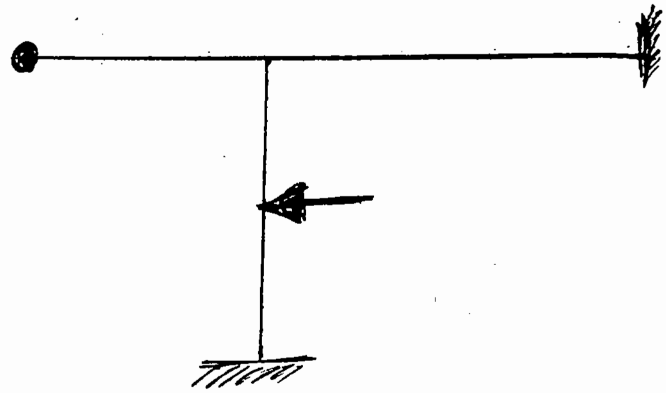
Date 14/9/18

Non Sway Frame :

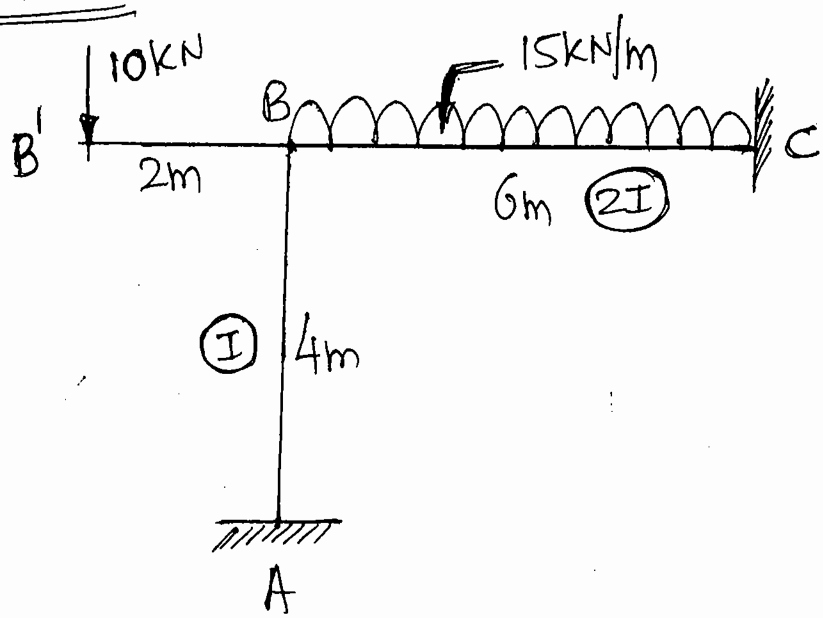








Eg:-1] Analyse the frame shown by S.D. method and draw BMD, SFD & EC.

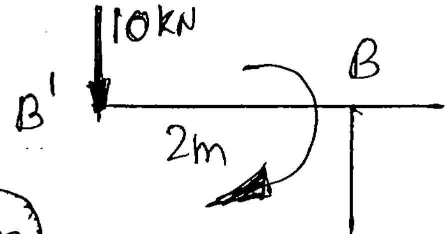


(a) FEM :

$M_{FAB} = M_{FBA} = 0$

$M_{FBC} = -\frac{\omega l^2}{12} = -\frac{15(6)^2}{12} = -45 \text{ kN-m}$

$M_{FCB} = +45.$



$M_{BB'} = +10 \times 2 = +20 \text{ kN-m}$

+ve sign for clockwise resisting moment.

(b) S.D. equation :-

$\theta_A = 0, \quad \theta_C = 0$ (∵ Fixed)

$\delta = 0$ (∵ Non-sway)

There is no equation for overhang BB'

$M_{AB} = \frac{2(EI)}{4}[\theta_B] = 0.5EI(\theta_B)$ —(i)

$M_{BA} = \frac{2EI}{4}[2\theta_B] = EI(\theta_B)$ —(ii)

$M_{BC} = \frac{2(2EI)}{6}[2\theta_B] - 45 = 1.33EI(\theta_B) - 45$ —(iii)

$M_{CB} = \frac{2(2EI)}{6}[\theta_B] + 45 = 0.666EI(\theta_B) + 45$ —(iv)

(c) Equillibrium Condition

At Intermediate joint "B"

$$\boxed{M_{BA} + M_{BC} + M_{BB'} = 0}$$

$$[EI\theta_B] + [1.33EI\theta_B - 45] + [20] = 0$$

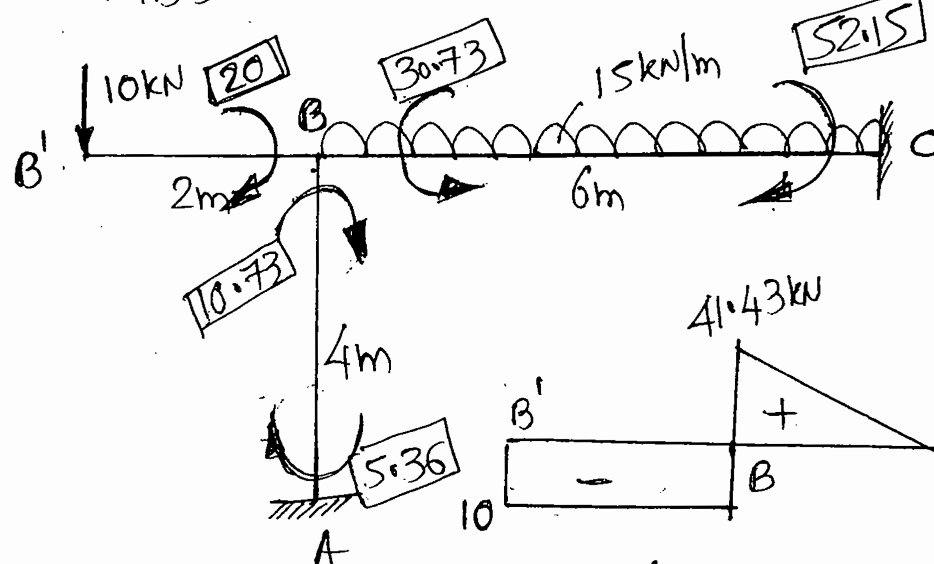
$$\therefore \theta_B = \frac{10.73}{EI}$$

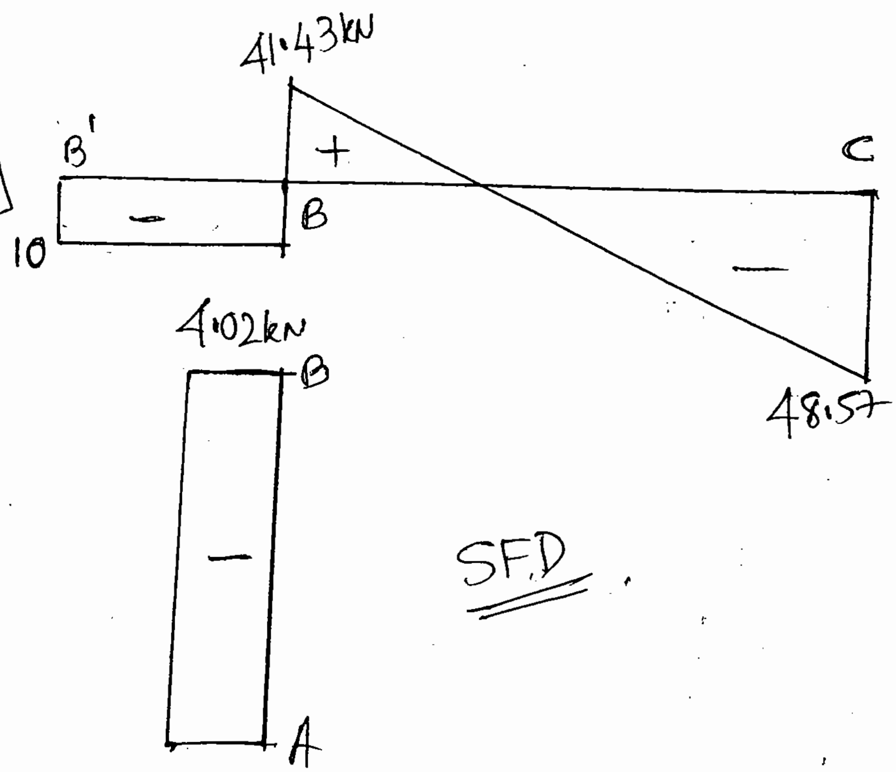
(d) Final Moment :- Substitute in eqⁿ (i) to (iv)

$M_{AB} = 5.36$ kN-m $M_{BC} = -30.73$ kN-m

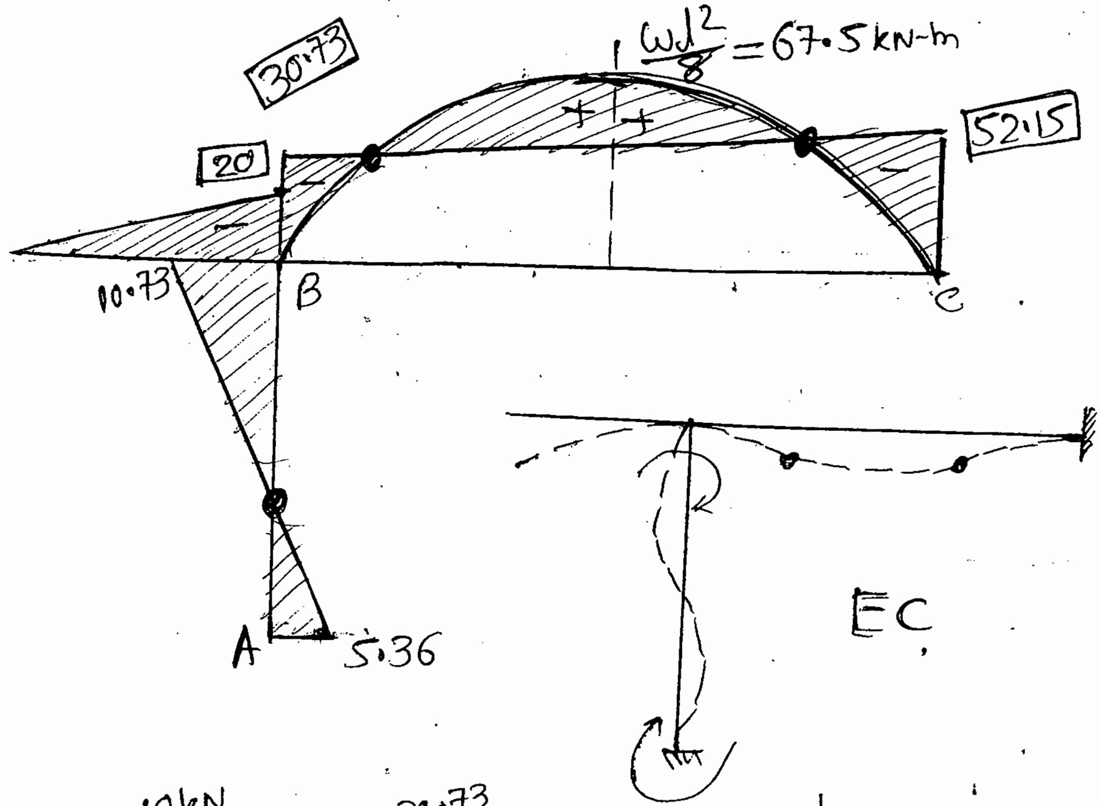
$M_{BA} = 10.73$ kN-m $M_{CB} = 52.15$ kN-m.

$M_{BB'} = +20$ kN-m

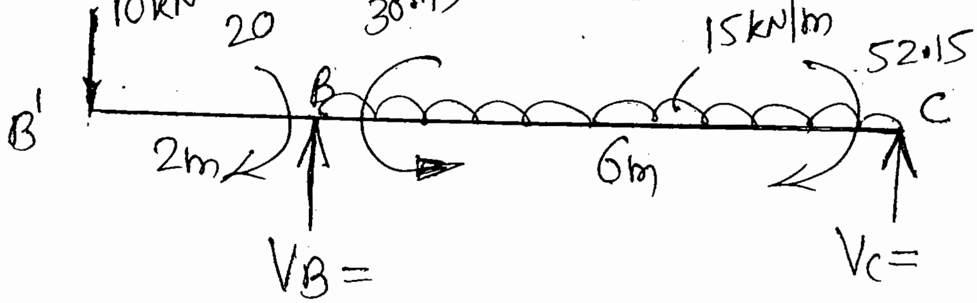




SFD



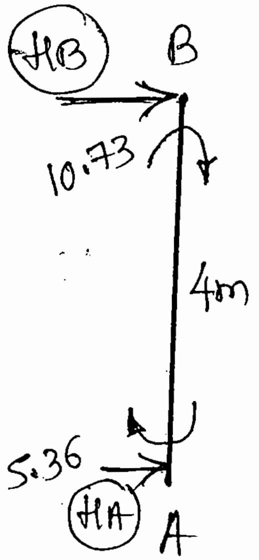
SFD



$V_A + V_B = 10 + 15 \times 6 = 100$ — (i)

$\Sigma M_C = 0, \quad -10 \times 8 + V_B \times 6 - 15 \times 6 \times 6/2 + 20 - 30.73 + 52.15 = 0$

$\boxed{V_B = 51.43}$ & $\boxed{V_C = 48.57}$



$\Sigma H = 0, \quad H_A + H_B = 0$

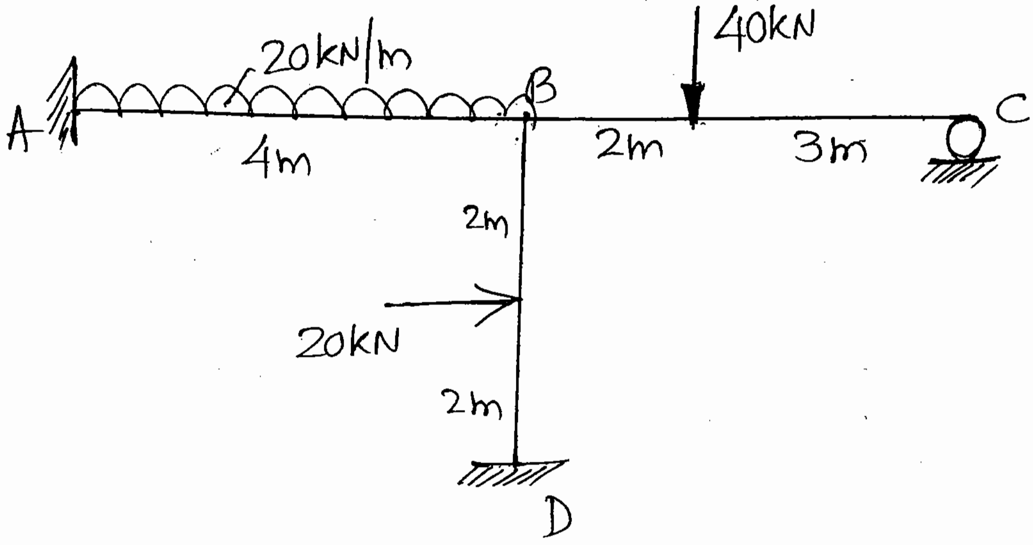
$H_B \times 4 + 10.73 + 5.36 = 0$

$\boxed{H_B = -4.02}$

& $\boxed{H_A = +4.02}$

—x—

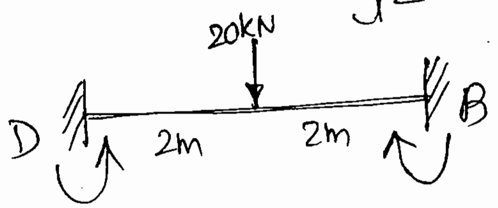
Eg:- 2] Analyse the frame shown by S.D. method. Draw BMD, SFD, EC



Sol^n (a) FEM

$M_{FAB} = -\frac{\omega l^2}{12} = -26.67$, $\quad M_{FBA} = +26.67$

$M_{FBC} = -\frac{Wab^2}{l^2} = -28.8$, $\quad M_{FCB} = +\frac{Wa^2b}{l^2} = +19.2$



$M_{FDB} = -\frac{Wl}{8} = -10$

$M_{FBD} = +10$

(b) S.D. Equation :

$\theta_A = \theta_D = 0$ (Fixed)

$\delta = 0$ (Non-Sway)

$M_{AB} = \frac{2EI}{4}\left[\theta_B\right] - 26.67 = 0.5EI\theta_B - 26.67$ —(i)

$M_{BA} = \frac{2EI}{4}\left[2\theta_B\right] + 26.67 = EI\theta_B + 26.67$ —(ii)

$$M_{BC} = \frac{2EI}{5}\left[2\theta_B + \theta_C\right] - 28.8$$

$$= 0.8EI(\theta_B) + 0.4EI\,\theta_C - 28.8 \quad \text{--- (III)}$$

$$M_{CB} = \frac{2EI}{5}\left[2\theta_C + \theta_B\right] + 19.2$$

$$= 0.8EI\,\theta_C + 0.4EI\,\theta_B + 19.2 \quad \text{--- (IV)}$$

$$M_{BD} = \frac{2EI}{4}\left[2\theta_B\right] + 10 = EI(\theta_B) + 10 \quad \text{--- (V)}$$

$$M_{DB} = \frac{2EI}{4}\left[\theta_B\right] - 10 = 0.5EI(\theta_B) - 10 \quad \text{--- (VI)}$$

(c) <u>Equillibrium Condition</u> :—

at "B" $\boxed{M_{BA} + M_{BC} + M_{BD} = 0}$

$$2.8EI(\theta_B) + 0.4EI(\theta_C) = -7.87 \rightarrow \text{(I)}$$

At "C" $\boxed{M_{CB} = 0}$

$$0.4EI(\theta_B) + 0.8EI(\theta_C) = -19.2 \rightarrow \text{(II)}$$

solving

$$\theta_B = \frac{0.67}{EI}$$

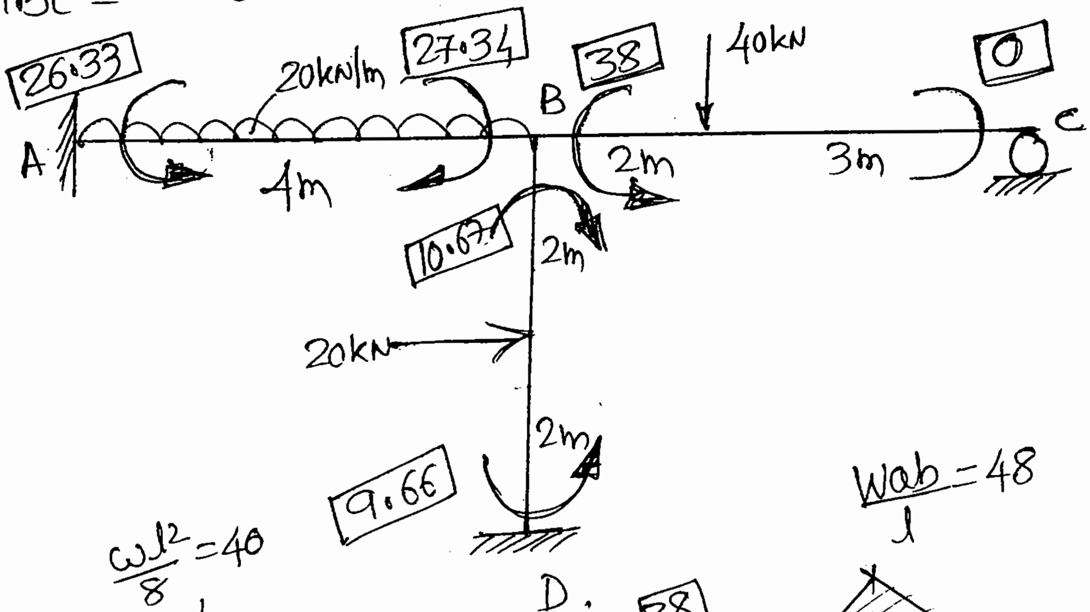
$$\theta_C = -\frac{24.33}{EI}$$

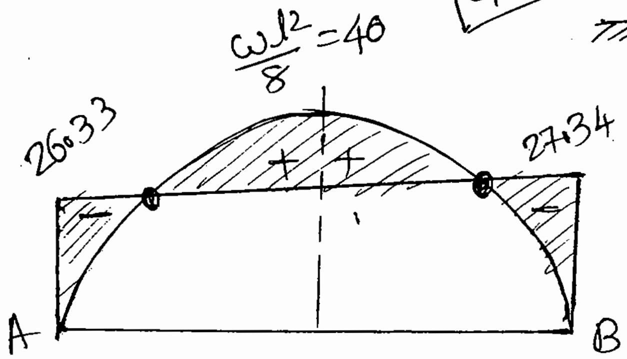
(d) Final Moment

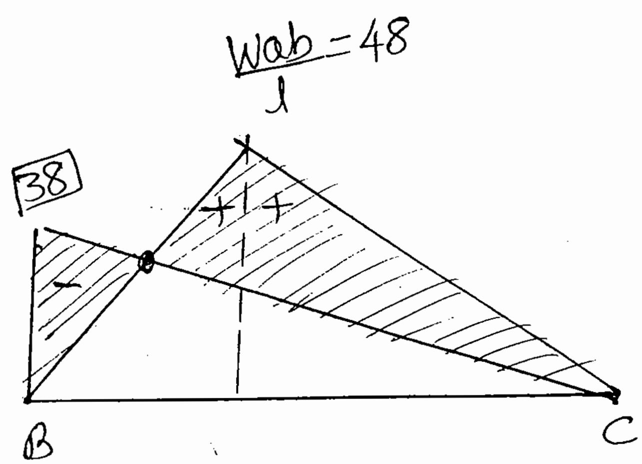
$M_{AB} = -26.33$ kN-m ↻ $M_{CB} = 0$

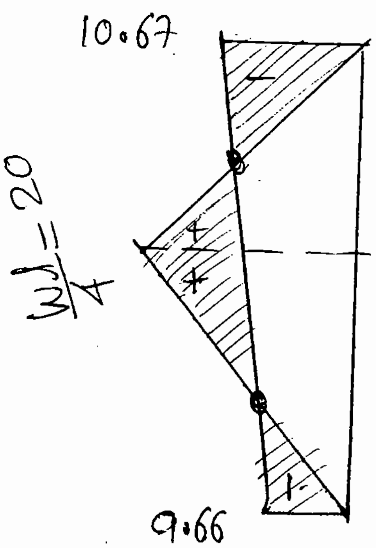
$M_{BA} = 27.34$ kN-m ↺ $M_{BD} = 10.67$ kN-m ↺

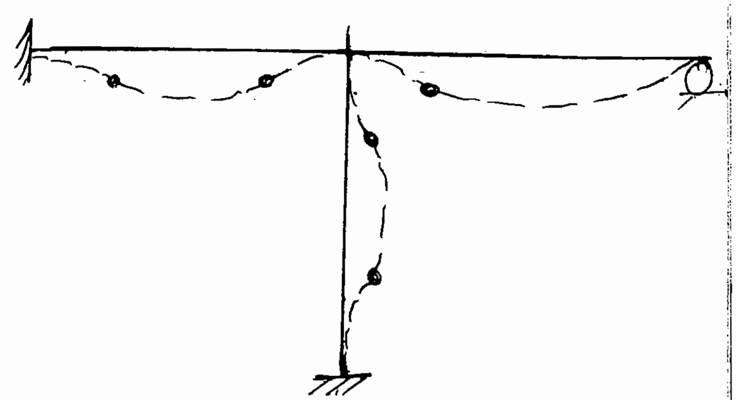
$M_{BC} = -38.00$ kN-m ↻ $M_{DB} = -9.66$ kN-m ↻









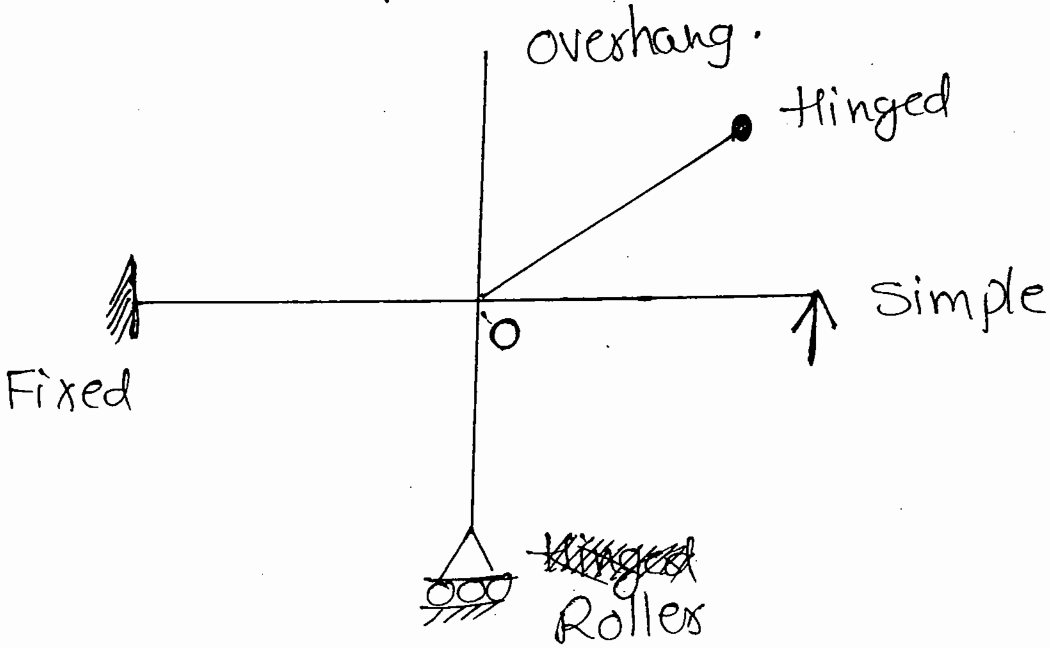


Date 21/9/18

(III) Moment Distribution Method

Relative stiffness = $\boxed{K = \frac{I}{l}}$

"The ratio of M.I to the span of beam is called relative stiffness."

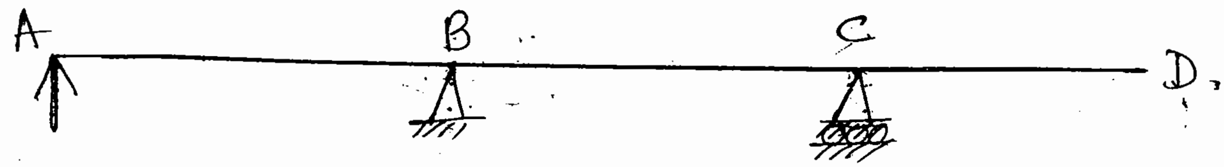


(a) For "Fixed end" or "Continuous" support } $\boxed{K = \frac{I}{l}}$

(b) For 'Simple' or 'Hinge' or "Roller" support } $\boxed{K = \frac{3}{4}\frac{I}{l}}$

(c) For "Over hang" $\rightarrow$ $\boxed{K = 0}$

Continuous support :



(i) w.r.t "B" $\longrightarrow$ 'A' is Not continuous $K = \frac{3}{4}\frac{I}{l}$

'C' is Not Continuous $K = \frac{3}{4}\frac{I}{l}$.

ii) w.r.to 'C' $\longrightarrow$ 'B' is Continuous $k = I/l$

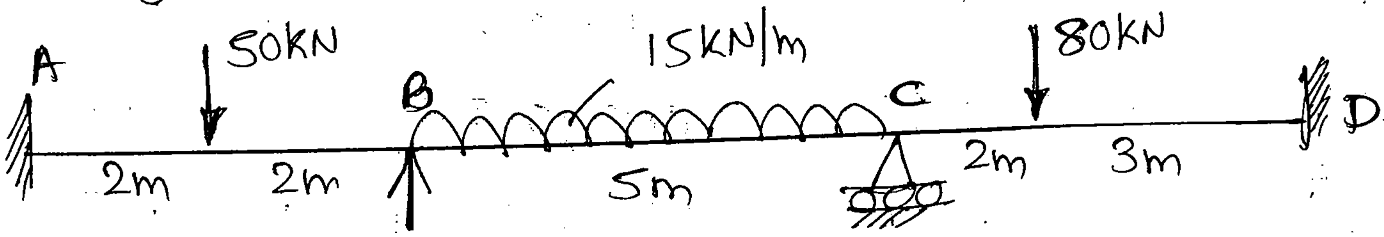
'D' is overhang $k = 0$

Carry over of Moments:-

(i) If the far end is "fixed" or "Continuous" 50% of moment with take or carry same sign.

(ii) If far end is Not continuous, then there is no Transfer of moment.

Eg:- 1 Analyse the continuous beam shown by MD method. Draw SFD, BMD & EC.



Sol^n (a) FEM

$M_{FAB} = -\frac{Wl}{8} = -25$ kN-m, $M_{FBA} = +25$ kN-m

$M_{FBC} = -\frac{\omega l^2}{12} = -31.25$, $M_{FCB} = +\frac{\omega l^2}{12} = 31.25$

$M_{FCD} = -\frac{Wab^2}{l^2} = -57.6$, $M_{FDC} = +\frac{Wa^2b}{l^2} = 38.4$

(b) Distribution Factor (For Intermediate support)

Joint	Member	Relative stiffness = K	Sum ΣK	$DF = \frac{K}{\Sigma K}$
B	BA	$\frac{I}{l} = \frac{I}{4} = 0.25I$	0.45I	$\frac{0.25}{0.45} = 0.56$
	BC	$\frac{I}{l} = \frac{I}{5} = 0.20I$		$\frac{0.2}{0.45} = 0.44$
C	CB	$\frac{I}{l} = \frac{I}{5} = 0.20I$	0.4I	0.5
	CD	$\frac{I}{l} = \frac{I}{5} = 0.2I$		0.5

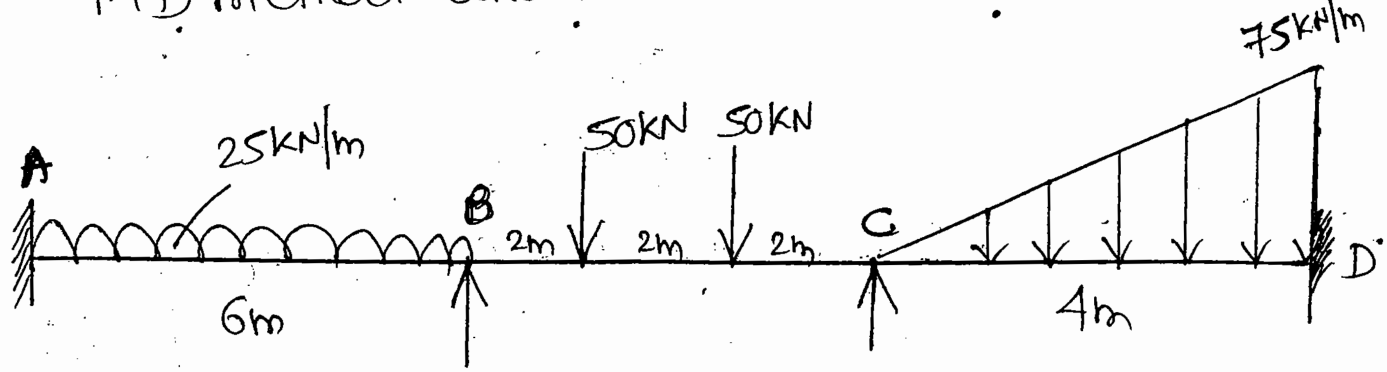
(c) Moment Distribution Table

A (Fixed) B C D (Fixed)

AB	BA	BC	CB	CD	DC	Member
-	0.56	0.44	0.5	0.5		DF
-25	25	-31.25	31.25	-57.6	38.4	FEM
	3.5	2.75	13.18	13.18		Balance
1.75		6.59	1.37		6.59	Carry over
	-3.69	-2.90	-0.68	-0.68		Bal
-1.84		-0.34	-1.45		-0.34	C.O
	0.19	0.15	0.73	0.73		Bal
0.09		0.36	0.075		0.36	C.O
	-0.20	-0.16	-0.037	-0.037		Bal
-0.10		-0.018	-0.08		-0.018	C.O
	0.01	0.008	0.04	0.04		Bal
25.10	24.81	-24.81	44.40	-44.40	44.992	Final Moments,

Draw SFD, BMD and EC.

Eg:- 2] Analyse the beam shown by MD method and draw BMD, SFD & EC.



Sol/

(a) FEM

$M_{FAB} = -\frac{\omega l^2}{12} = -75$, $M_{FBA} = +75$

$M_{FBC} = -\frac{Wab^2}{l^2} = -\left[\frac{50\times2\times4^2}{6^2} + \frac{50\times4\times2^2}{6^2}\right] = -66.67$

$M_{FCB} = +\frac{Wa^2b}{l^2} = +66.67$

$M_{FCD} = -\frac{\omega l^2}{30} = \frac{-75\times4^2}{30} = -40$

$M_{FDC} = +\frac{\omega l^2}{20} = +60$

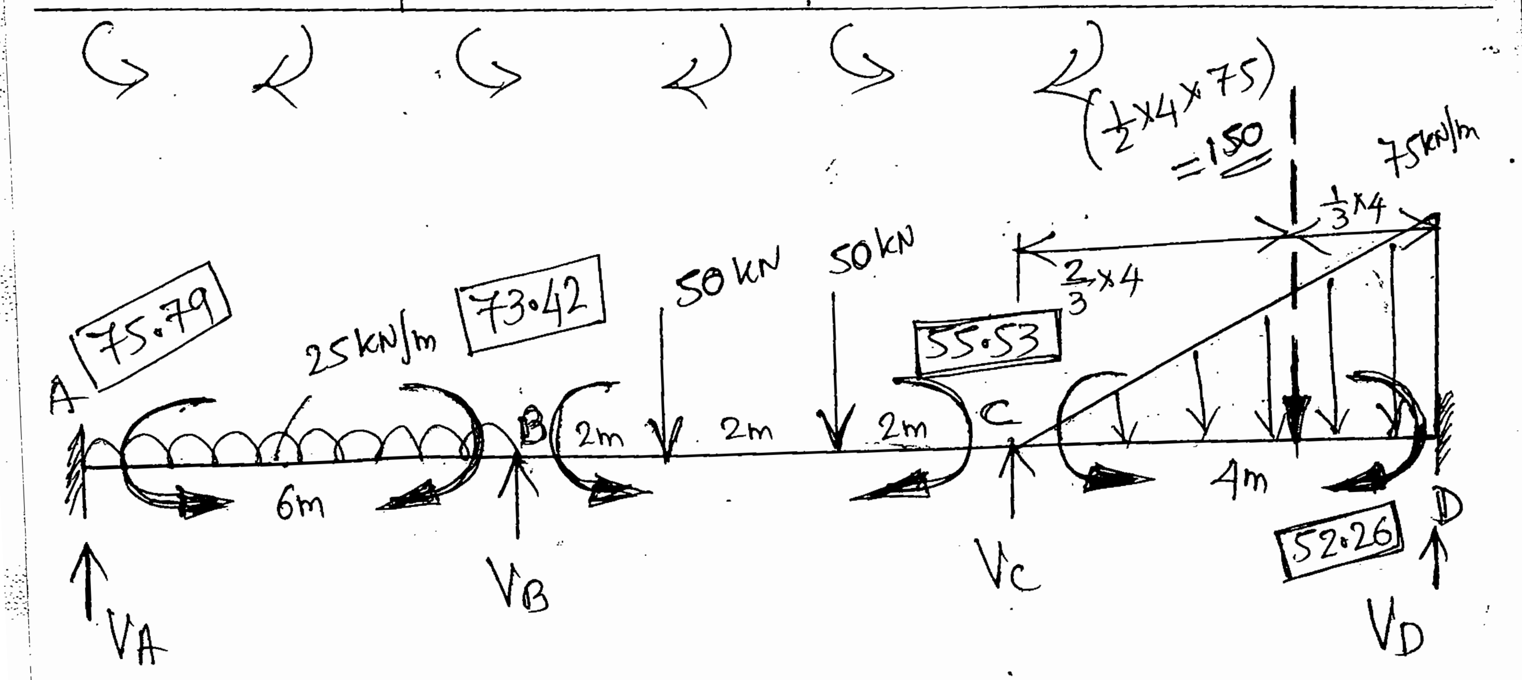
(b) D.F : (For Intermediate Support)

	Member	Relative stiffness = K	ΣK	$DF = \frac{K}{\Sigma K}$
B	BA	$I/l = I/6 = 0.167I$	0.333I	0.5
	BC	$I/l = I/6 = 0.167I$		0.5
C	CB	$I/l = I/6 = 0.167I$	0.417I	0.4
	CD	$I/l = I/4 = 0.25I$		0.6

(c) M.D. Table

A (Fixed) — B — C — D (Fixed)

AB	BA	BC	CB	CD	DC	Member
	0.5	0.5	0.40	0.60		DF
-75	75	-66.67	+66.67	-40	60	FEM
	-4.17	-4.17	-10.67	-16.0		Balance
-2.09		-5.34	-2.09		-8	C.O
	2.67	2.67	0.84	1.25		Bal
1.34		0.42	1.34		0.63	C.O
	-0.21	-0.21	-0.54	-0.80		Bal
-0.11		-0.27	-0.11		-0.4	C.O
	0.14	0.14	+0.105	+0.06		Bal
0.07		0.02	0.07		0.03	C.O
	-0.01	-0.01	-0.03	-0.04		Bal
-75.79	73.42	-73.42	55.53	-55.53	52.26	Final Values,



Reactions

$\Sigma V = 0,\ V_A + V_B + V_C + V_D = 25\times6 + 2\times50 + 150$

$= 400 \quad \text{---(i)}$

$\Sigma M_B = 0\ (LHS)$

$V_A\times6 - 25\times6\times6/2 - 75.79 + 73.42 = 0 \quad \boxed{V_A = 75.4}$

$\Sigma M_C = 0\ (RHS)$

$-V_D\times4 + \left(150\times\frac{2}{3}\times4\right) - 55.53 + 52.26 = 0$

$\boxed{V_D = 99.18}$

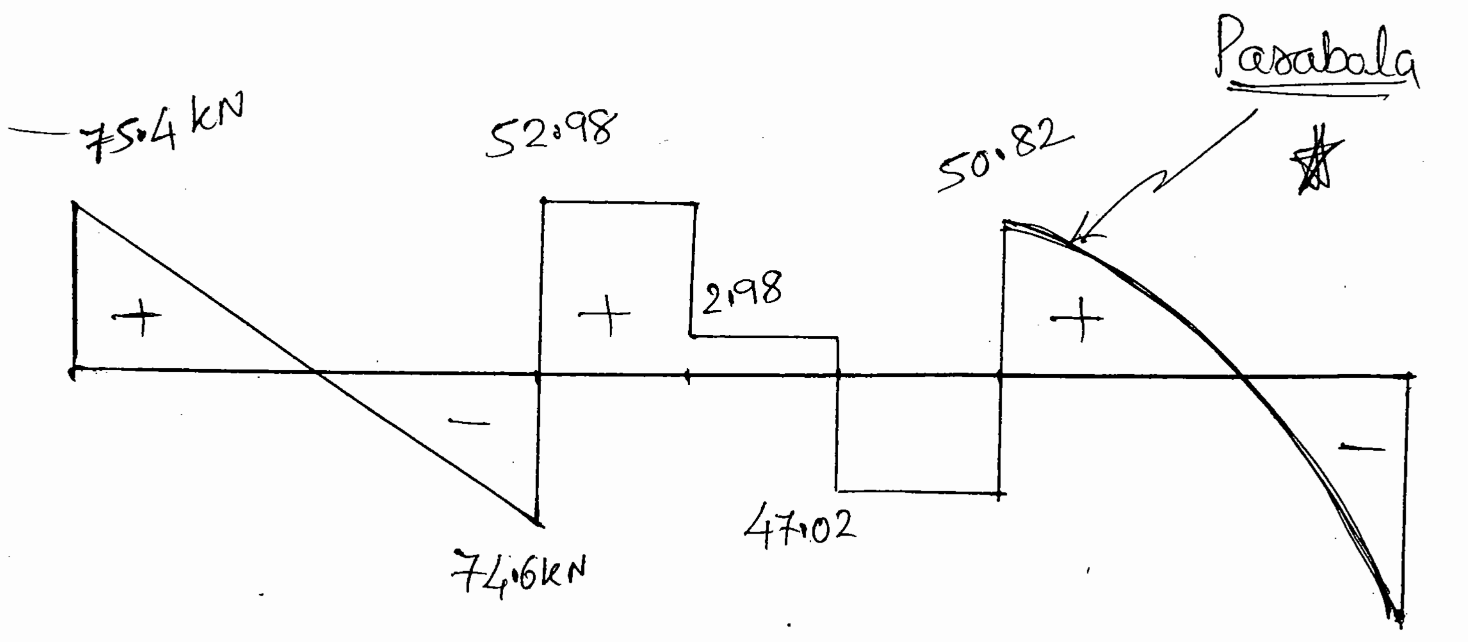
$\Sigma M_C = 0\ (LHS)$

$75.4\times12 + V_B\times6 - 25\times6\times9 - 50\times2 - 50\times4$

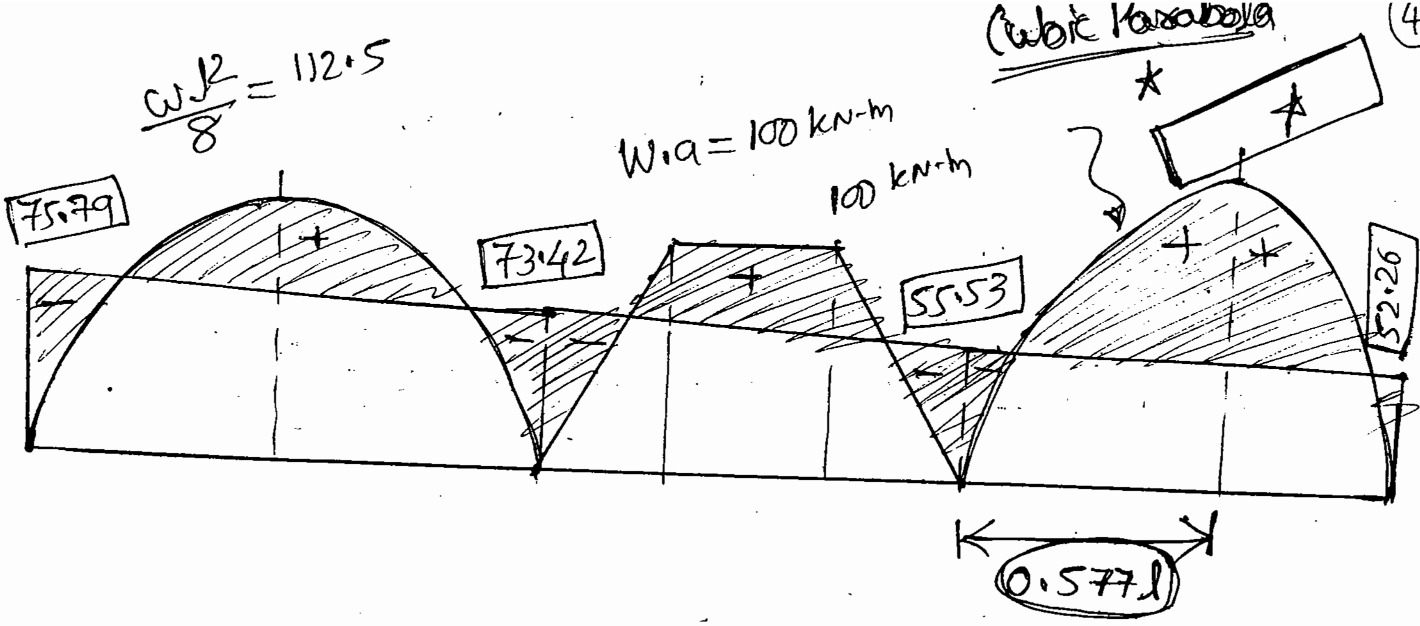
$-75.79 + \cancel{73.42} - \cancel{73.42} + 55.53 = 0$

$\boxed{V_B = 127.58}$

From (i) $\boxed{V_C = 97.84}$



SFD



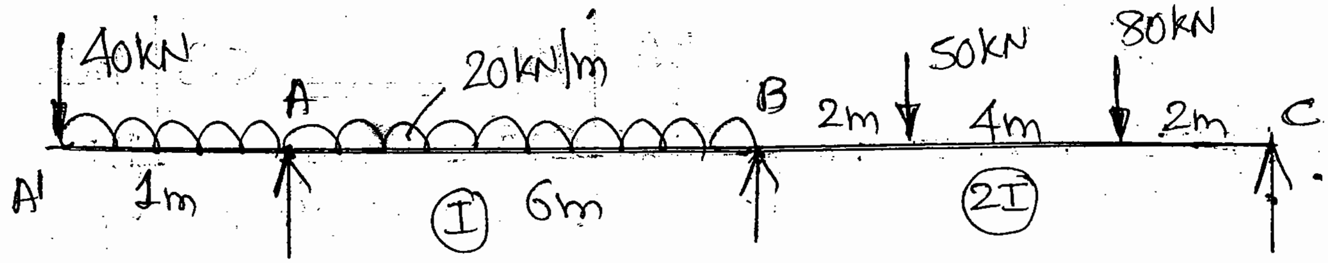
$= 0.577 \times 4$
$= 2.308m$

$\boxed{M = 0.06415 \omega l^2}$

$= 0.06415 \times 75 \times (4)^2 = 77$ kN-m

—x—

Eg:- 3 Analyse the beam shown by M.D. method. Draw SFD, BMD & EC.

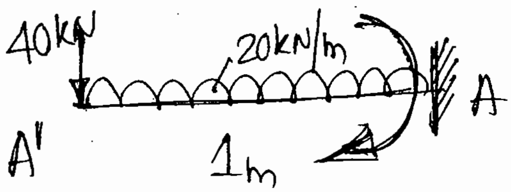


(a) FEM

$$M_{FAB} = -\frac{wl^2}{12} = -60 \text{ kN-m}, \quad M_{FBA} = +60 \text{ kN-m}$$

$$M_{FBC} = -\frac{Wab^2}{l^2} = -\left[\frac{50\times2\times6^2}{8^2} + \frac{80\times6\times2^2}{8^2}\right] = -86.25 \text{ kN-m}$$

$$M_{FCB} = +\frac{Wa^2b}{l^2} = +\left[\frac{50\times2^2\times6}{8^2} + \frac{80\times6^2\times2}{8^2}\right] = +108.75 \text{ kN-m}$$



$$M_{AA'} = +40\times1 + 20\times1\times\tfrac{1}{2}$$
$$= +50 \text{ kN-m}.$$

(b) D.F. (For Intermediate)

	Member	K	ΣK	$DF = \frac{K}{\Sigma K}$
A	AA'	0 ($\because$ Overhang)	0.167I	0
	AB	$\left(\frac{I}{l}\right) = \frac{I}{6} = 0.167\,I$		1
B	BA	$\frac{3}{4}\left(\frac{I}{l}\right) = \frac{3}{4}\left(\frac{I}{6}\right) = 0.125\,I$	0.3125I	0.40
	BC	$\frac{3}{4}\left(\frac{I}{l}\right) = \frac{3}{4}\left(\frac{2I}{8}\right) = 0.1875\,I$		0.60

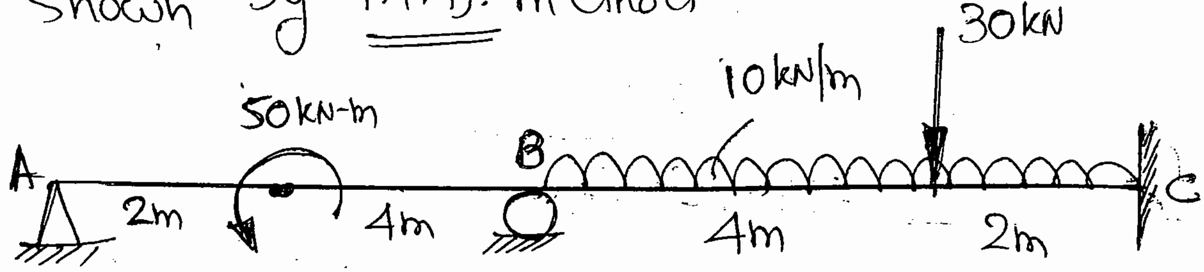
M.D. Table

A, B, C — Simple Hinge Roller

AA'	AB	BA	BC	CB	Members
0	1	0.4	0.6		DF
50	-60	60	-86.25	108.75	FEM
				-108.75	Release (C)
			-54.37		C.O
50	-60	60	-140.62	0	Initial Values
0	10	32.25	48.37		Bal
	0	5		0	C.O
		-2	-3		Bal
	0			0	C.O
50	-50	95.25	-95.25	0	Final Values

Refer S.D. Notes for SFD, BMD.

Eg:-4] Analyse the continuous beam shown by M.D. method.



Sol^n (a) FEM

$$M_{FAB} = -\frac{Mb}{l^2}(2a-b) = \frac{-50\times4(2\times2-4)}{6^2} = 0$$

$$M_{FBA} = -\frac{Ma}{l^2}(2b-a) = -\frac{50\times2}{6^2}(2\times4-2) = -16.67 \text{ kN-m}$$

$$M_{FBC} = -\frac{wl^2}{12} - \frac{Wab^2}{l^2} = -43.33 \text{ kN-m}$$

$$M_{FCB} = +\frac{wl^2}{12} + \frac{Wa^2b}{l^2} = 56.67 \text{ kN-m.}$$

(b) D.F. (For Intermediate Support)

		k	Σk	$DF=\frac{k}{\Sigma k}$
B	BA	$\frac{3}{4}\left(\frac{I}{l}\right) = \frac{3}{4}\left(\frac{I}{6}\right) = 0.125I$	0.292I	0.43
	BC	$\left(\frac{I}{l}\right) = \frac{I}{6} = 0.167I$		0.57

M.D. Table

A (Hinge) | B | C (Fixed)

AB	BA	BC	CB	Member
-	0.43	0.57		DF
0	-16.67	-43.33	56.67	FEM
0 →	0			Release (A) C.O.
0	-16.67	-43.33	56.67	Initial Values.
0 ←	25.80	34.20 →	17.10	Bal C.O
0	9.13	-9.13	73.77	Final Values

Draw BMD, SFD.

Sinking and Rotation of Support

Additional Moment due to Rotation

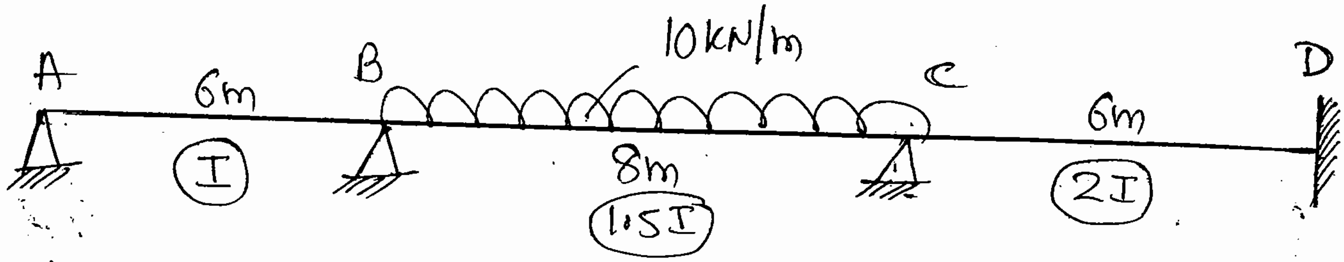
(i) Additional Moment at Near end $\Big\} = \dfrac{4EI\theta}{l}$

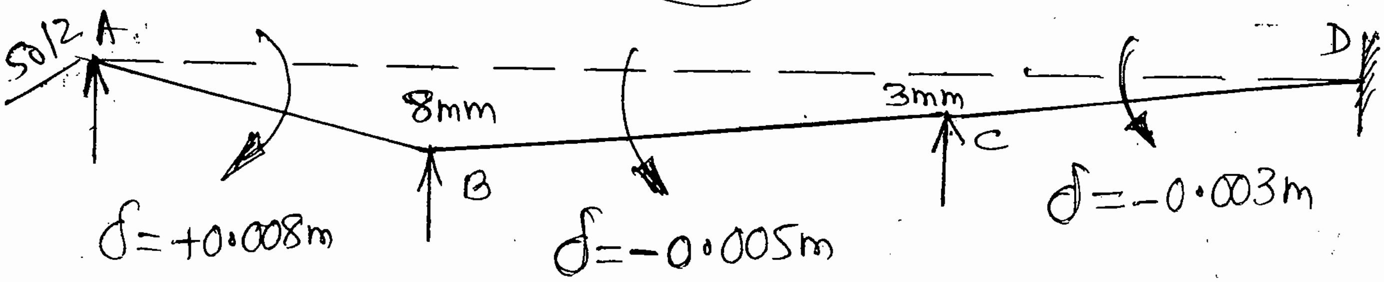
(ii) Additional moment at Far end $\Big\} = \dfrac{2EI\theta}{l}$.

(iii) Additional moment due to Sinking $\Big\} = -\dfrac{6EI\delta}{l^2}$

☆ ∴ The above additional moments are added to F.E.M ☆

Eg:– 5] Analyse the continuous beam shown by M.D. method and draw SFD, BMD. Support 'B' and 'C' settle by 8mm and 3mm respt. $EI = 2\times10^4\ kN/m^2$





(a) FEM

$$M_{FAB} = 0 - \frac{6EI\delta}{l^2} = 0 - \frac{6(1\times2\times10^4)(0.008)}{6^2} = -26.67$$

$$M_{FBA} = 0 - \frac{6EI\delta}{l^2} = 0 - \frac{6(1\times2\times10^4)(0.008)}{6^2} = -26.67$$

$$M_{FBC} = -\frac{\omega l^2}{12} - \frac{6EI\delta}{l^2} = \frac{-10\times8^2}{12} - \frac{6(1.5\times2\times10^4)(-0.005)}{8^2}$$

$$= -39.27\ kN\text{-}m$$

$$M_{FCB} = +\frac{\omega l^2}{12} - \frac{6EI\delta}{l^2} = \frac{10\times8^2}{12} - \frac{6(1.5\times2\times10^4)(-0.005)}{8^2}$$

$$= +67.40\ kN\text{-}m$$

$$M_{FCD} = 0 - \frac{6EI\delta}{l^2} = 0 - \frac{6(2\times 2\times 10^7)(-0.003)}{6^2}$$

$$= +20 \text{ kN-m}$$

$$M_{FDC} = 0 - \frac{6EI\delta}{l^2} = +20 \text{ kN-m}$$

(b) D.F

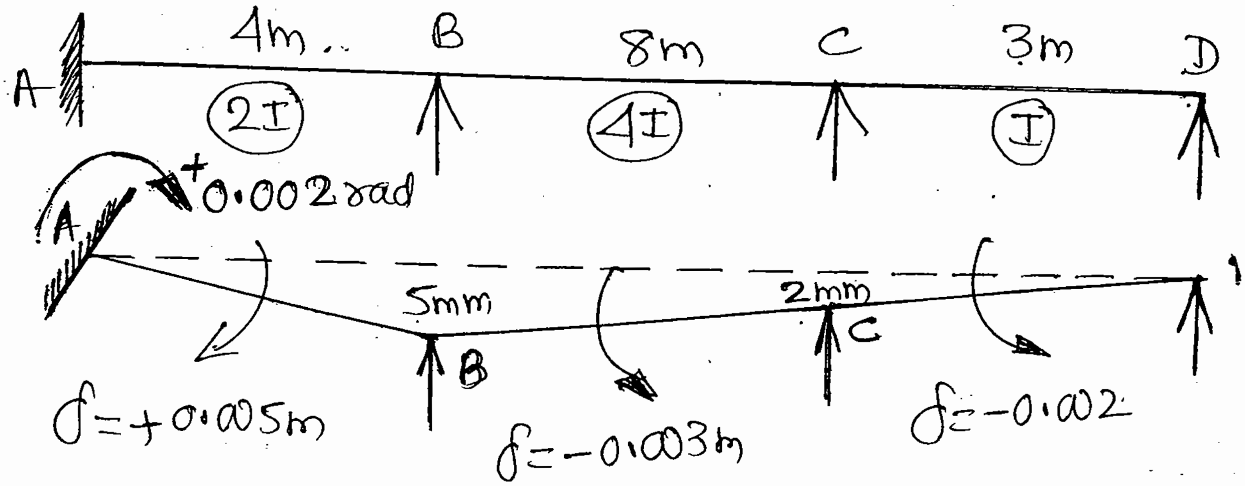
		k	Σk	$DF = \frac{k}{\Sigma k}$
B	BA	$\frac{3}{4}\left(\frac{I}{l}\right) = \frac{3}{4}\times\frac{I}{6} = 0.125I$	$0.3125I$	0.4
	BC	$I/l = \frac{1.5I}{8} = 0.1875I$		0.6
C	CB	$I/l = \frac{1.5I}{8} = 0.1875I$	$0.5200I$	0.36
	CD	$I/l = \frac{2I}{6} = 0.333I$		0.64

(c) M.D Table

AB	BA	BC	CB	CD	DC (Fixed)	Member
	0.4	0.6	0.36	0.64		DF
−26.67	−26.67	−39.27	67.40	20	20	FEM
+26.67	13.33					Release (A) C.O
0	−13.33	−39.27	67.40	20	20	Initial
0	21.04	31.56 −15.88	−31.46 15.76	−55.94	−27.97	Bal C.O
0	6.35	9.53 −2.83	−5.67 4.76	−10.08	−5.04	Bal C.O
0	1.13	1.70 −0.85	−1.71 0.85	−3.04	−1.52	Bal C.O
0	0.34	0.51 −0.15	−0.30 0.25	−0.54	−0.27	Bal C.O
0	+0.06	0.09	−0.09	−0.16		Bal
	15.59	−15.59	49.79	−49.79	−14.80	Final

Eg:- 6 fig shows a continuous beam ABCD. Analyse the beam by M.D method. If the End "A" rotates by 0.002 radians in the clockwise order & Support 'B' sinks by 5mm & 'C' by 2mm. Take

$EI = 18000\ kN\text{-}m^2$



(a) FEM

$$M_{FAB} = 0 + \underbrace{\frac{4EI\theta}{l}}_{\text{Near End}} - \underbrace{\frac{6EI\delta}{l^2}}_{\text{Sinking}}$$

$$= 0 + \frac{4(2\times18000)(0.002)}{4} - \frac{6(2\times18000)(0.005)}{4^2} = 4.5$$

$$M_{FBA} = 0 + \underbrace{\frac{2EI\theta}{l}}_{\text{Far end}} - \frac{6EI\delta}{l^2}$$

$$= 0 + \frac{2(2\times18000)(0.002)}{4} - \frac{6(2\times18000)(0.005)}{4^2} = -31.5$$

$$M_{FBC} = 0 - \frac{6EI\delta}{l^2} = 0 - \frac{6(4\times18000)(-0.003)}{8^2} = 20.25$$

$$M_{FCB} = 0 - \frac{6EI\delta}{l^2} = 20.25$$

$$M_{FCD} = 0 - \frac{6EI\delta}{l^2} = -\frac{6(1\times18000)(-0.002)}{3^2}$$

$$= 24 \text{ kN-m}$$

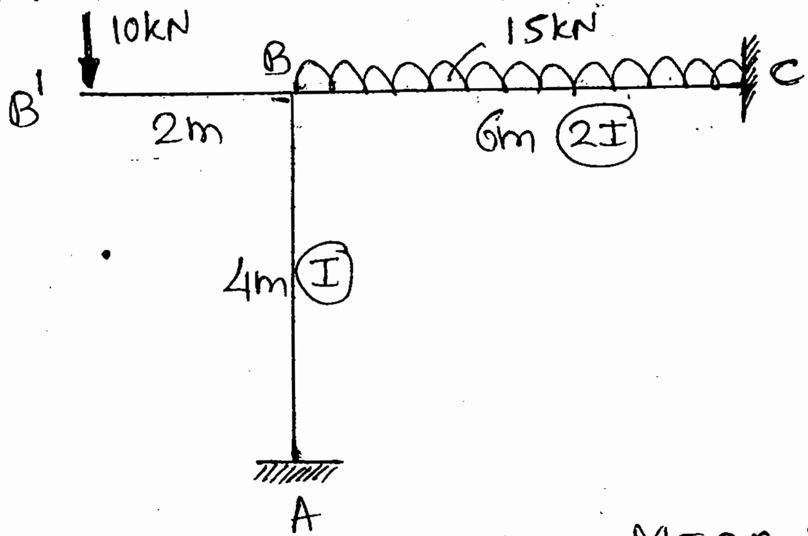
$$M_{FDC} = 0 - \frac{6EI\delta}{l^2} = 24 \text{ kN-m}$$

(b)

Date 05/10/18

: Non-Sway Frames :-

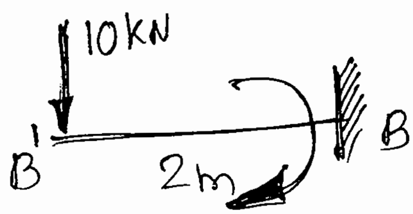
Eg:- 1] Analyse the rigid frame by M.D. method. Draw SFD, BMD & EC.



(a) FEM:

$M_{FAB} = M_{FBA} = 0$

$M_{FBC} = \frac{-wl^2}{12} = -45$, $M_{FCB} = +45$ kN-m



$M_{BB'} = +10 \times 2 = +20$ kN-m

(clockwise resisting moment).

(b) D.F (For Intermediate)

		K	ΣK	$D.F = \frac{K}{\Sigma K}$
B	BA	$I/l = I/4 = 0.25I$		0.43
	BC	$I/l = \frac{2I}{6} = 0.33I$	$0.58I$	0.57
	BB'	0		0

(C) M.D. Table

BB'	AB	BA	BC	CB	Member
0		0.43	0.57		DF
20	0	0	-45	45	FEM
0		10.75	14.25		Bal
—	5.37	—	—	7.13	C.O
20	5.37	10.75	-30.75	52.13	Final Values.

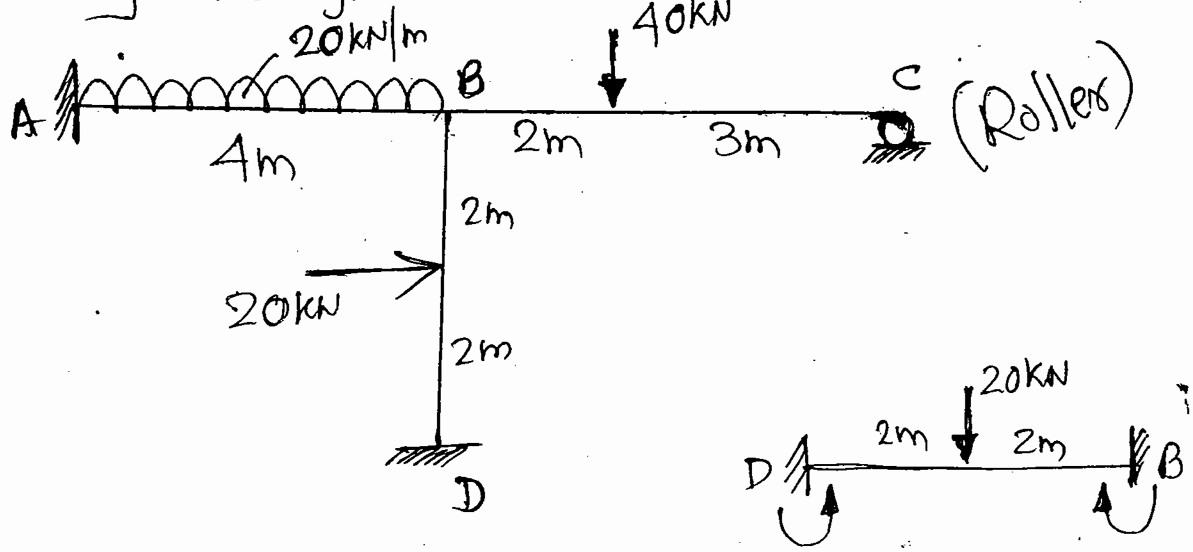
At "B" $\boxed{M_{BA} + M_{BC} + M_{BB'} = 0}$

$0 - 45 + 20 = -25$

Refer S.D. method for Reaction, SFD & BMD.

=x=

Eg:- 2] Analyse the frame by M.D. method



(a) **FEM**:

$M_{FAB} = -\frac{wl^2}{12} = -26.67$, $M_{FBA} = +26.67$

$M_{FBC} = -\frac{Wab^2}{l^2} = -28.8$, $M_{FCB} = +\frac{Wa^2b}{l^2} = +19.2$

$M_{FDB} = -\frac{Wl}{8} = -10$; $M_{FBD} = +10$

(b) **D.F**: (For Intermediate)

		K	ΣK	$DF = \frac{K}{\Sigma K}$
B	BA	$I/l = I/4 = 0.25I$		0.38
	BC	$\frac{3}{4}(I/l) = \frac{3}{4}(I/5) = 0.15I$	0.65I	0.24
	BD	$I/l = I/4 = 0.25I$		0.38

"C" Roller

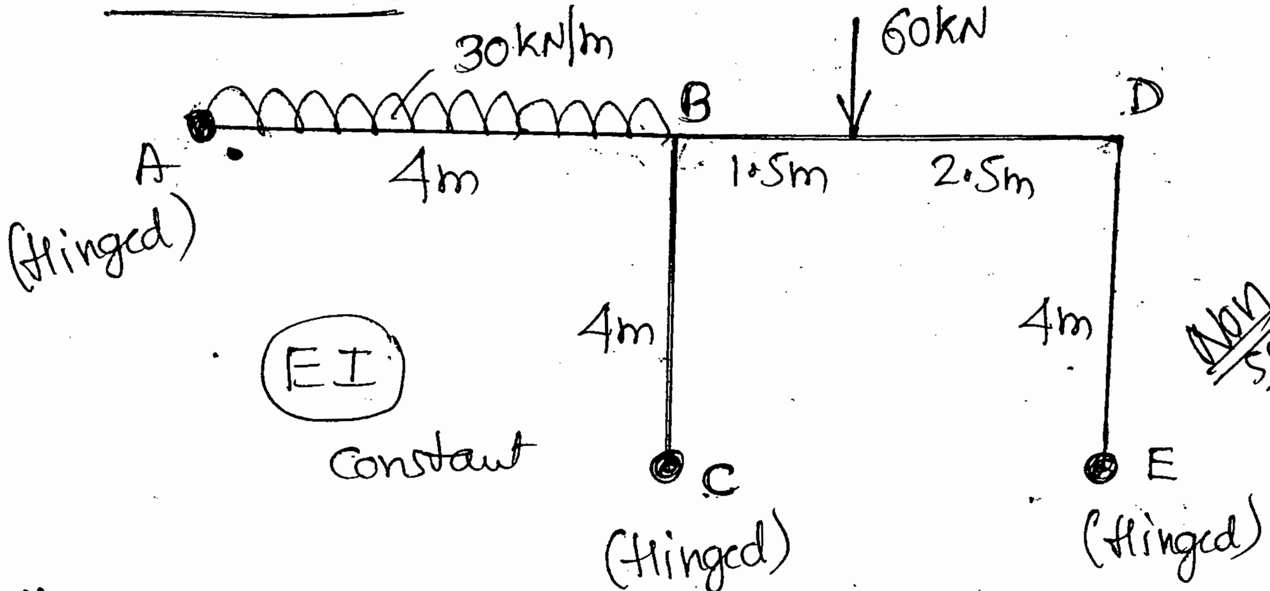
(c)

AB	BA	BD	DB	BC	CB	Member
	0.38	0.38		0.24		DF
−26.67	26.67	+10	−10	−28.8	19.2	FEM
					−19.2	Release (C)
				−9.60		C.O.
−26.67	26.67	10	−10	−38.40	0	Initial Values.
	0.66	0.66		0.41		Bal
0.33			0.33		0	C.O
−26.33	27.33	10.66	−9.67	−37.99	0	Final Values.
↻	↺	↺	↻	↻	0	

Refer S.D. Notes For BMD.

At "B" $M_{BA} + M_{BC} + M_{BD} = 0$

Eg:- 3) Analyse the frame shown by MD method.



Non sway

Sol^n (a) FEM

$M_{FAB} = -\frac{\omega l^2}{12} = -40$, $M_{FBA} = +40$

$M_{FBD} = -35.16$, $M_{FDB} = +21.10$ kN-m

(b) D.F (For Intermediate)

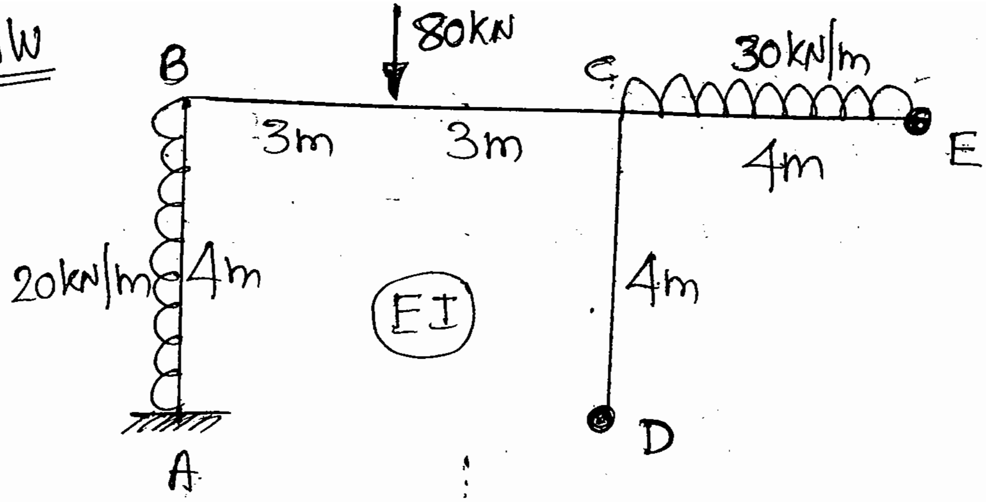
		K	Σk	$DF = \frac{K}{\Sigma k}$
B	BA	$\frac{3}{4}(I/4) = 0.187I$	0.625I	0.30
	BC	$\frac{3}{4}(I/4) = 0.187I$		0.30
	BD	$I/4 = 0.25I$		0.40
D	DB	$I/4 = 0.25I$	0.437I	0.57
	DE	$\frac{3}{4}\left(\frac{I}{4}\right) = 0.187I$		0.43.

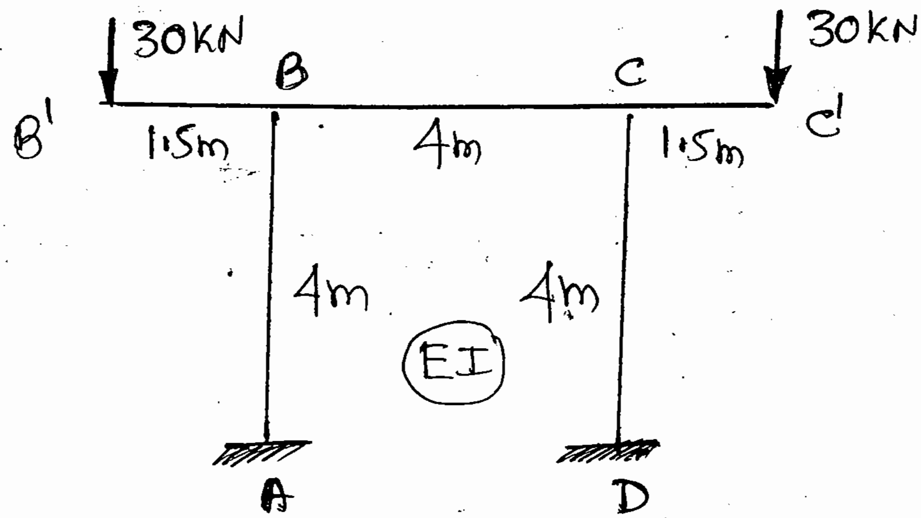
M.D. Table :

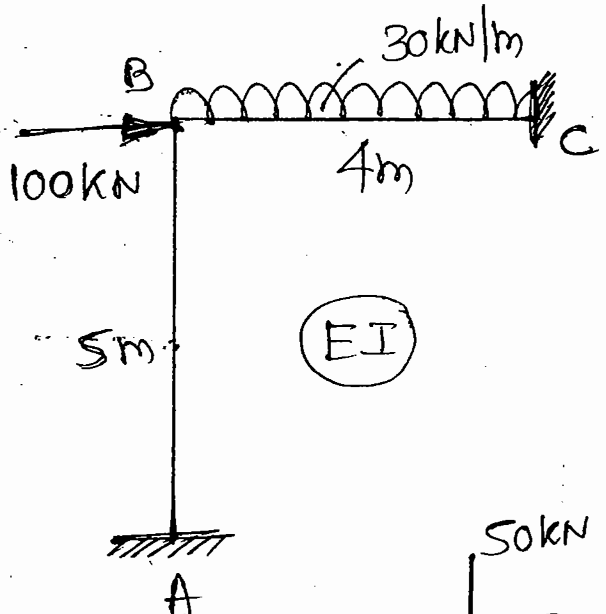
AB	BA	BC	CB	BD	DB	DE	ED	Member
	0.30	0.30		0.40	0.57	0.43		DF
-40	40	0	0	-35.16	21.10	0	0	FEM
+40	20							Release @ (A) C.O
0	60	0	0	-35.16	21.10	0	0	Initial
	-7.45	-7.45		-9.94	-12.03	-9.07		Bal
0			0	-6.01	-4.97		0	C.O
	1.80	1.80		2.40	2.83	2.14		Bal
0			0	1.42	1.20		0	C.O
	-0.43	-0.43		-0.57	-0.68	-0.52		Bal.
0			0	-0.34	-0.28		0	C.O
	0.10	0.10		0.14	0.16	0.12		Bal
0			0	0.08	0.07		0	C.O
	-0.02	-0.02		-0.04	-0.04	-0.03		Bal
0	54	-6	0	-48.02	7.36	-7.36	0	Final

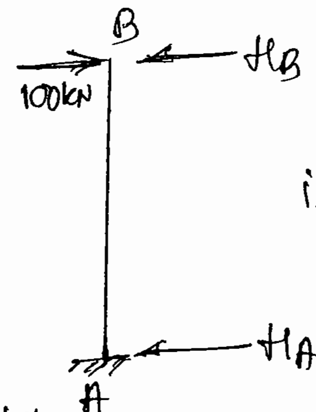
At "B" $\rightarrow$ $M_{BA} + M_{BC} + M_{BD} = 0$ | At "D" $M_{DB} + M_{DE} = 0$

HW

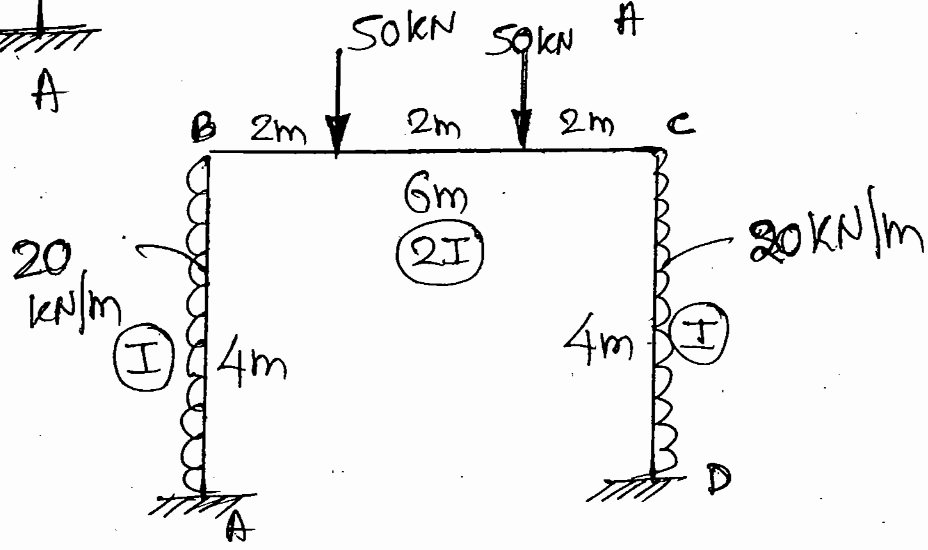








100 kN load is used only at the time of SFD.



Part-B

Kani's Method

(i) Rotation Factor = $\boxed{U = \left(-\frac{1}{2}\right)\frac{K}{\Sigma K}}$

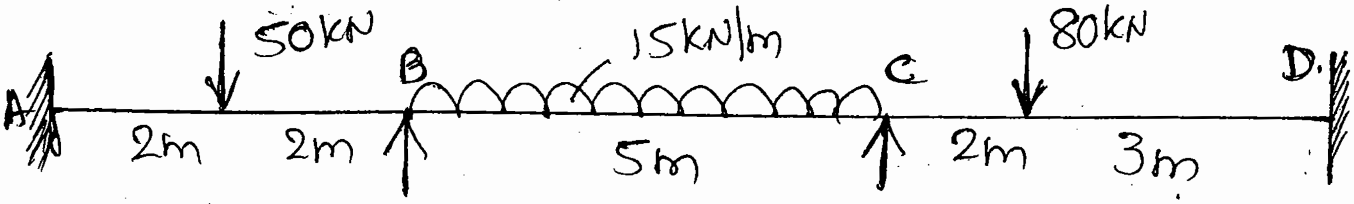
(ii) Rotation Moment :

$$\boxed{m'_{AB} = U\left[\Sigma M_F + \Sigma \text{ Far end Rotation Moment}\right]}$$

(iii) Final Moment

$$\boxed{M = F.E.M + 2\left(\text{Near End Rotation Moment}\right) + \left(\text{Far end Rotation Moment}\right)}$$

Eg:- 1] Analyse the beam shown by Kani's method, Draw BMD.



Sol^{n} (a) FEM

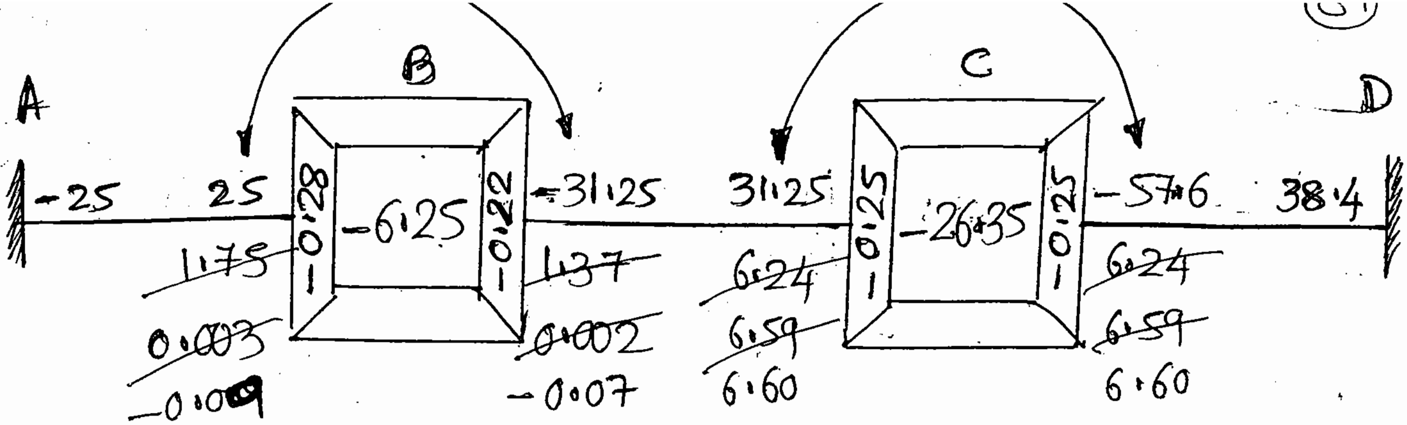
$M_{FAB} = -25$ kN-m, $M_{FBA} = +25$

$M_{FBC} = -31.25$, $M_{FCB} = +31.25$

$M_{FCD} = -57.6$, $M_{FDC} = 38.4$.

(b) Rotation Factor (For Intermediate)

		K	ΣK	$U = \left(-\frac{1}{2}\right)\frac{K}{\Sigma K}$
B	BA	$I/4 = 0.25I$	$0.45I$	-0.28
	BC	$I/5 = 0.20I$		-0.22
C	CB	$I/5 = 0.20I$	$0.4I$	-0.25
	CD	$I/5 = 0.20I$		-0.25



Rotation Moment $\boxed{m'_{AB} = U\left[\Sigma M_F + \Sigma \text{ Far end Rotation moment}\right]}$

Trial (1)

$m'_{BA} = -0.28(-6.25 + 0) = 1.75$

$m'_{BC} = -0.22(-6.25 + 0) = 1.37$

$m'_{CB} = -0.25(-26.35 + 1.37) = 6.24$

$m'_{CD} = -0.25(-26.35 + 1.37) = 6.24$

Trial (2)

$m'_{BA} = -0.28(-6.25 + 6.24) = 0.003$

$m'_{BC} = -0.22(-6.25 + 6.25) = 0.002$

$m'_{CB} = -0.25(-26.35 + 0.002) = 6.59$

$m'_{CD} = -0.25(-26.35 + 0.002) = 6.59$

Trial (3)

$m'_{BA} = -0.28(-6.25 + 6.59) = -0.09$

$m'_{BC} = -0.22(-6.25 + 6.59) = -0.07$

$m'_{CB} = -0.25(-26.35 - 0.07) = 6.60$

$m'_{CD} = -0.25(-26.35 - 0.07) = 6.60$

Final Moment

$$M = FEM + 2\left(\begin{array}{c}\text{Near End}\\ \text{Rotation}\\ \text{moment}\end{array}\right) + 1\left(\begin{array}{c}\text{Far end}\\ \text{Rotation}\\ \text{moment}\end{array}\right)$$

$M_{AB} = -25 + 2(0) - 0.09 = -25.09$ kN-m

$M_{BA} = +25 + 2(-0.09) + 0 = 24.82$ kN-m

$M_{BC} = -31.25 + 2(-0.07) + 6.60 = -24.79$ kN-m

$M_{CB} = +31.25 + 2(6.60) - 0.07 = 44.38$ "

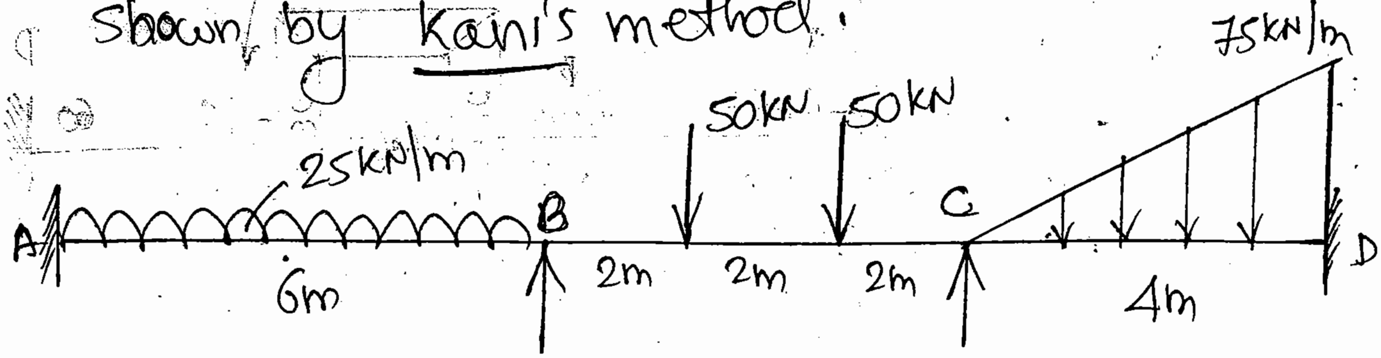
$M_{CD} = -57.6 + 2(6.60) - 0 = -44.40$ "

$M_{DC} = 38.4 + 2(0) + 6.60 = 45$ kN-m

Draw SFD, BMD & EC.

Refer S.D. Notes.

Eg:- Analyse the continuous beam shown by Kani's method.



(a) FEM

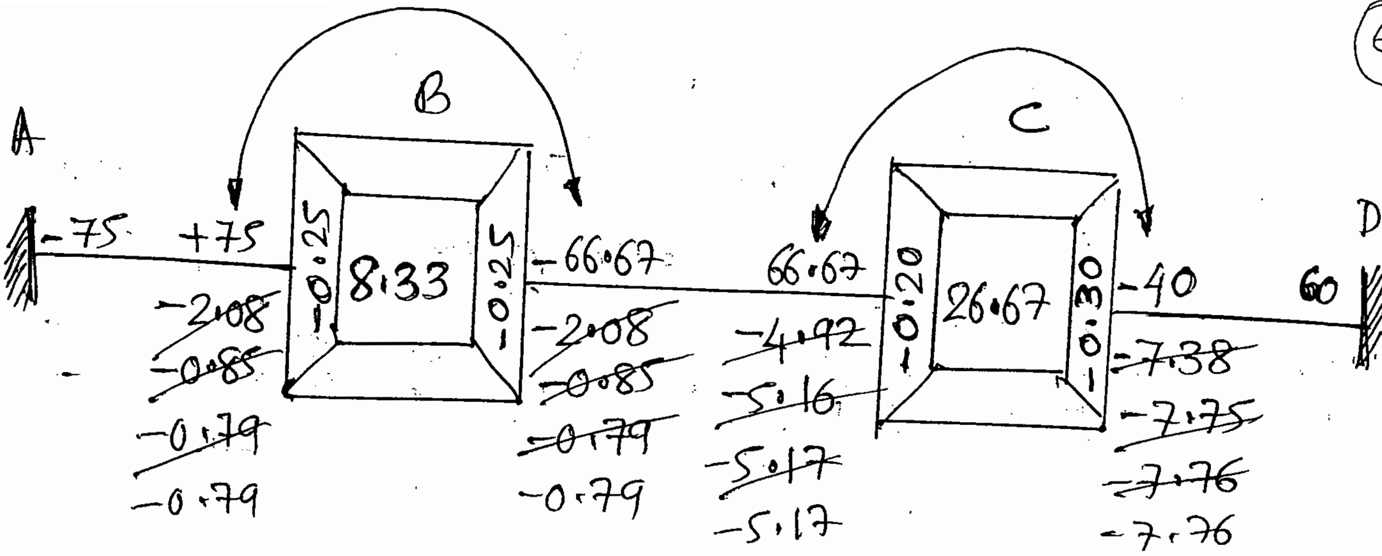
$M_{FAB} = \frac{-wl^2}{12} = -75$, $M_{FBA} = +75$

$M_{FBC} = -\frac{Wab^2}{l^2} = -66.67$ kN-m, $M_{FCB} = +66.67$ kN-m

$M_{FCD} = \frac{-Wl^2}{30} = -40$, $M_{FDC} = \frac{+wl^2}{20} = +60$

(b) Rotation Factor (For Intermediate)

		K	ΣK	$U = \left(\frac{-1}{2}\right)\frac{K}{\Sigma K}$
B	BA	$I/l = I/6 = 0.167I$	0.334I	−0.25
	BC	$I/l = I/6 = 0.167I$		−0.25
C	CB	$I/l = I/6 = 0.167I$	0.417I	−0.20
	CD	$I/l = \frac{I}{4} = 0.25I$		−0.30



Rotation Moment

$$\boxed{m = U\left[\Sigma M_F + \Sigma \text{ Far end Rotation Moment}\right]}$$ ✓

Trial (1)

$$m_{BA} = -0.25\,(8.33 + 0) = -2.08$$

$$m_{BC} = -0.25\,(8.33 + 0) = -2.08$$

$$m_{CB} = -0.20\,(26.67 - 2.08) = -4.92$$

$$m_{CD} = -0.30\,(26.67 - 2.08) = -7.38$$

Trial (2)

$$m_{BA} = -0.25(8.33 - 4.92) = -0.85$$

$$m_{BC} = -0.25(8.33 - 4.92) = -0.85$$

$$m_{CB} = -0.20(26.67 - 0.85) = -5.16$$

$$m_{CD} = -0.30(26.67 - 0.85) = -7.75$$

Trial (3)

$M_{BA} = -0.25\,(8.33 - 5.16) = -0.79$

$M_{BC} = -0.25\,(8.33 - 5.16) = -0.79$

$M_{CB} = -0.20\,(26.67 - 0.79) = -5.17$

$M_{CD} = -0.30\,(26.67 - 0.79) = -7.76$

Trial (4)

$M_{BA} = -0.79$ kN-m

$M_{BC} = -0.79$

Final Moment :-

$$\boxed{M = FEM + 2\begin{bmatrix}\text{Near end}\\ \text{Ro. Moment}\end{bmatrix} + \begin{bmatrix}\text{Far End}\\ \text{Ro. moment}\end{bmatrix}}$$

$M_{AB} = -75 + 2(0) - 0.79 = -75.79$ kN-m

$M_{BA} = +75 + 2(-0.79) + 0 = 73.42$ kN-m

$M_{BC} = -66.67 + 2(-0.79) - 5.17 = -73.42$ kN-m

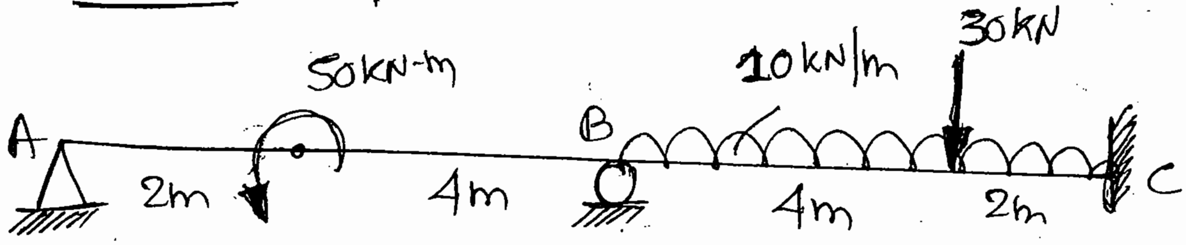
$M_{CB} = +66.67 + 2(-5.17) - 0.79 = 55.54$ kN-m

$M_{CD} = -40 + 2(-7.76) + 0 = -55.52$ kN-m

$M_{DC} = +60 + 2(0) - 7.76 = 52.24$ kN-m

Refer M.D. Notes for SFD & BMD.

Eg:-3] Analyse the beam shown by Kani's method



Sol^n (a) FEM

$$M_{FAB} = -\frac{Mb(2a-b)}{l^2} = -\frac{50\times4(2\times2-4)}{6^2} = 0$$

$$M_{FBA} = -\frac{Ma(2b-a)}{l^2} = -\frac{50\times2(2\times4-2)}{6^2} = -16.67$$

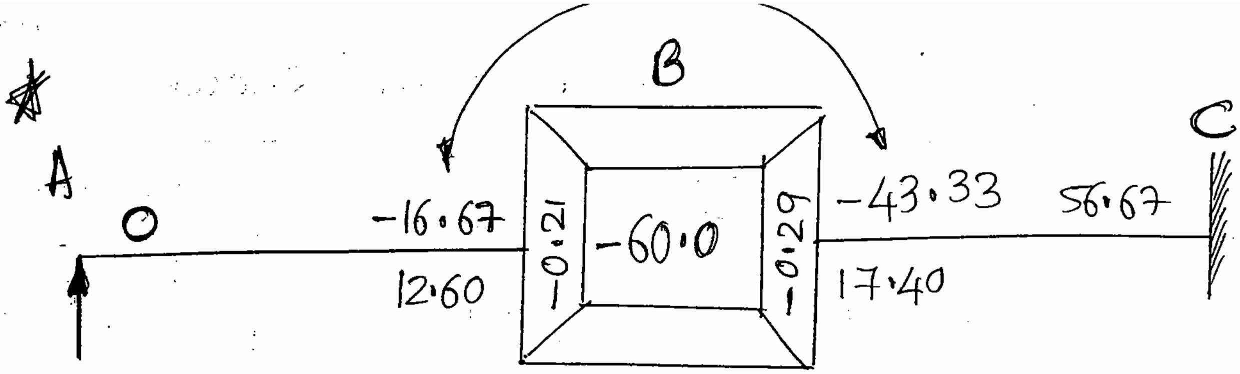
$$M_{FBC} = -\frac{wl^2}{12} - \frac{Wab^2}{l^2} = -43.33 \text{ kN-m}$$

$$M_{FCB} = +\frac{wl^2}{12} + \frac{Wa^2b}{l^2} = +56.67 \text{ ''}$$

(b) Rotation Factor : (For Intermediate)

		k	Σk	$U = \left(-\frac{1}{2}\right)\frac{k}{\Sigma k}$
B	BA	$\frac{3}{4}\left(\frac{I}{l}\right) = \frac{3}{4}\left(\frac{I}{6}\right) = 0.125I$	$0.292I$	-0.21
	BC	$\frac{I}{l} = \frac{I}{6} = 0.167I$		-0.29

(OT)



Rotation Moment

$$\boxed{M = U\left[\Sigma M_F + \Sigma \text{ Rotation far end moment}\right]}$$

$$M_{BA} = -0.21\,(-60 + 0) = 12.60$$

$$M_{BC} = -0.29\,(-60 + 0) = 17.40$$

Final Moment :

$$\boxed{M = FEM + 2\left(\text{Near end Ro, moment}\right) + \left(\text{Far end Ro moment}\right)}$$

$M_{AB} = 0$ ★ If last support is Simple or Hinge or Roller the above eq^n is not applicable.

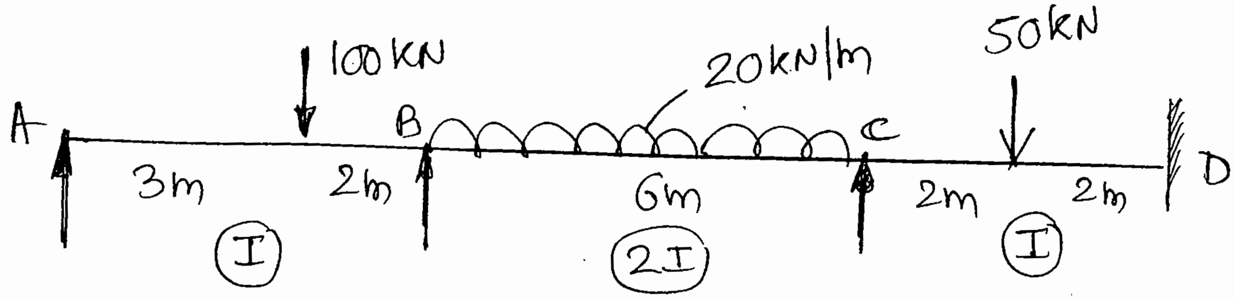
$$M_{BA} = -16.67 + 2(12.60) + 0 = 8.53$$

$$M_{BC} = -43.33 + 2(17.40) + 0 = -8.53 \text{ kN-m}$$

$$M_{CB} = 56.67 + 2(0) + 17.40 = 74.07 \text{ kN-m}$$

Draw BMD & SFD.

Eg:- 4 Analyse the beam shown by kani's method



Sol:

(a) FEM

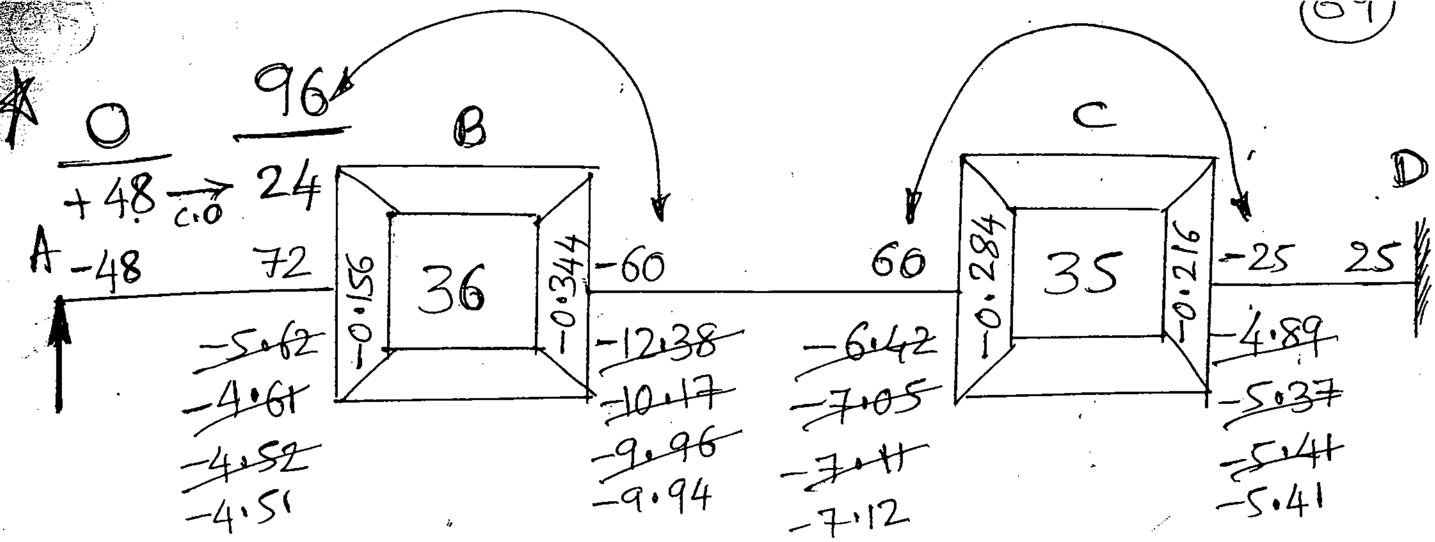
$M_{FAB} = -48$, $M_{FBA} = 72$

$M_{FBC} = -60$, $M_{FCB} = +60$

$M_{FCD} = -25$, $M_{FDC} = 25$.

(b) Rotation Factor :-

		K	ΣK	$U = \left(-\frac{1}{2}\right)\frac{K}{\Sigma K}$
B	BA	$\frac{3}{4}\left(\frac{I}{5}\right) = 0.15I$	0.48I	-0.156
	BC	$\frac{2I}{6} = 0.33I$		-0.344
C	CB	$\frac{2I}{6} = 0.33I$	0.58I	-0.284
	CD	$\frac{I}{4} = 0.25I$		-0.216



Rotation Moment

$$\boxed{M = U\left[\Sigma M_F + \Sigma \text{ Far End Ro. Moment}\right]}$$

Trial-1

$M_{BA} = -0.156\,(36 + 0) = -5.62$

$M_{BC} = -0.344\,(36 + 0) = -12.38$

$M_{CB} = -0.284\,(35 - 12.38) = -6.42$

$M_{CD} = -0.216\,(35 - 12.38) = -4.89$

Final Moment : $\boxed{M = FEM + 2(\text{Near}) + (\text{Far})}$

$M_{AB} = 0$ * The above eq^n is not applicable *

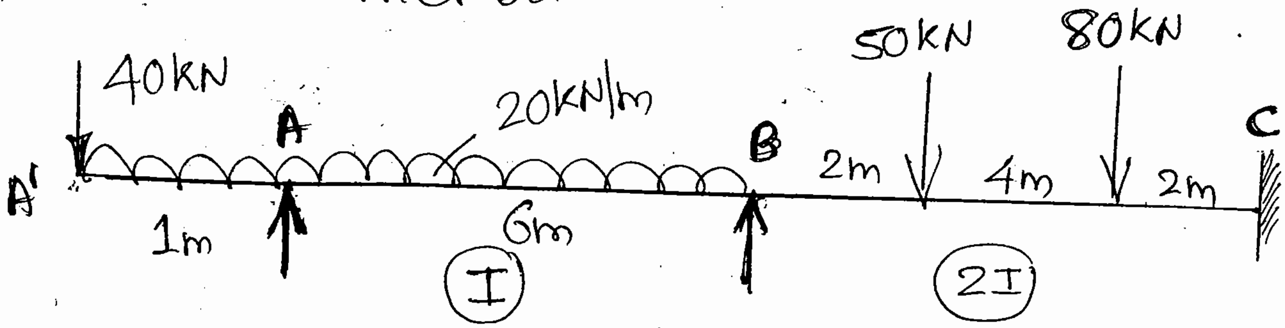
$M_{BA} = 96 + 2(-4.51) + 0 = 86.98$ kN-m

$M_{BC} = -60 + 2(-9.94) - 7.12 = -87.00$ kN-m

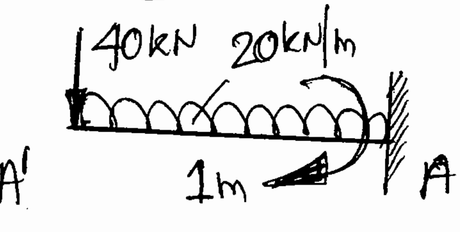
$M_{CB} = +60 + 2(-7.12) - 9.94 = 35.82$ kN-m

$M_{CD} = -35.82$ kN-m, $M_{DC} = 19.59$ kN-m

Eg: 5] Analyse the beam shown by Kani's method.



Sol^n (a) FEM



$M_{AA'} = +40 \times 1 + 20 \times 1 \times 1/2 = +50$ kN-m (Final moment)

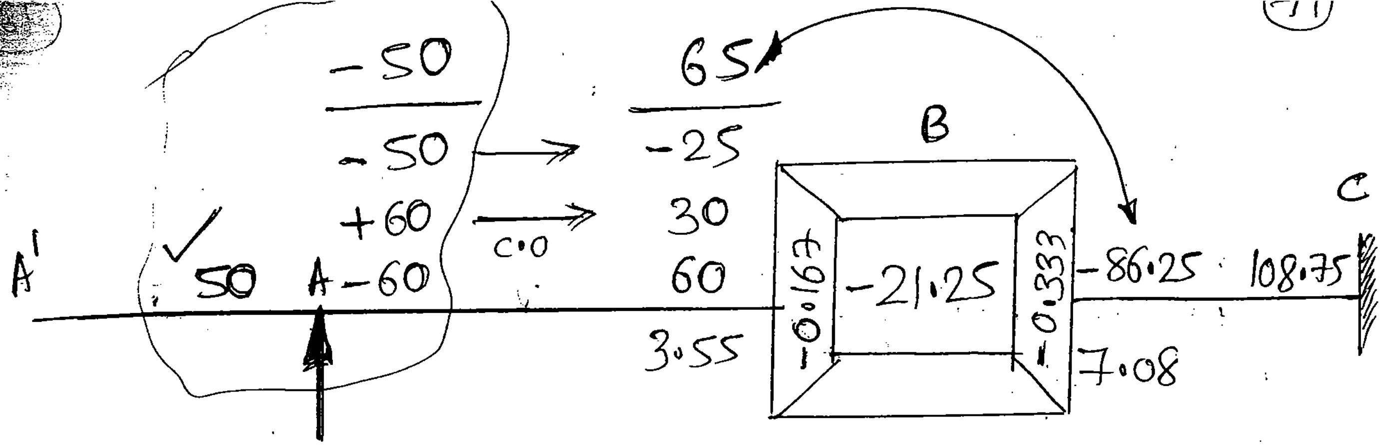
$M_{FAB} = -60$, $M_{FBA} = +60$

$M_{FBC} = -86.25$, $M_{FCB} = 108.75$.

(b) Rotation Factor :

★ The support with overhang portion, ~~ration~~ rotation factor and Kani's Box both are not required. ★

		K	ΣK	$U = \left(-\frac{1}{2}\right)\frac{K}{\Sigma K}$
B	BA	$\frac{3}{4}\left(\frac{I}{l}\right) = \frac{3}{4}\left(\frac{I}{6}\right) = 0.125I$	0.375I	-0.167
	BC	$\frac{I}{l} = \frac{2I}{8} = 0.25I$		-0.333



Rotation moment

$M_{BA} = -0.167(-21.25+0) = 3.55 \text{ kN-m}$

$M_{BC} = -0.333(-21.25+0) = 7.08 \text{ kN-m}$

Final Moment: $\boxed{M = FEM + 2(\text{Near}) + (\text{Far})}$

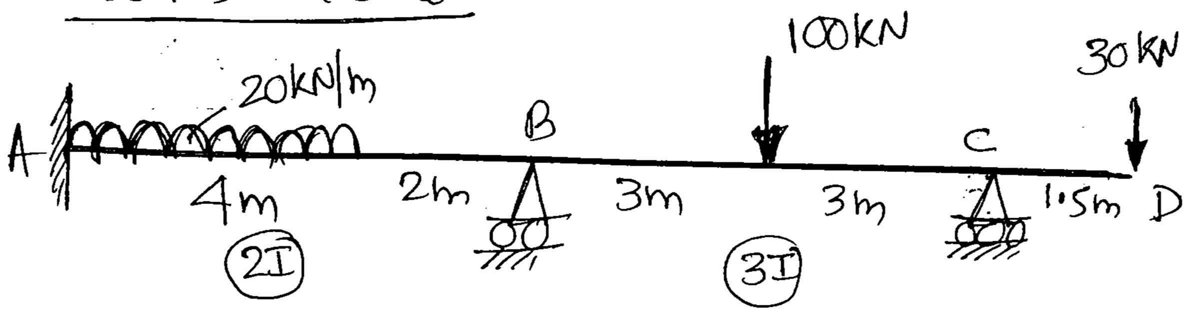
$M_{AA'} = 50 \text{ kN-m}$
$M_{AB} = -50 \text{ kN-m}$
★ For these two the eq^n is not applicable. They are final moments.

$M_{BA} = 65 + 2(3.55) + 0 = 72.1 \text{ kN-m}$

$M_{BC} = -86.25 + 2(7.08) + 0 = -72.10 \text{ kN-m}$

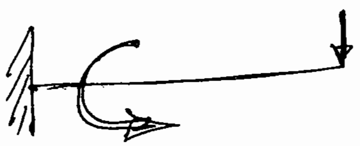
$M_{CB} = 108.75 + 2(0) + 7.08 = 115.83 \text{ kN-m}$

Eg:-6] Analyse the beam shown by Kani's method



Sol: (a) $M_{FAB} = -53.33$, $M_{FBA} = +35.56$

$M_{FBC} = -75$, $M_{FCB} = +75$



$M_{CD} = -30 \times 1.5 = -45$ kN-m.

(b) R.F: (Only at "B").

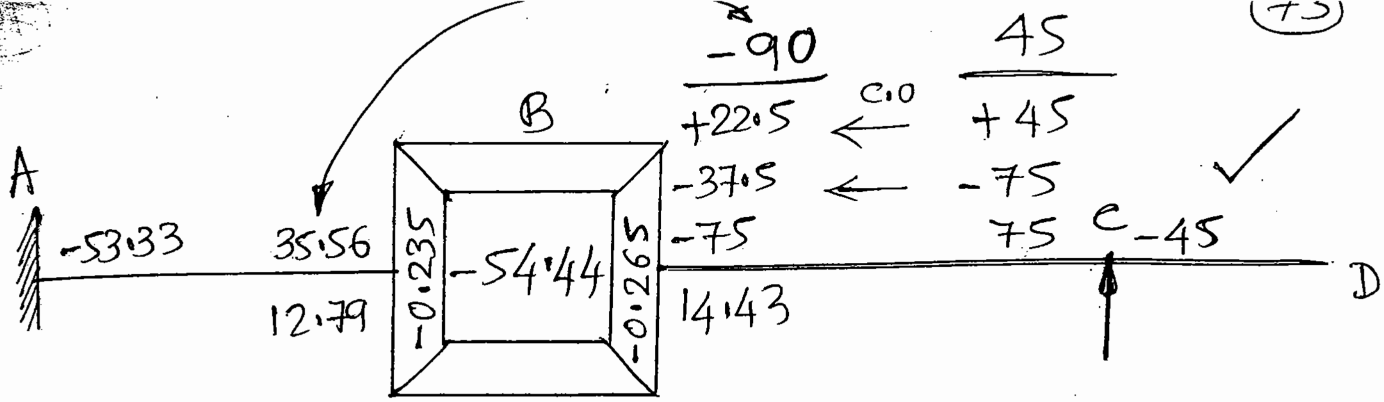
		K	ΣK	U
B	BA	$I/l = 2I/6 = 0.333I$	$0.708I$	-0.235
	BC	$\frac{3}{4}\left(\frac{I}{l}\right) = \frac{3}{4}\left(\frac{3I}{6}\right) = 0.375I$		-0.265

☆ Any overhanging moments are final.

$\therefore M_{CD} = -45$ kN-m

$\therefore$ from equilibrium point of view

"M_{CB}" should be $+45$ kN-m



Rotation Moment

$$m = U\left[\Sigma FEM + \Sigma \text{ Farend Ro. Moment}\right]$$

$$m_{BA} = -0.235(-54.44+0) = 12.79$$

$$m_{BC} = -0.265(-54.44+0) = 14.43$$

Final Moments :

$$M = FEM + 2(\text{Near}) + (\text{Far})$$

$$M_{AB} = -53.33 + 2(0) + 12.79 = -40.54 \text{ kN-m}$$

$$M_{BA} = 35.56 + 2(12.79) + 0 = 61.14 \text{ kN-m}$$

$$M_{BC} = -90 + 2(14.43) + 0 = -61.14 \text{ kN-m}$$

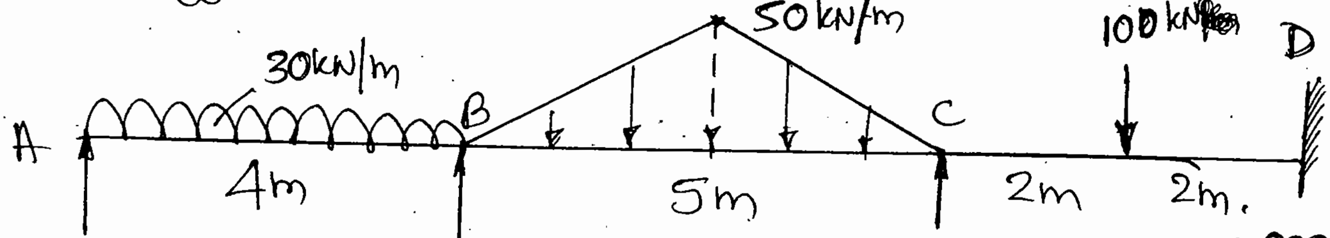
$$M_{CB} = +45 \star$$

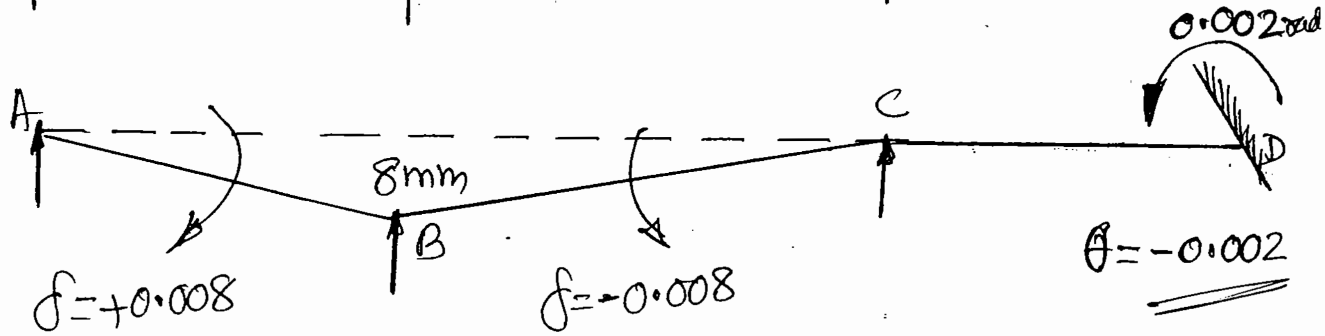
$$M_{CD} = -45 \star$$

Sinking of Support

Eg:- Analyse the beam shown by kani's method. The support 'D' rotates by 0.002 rad. anti-clockwise. The support 'B' sinks by 8mm.

Take $E = 210 GPa$, $I = 0.1 G\ mm^4$.





$$E = 210 \times 10^9 \times 10^{-6} = 210 \times 10^3\ N/mm^2$$

$$I = 0.1 \times 10^9\ mm^4$$

$$EI = \frac{(210 \times 10^3)(0.1)10^9}{(10^3)\ (10^3)^2}\ N\text{-}mm^2 = 21000\ kN\text{-}m^2$$

(a) FEM :

$$\text{Additional moment} = \frac{-6EI\delta}{l^2} \quad \text{(sinking)}$$

$$= \frac{4EI\theta}{l} \longrightarrow \text{Near end Rotation}$$

$$= \frac{2EI\theta}{l} \rightarrow \text{Far end rotation}$$

$$M_{FAB} = -\frac{wl^2}{12} - \frac{6EI\delta}{l^2} = \frac{-30\times4^2}{12} - \frac{6(21000)(0.008)}{4^2}$$

$$= -103.0 \text{ kN-m}$$

$$M_{FBA} = +\frac{wl^2}{12} - \frac{6EI\delta}{l^2} = -23 \text{ kN-m}.$$

$$M_{FBC} = -\frac{5wl^2}{96} - \frac{6EI\delta}{l^2} = -\frac{5(50)5^2}{96} - \frac{6(21000)(-0.008)}{5^2}$$

$$= -24.78$$

$$M_{FCB} = +\frac{5wl^2}{96} - \frac{6EI\delta}{l^2} = 105.42 \text{ kN-m}.$$

Far end

$$M_{FCD} = -\frac{Wl}{8} + \frac{2EI\theta}{l} = \frac{-100\times4}{8} + \frac{2(21000)(-0.002)}{4}$$

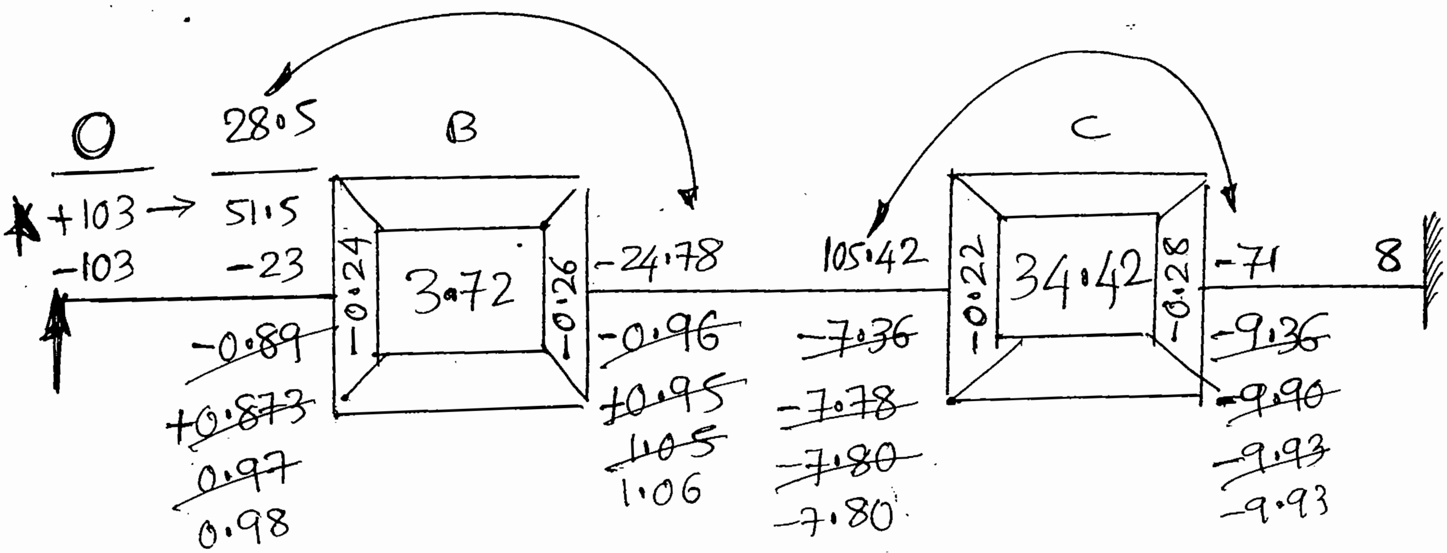
$$= -71 \text{ kN-m}$$

Near

$$M^F_{DC} = +\frac{Wl}{8} + \frac{4EI\theta}{l} = \frac{100\times4}{8} + \frac{4(21000)(-0.002)}{4}$$

$$= +8$$

		K	ΣK	U
B	BA	$\frac{3}{4}(I/4) = 0.1875I$	$0.3875I$	-0.24
	BC	$I/5 = 0.2I$		-0.26
C	CB	$I/5 = 0.2I$	$0.45I$	-0.22
	CD	$I/4 = 0.25I$		-0.28



$$m = U\left[\Sigma FEM + \Sigma \begin{array}{c}\text{Far end} \\ \text{Ro. moment}\end{array}\right]$$

Trial (1)

$$m_{BA} = -0.24\,(3.72 + 0) = -0.89$$

$$m_{BC} = -0.26\,(3.72 + 0) = -0.96$$

$$m_{CB} = -0.22\,(34.42 - 0.96) = -7.36$$

$$m_{CD} = -0.28\,(34.42 - 0.96) = -9.36$$

Final Moment

$M_{AB} = 0$ ★

$M_{BA} = 28.5 + 2(0.98) + 0 = 30.46$ kN-m

$M_{BC} = \qquad = -30.46$ kN-m

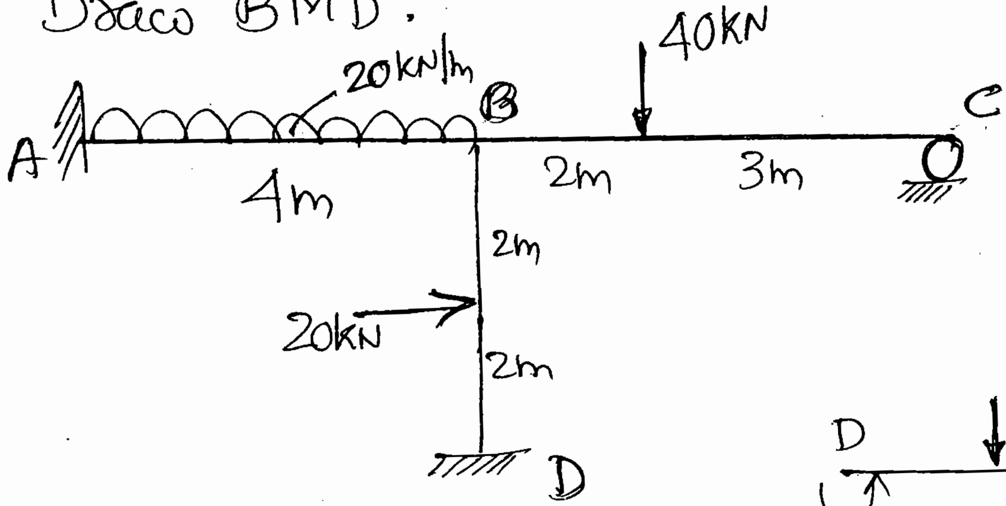
$M_{CB} = \qquad = 90.88$ kN-m

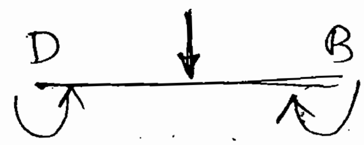
$M_{CD} = \qquad = -90.86$ kN-m

$M_{DC} = \qquad = -1.93$ kN-m

Non Sway Frames

Eg:- Analyse the frame by Kani's method. Draw BMD.





Sol^n

(a) FEM

$M_{FAB} = -26.67$, $M_{FBA} = +26.67$

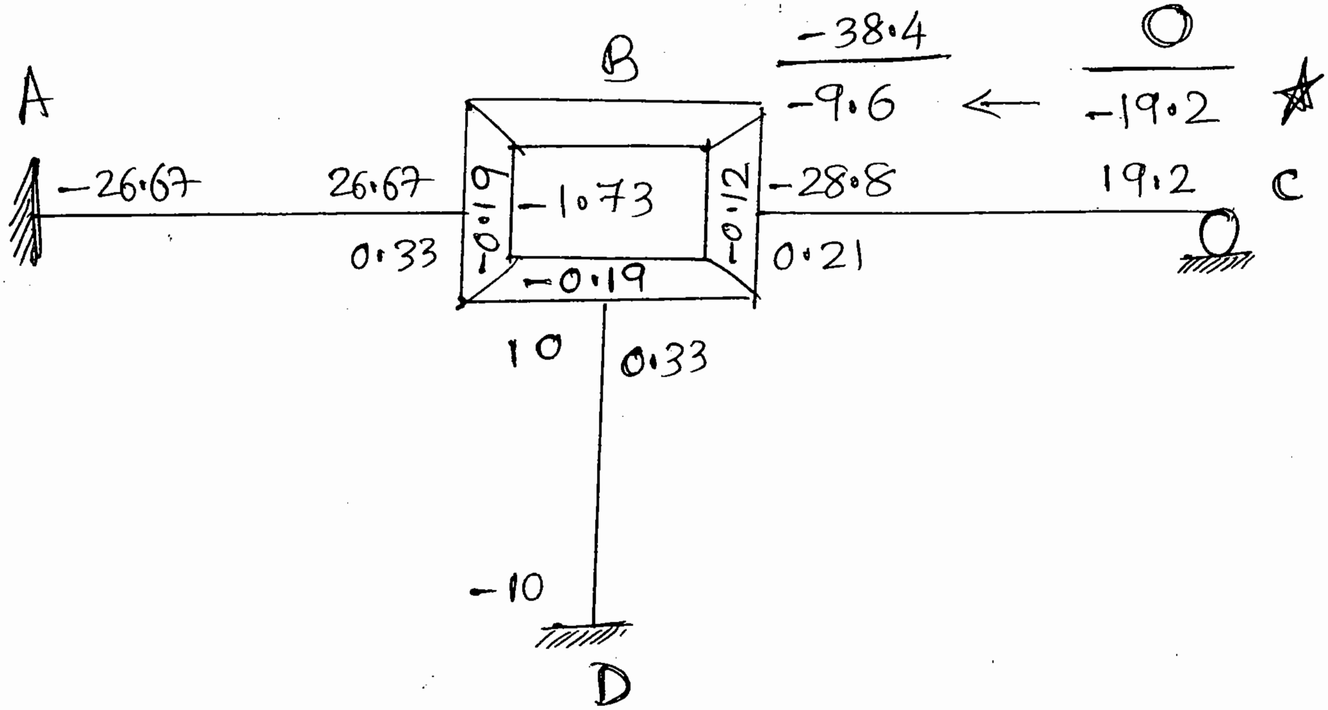
$M_{FBC} = -28.8$, $M_{FCB} = +19.20$

$M_{FDB} = -10$, $M_{FBD} = +10$

(b) R.F

		K	ΣK	U
B	BA	$I/4 = 0.25I$	$0.65I$	-0.19
	BC	$\frac{3}{4}\left(I/5\right) = 0.15I$		-0.12
	BD	$I/4 = 0.25I$		-0.19

At B → 26.67 − 38.4 + 10 = (−1.73)
BA BC BD



$$m_{BA} = -0.19(-1.73+0) = 0.33$$

$$m_{BC} = -0.12(-1.73+0) = 0.21$$

$$m_{BD} = -0.19(-1.73+0) = 0.33$$

Final :-

$$M_{AB} = -26.67 + 2(0) + 0.33 = -26.34 \text{ KN-m}$$

$$M_{BA} = 26.67 + 2(0.33) + 0 = 27.33 \text{ ''}$$

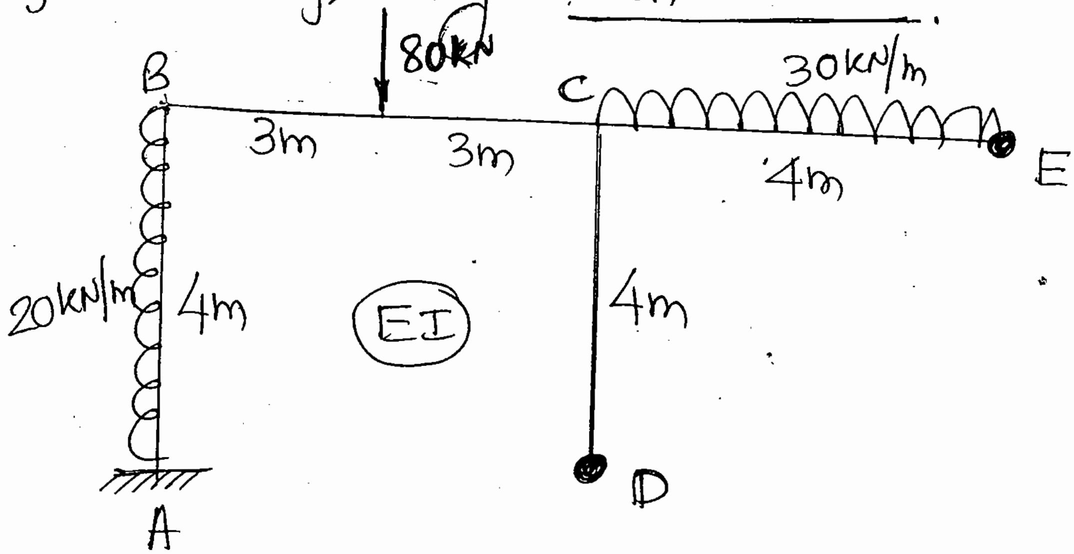
$$M_{BC} = -38.4 + 2(0.21) + 0 = -37.98 \text{ ''}$$

$$M_{CB} = 0$$

$$M_{BD} = 10 + 2(0.33) + 0 = 10.66 \text{ ''}$$

$$M_{DB} = -10 + 2(0) + 0.33 = -9.67 \text{ ''}$$

Eg:- Analyse by Kani's Method



Sol^n

(a) FEM

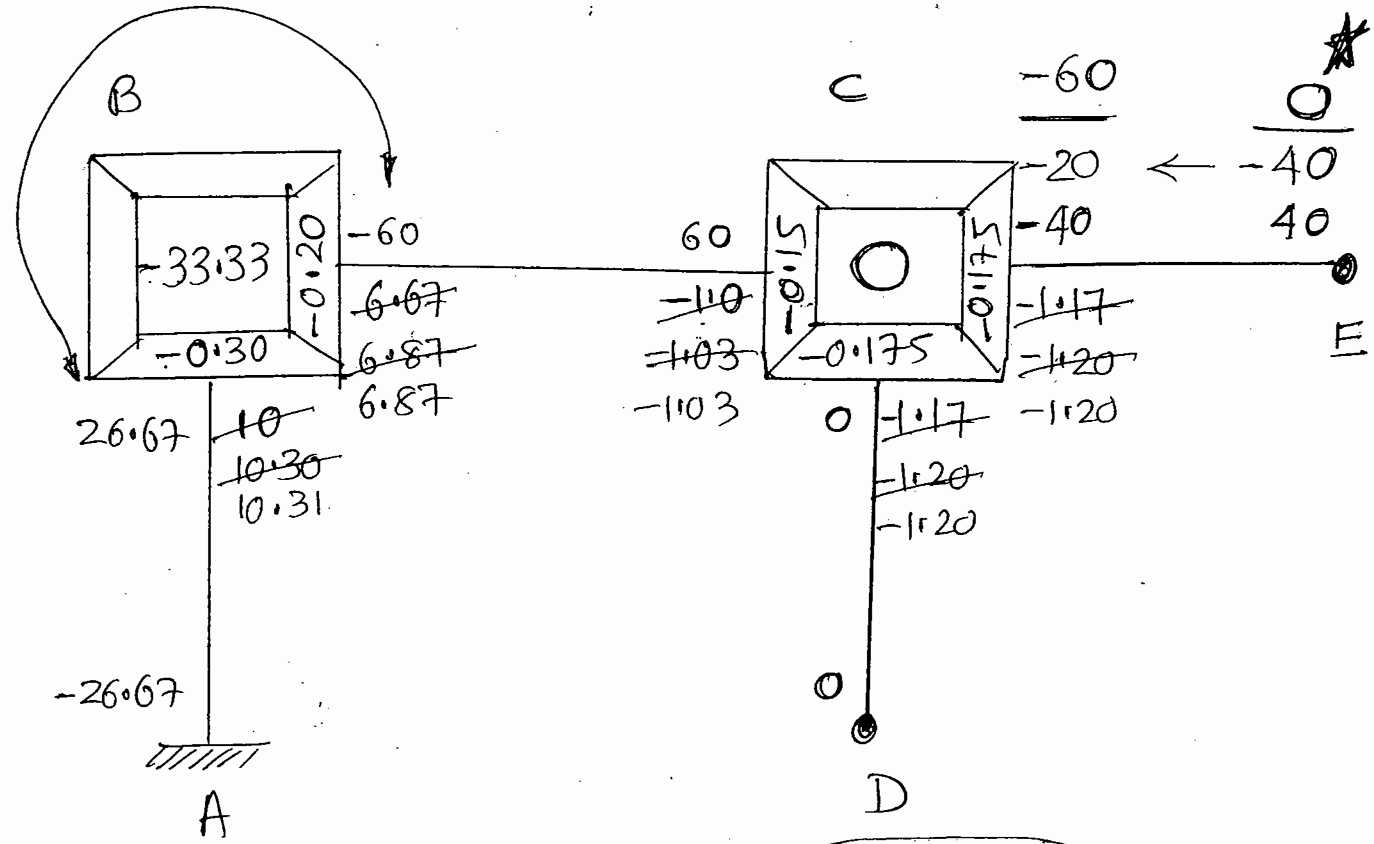
$M_{FAB} = -26.67$, $M_{FBA} = +26.66$ kN

$M_{FBC} = -60$, $M_{FCB} = +60$

$M_{FCE} = -40$, $M_{FEC} = +40$

(b) Rotation Factor (For "B" & "C")

		K	ΣK	$U = \left(-\frac{1}{2}\right)\frac{K}{\Sigma K}$
B	BA	$I/4 = 0.25I$	$0.416I$	-0.3
	BC	$I/6 = 0.167I$		-0.2
C	CB	$I/6 = 0.167I$	$0.542I$	-0.15
	CD	$\frac{3}{4}\left(\frac{I}{4}\right) = 0.1875I$		-0.175
	CE	$\frac{3}{4}\left(\frac{I}{4}\right) = 0.1875I$		-0.175



$$\boxed{m = U\left[\sum FEM + \sum \text{Far end Rot. moment}\right]}$$

Final Moment

$M_{AB} = -26.67 + 2(0) + 10.31 = -16.36$ ↻

$M_{BA} = 26.67 + 2(10.31) + 0 = 47.29$ ↺

$M_{BC} = -60 + 2(6.87) - 1.03 = -47.29$ ↻

$M_{CB} = 60 + 2(-1.03) + 6.87 = 64.81$ ↺

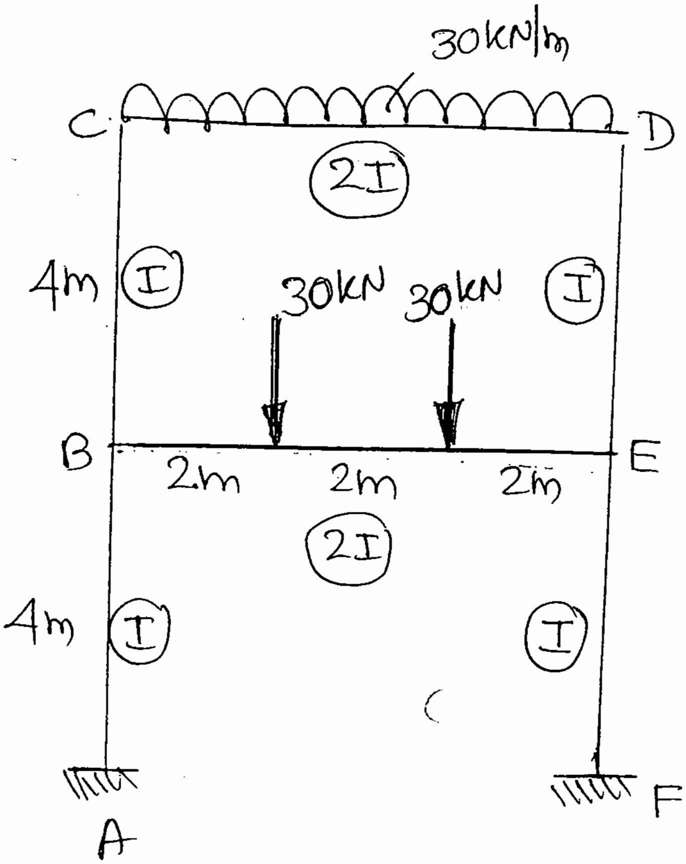
$M_{CD} = 0 + 2(-1.20) + 0 = -2.40$ ↻

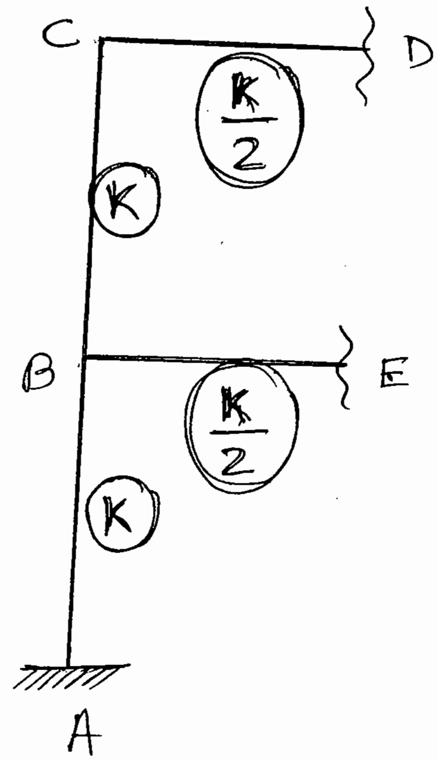
$M_{DC} = 0$

$M_{CE} = -60 + 2(-1.20) + 0 = -62.40$ kN-m ↻

Draw BMD.

Eg:- Analyse the frame shown by Kani's method





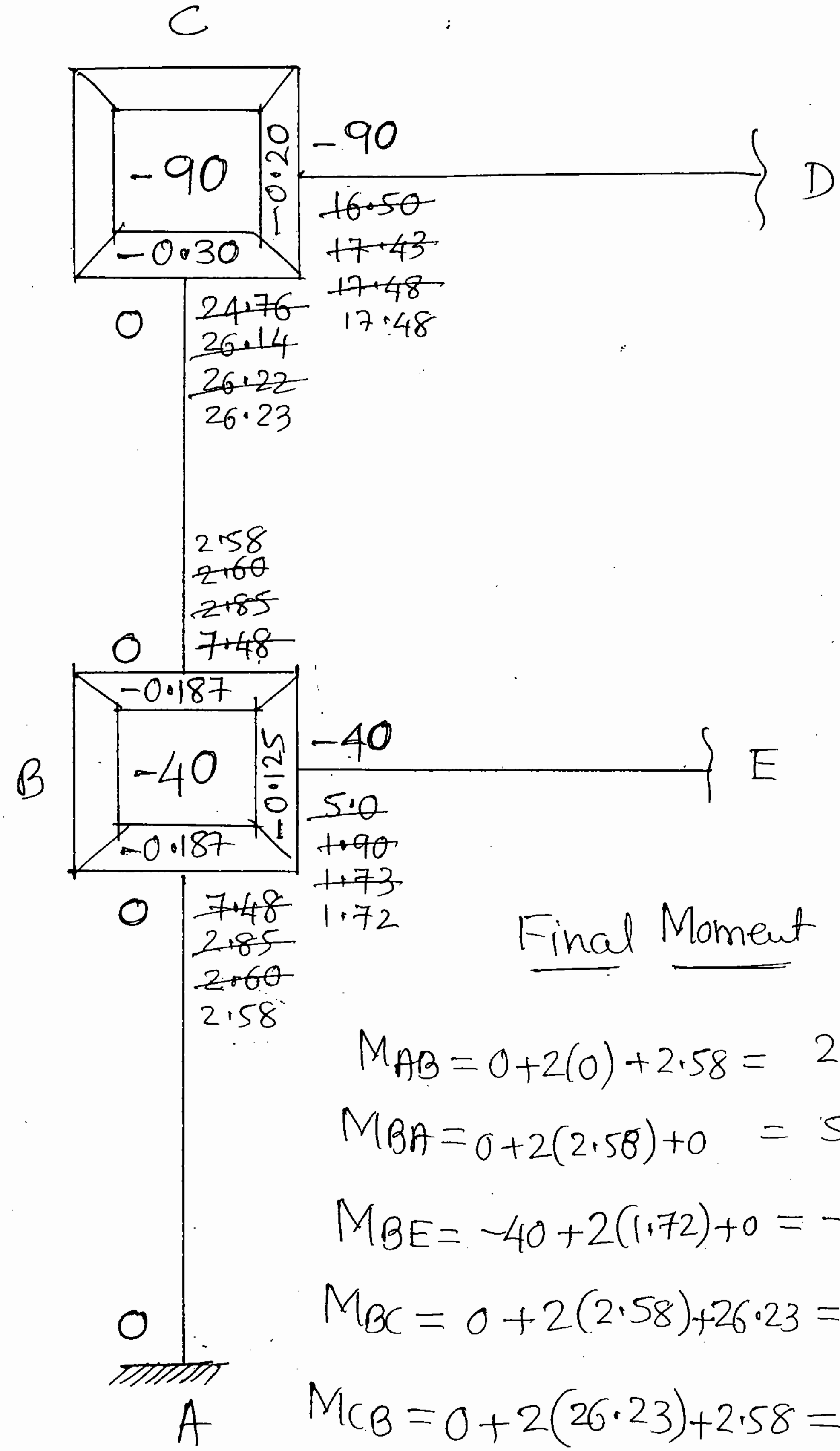
(a) FEM

$$M_{FBE} = \frac{-Wab^2}{l^2} = -\left[\frac{30\times2\times4^2}{6^2} + \frac{30\times4\times2^2}{6^2}\right] = -40$$

$$M_{FCD} = \frac{-wl^2}{12} = -90$$

(b) R.F (Only For "B" & "C")

		K	ΣK	U
B	BA	$K = I/l = I/4 = 0.25I$	0.667I	−0.187
	BC	$K = I/l = I/4 = 0.25I$		−0.187
	BE	$\frac{K}{2} = \frac{1}{2}\left(\frac{I}{l}\right) = \frac{1}{2}\left(\frac{2I}{6}\right) = 0.167I$		−0.125
C	CB	$K = I/l = I/4 = 0.25I$	0.417I	−0.30
	CD	$\frac{K}{2} = \frac{1}{2}\left(\frac{I}{l}\right) = \frac{1}{2}\left(\frac{2I}{6}\right) = 0.167I$		−0.20



Final Moment

$$M_{AB} = 0 + 2(0) + 2.58 = 2.58 \text{ kN-m}$$

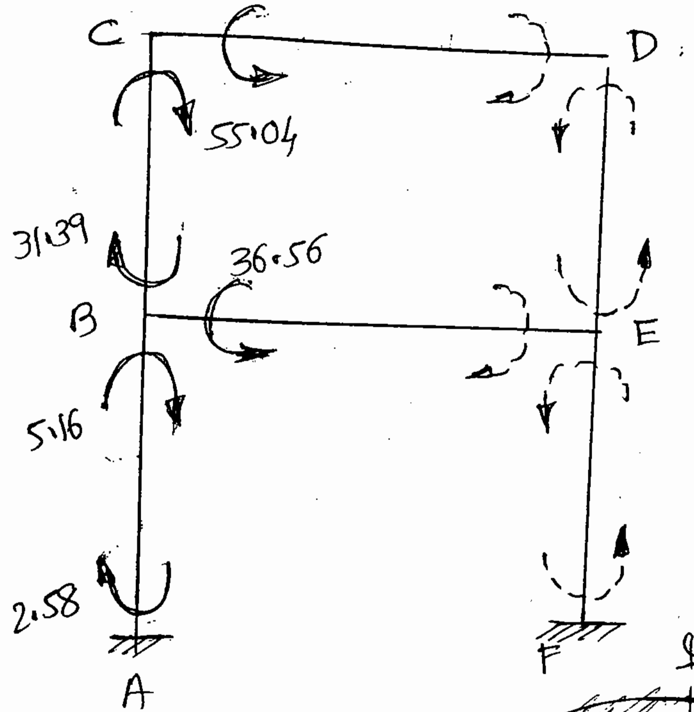
$$M_{BA} = 0 + 2(2.58) + 0 = 5.16 \text{ kN-m}$$

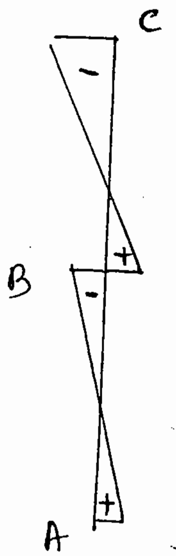
$$M_{BE} = -40 + 2(1.72) + 0 = -36.56 \text{ "}$$

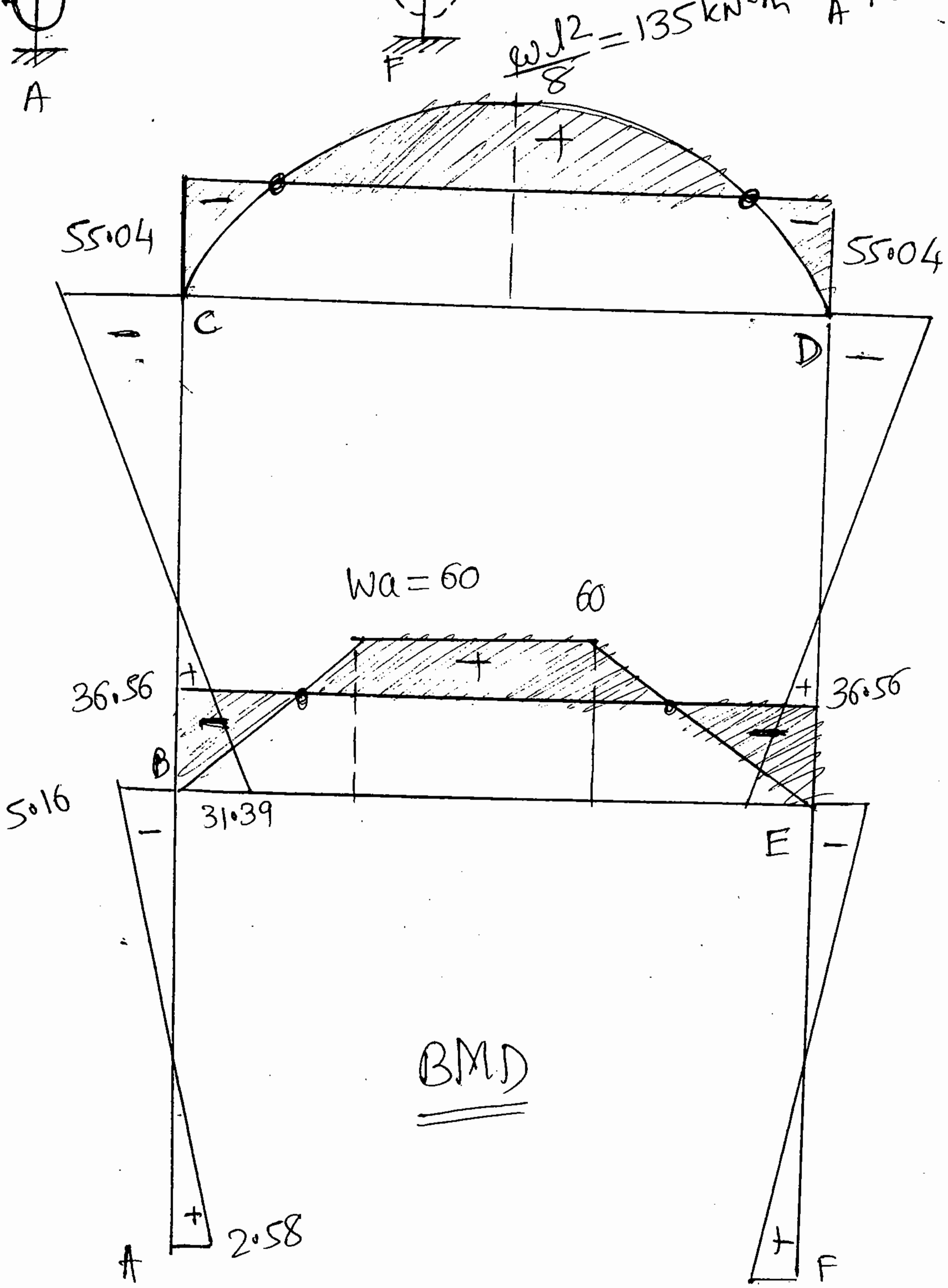
$$M_{BC} = 0 + 2(2.58) + 26.23 = 31.39 \text{ "}$$

$$M_{CB} = 0 + 2(26.23) + 2.58 = 55.04 \text{ "}$$

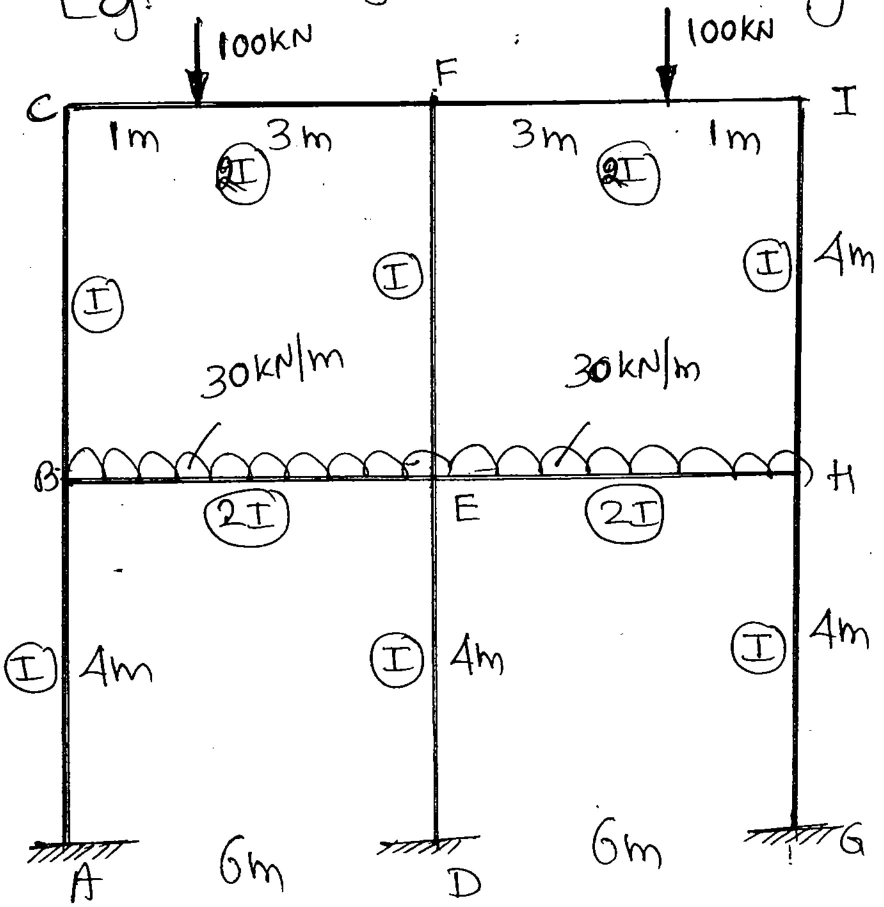
$$M_{CD} = -90 + 2(17.48) + 0 = -55.04 \text{ "}$$

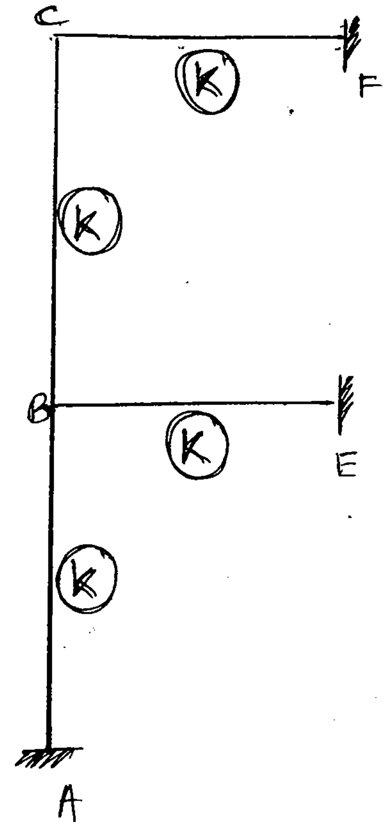

C
D
55.04
31.39
36.56
B
E
5.16
2.58
A
F


C
-
+
B
-
+
A


$\frac{wl^2}{8} = 135$ kN-m
+
-
55.04
55.04
C
D
-
-
Wa = 60
60
+
36.56
36.56
+
+
-
-
B
5.16
31.39
E
-
-
BMD
+
2.58
A
+
F

Eg:- Analyse the frame by Kani's



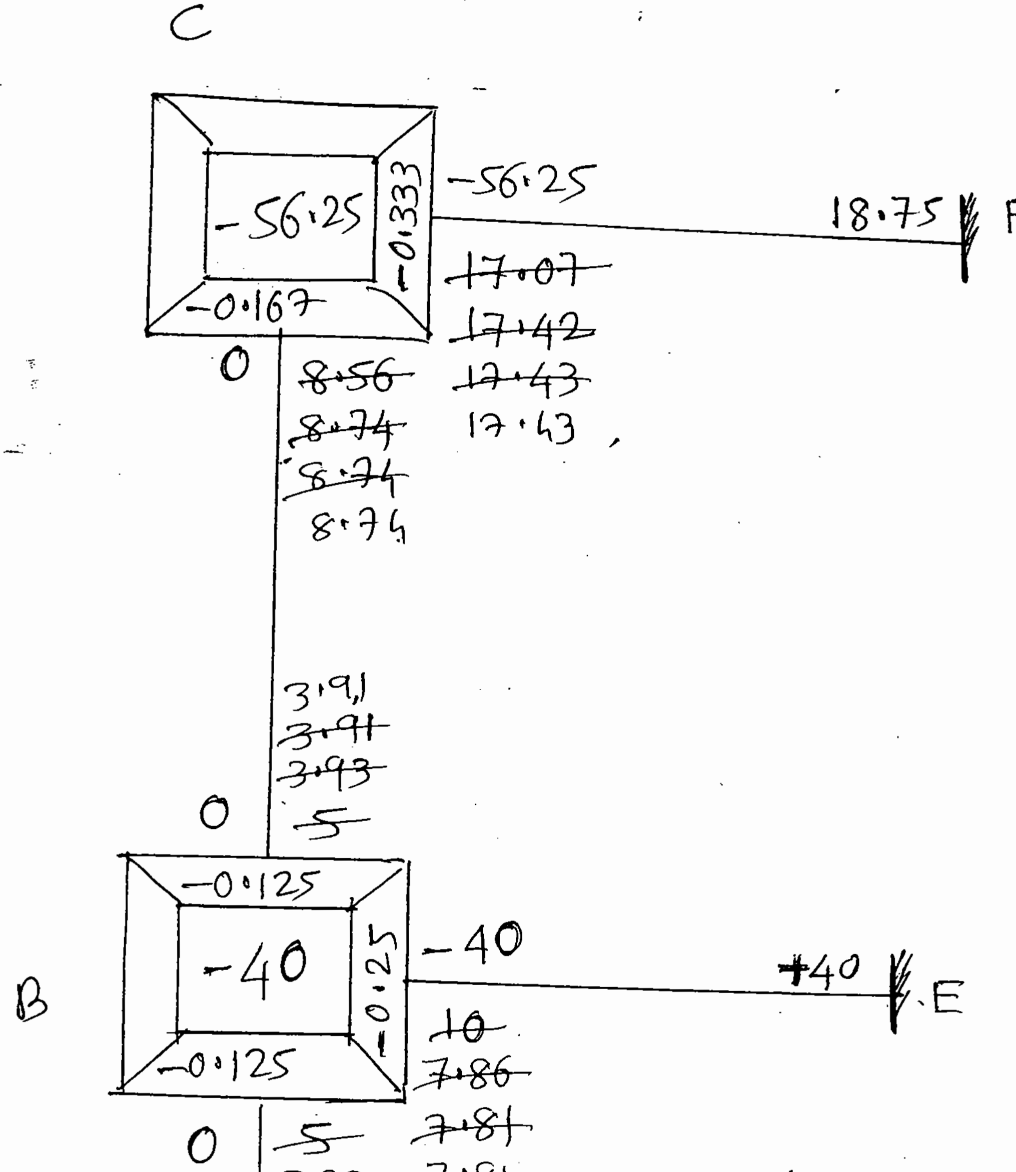


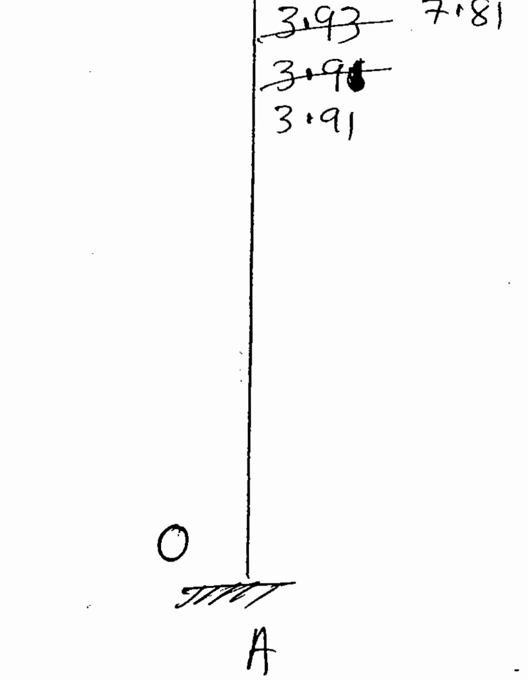
Sol^n (a) FEM

$M_{FBE} = -40$ kN-m, $M_{FCF} = \frac{-Wab^2}{l^2} = -56.25$ kN-m

(b) R.F (Only at B & C) $M_{FFC} = \frac{Wa^2b}{l^2} = 18.75$

		K	ΣK	U
B	BA	$K = I/4 = 0.25I$	1.0 I	−0.125
	BE	$K = 2I/4 = 0.5I$		−0.25
	BC	$K = I/4 = 0.25I$		−0.125
C	CB	$K = I/4 = 0.25I$	0.75I	−0.167
	CF	$K = 2I/4 = 0.5I$		−0.333





Final

$M_{AB} = 3.91\ kN\text{-}m$

$M_{BA} = 7.82\ kN\text{-}m$

$M_{BE} = -24.38$ "

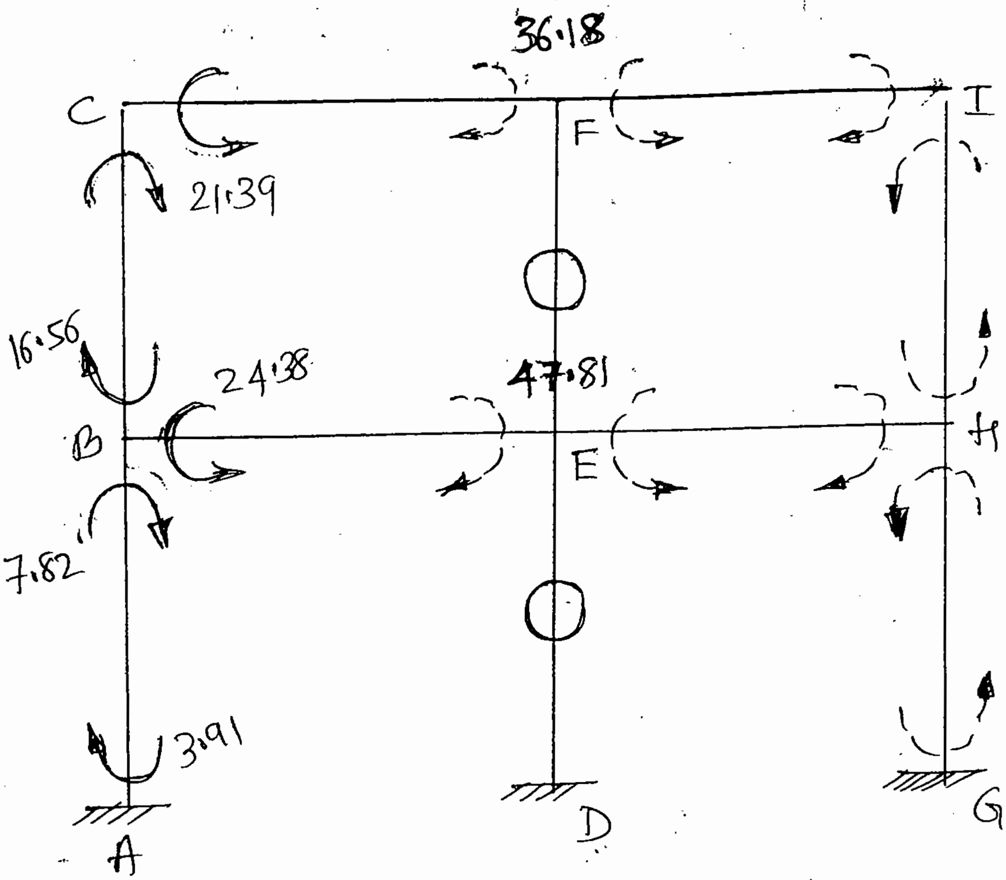
$M_{BC} = 16.56$ "

$M_{CB} = 21.39$ "

$M_{CF} = -21.39\ kN\text{-}m$

$M_{FC} = 18.75 + 2(0) + 17.43 = \underline{\underline{36.18}}$

$M_{EB} = 40 + 2(0) + 7.81 = \underline{\underline{47.81}}$


36.18
C
F
I
21.39
16.56
24.38
47.81
B
E
H
7.82
3.91
A
D
G

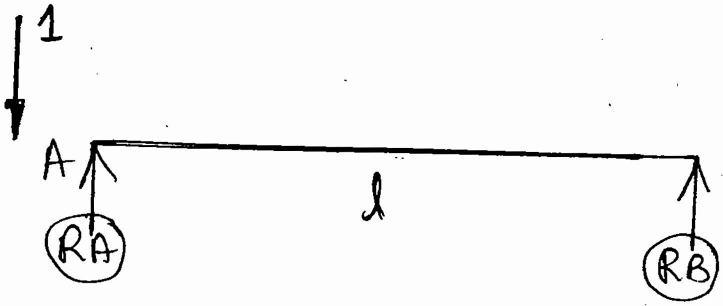
Date
02/11/08

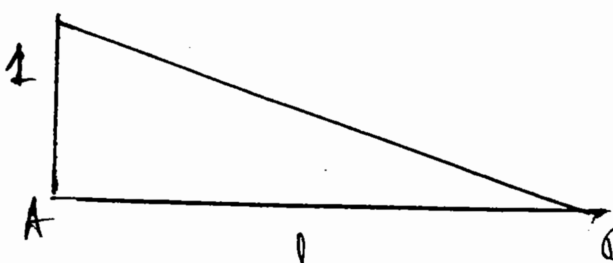
: Influence Line Diagram :

Rolling Loads

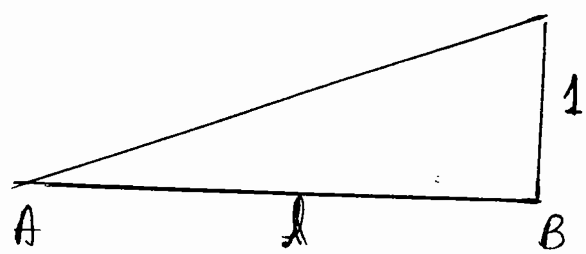
A curve or graph that represents a function like a reaction at a support, the shear force & bending moment at a section of a structure for various position of a unit load on the span is called ILD.

a) ILD for Reactions



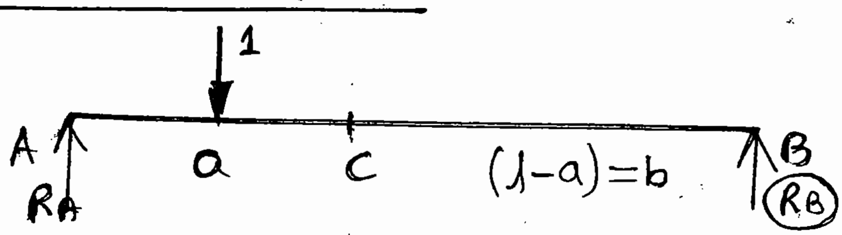


ILD for RA



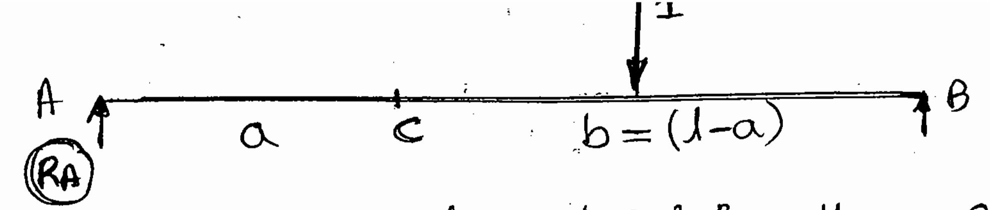
ILD for RB

b) ILD for "SF" at a section



(i) when load is in between AC, then $SF)_C = -R_B$

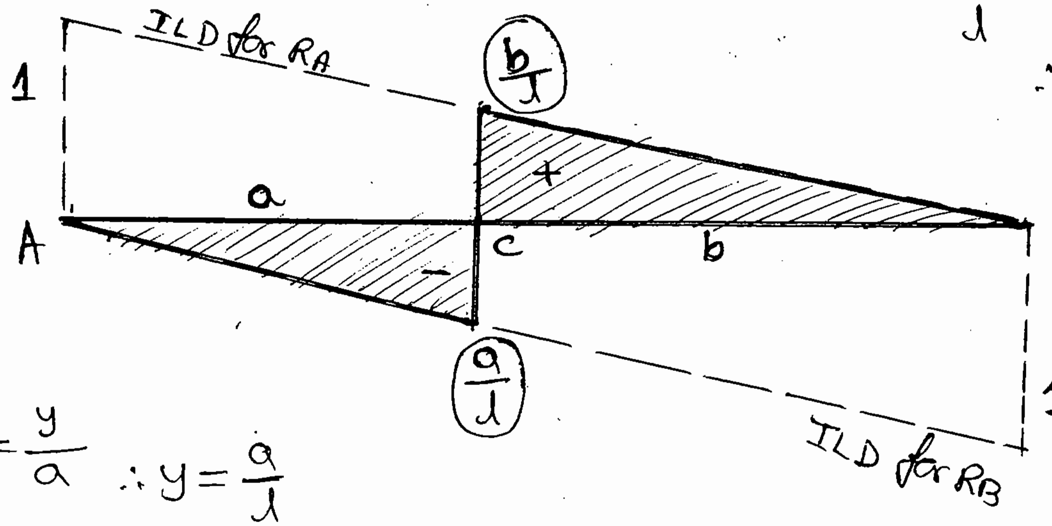
Draw ILD for V_B & consider part of ILD from A to C



when load is in between C & B, then $S.F)_C = +R_A$

$\therefore$ Draw ILD for V_A & consider part of the ILD from C to B.

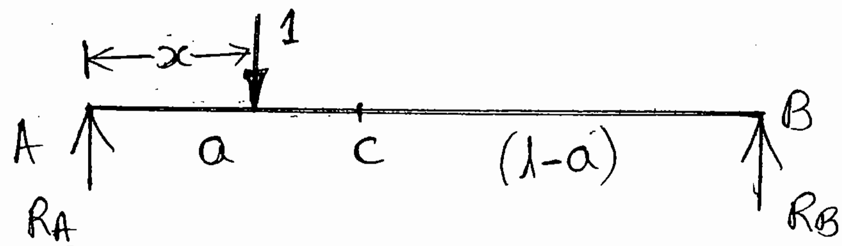
$$\frac{1}{l} = \frac{y}{b} \quad \therefore y = \frac{b}{l}$$



$$\frac{1}{l} = \frac{y}{a} \quad \therefore y = \frac{a}{l}$$

c) <u>ILD for BM at a section :-</u>

(i) When the unit load in bet^n A & C



$R_B = \frac{x}{l}$

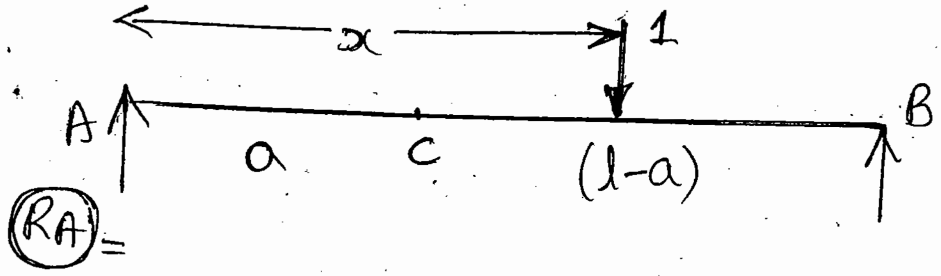
$\Sigma M_A = 0$, $1 \times x - R_B \times l = 0 \quad \therefore R_B = \frac{x}{l}$

$$\therefore M_C = R_B \times (l-a) = \frac{x}{l}(l-a)$$

When point load is at 'A' $\therefore x = 0 \quad \therefore M_C = 0$

When point load is at "C" $\therefore x = a \quad \therefore M_C = \frac{a(l-a)}{l}$

(ii) When unit load is betn C & B :



RA =

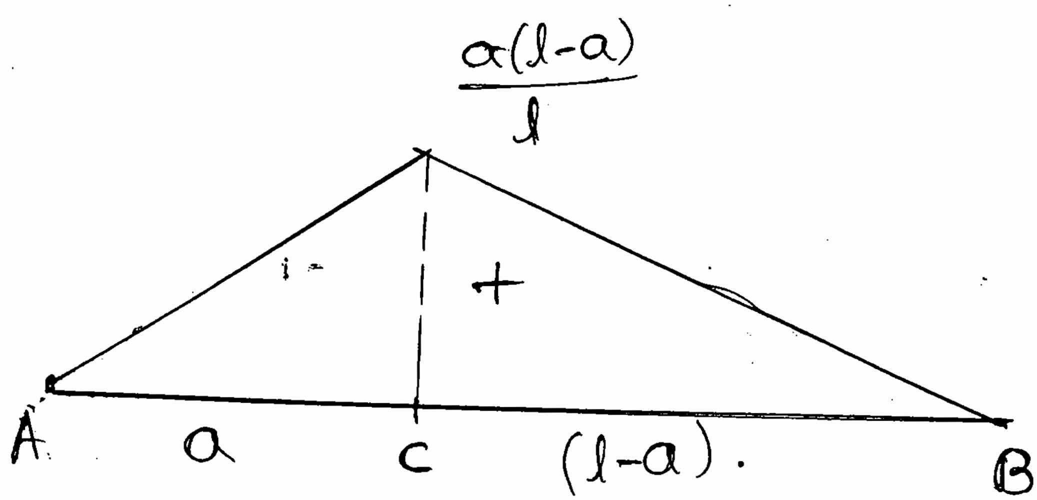
$$\Sigma M_B = 0, \quad R_A \times l - 1(l-x) = 0 \qquad R_A = \frac{(l-x)}{l}$$

$$M_C = R_A \times a = \frac{a(l-x)}{l}$$

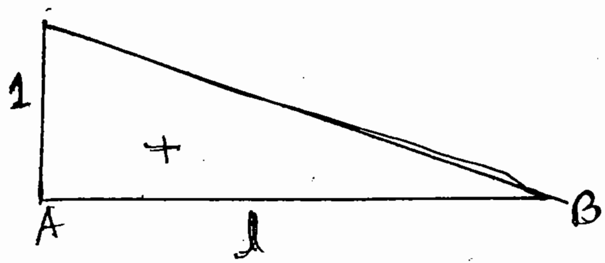
(i) When point load is at "C", $\therefore x = a$

$$\therefore M_C = \frac{a(l-a)}{l}$$

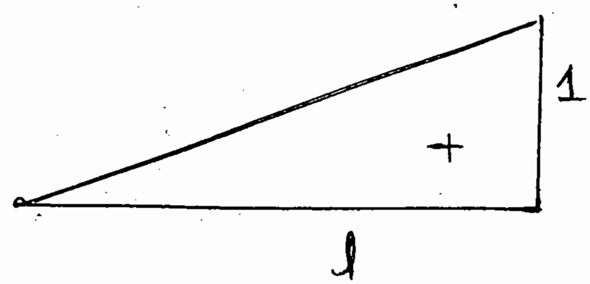
When point load at 'B' $x = l$ $\quad \therefore M_C = 0$



(1) ILD for "Reaction"

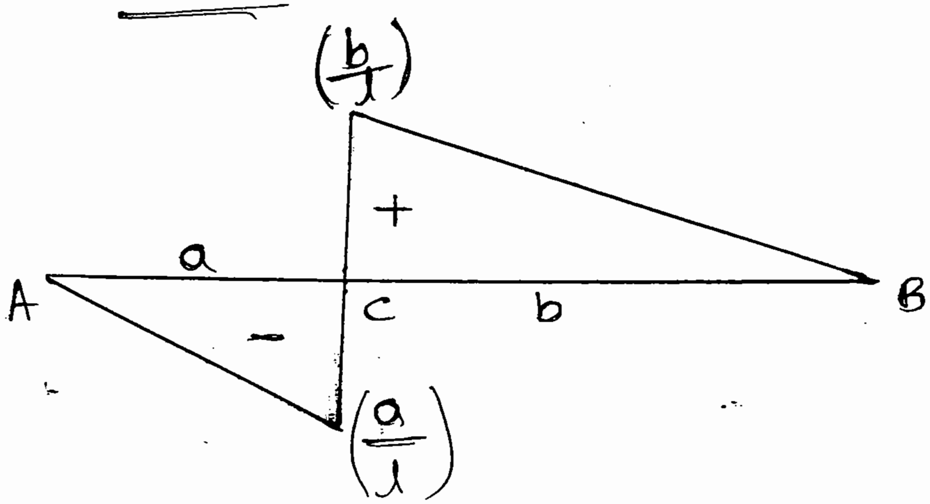


ILD for R_A

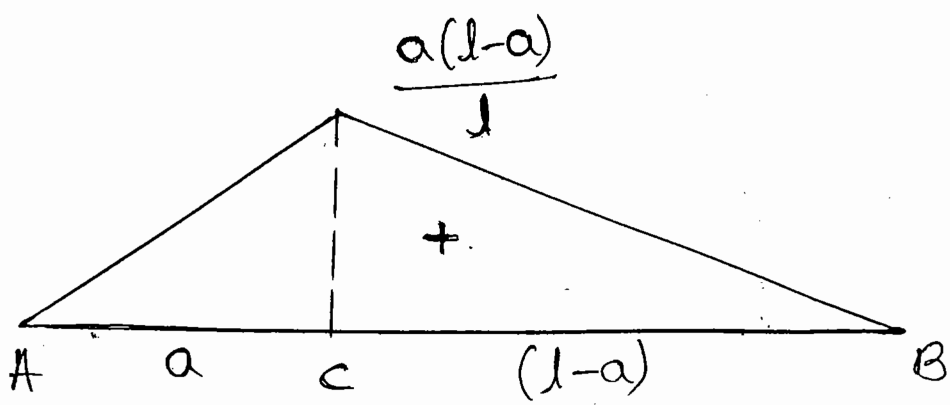


ILD for R_B

(2) ILD for "SF" at a section

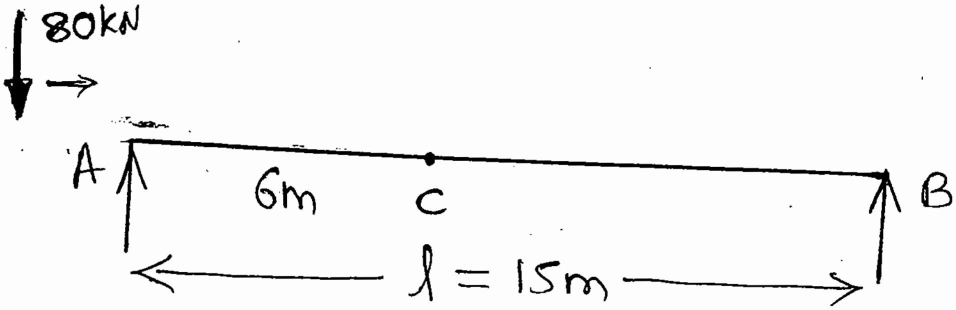


(3) ILD for "B.M." at a Section

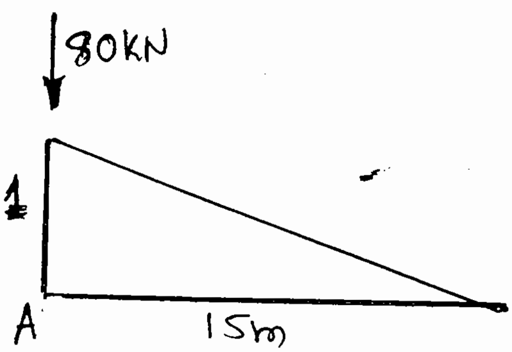


Eg:-1] A point load 80kN crosses a girder of span 15m from left to right calculate (i) Max. Reaction
(ii) SF and BM at a section 6m from left.

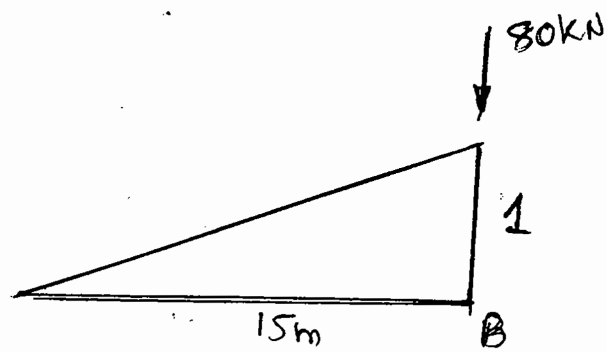
Sol:



(a) Reaction



ILD for (RA)

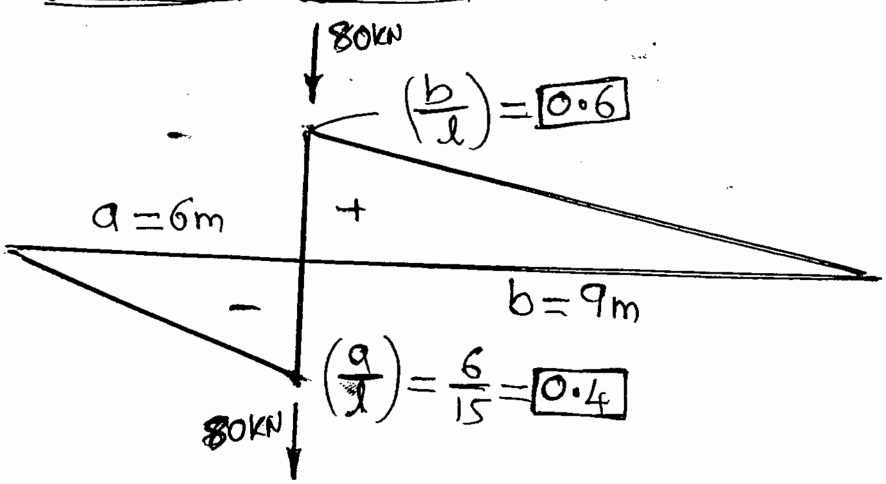


ILD for (RB)

$\therefore R_A = (\text{Load})(\text{Height}) = 80 \times 1 = 80\,kN$

$R_B = 80 \times 1 = 80\,kN$

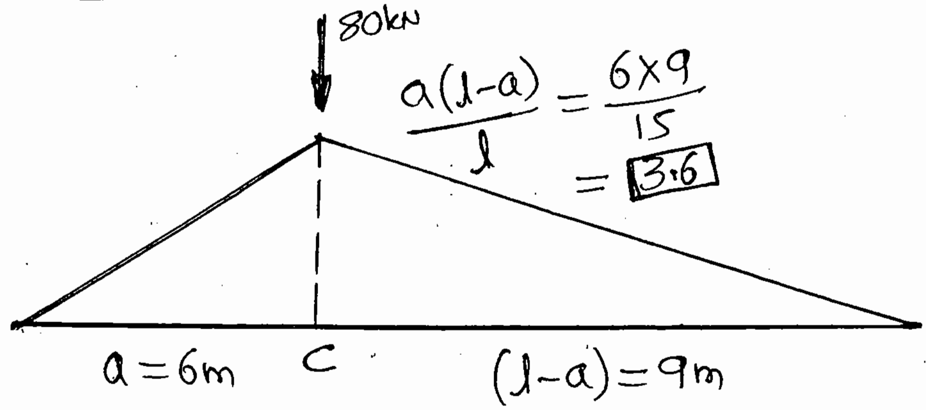
(b) SF at a section "6m" from left



$\therefore -ve\ SF)_c = 80 \times 0.4 = 32\,kN$

$+ve\ SF)_c = 80 \times 0.6 = 48\,kN$

(c) B.M. at a section "C" (at 6m)



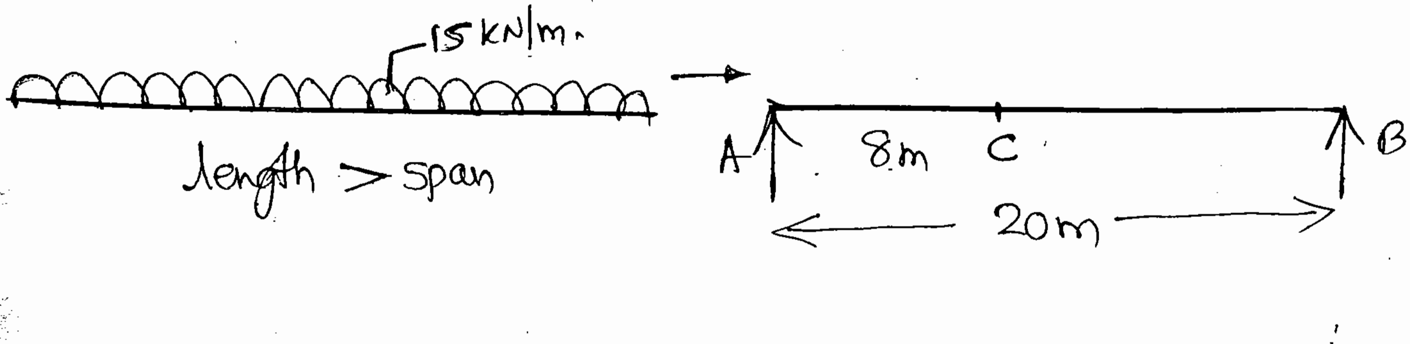
$\therefore$ Max. BM$)_c = 80 \times 3.6 = 288\,kN\text{-}m$

— x —

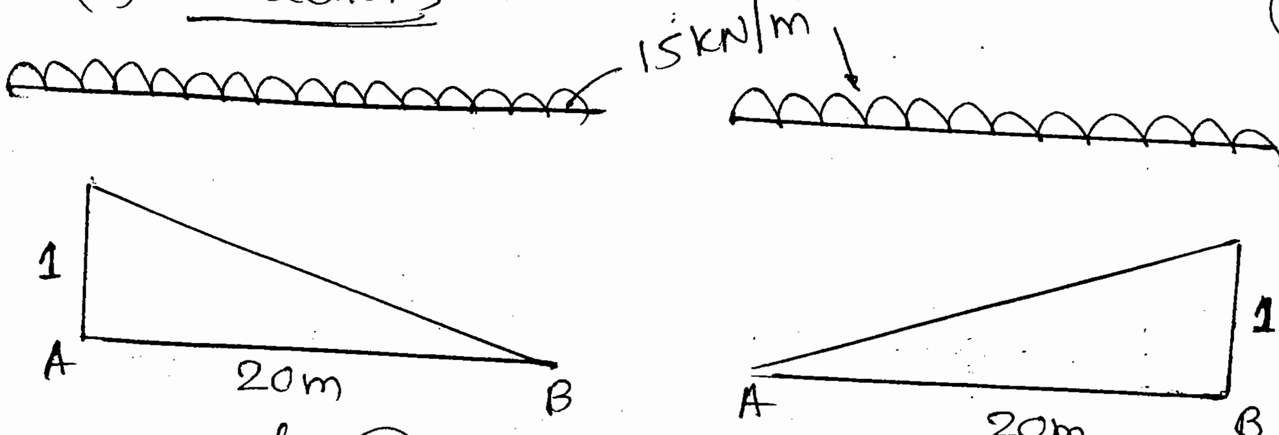
Eg:- 2] A UDL of intensity 15 kN/m longer than the span crosses a girder of span 20m from left to right. Calculate

(i) Max. Reaction

(ii) S.F and BM at a section 8m from "A"



(a) Reactions

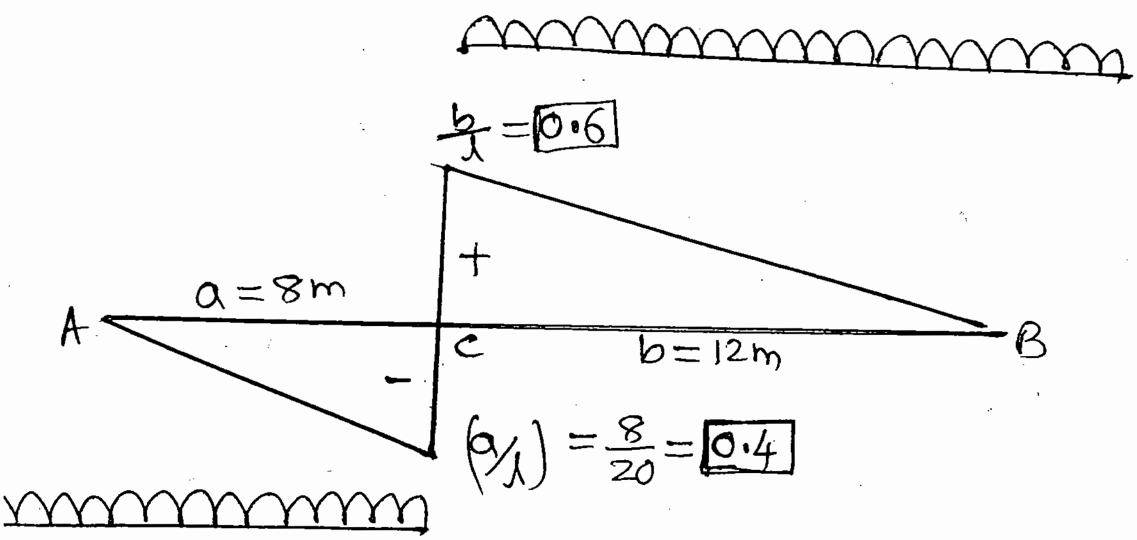


ILD for (RA) ILD for (RB)

$$R_A = R_B = (\text{Load intensity})(\text{Area})$$

$$= (15)\left(\frac{1}{2} \times 20 \times 1\right) = 150\,\text{kN}$$

(b) SF at a section (at 8m)

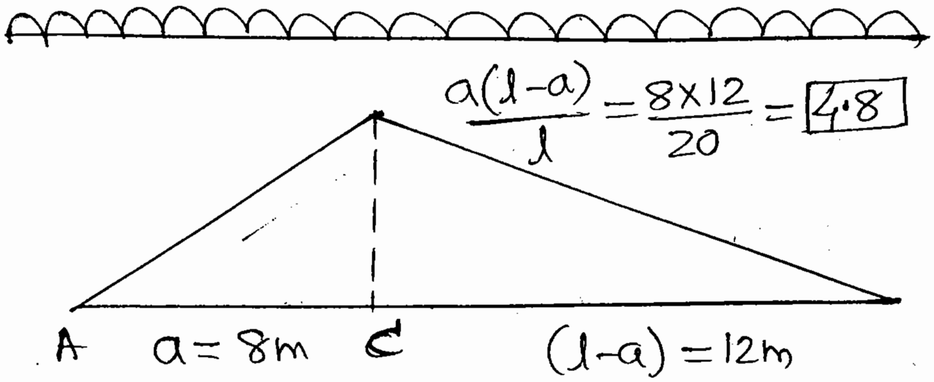


$$\therefore \; -\text{ve SF})_c = 15[-\text{ve area}] = 15\left[\left(\frac{1}{2} \times 8 \times 0.4\right)\right]$$

$$= 24\,\text{kN}$$

$$+\text{ve SF})_c = 15\left[\frac{1}{2} \times 12 \times 0.6\right] = 54\,\text{kN}$$

(c) BM. at a Section C



$$M_{max})_C = 15\left[\frac{1}{2} \times 20 \times 4.8\right] = 720 \text{ kN-m}$$

═══ X ═══

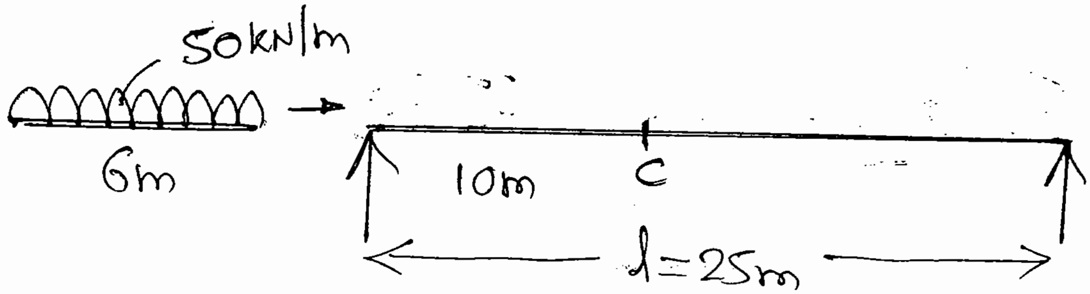
Eg:-3 A UDL of intensity 50 kN/m and length 6m crosses a girder of span 25m from left to right.

Calculate (1) Reaction

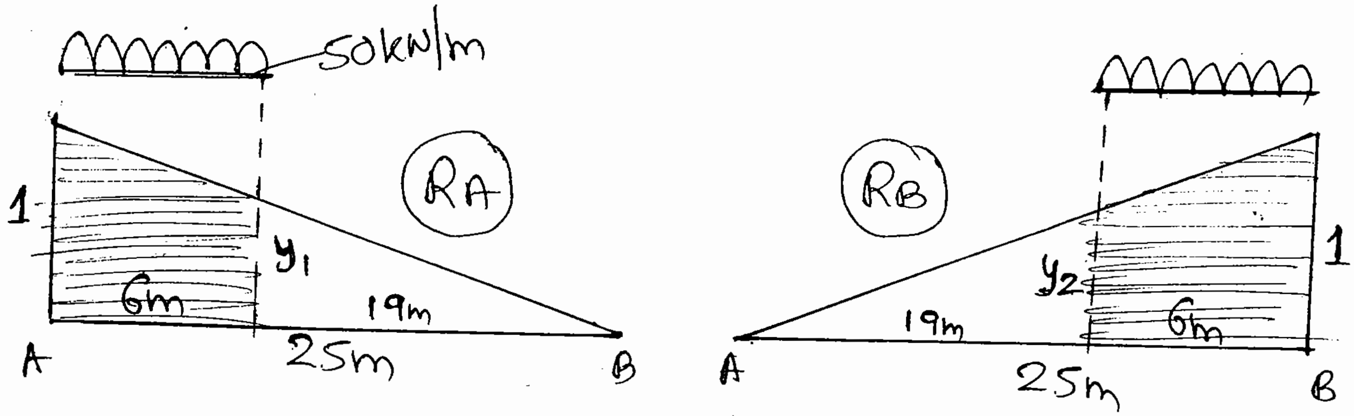
(2) Max. SF and Max BM at a section 10m from left.

(3) Absolute max. BM.

Sol^n



(a) Reaction :



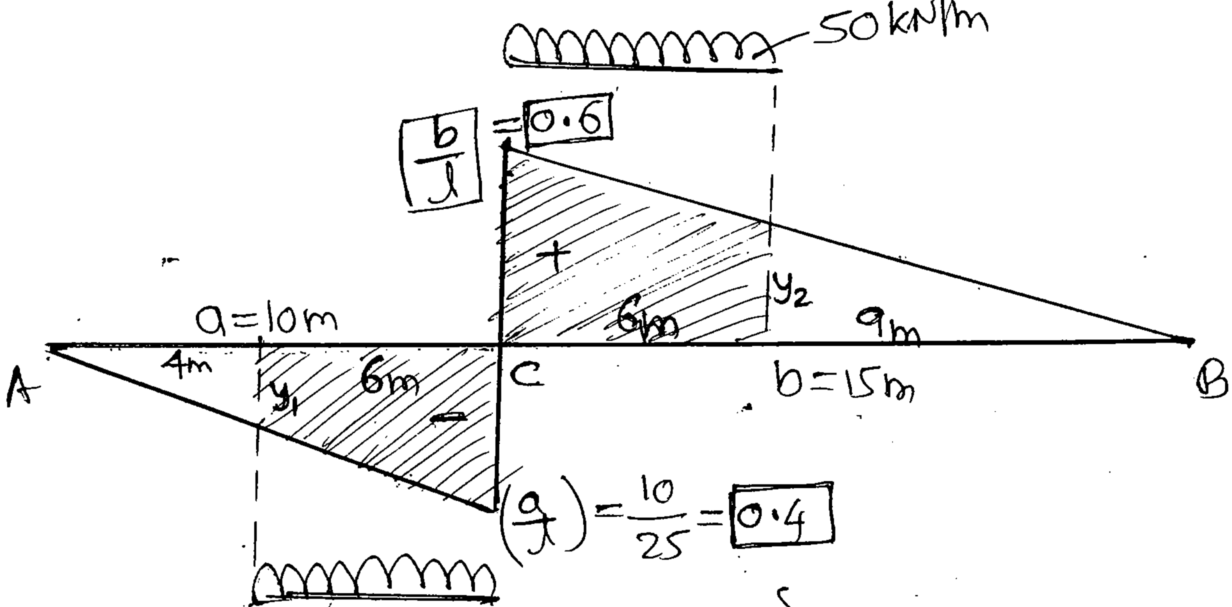
$$\frac{1}{25} = \frac{y_1}{19} \qquad \therefore \boxed{y_1 = 0.76 = y_2}$$

$\therefore$ $R_A = R_B =$ (Load intensity)(Shaded area)

$$= 50\left[\left(\frac{1+y_1}{2}\right)6\right] = 50\left[\left(\frac{1+0.76}{2}\right)6\right] = 264 \text{ kN}$$

↑ Area of Trapezoidal.

(b) Max. SF at a Section C



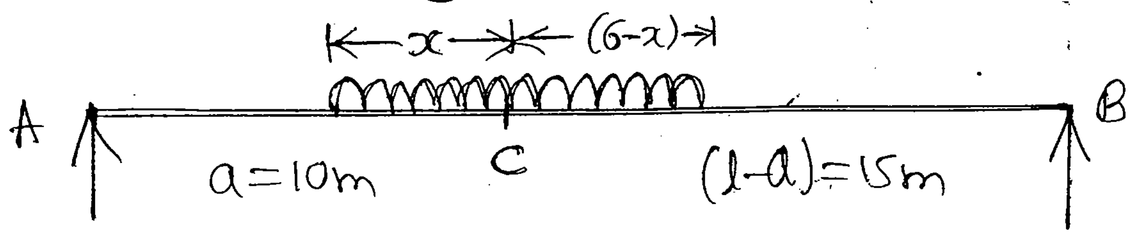
$$\frac{0.4}{10} = \frac{y_1}{4} \qquad \therefore \boxed{y_1 = 0.16}$$

$$\frac{0.6}{15} = \frac{y_2}{9} \qquad \boxed{y_2 = 0.36}$$

$$\therefore \text{Max -ve SF} = 50\left[\left(\frac{0.4+0.16}{2}\right)6\right] = 84\text{ kN}$$

$$\text{Max. +ve SF} = 50\left[\left(\frac{0.6+0.36}{2}\right)6\right] = 144\text{ kN}$$

(c) Max. Bending Moment at a Section

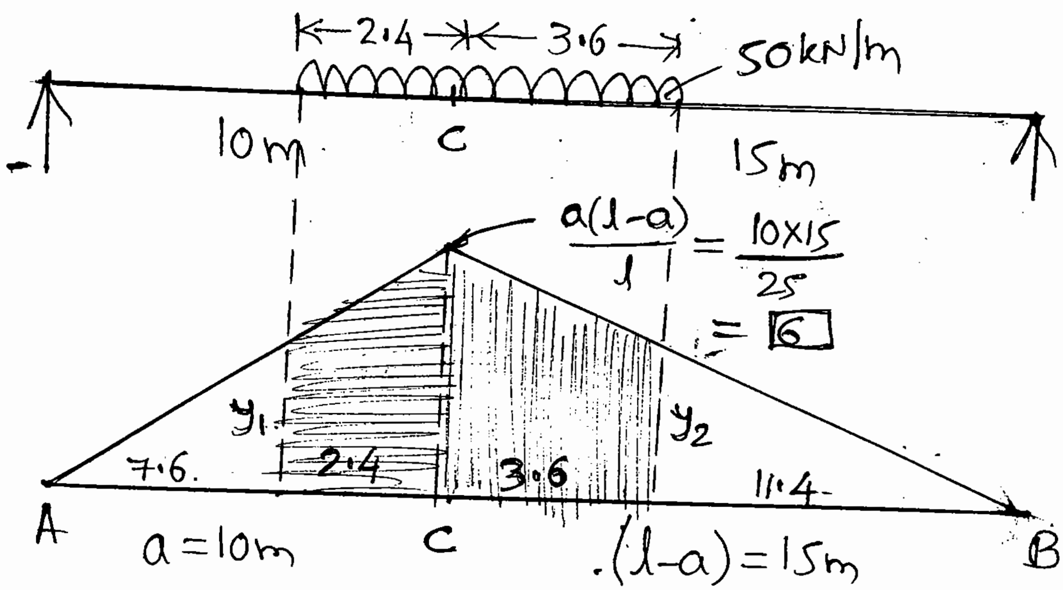


The load position for max. BM is,

" The ratio of span and the ratio of load should be same. "

$$\therefore \frac{10}{15} = \frac{x}{(6-x)} \qquad \therefore \boxed{x = 2.4m}$$

& Remaining $= 3.6m$



$$\frac{6}{10} = \frac{y_1}{7.6} \qquad \boxed{y_1 = 4.56}$$

$$\frac{6}{15} = \frac{y_2}{11.4} \qquad \boxed{y_2 = 4.56}$$

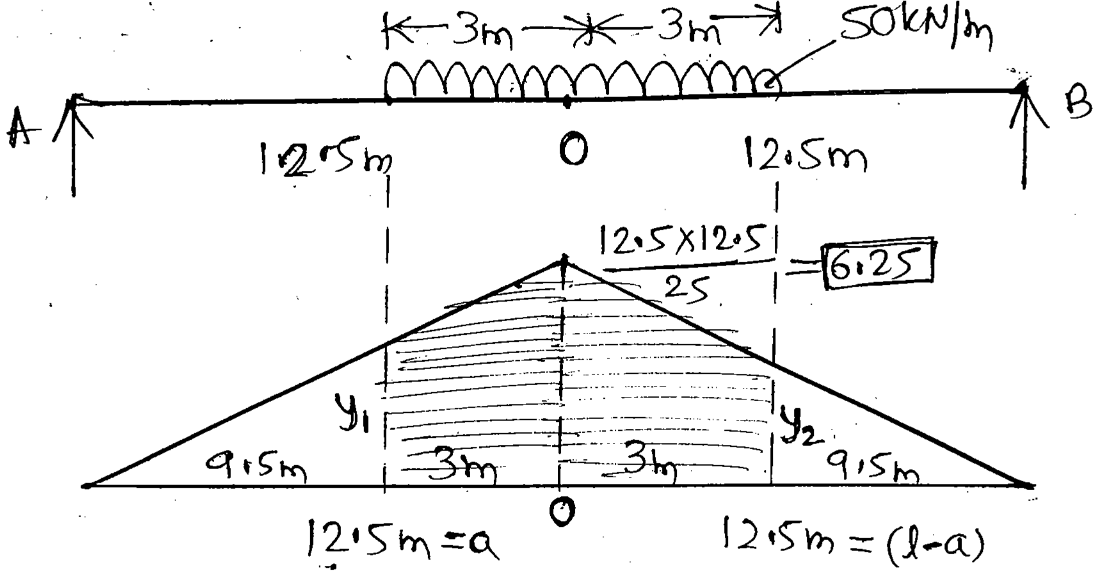
$\therefore$ Max. BM$)_C$ = (Load)(Shaded area)

$$= 50\left[\left(\frac{6+4.56}{2}\right)2.4 + \left(\frac{6+4.56}{2}\right)3.6\right] = 1584 \text{ KN-m}$$

(d) **Absolute max. BM :-**

Maximum of maximum value along the beam is called absolute max. BM.

and this occurs at mid span



$$\frac{6.25}{12.5} = \frac{y_1}{9.5} \qquad \boxed{y_1 = 4.75 = y_2}$$

$\therefore$ <u>Absolute max. BM</u>

$$= 50\left[\left(\frac{6.25+4.75}{2}\right)3 + \left(\frac{6.25+4.75}{2}\right)3\right]$$

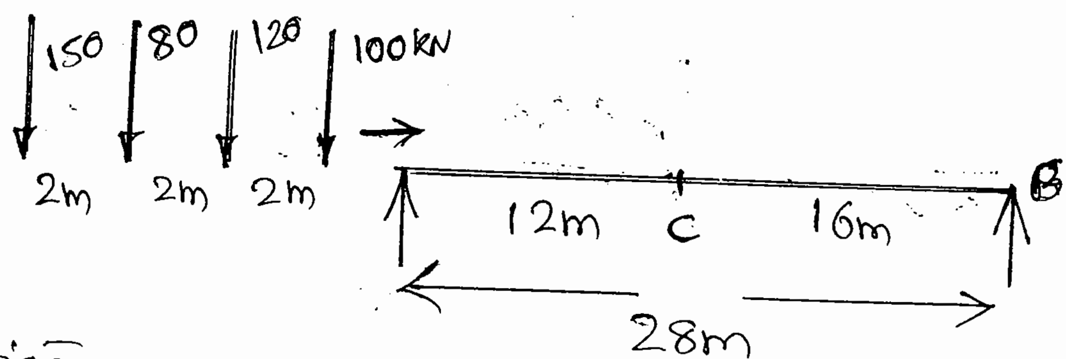
$$= 1650 \text{ kN-m}$$

$= x =$

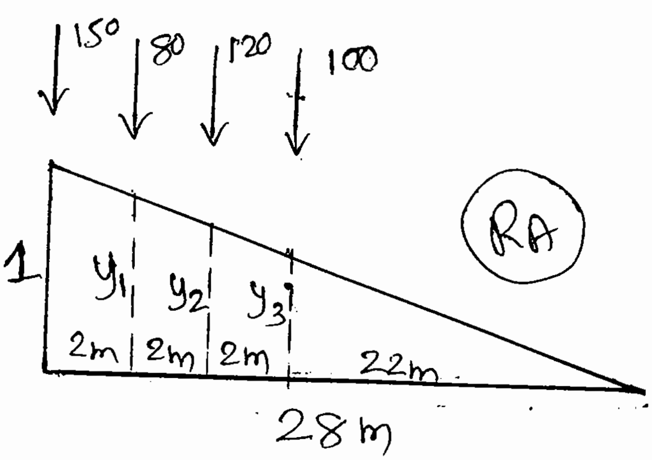
Eg] Multiple Concentrated Loads

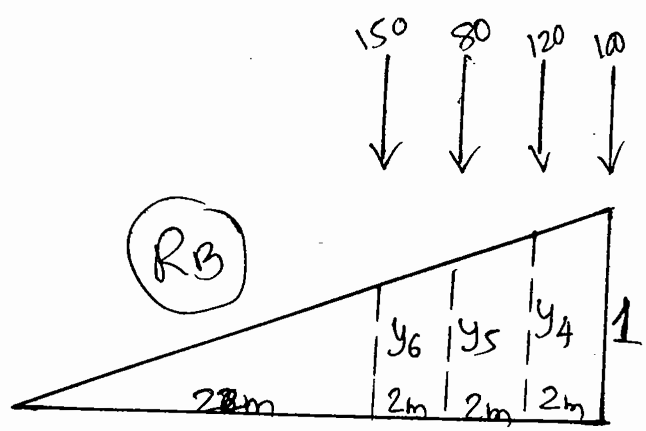
The multiple point loads 100 kN, 120 kN, 80 kN and 150 kN with a spacing 2m crosses a girder of span 28m from left to right with 100 kN load leading.
Calculate (1) Reactions
(2) Max. SF at a section 12m from left
(3) Max. BM at a section 12m —"—

Sol2



(a) Reaction





$\frac{1}{28} = \frac{y_1}{26}$ $\boxed{y_1 = 0.928} = y_4$

$\frac{1}{28} = \frac{y_2}{24}$ $\boxed{y_2 = 0.857} = y_5$

$\frac{1}{28} = \frac{y_3}{22}$

$\boxed{y_3 = 0.786} = y_6$

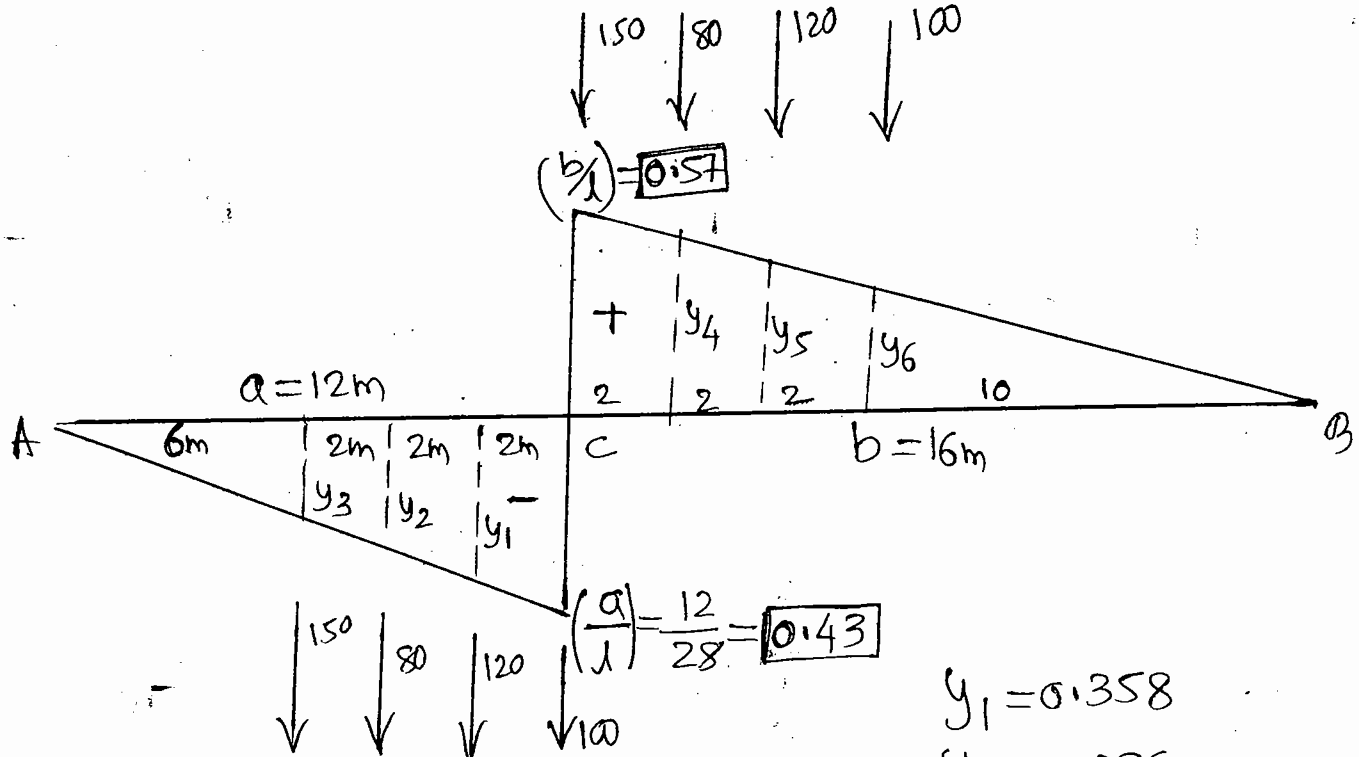
$\therefore R_A = 100 \times y_3 + 120 \times y_2 + 80 \times y_1 + 150 \times 1$

$= 405.68\ kN$

$R_B = 100 \times 1 + 120 \times y_4 + 80 \times y_5 + 150 \times y_6$

$= 397.82\ kN.$

(b) Max. SF at a section



$y_1 = 0.358$
$y_2 = 0.286$
$y_3 = 0.215$

$y_4 = 0.498$
$y_5 = 0.427$
$y_6 = 0.356$

$\therefore$ Max −ve SF $= 100 \times 0.43 + 120 \times y_1 + 80 \times y_2 + 150 \times y_3$

$= 141.09\ kN$

Max. +ve $= 100 \times y_6 + 120 \times y_5 + 80 \times y_4 + 150 \times 0.57$

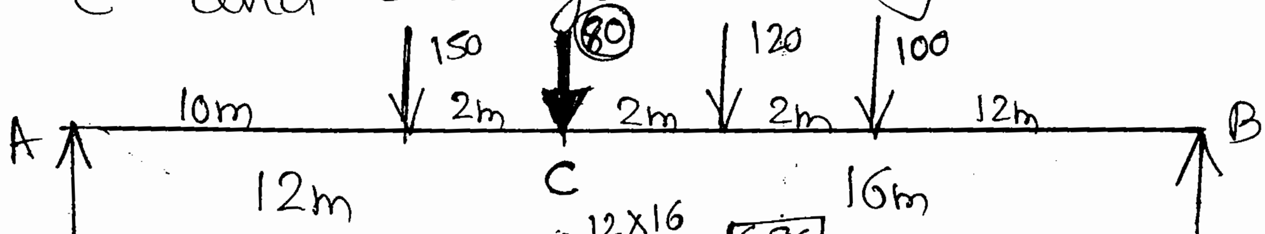
$= 212.18\ kN$

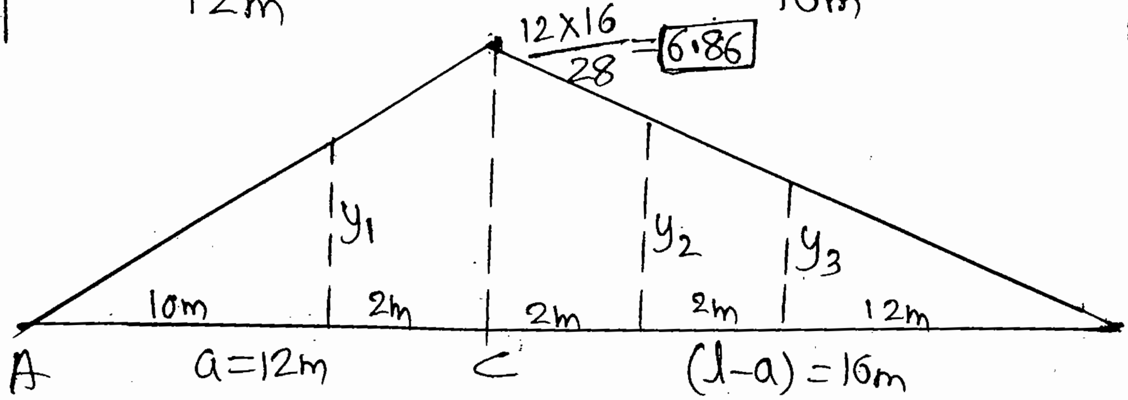
(C) Max. BM. at a section

Cross one by one point load across the section and calculate average load on each side.

Load crossing the section	Average load in AC	Average load in CB	Remark.
100 kN	$\frac{120+80+150}{12} = 29.17$	$\frac{100}{16} = 6.25$	AC > CB
120 kN	$\frac{80+150}{12} = 19.17$	$\frac{100+120}{16} = 13.75$	AC > CB
$\boxed{80 kN}$	$\frac{150}{12} = 12.5$	$\frac{100+120+80}{16} = 18.75$	$\boxed{AC < CB}$

The load (80 kN) which causes change in sign is kept exactly above the section 'C' and arrange remaining loads.





$y_1 = 5.716$

$y_2 = 6.00$

$y_3 = 5.14$

$\therefore$ Max. BM$)_C = 100 \times y_3 + 120 \times y_2$

$+ 80 \times 6.86 + 150 \times y_1$

$= 2640.2$ kN-m

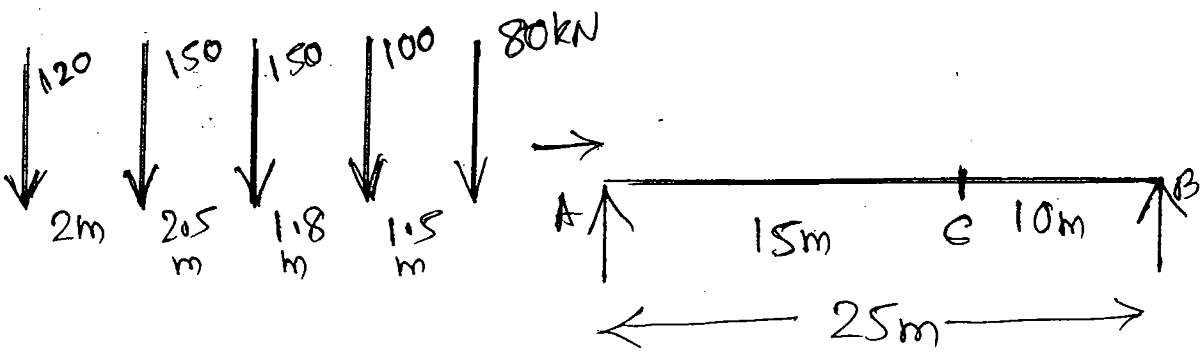
═══ × ═══ × ═══

Eg:-] the multiple points 120kN, 150kN, 150kN, 100kN and 80kN with spacing 2m, 2.5m, 1.8m and 1.5m crosses a girder of 25m from left to right with "80 KN load leading". Calculate

(i) Reaction

(ii) Max. SF & Max. BM at a section 15m from left.

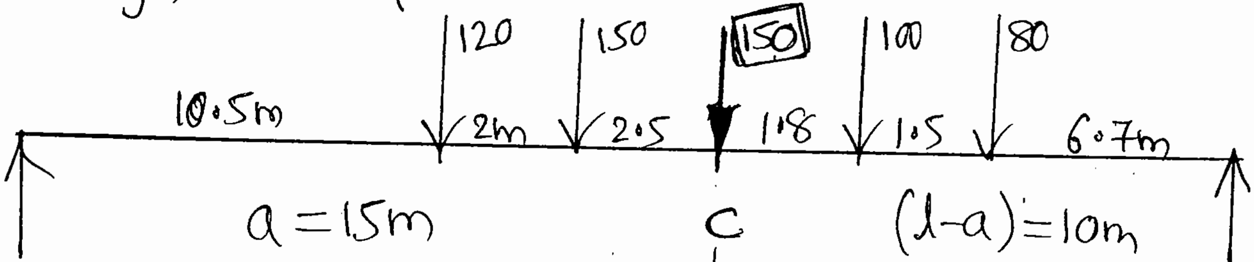
Sol^n

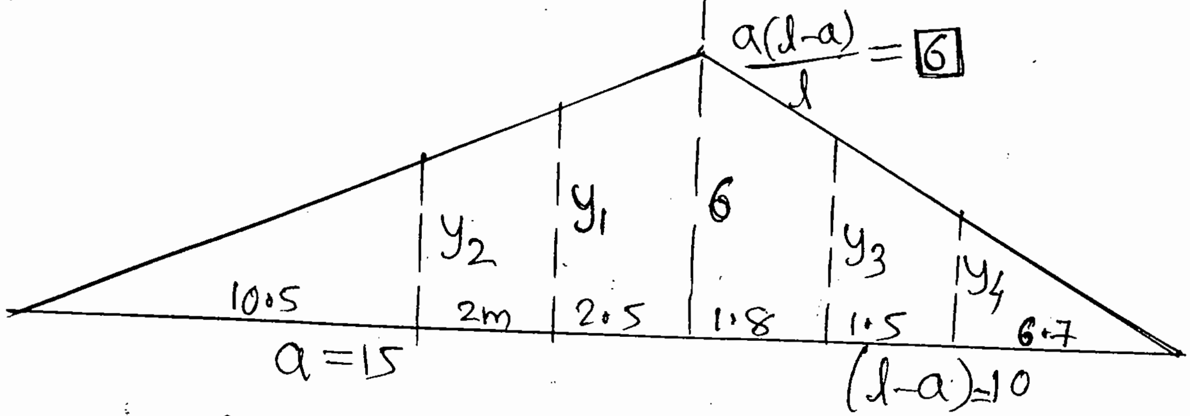


(a) Max. B.M. at a section "C"

Load crossing Section	Average load on "AC"	Average load in "CB"	Remark
80 kN	$\frac{100+150+150+120}{15} = 34.67$	$\frac{80}{10} = 8$	AC > CB
100 kN	$\frac{150+150+120}{15} = 28$	$\frac{80+100}{10} = 18$	AC > CB
$\boxed{150\text{ kN}}$	$\frac{150+120}{15} = 18$	$\frac{80+100+150}{10} = 33$	$\boxed{AC < CB}$

The load (150 kN) which causes change in sign is kept exactly above the section

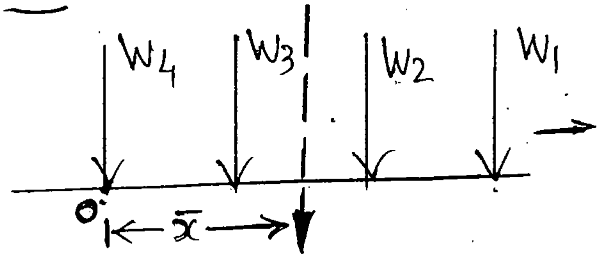




$y_1 = 5$
$y_2 = 4.2$
$y_3 = 4.92$
$y_4 = 4.02$

$$M_{max})_C = 120 \times 4.2 + 150 \times 5 + 150 \times 6 + 100 \times 4.92 + 80 \times 4.02 = 2967.6 \text{ kN-m}$$

Absalute Max. B.M. :-



Procedure

① calculate Resultant $R = W_1 + W_2 + W_3 + W_4$

② Calculate location of "R" from one end

$$\bar{x} = \frac{W_1 x_1 + W_2 x_2 + \cdots}{W_1 + W_2 + \cdots}$$

③ Selection of Load :

Now observe loads which is adjacent to resultants (W_2 & W_3)

Select a load which "Nearer" or "heavier"

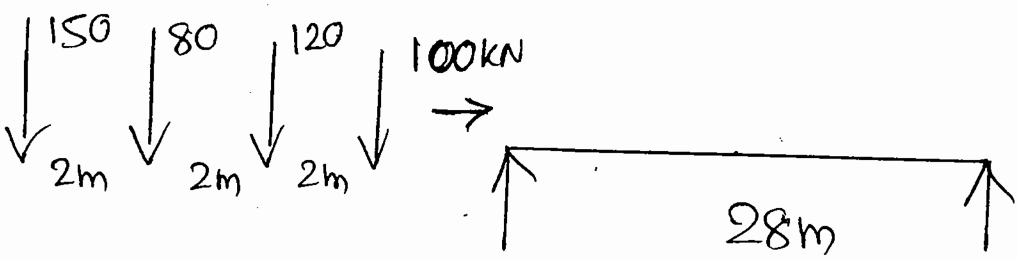
④ Arrangement of loads

The selected load and resultant should be equidistance from mid span.

Then draw ILD below the "Selected load" and calculate the Absolute max. BM.

Eg:- 1) The multiple point loads 100kN, 120kN, 80kN & 150kN with a spacing 2m crosses a girder of span 28m from left to right with 100kN load leading. Calculate "Absolute B.M" and "Ab. max. S.F"

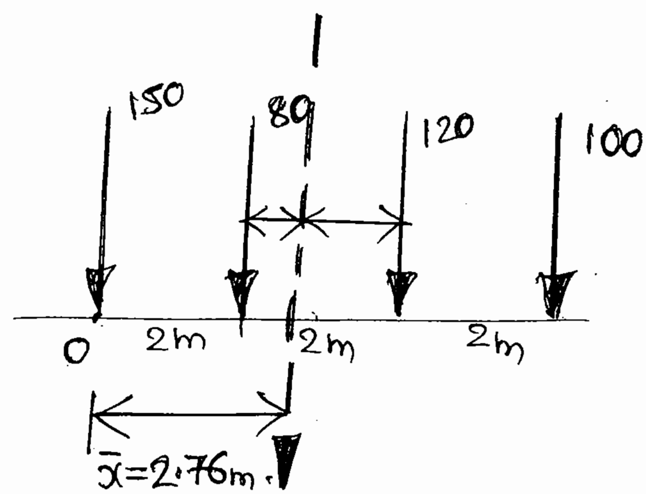
Sol^{n}



(a) Absolute Max. BM :- (Any where in the beam).

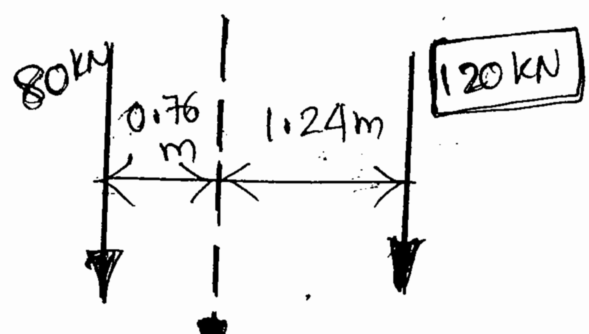
(1) Resultant $R = 100 + 120 + 80 + 150 = 450\,kN$

(2) Location of "R"



$$\bar{x} = \frac{W_1 x_1 + W_2 x_2 + \cdots}{W_1 + W_2 + \cdots}$$

$$\bar{x} = \frac{100 \times 6 + 120 \times 4 + 80 \times 2 + 150 \times 0}{450} = 2.76\,m$$



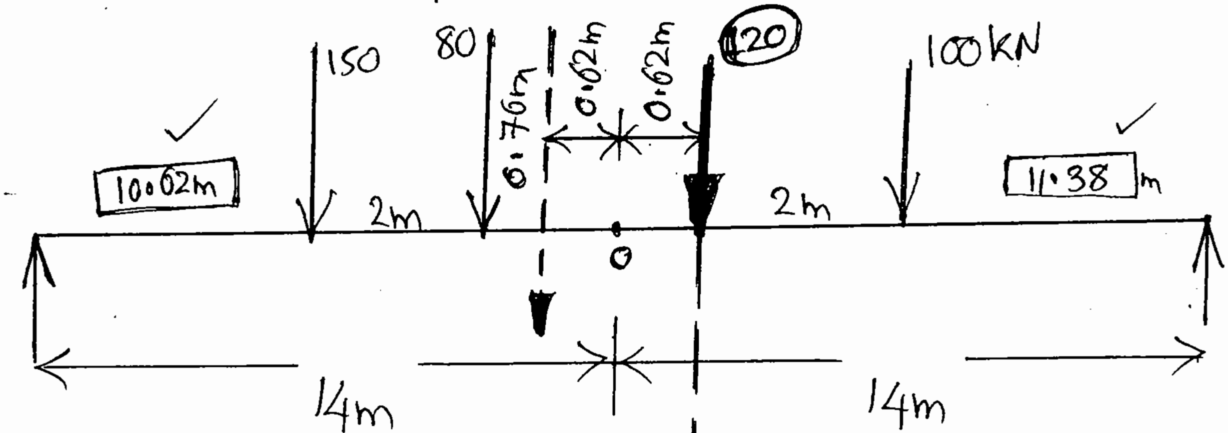
(3) Selection of loads

By observing only adjacent loads (80kN & 120kN)

Select "heavier Load" (i.e **120kN**)

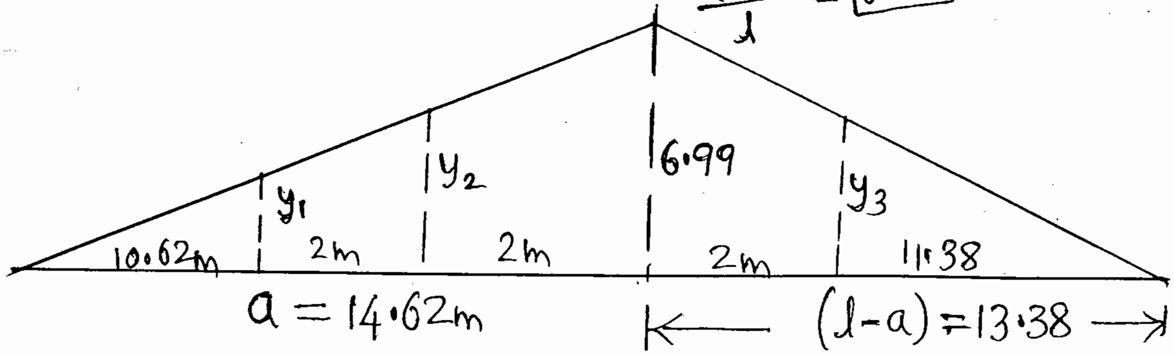
(4) Arrangement of loads :

The selected load (120kN) and resultant should be equidistance from mid span.



Draw ILD below the selected load

$\frac{a(l-a)}{l} = \boxed{6.99}$



$$\left.\begin{array}{l} y_1 = 5.08 \\ y_2 = 6.03 \end{array}\right\} \quad y_3 = 5.95.$$

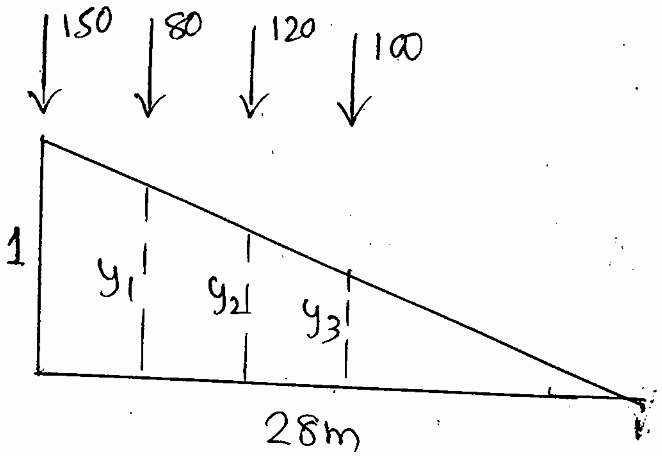
$\therefore$ Ab. max. BM $= 150 \times 5.08 + 80 \times 6.03 + 120 \times 6.99 + 100 \times 5.95 = 2678.2$ kN-m

= x =

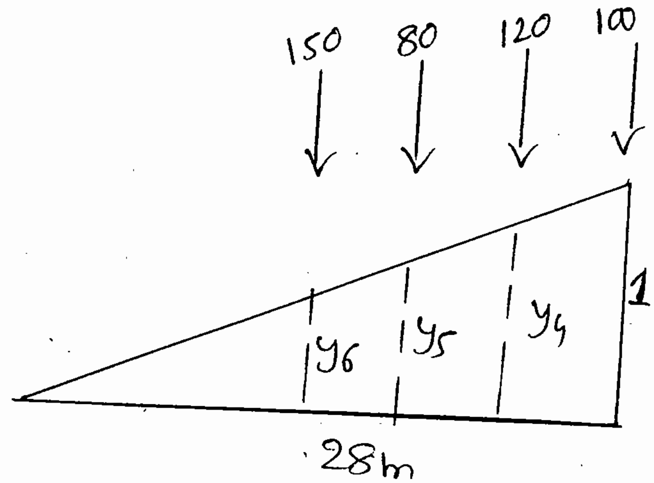
Eg:-

(b) Ab. max. Shear :-

The maximum reaction R_A or R_B is called ab. max. shear force



ILD for (R_A)



ILD for (R_B)

$\therefore$ $R_A = 405.68$ kN

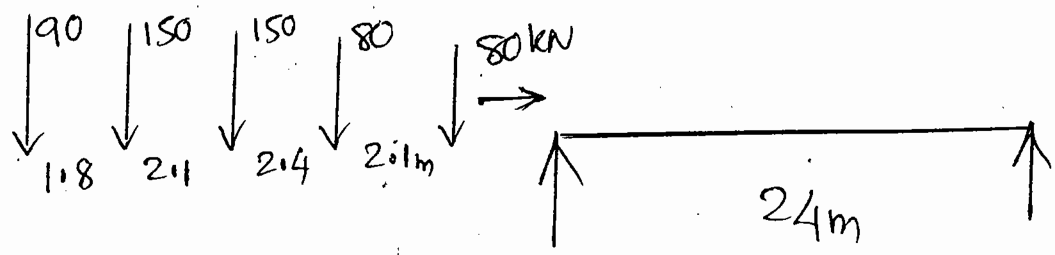
$R_B = 397.82$ kN

$\therefore$ Ab. max. Shear Force $= 405.68$ kN

=X=

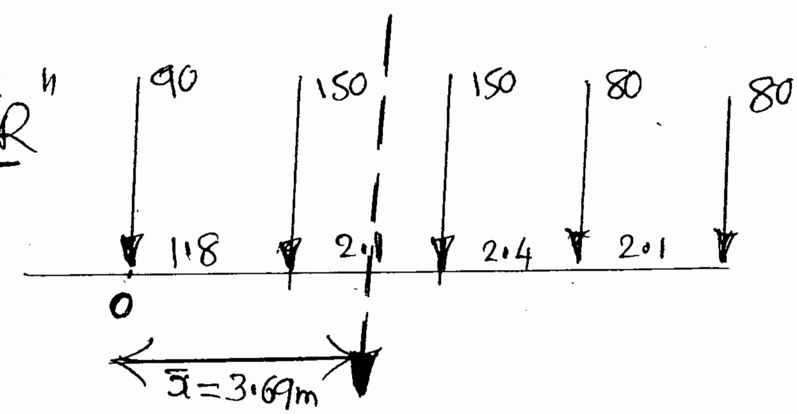
80KN, 150KN, 150KN & 90KN spaced at 2-1m, 2-4m, 2-1m & 1-8m in order crosses a girder of 24m span from left to right with 80KN load is leading. calculate

(i) Abs. Max^m B.M.



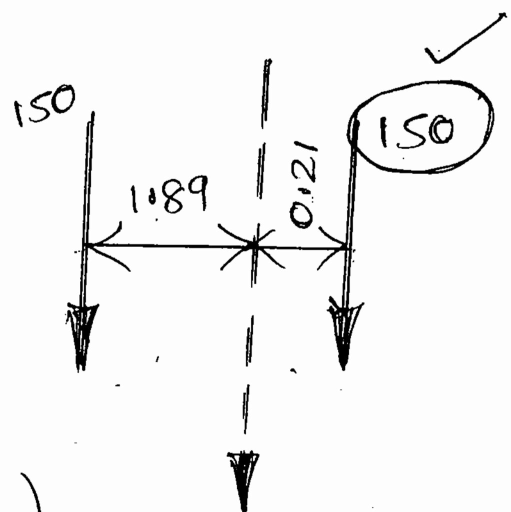
(1) Resultants $R = 80 + 80 + 150 + 150 + 90 = \boxed{550}$

(2) Location of "R"



$$\bar{x} = \frac{80 \times 8.4 + 80 \times 6.3 + 150 \times 3.9 + 150 \times 1.8 + 90 \times 0}{550}$$

$\boxed{\bar{x} = 3.69m}$



(3) Selection of Load:

Observing only adjacent loads.

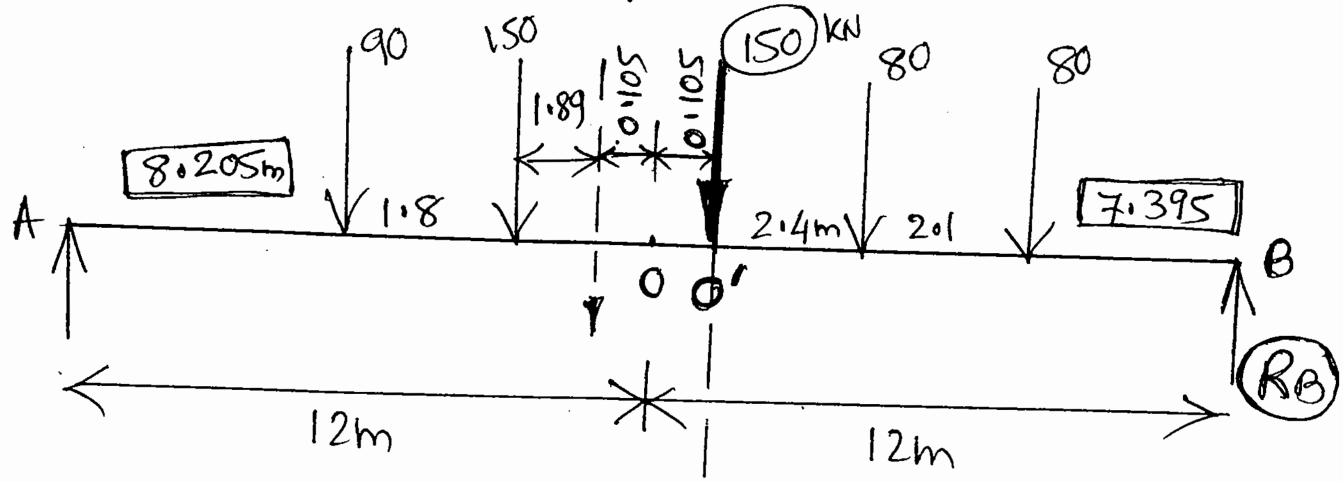
Select "Nearer Load"

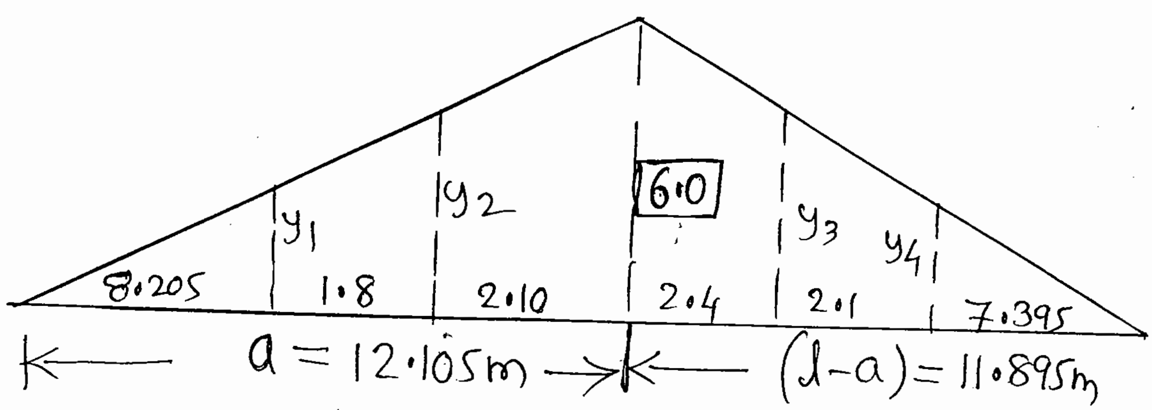
(∵ Both magnitudes are same)

∴ Select 3rd 150 kN load

(4) Arrangement

The selected load and resultant are equi-distance from mid span.





$y_1 = 4.07$, $y_2 = 4.96$, $y_3 = 4.79$, $y_4 = 3.73$

∴ Ab. max. BM

$= 90 \times 4.07 + 150 \times 4.96 + 150 \times 6$

$+ 80 \times 4.79 + 80 \times 3.73$

$= 2691.9$ kN-m

OR By Rolling Load Method

For the above arrangement of load Calculate reactions and take moment at "selected load"

$\therefore \sum M_A = 0,$

$$-R_B \times 24 + 90 \times 8.205 + 150(1.8 + 8.205)$$
$$+ 150(2.1 + 1.8 + 8.205) + 80(2.4 + 2.1 + 1.8 + 8.205)$$
$$+ 80(2.1 + 2.4 + 2.1 + 1.8 + 8.205) = 0$$

$$R_B = 272.66 \text{ kN}$$

$$\therefore \text{Ab. max. BM} = M)_{150kN} = 272.66 \times 11.895$$
$$- 80 \times 4.5 - 80 \times 2.4 = 2691.9 \text{ kN-m}$$

=== x ===

HW

1) Four loads of 100KN, 150KN, 175KN & 200KN spaced @ 1·0m, 1·5m and 2·0m in order move on a beam of span 20m. The load 200KN is to leading. Calculate (i) Ab. S.F.

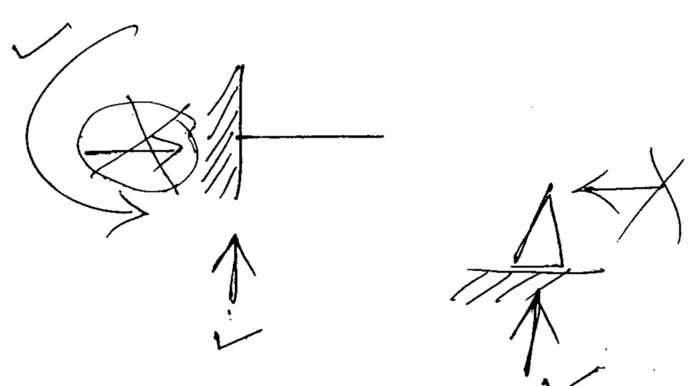
(ii) Ab. Max^m B.M.

2) A UDL 12KN/m of length 8m crosses a girder of span 15m from L to R. Calculate (i) Reaction.

(ii) S.F & B.M @ 6m from left.

(iii) Ab. max^m. BM.

3) Determine max^m B.M @ a section 10m from left for a span 25m, when a series of point loads 100KN, 200KN, 200KN, 200KN, 200KN with spacing 4m, 2·5m, 2·5m & 2·5m crosses the girder from each direction (either side). 100KN load is leading in each case.



23/11/18

Flexibility Matrix :

$$[P] = [F]^{-1}\{[\Delta] - [\Delta_L]\}$$

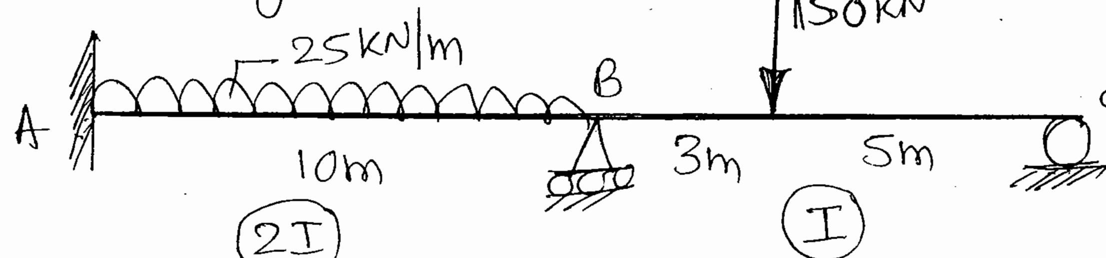
P → Redundants. (excess unknowns)

F → Flexibility matrix

Δ → Displacements or sinking of supports.

Δ_L → Displacement due to loads.

Eg:-1] Analyse the beam shown by Flexibility method and draw BMD.



No. of unknowns = 4 (V_A, V_B, V_C, M_A)

Available equilibrium equations = 2 ($\Sigma V = 0, \Sigma M = 0$)

$\overline{2}$

2

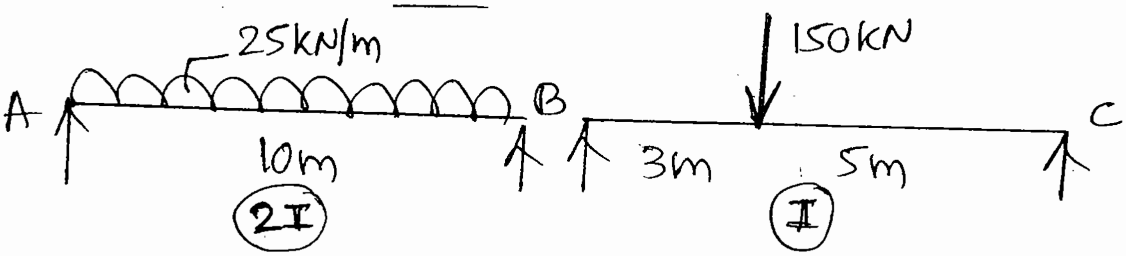
∴ The above beam is Two degree Redundant

★ Note :- (i) If there is "no" sinking of supports, then consider "moments" as redundants.

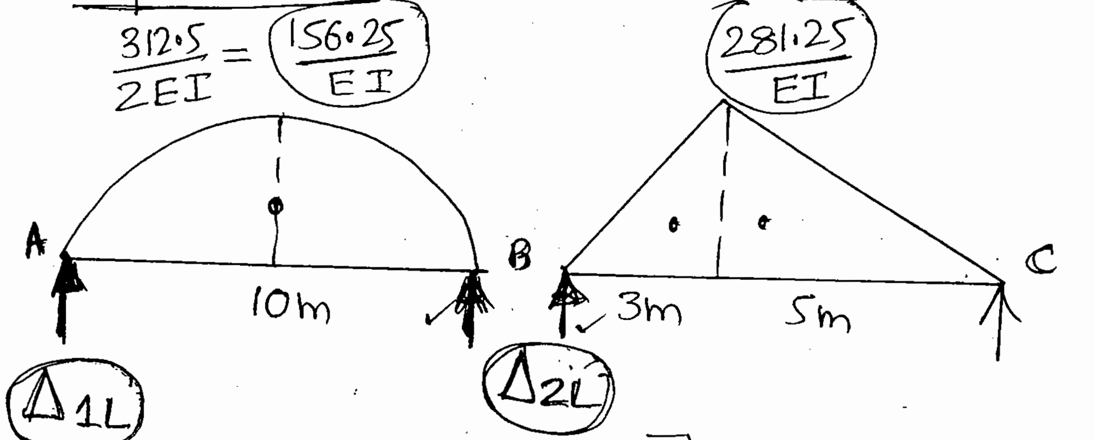
(ii) If there is a sinking of supports, then consider "Reactions" as redundants. ★

a)

∴ Consider "M_A" and "M_B" as Redundants



Displacements due to Loads



BMD & Conjugate Beam

$$\Delta_{1L} = \frac{1}{2}\left[\frac{2}{3} \times 10 \times \frac{156.25}{EI}\right] = \frac{520.83}{EI}$$

$$\Delta_{2L} = \frac{1}{2}\left[\frac{2}{3} \times 10 \times \frac{156.25}{EI}\right] + \frac{1}{8}\left[\left(\frac{1}{2} \times 5 \times \frac{281.25}{EI}\right)\frac{2}{3} \times 5 + \left(\frac{1}{2} \times 3 \times \frac{281.25}{EI}\right)\left(\frac{1}{3} \times 3 + 5\right)\right]$$

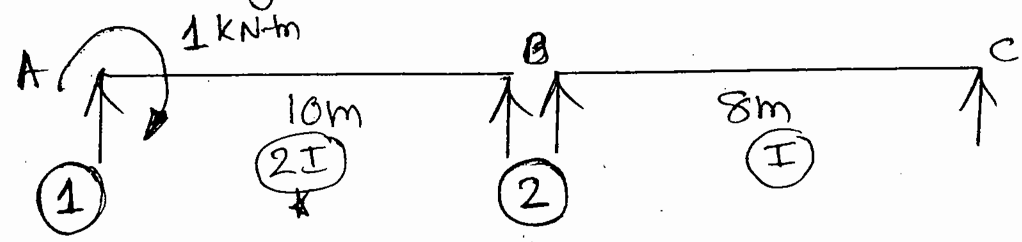
$$= \frac{1130.20}{EI}$$

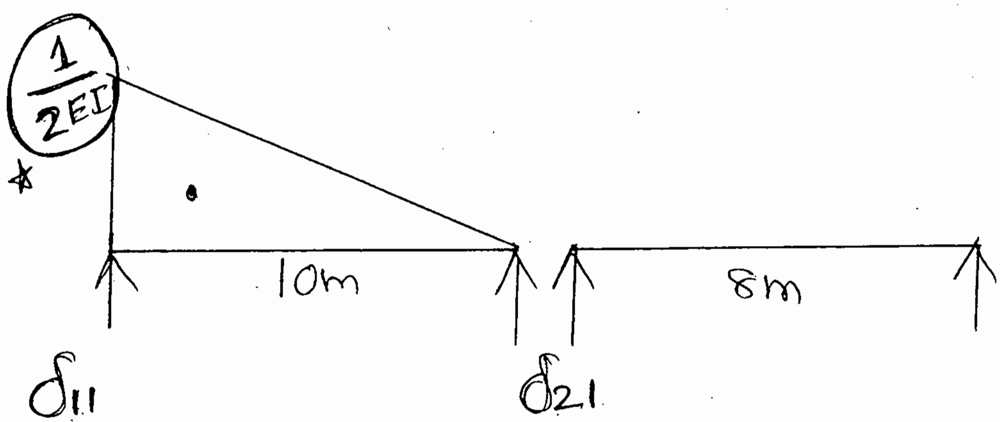
(b) Flexibility Matrix :-

$$[F] = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}$$

Since 'Moment' are taken as Redundants apply unitmoment at "A" and "B"

(1) Apply unit moment at "A" :

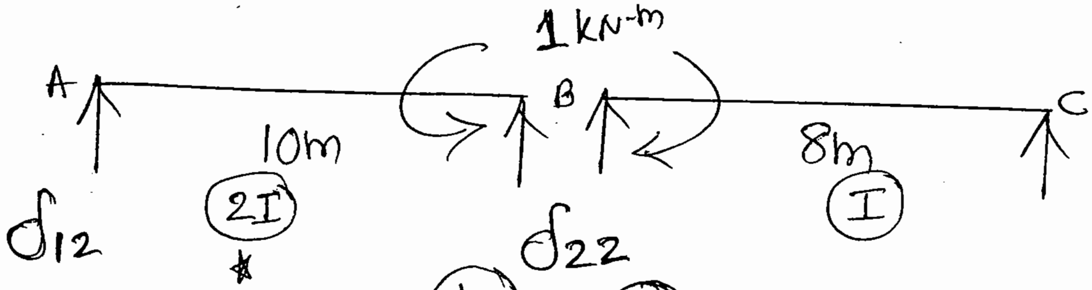


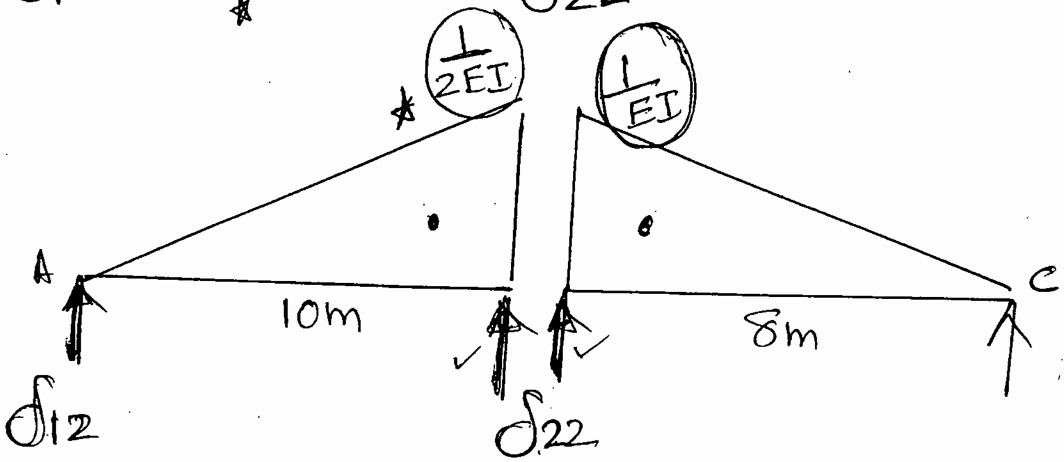


$$\delta_{11} = \frac{1}{10}\left[\left(\frac{1}{2} \times 10 \times \frac{1}{2EI}\right)\frac{2}{3} \times 10\right] = \frac{1.67}{EI}$$

$$\delta_{21} = \frac{1}{10}\left[\left(\frac{1}{2} \times 10 \times \frac{1}{2EI}\right)\frac{1}{3} \times 10\right] = \frac{0.833}{EI}$$

② Apply unit moment at "B"





$$\delta_{12} = \frac{1}{10}\left[\left(\frac{1}{2} \times 10 \times \frac{1}{2EI}\right)\frac{1}{3} \times 10\right] = \frac{0.833}{EI}$$

$$\delta_{22} = \frac{1}{10}\left[\left(\frac{1}{2} \times 10 \times \frac{1}{2EI}\right)\frac{2}{3} \times 10\right] + \frac{1}{8}\left[\left(\frac{1}{2} \times 8 \times \frac{1}{EI}\right)\frac{2}{3} \times 8\right]$$

$$= \frac{4.33}{EI}$$

$$[F] = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix} = \begin{bmatrix} \frac{1.67}{EI} & \frac{0.833}{EI} \\ \frac{0.833}{EI} & \frac{4.33}{EI} \end{bmatrix}$$

$$= \frac{1}{EI}\begin{bmatrix} 1.67 & 0.833 \\ 0.833 & 4.33 \end{bmatrix}$$

$$\therefore [F]^{-1} = \frac{EI}{1.67 \times 4.33 - 0.833 \times 0.833}\begin{bmatrix} 4.33 & -0.833 \\ -0.833 & 1.67 \end{bmatrix}$$

$$[F]^{-1} = \frac{EI}{6.54}\begin{bmatrix} 4.33 & -0.833 \\ -0.833 & 1.67 \end{bmatrix}$$

$$\therefore \quad [P] = [F]^{-1}\left\{ \underbrace{\cancel{[\Delta]}}_{=0} - [\Delta_L] \right\}$$

$=0$ ∵ No sinking

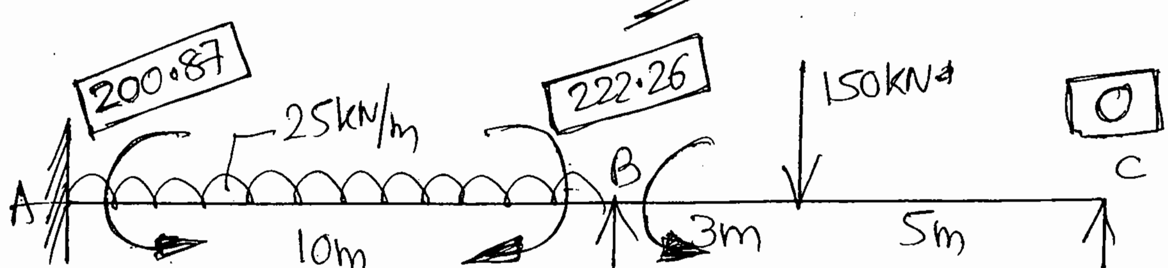
$$\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \frac{\cancel{EI}}{6.54}\begin{bmatrix} 4.33 & -0.833 \\ -0.833 & 1.67 \end{bmatrix}\left\{ \begin{bmatrix} 0 \end{bmatrix} - \begin{bmatrix} \frac{520.83}{\cancel{EI}} \\ \frac{1130.20}{\cancel{EI}} \end{bmatrix} \right\}$$

$$= \frac{1}{6.54}\begin{bmatrix} 4.33 & -0.833 \\ -0.833 & 1.67 \end{bmatrix}\begin{bmatrix} -520.83 \\ -1130.20 \end{bmatrix}$$

$$= \frac{1}{6.54}\begin{bmatrix} 4.33(-520.83) + (-0.833)(-1130.20) \\ (-0.833)(-520.83) + (1.67)(-1130.20) \end{bmatrix}$$

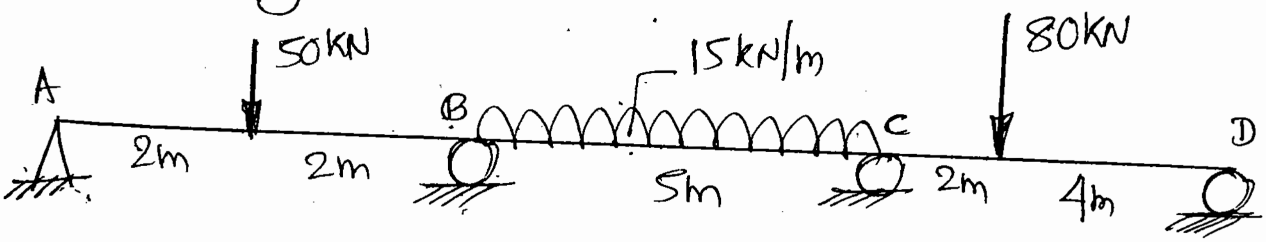
$$\therefore \begin{bmatrix} M_A \\ M_B \end{bmatrix} = \begin{bmatrix} -200.87 \\ -222.26 \end{bmatrix}$$

$\therefore$ $M_A = -200.87$ kN-m, (Hogg) $\quad M_B = -222.26$ kN-m (Hogg).



Draw BMD

Eg:-2] Analyse the beam shown by Flexibility matrix Draw BMD. (118)



Sol:

No. of Unknowns $= 4\ (V_A, V_B, V_C, V_D)$

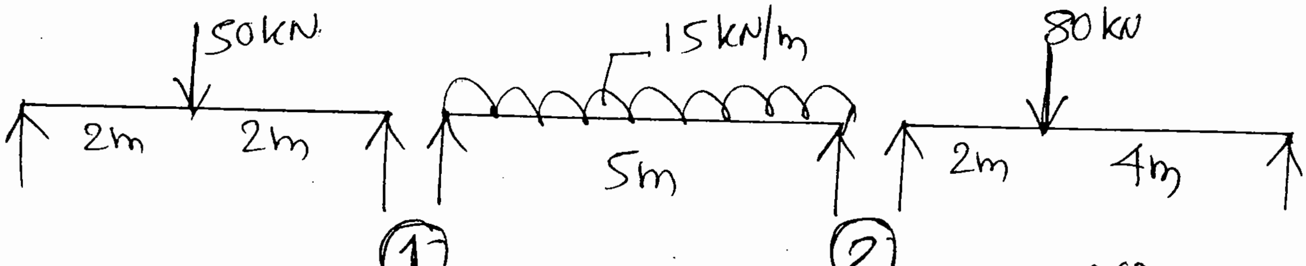
Available equillibrium eq^n $= 2\ (\Sigma V = 0, \Sigma M = 0)$

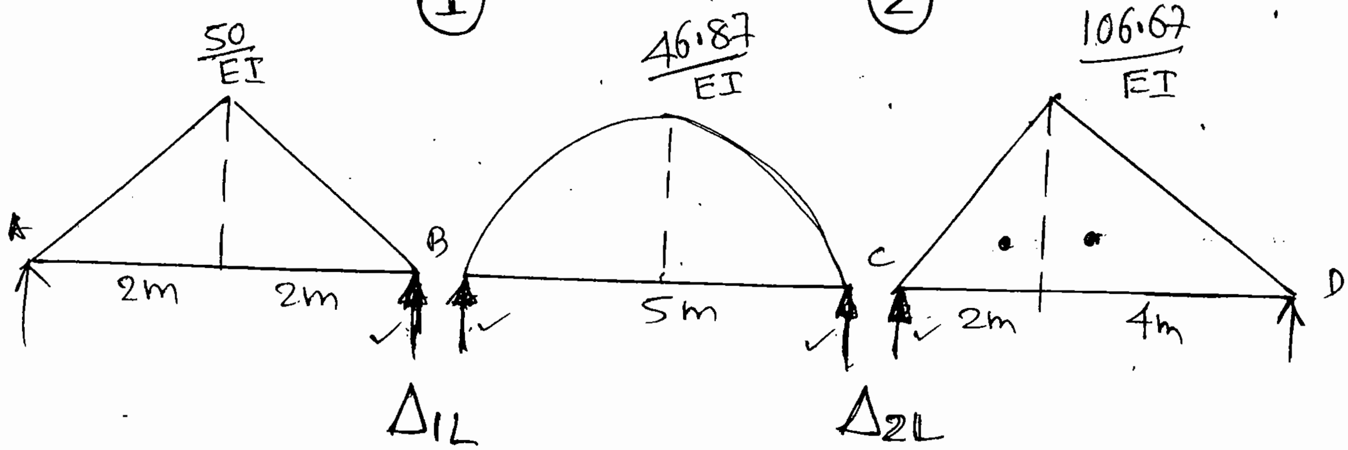
$\overline{\quad 2 \quad}$

Consider Moment M_B and M_C as Redundants

[Since there is no sinking, moment is taken as Redu]

(a) Displacement due to Loads





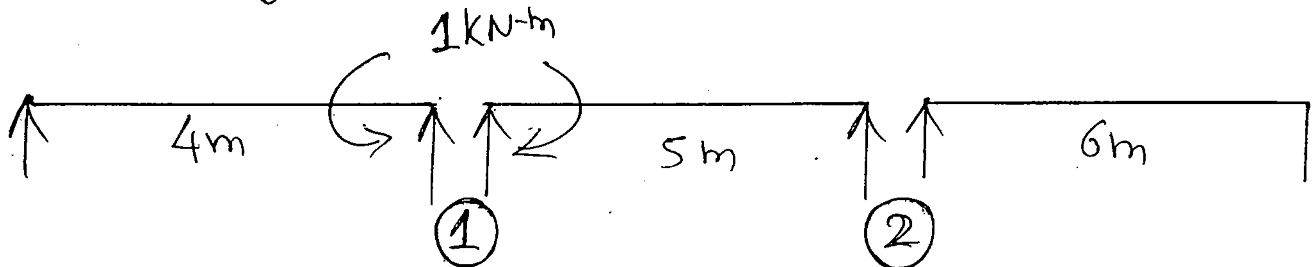
$$\Delta_{1L} = \frac{1}{2}\left[\frac{1}{2}\times 4\times\frac{50}{EI}\right] + \frac{1}{2}\left[\frac{2}{3}\times 5\times\frac{46.87}{EI}\right] = \frac{128.11}{EI}$$

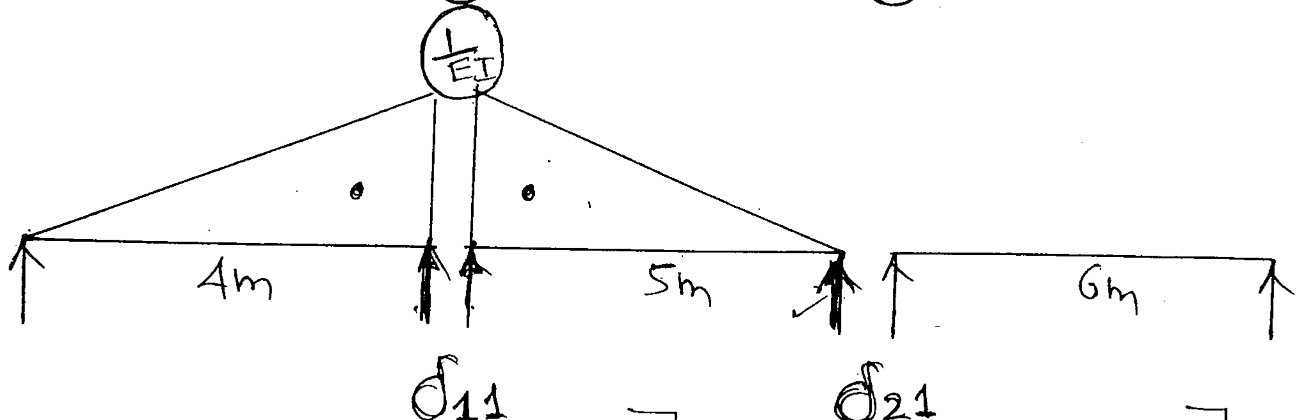
$$\Delta_{2L} = \frac{1}{2}\left[\frac{2}{3}\times 5\times\frac{46.87}{EI}\right] + \frac{1}{6}\left[\left(\frac{1}{2}\times 4\times\frac{106.67}{EI}\right)\frac{2}{3}\times 4 + \left(\frac{1}{2}\times 2\times\frac{106.67}{EI}\right)\left(\frac{1}{3}\times 2+4\right)\right]$$

$$= \frac{255.90}{EI}$$

(b) Flexibility Matrix :

(1) Apply "unit moment" at (B)

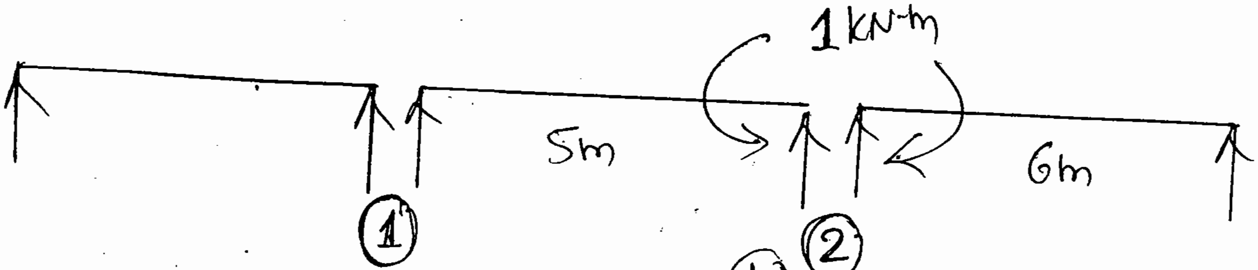


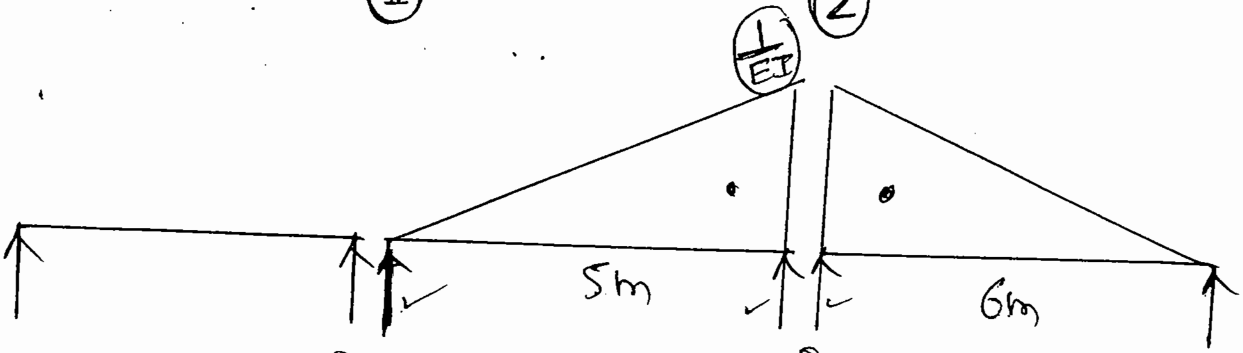


$$\delta_{11} = \frac{1}{4}\left[\left(\frac{1}{2}\times 4\times\frac{1}{EI}\right)\frac{2}{3}\times 4\right] + \frac{1}{5}\left[\left(\frac{1}{2}\times 5\times\frac{1}{EI}\right)\frac{2}{3}\times 5\right] = \frac{3}{EI}$$

$$\delta_{21} = \frac{1}{5}\left[\left(\frac{1}{2}\times 5\times\frac{1}{EI}\right)\frac{1}{3}\times 5\right] = \frac{0.833}{EI}$$

(2) Unit moment is applied at C





$$\delta_{12} = \frac{1}{5}\left[\left(\frac{1}{2}\times 5\times\frac{1}{EI}\right)\frac{1}{3}\times 5\right] = \frac{0.833}{EI}$$

$$\delta_{22} = \frac{1}{5}\left[\left(\frac{1}{2}\times 5\times\frac{1}{EI}\right)\frac{2}{3}\times 5\right] + \frac{1}{6}\left[\left(\frac{1}{2}\times 6\times\frac{1}{EI}\right)\frac{2}{3}\times 6\right] = \frac{3.67}{EI}$$

$$\therefore [F] = \frac{1}{EI}\begin{bmatrix} 3 & 0.833 \\ 0.833 & 3.67 \end{bmatrix}$$

$$[F]^{-1} = \frac{EI}{3\times 3.67 - 0.833^2}\begin{bmatrix} 3.67 & -0.833 \\ -0.833 & 3 \end{bmatrix}$$

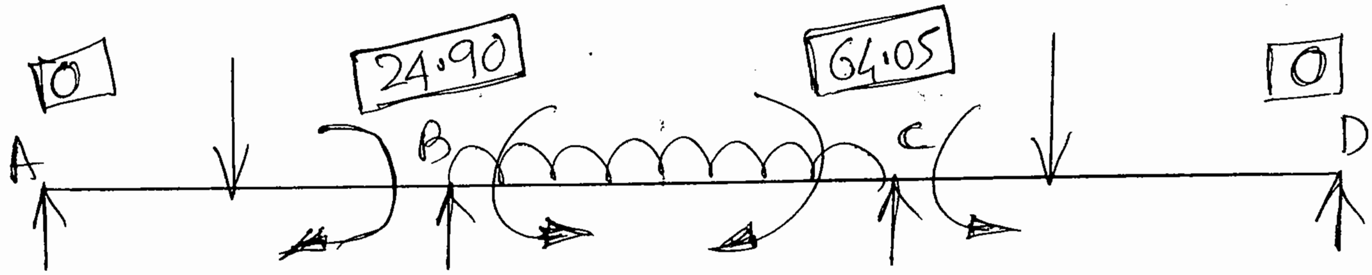
$$= \frac{EI}{10.32}\begin{bmatrix} 3.67 & -0.833 \\ -0.833 & 3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} M_B \\ M_C \end{bmatrix} = \frac{\cancel{EI}}{10.32} \begin{bmatrix} +3.67 & -0.833 \\ -0.833 & 3 \end{bmatrix} \left\{ (0) - \begin{bmatrix} 128.11/\cancel{EI} \\ 255.90/\cancel{EI} \end{bmatrix} \right.$$

$$= \frac{1}{10.32} \begin{bmatrix} 3.67 & -0.833 \\ -0.833 & 3 \end{bmatrix} \begin{bmatrix} -128.11 \\ -255.90 \end{bmatrix}$$

$\therefore$ $M_B = -24.90$ kN-m (Hog)

$M_C = -64.05$ kN-m (Hog)



Draw BMD

═══ x ═══

FLEXIBILITY MATRIX METHOD.

(FORCE METHOD).

$$\{D_Q\} = \{D_{QL}\} + [F]\{Q\} \longrightarrow \text{(I)}$$

Where $\{D_Q\}$ → Actual displacement existing in the str.

$\{D_{QL}\}$ → Displacement due to loads in the released structure.

$[F]$ → Flexibility matrix for the released str.

$\{Q\}$ → Redundants.

∴ The Redundants.

$$\{Q\} = [F]^{-1}\left[\{D_Q\} - \{D_{QL}\}\right] \longrightarrow \text{(II)}$$

If displacement at the redundants points are zero

then $\{D_Q\} = \{0\}$

$$\therefore \boxed{\{Q\} = -[F]^{-1}\{D_{QL}\}} \longrightarrow \text{(III)}$$

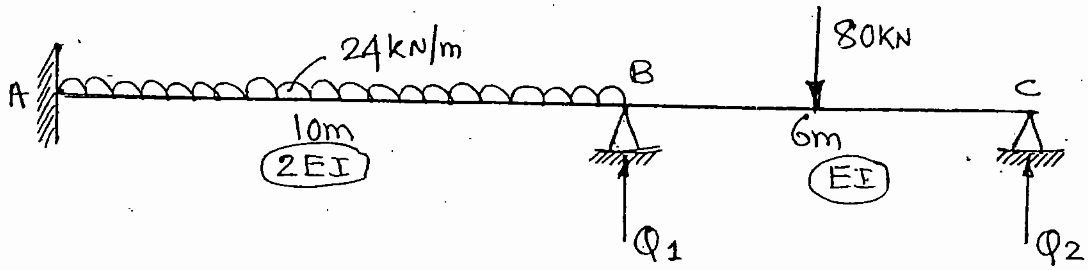
or

$$[P] = [F]^{-1}[(\Delta) - (\Delta_L)]$$

Redundant

Eg:-1] Analyse the continuous beam shown by F.M. method and draw BMD & SFD.

What will be the change in the forces if supports "B" & "C" settle down by $\frac{200}{EI}$ and $\frac{100}{EI}$.

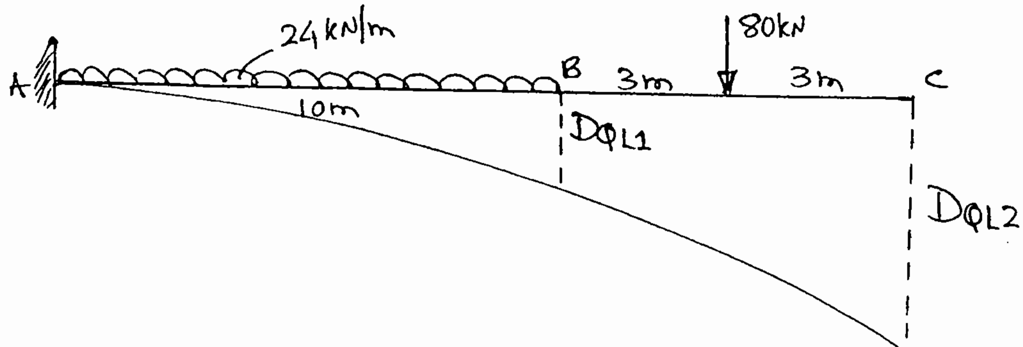


Degree of static Indeterminacy

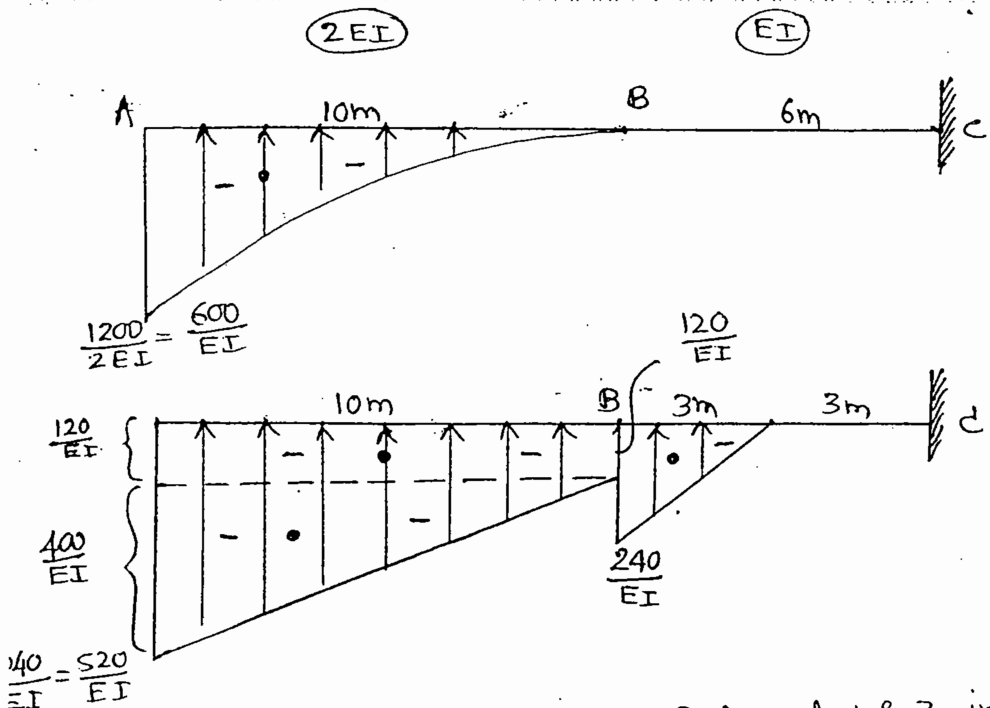
$= 4 - 2 = 2$

∴ Vertical reactions at B and C will be chosen as the redundant reaction

Released or Basic Determinate Str:-



Deflection due to external loads DQL1 & DQL2 can be calculated by Conjugate beam method



Conjugate Beam

Deflection at 1 & 2 = B.M. at 1 & 2 in C.Beam.

$$D_{QL1} = -\left[\left(\frac{1}{3}\times 10\times\frac{600}{EI}\right)\frac{3}{4}\times 10 + \left(\frac{1}{2}\times 10\times\frac{400}{EI}\right)\frac{2}{3}\times 10 + \left(\frac{120}{EI}\times 10\times 5\right)\right]$$

(at B)

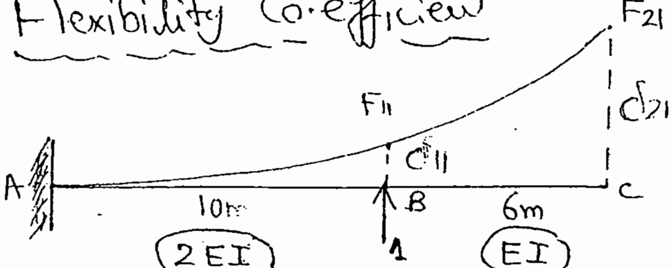
$$\Delta_{1L} = -\frac{34333.33}{EI}\ (\downarrow)$$

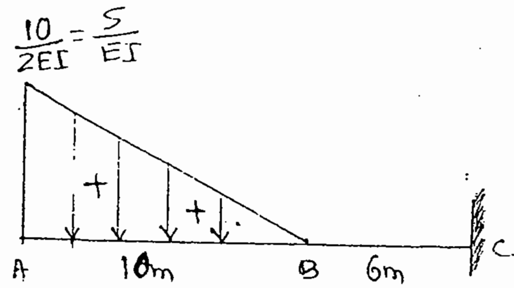
$$D_{QL2} = -\left[\left(\frac{1}{3}\times 10\times\frac{600}{EI}\right)\left(\frac{3}{4}\times 10+6\right) + \left(\frac{1}{2}\times 3\times\frac{240}{EI}\right)\left(\frac{2}{3}\times 3+3\right) + \left(\frac{1}{2}\times 10\times\frac{400}{EI}\right)\left(\frac{2}{3}\times 10+6\right) + \left(10\times\frac{120}{EI}\right)11\right]$$

(at C)

$$\Delta_{2L} = -\frac{67333.33}{EI}\ (\downarrow)$$

Flexibility Co-efficient



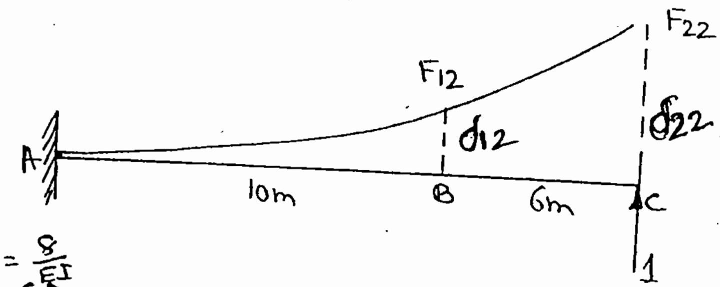


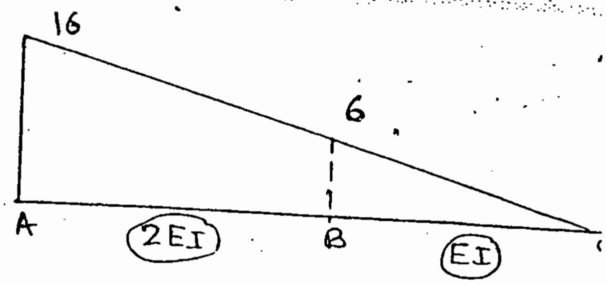
$$F_{11} = \left(\frac{1}{2}\times 10\times\frac{5}{EI}\right)\frac{2}{3}\times 10 = \frac{166.67}{EI}$$

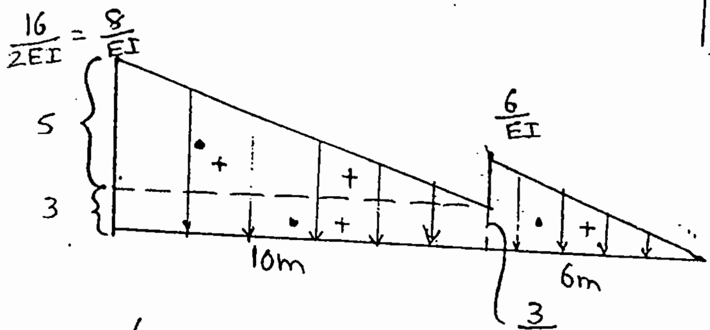
δ_{11}

$$F_{21} = \left(\frac{1}{2}\times 10\times\frac{5}{EI}\right)\left(\frac{2}{3}\times 10+6\right) = \frac{316.67}{EI}$$

δ_{21}







$$F_{12} \; (\delta_{12}) = +\left(\frac{1}{2}\times 10\times\frac{5}{EI}\right)\frac{2}{3}\times 10 + \left(10\times\frac{3}{EI}\right)5 = \frac{316.67}{EI}$$

$$F_{22} \; (\delta_{22}) = \left(\frac{1}{2}\times 6\times\frac{6}{EI}\right)\frac{2}{3}\times 6 + \left(\frac{1}{2}\times 10\times\frac{5}{EI}\right)\left(\frac{2}{3}\times 10+6\right) + \left(10\times\frac{3}{EI}\right)11 = \frac{718.67}{EI}$$

$$\therefore [F] = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} = \begin{bmatrix} 166.67 & 316.67 \\ 316.67 & 718.67 \end{bmatrix}\frac{1}{EI}$$

$$\therefore [F]^{-1} = \frac{EI}{166.67\times 718.67-(316.67)^2}\begin{bmatrix} 718.67 & -316.67 \\ -316.67 & +166.67 \end{bmatrix}$$

$$= \frac{EI}{19500.84}\begin{bmatrix} 718.67 & -316.67 \\ -316.67 & 166.67 \end{bmatrix}$$

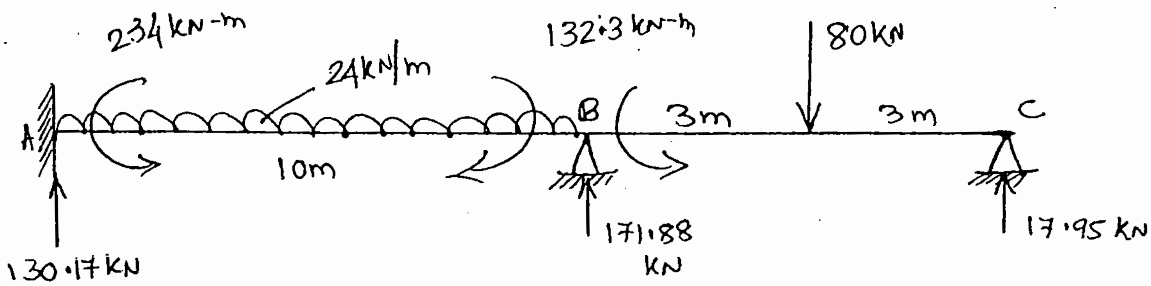
Without Sinking of Supports.

$$\{Q\} = [F]^{-1}\left[\{D_Q\} - \{D_{QL}\}\right]$$

$$\begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} = \frac{EI}{19500.84}\begin{bmatrix} 718.67 & -316.67 \\ -316.67 & 166.67 \end{bmatrix}\left[\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - \frac{1}{EI}\begin{Bmatrix} -34333.33 \\ -67333.33 \end{Bmatrix}\right]$$

$$= \frac{1}{19500.84}\left[\text{— '' —}\right]\begin{Bmatrix} 34333.33 \\ 67333.33 \end{Bmatrix}$$

$$= \frac{1}{19500.84}\begin{Bmatrix} 3351888.66 \\ 350110.50 \end{Bmatrix} \qquad \therefore \begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} = \begin{Bmatrix} 171.88 \\ 17.95 \end{Bmatrix}$$



$\therefore\ M_A = 234\ \text{kN-m}$

$M_B = 132.3\ \text{kN-m}.$

Draw BMD & SFD.

— x —

With Settlement of Supports.

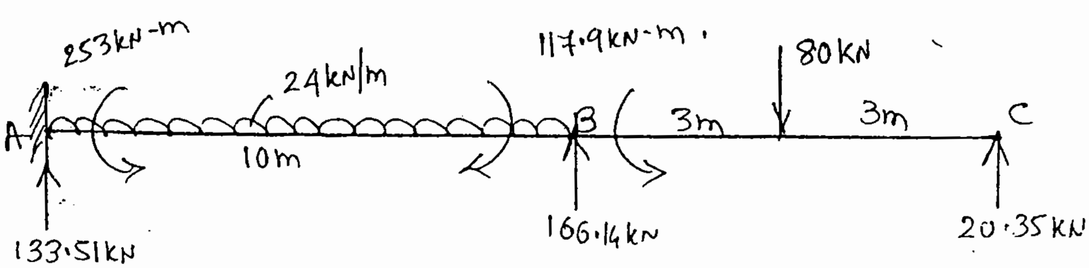
Downward sinking is taken -ve.

$$\{\phi\} = [F]^{-1}\left[\{D_\phi\} - \{D_{\phi L}\}\right]$$

$$= \begin{bmatrix} 718.67 & -316.67 \\ -316.67 & 166.67 \end{bmatrix} \frac{EI}{19500.84} \left[\begin{Bmatrix} -200/EI \\ -100/EI \end{Bmatrix} - \begin{Bmatrix} -34333.33 \\ -67333.33 \end{Bmatrix} \frac{1}{EI} \right]$$

$$= \frac{EI}{19500} \left[\text{— '' —} \right] \left[\frac{1}{EI} \begin{Bmatrix} 34133.33 \\ 67233.33 \end{Bmatrix} \right]$$

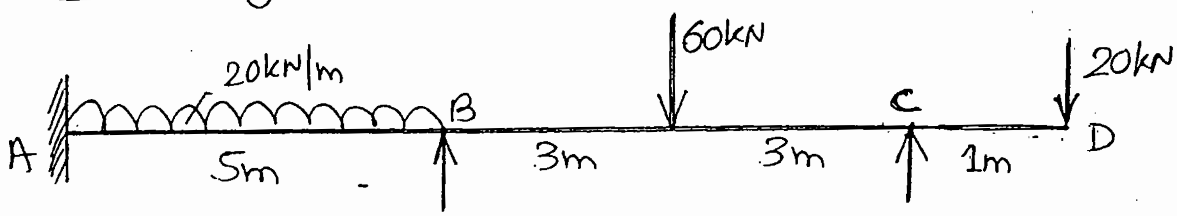
$$\therefore \begin{matrix} R_B \\ R_C \end{matrix} \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix} = \begin{Bmatrix} 166.14 \\ 20.35 \end{Bmatrix} \text{kN}$$



Draw BMD & SFD.

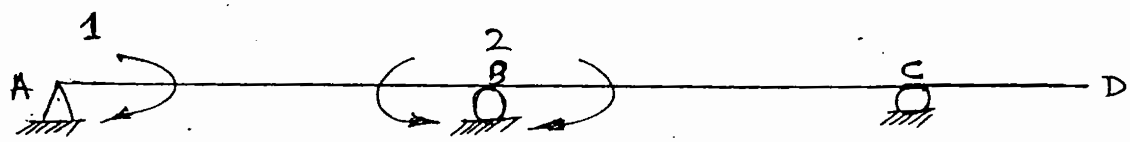
— x —

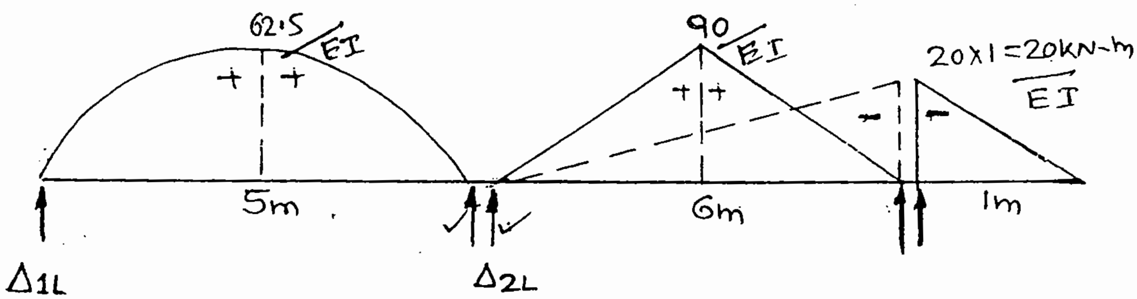
Eg:- 2] Analyse the beam shown by Flexibility matrix. Draw BMD.



Sol^n Degree of static Indeterminacy $= 4-2 = 2.$

Select (MA) & (MB) as redundant forces.



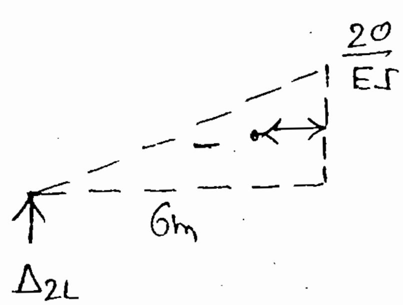


Displacement due to loads

$$\Delta_{1L} = \frac{1}{2}\left[\frac{2}{3} \times 5 \times \frac{62.5}{EI}\right] = \frac{104.17}{EI}$$

$$\Delta_{2L} = \frac{1}{2}\left[\frac{2}{3} \times 5 \times \frac{62.5}{EI}\right] + \frac{1}{2}\left[\frac{1}{2} \times 6 \times \frac{90}{EI}\right] - \frac{1}{6}\left[\frac{1}{2} \times 6 \times \frac{20}{EI}\right]\frac{1}{3} \times 6$$

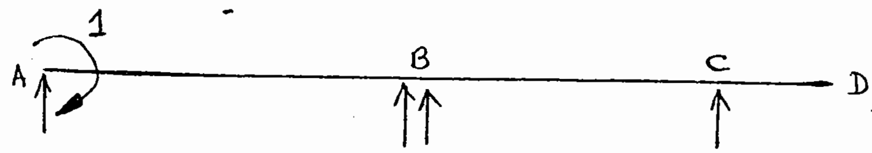
$$= \frac{219.17}{EI}$$

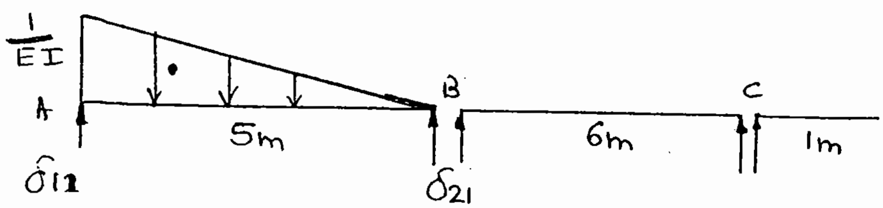


Flexibility Matrix :

Unit force or moment is applied at "A" & "B".

(i) Unit moment is applied at "A" :

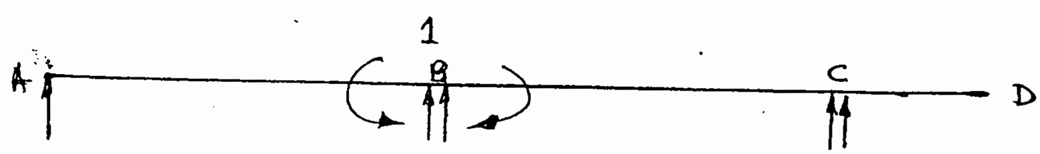


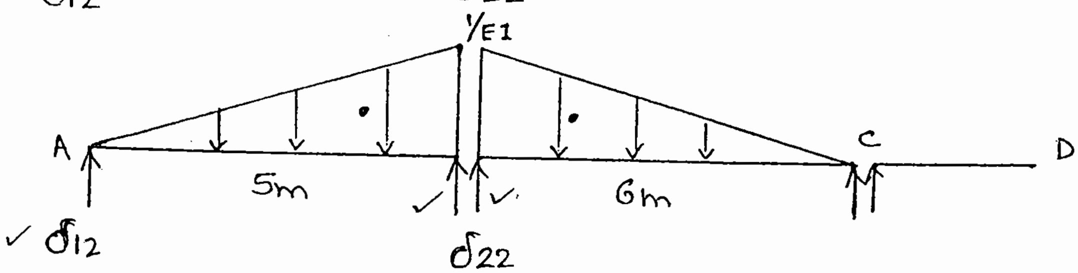


$$\delta_{12} = \frac{1}{5}\left[\frac{1}{2} \times 5 \times \frac{1}{EI}\right]\frac{2}{3} \times 5 = \frac{5}{3EI}$$

$$\delta_{21} = \frac{1}{5}\left[\frac{1}{2} \times 5 \times \frac{1}{EI}\right]\frac{1}{3} \times 5 = \frac{5}{6EI}$$

ii) Unit moment is applied at "B"





$$\delta_{12} = \frac{1}{5}\left[\frac{1}{2} \times 5 \times \frac{1}{EI}\right]\frac{1}{3} \times 5 = \frac{5}{6EI}$$

$$\delta_{22} = \frac{1}{5}\left[\frac{1}{2} \times 5 \times \frac{1}{EI}\right]\frac{2}{3} \times 5 + \frac{1}{6}\left[\frac{1}{2} \times 6 \times \frac{1}{EI}\right]\frac{2}{3} \times 6 = \frac{11}{3EI}$$

$$\therefore [F] = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix} = \begin{bmatrix} \frac{5}{3EI} & \frac{5}{6EI} \\ \frac{5}{6EI} & \frac{11}{3EI} \end{bmatrix}$$

$$= \frac{1}{3EI}\begin{bmatrix} 5 & 2.5 \\ 2.5 & 11 \end{bmatrix}$$

$$\therefore [F]^{-1} = \frac{3EI}{5\times 11 - 2.5\times 2.5}\begin{bmatrix} 11 & -2.5 \\ -2.5 & 5 \end{bmatrix} = \frac{4EI}{65}\begin{bmatrix} 11 & -2.5 \\ -2.5 & 5 \end{bmatrix}$$

$\therefore$ Redundants $\boxed{[p] = [F]^{-1}\left[(\Delta) - (\Delta_L)\right]}$

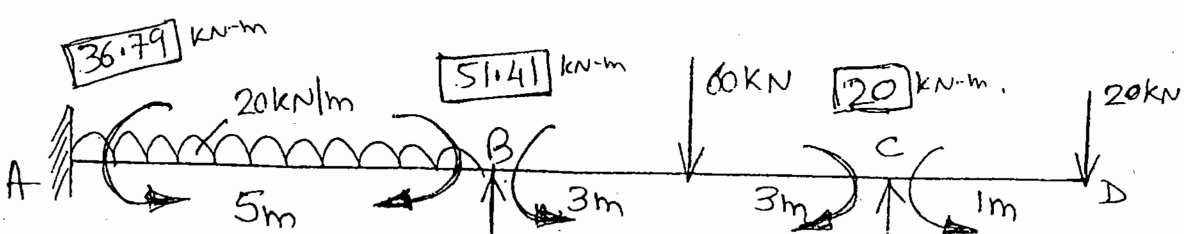
$\Delta = 0$ ($\because$ There is <u>No</u> sinking)

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \frac{4E\cancel{I}}{65}\begin{bmatrix} 11 & -2.5 \\ -2.5 & 5 \end{bmatrix}\left\{\begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 104.17/\cancel{EI} \\ 219.17/\cancel{EI} \end{bmatrix}\right\}$$

$$= \frac{4}{65}\begin{bmatrix} 11 & -2.5 \\ -2.5 & 5 \end{bmatrix}\begin{bmatrix} -104.17 \\ -219.17 \end{bmatrix}$$

$$\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} -36.79 \\ -51.41 \end{bmatrix}.$$

$\therefore$ $M_A = -36.79$ kN-m (Hog)
$M_B = -51.41$ kN-m (〃)
$M_C = -20$ kN-m (〃)



Draw BMD & SFD.