Module-2

MOMENT DISTRIBUTION METHOD

It is a displacement method of analysis of kinematically indeterminate structure. It is based on the stiffness approach.

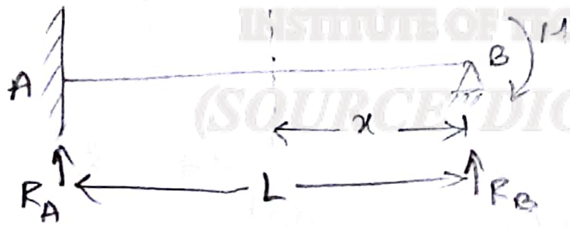
It is also known as hardy cross method of successive approximation.

Stiffness:- Stiffness for a member at a joint is the moment (force) required to produce unit rotation (displacement) at that joint represented in 'k'. $K = \frac{F}{d} = \frac{M}{\theta}$

Distribution factor:-

DF for a member at a joint is the ratio of the stiffness of the member to the total stiffness of all the members meeting at that joint.

To find the members stiffness factor of a beam fixed at one end & hinged at the other.



$$EI\frac{d^2y}{dx^2} = M_x$$

$$EI\frac{d^2y}{dx^2} = R_B x - M \rightarrow (1)$$

Diff w.r. to x

$$EI\frac{dy}{dx} = R_B\frac{x^2}{2} - Mx + C_1 \rightarrow (2)$$

Diff w.r.to. x

$$EIy = \frac{R_B x^3}{6} - \frac{Mx^2}{2} + C_1 x + C_2 \rightarrow (3)$$

Boundary conditions:-

At $x=0$, $y=0 \rightarrow (3)$

$$\boxed{C_2 = 0}$$

At $x=L$, $y=0 \rightarrow (3)$

$$0 = \frac{R_B L^3}{6} - \frac{ML^2}{2} + C_1 L \rightarrow (4)$$

@ $x=L$, $\frac{dy}{dx} = 0 \rightarrow (2)$

$$0 = \frac{R_B L^2}{2} - ML + C_1$$

$$C_1 = ML - \frac{R_B L^2}{2} \rightarrow (5)$$

put (5) in eq^n (4)

$$\frac{R_B L^3}{6} - \frac{ML^2}{2} + \left(ML - \frac{R_B L^2}{2}\right) L = 0$$

$$\frac{R_B L^3}{6} - \frac{ML^2}{2} + ML^2 - \frac{R_B L^3}{2} = 0$$

$$-\frac{2 R_B L^3}{6} + \frac{ML^2}{2} = 0$$

$$\boxed{R_B = \frac{3M}{2L}}$$

Substitute R_B in eq (2)

$$EI\frac{dy}{dx} = \frac{3M}{2L}\frac{x^2}{2} - Mx + C_1$$

put $R_B \rightarrow (5)$

$$C_1 = ML - \frac{3ML^2}{4L}$$

$$= ML - \frac{3ML}{4}$$

$$\boxed{C_1 = \frac{ML}{4}}$$

$$\therefore EI\frac{dy}{dx} = \frac{3Mx^3}{4L} - Mx + \frac{ML}{4}$$

$\therefore x = a$

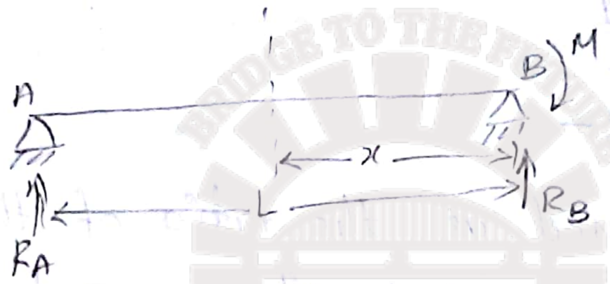
$$EI\frac{dy}{dx}\Big)_B = \frac{ML}{4}$$

$$EI\theta_B = \frac{ML}{4}$$

$$\frac{M}{\theta_B} = \frac{4EI}{L}$$

$\boxed{K = \frac{4EI}{L}}$ The stiffness for a member whose farer end is fixed is given by $\frac{4EI}{L}$.

Case (ii) :-



$$EI\frac{d^2y}{dx^2} = M_x = R_B x - M$$

$\Sigma V = 0$, $R_A + R_B = 0$

$\Sigma M_A = 0$, $(-R_B \times L) + M = 0$

$$\boxed{R_B = \frac{M}{L}}$$

$$EI\frac{d^2y}{dx^2} = \frac{Mx}{L} - M \rightarrow (1)$$

$$EI\frac{dy}{dx} = \frac{Mx^2}{2L} - Mx + C_1 \rightarrow (2)$$

$$EIy = \frac{Mx^3}{6L} - \frac{Mx^2}{2} + C_1x + C_2 \rightarrow (3)$$

Boundary conditions :-

@ $x=0$, $y=0$

eq^n(3) $\boxed{C_2 = 0}$

@ $x = L$, $y = 0$

eq^n(3), $0 = \frac{ML^2}{6} - \frac{ML^2}{2} + C_1L$

$$C_1 = \frac{ML}{2} - \frac{ML}{6}$$

$$\boxed{C_1 = \frac{ML}{3}}$$

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Put C_1 in eq^n(2)

$$EI\frac{dy}{dx} = \frac{Mx^2}{2L} - Mx + \frac{ML}{3}$$

@ $x=0$

$$EI\left.\frac{dy}{dx}\right|_B = \frac{ML}{3}$$

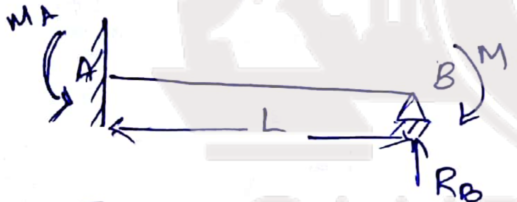
$$\frac{M}{\theta_B} = k = \frac{3EI}{L}$$

$\therefore$ stiffness of the member hinged at the farer end is given by $\frac{3EI}{L}$.

Carry over factor:-

"It is defined as the ratio of the moment at the fixed far end to the moment at the rotating near end."

Mathematically, $COF = \frac{\text{carry over moment at fixed end}}{\text{applied moment at near end}}$



$\Sigma M_A = 0$

$M - (R_B \times L) - M_A = 0$

$M - \frac{3M}{2} - M_A = 0$

$-0.5M - M_A = 0$

$M_A = -0.5M$

$$\boxed{M_A = -\frac{M}{2}}$$

$$COF = \frac{M_A}{M} = \frac{-M/2}{M}$$

$$\boxed{COF = -1/2}$$

Problems:-

General procedure:-

1. For the

Step-1:- find the fixed end moments for each member

Step-2:- find the distribution factors for all the members meeting at a joint [if there is an imbalance of the fixed end moments at the joints].

Step-3:- find the unbalanced moments & distribute it at each joint according to their distribution factor & transfer carry over moments to their farther end. if the farther end is fixed.

Step-4:- Repeat Step-3 untill an accuracy of 0.01 is reached.

Step-5:- find the final end moments at the ends when all the joints are balanced.

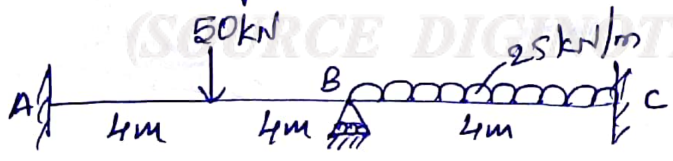
Step-6:- Draw BMD using the following sign conventions.

+ve moment = clockwise rotation / sagging

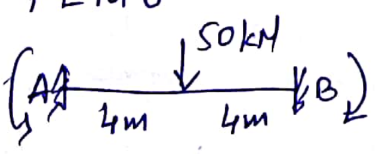
-ve moment = anticlockwise rotation / hogging.

Problems:- [without settlement]

1. For the beam as shown in the figure analyse and draw BMD using moment distribution method.

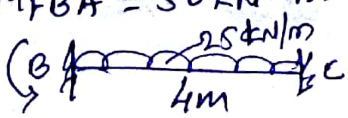


Step-1:- FEM's

* AB 

$M_{FAB} = -50 kN\text{-}m$

$M_{FBA} = 50 kN\text{-}m$

* BC 

$M_{FBC} = -33.33 kN\text{-}m$

$M_{FCB} = 33.33 kN\text{-}m$

Step-2 :- DF's

joint	member	stiffness	total stiffness	DF
B	AB	$k = \frac{4EI}{L} = 0.5EI$	$1.5EI$	0.3
	BC	$k = \frac{4EI}{L} = EI$		0.7

Step-3 :- EQ

	A	B	B	C
D.F		0.3	0.7	
* FEM's	-50	50	-33.33	33.33
* unbalanced moment		16.67		
← balanced moment		-16.67		
		-5.001	-11.669	
	-2.5005			-5.83
Σ	-52.5005	44.99	-44.99	27.5

Step-5 :- final moments

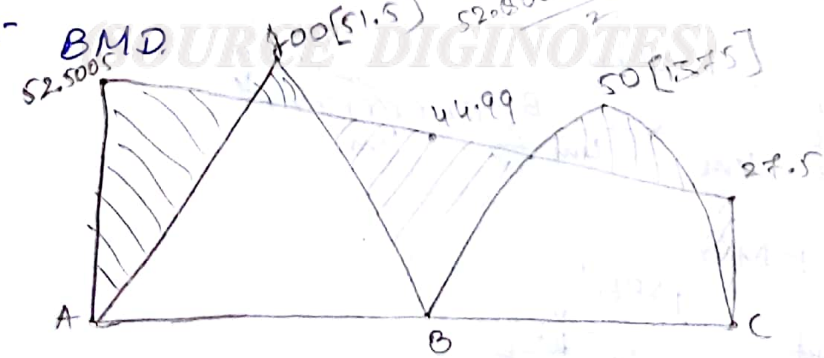
$M_{AB} = -52.005$ kN-m

$M_{BA} = 44.99$ kN-m

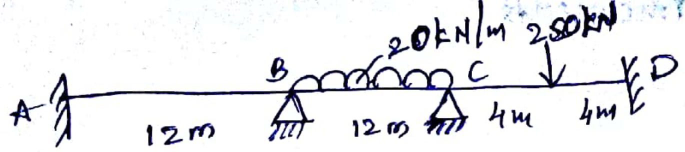
$M_{BC} = -44.99$ kN-m

$M_{CB} = 27.5$ kN-m

Step-6 :- BMD



2. Analyse the given contineous beam by moment distribution method. & draw the BM diagram. [Assume EI = constant.]



Step-1:- find FEM's

* AB, $M_{FAB} = M_{FBA} = 0$ kN-m

* BC, $M_{FBC} = -\frac{wL^2}{12} = \frac{-20\times12^2}{12} = -240$ kN-m

* CB, $M_{FCB} = 240$ kN-m.

* CD, $M_{FCD} = -\frac{PL}{8} = \frac{-250\times8}{8} = -250$ kN-m

$M_{FDC} = +250$ kN-m.

Step-2:- DF's

Joint	member	stiffness	T.S	DF
B	BA	$k = \frac{4EI}{L}$ $k = 0.3EI$	0.55EI	0.57
	BC	$k = \frac{3EI}{L}$ $k = 0.25EI$		0.43
C	CB	$k = \frac{3EI}{L}$ $k = 0.25EI$	0.75EI	0.33
	CD	$k = \frac{4EI}{L}$ $k = 0.5EI$		0.67

Step-3:-

	A	B		C		D
		0.57	0.43	0.33	0.67	
	0	0	-240	240	-250	250
		-240		-10		
		240		10		
		136.8	103.2	3.3	6.7	
	68.4					3.35
Σ	68.4	136.8	-136.8	243.3	-243.3	253.35

Step-5:- Final moments

$M_{AB} =$

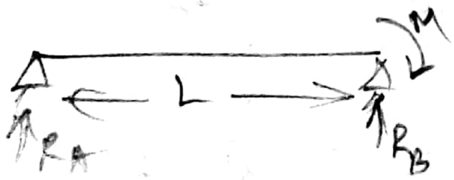
$M_{BA} =$

M,

Step-8:-

A	B		C		D
	0.57	0.43	0.33	0.67	
0	0	-240	240	-250	250
	-240		-10		
	240		10		
	136.8	103.2	3.3	6.7	
68.4		1.65	51.6		3.35
	-1.65		-51.6		
	-0.94	-0.71	-17.03	-34.57	
-0.47		-8.515	-0.35		-17.285
	8.515		0.35		
	4.85	3.66	0.12	0.24	
2.425		0.06	1.83		0.12
	-0.06		-1.83		
	-0.03	-0.03	-0.6	-1.23	
-0.015		-0.3	-0.016		-0.61
	0.3		0.015		
	0.171				

147.58
-140.980



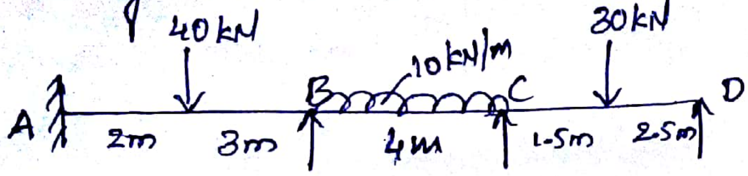
$\Sigma M_A = 0$

$-R_B \times L + M = 0$

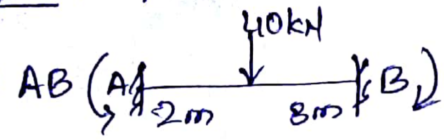
$R_B = \frac{M}{L}$ $\quad \frac{M}{L} \times L + M20$

$M = M$

Analyse the continuous beam loaded as shown in the figure. Sketch the BM & SF diagrams.

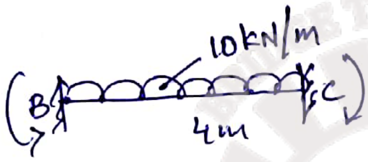


Step-1 :- FEM's



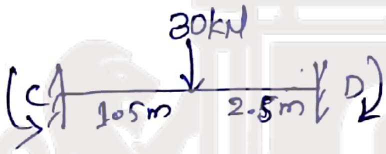
$$M_{FAB} = -\frac{Pb^2a}{L^2} = -\frac{40\times3^2\times2}{5^2} = -28.8\,kN\text{-}m$$

$$M_{FBA} = \frac{Pa^2b}{L^2} = \frac{40\times2^2\times3}{5^2} = 19.2\,kN\text{-}m$$



$$M_{FBC} = -\frac{wL^2}{12} = -\frac{10\times4^2}{12} = -13.3\,kN\text{-}m$$

$$M_{FCB} = \frac{wL^2}{12} = \frac{10\times4^2}{12} = 13.3\,kN\text{-}m$$



$$M_{FCD} = -\frac{Pb^2a}{L^2} = -17.57\,kN\text{-}m$$

$$M_{FDC} = \frac{Pa^2b}{L^2} = 10.57\,kN\text{-}m$$

Step-2 :- DF's

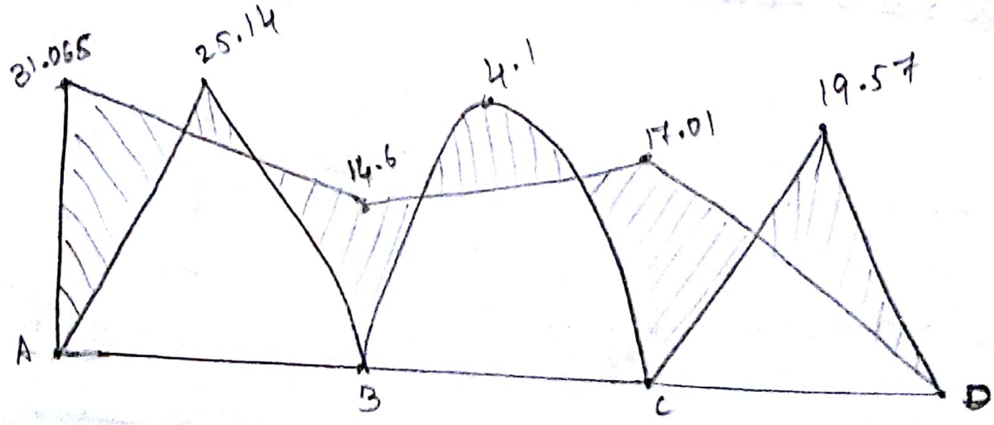
joint	member	stiffness	total S.	DF
B	BA	$\frac{4EI}{L}$	1.55EI	0.52
	BC	$\frac{3EI}{L}$		0.48
C	CB	$\frac{3EI}{L}$	1.5EI	0.5
	CD	$\frac{3EI}{L}$		0.5
D	DC	$\frac{3EI}{L}$	0.75EI	1

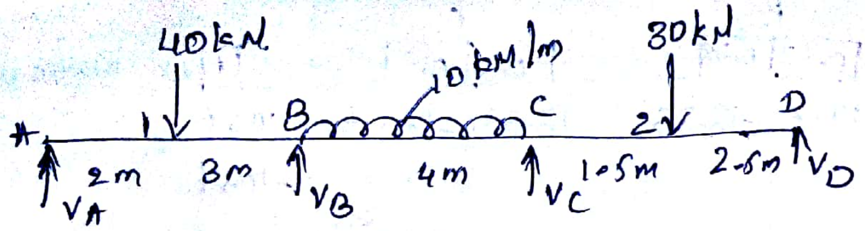
A	B		C		D
	0.52	0.48	0.5	0.5	1
-28.8	19.2	-13.33	13.33	-17.57	10.57
					-10.57
				-5.28	
-28.8	19.2	-13.33	13.33	-22.85	0
	5.87		-9.52		
	-5.87		9.52		
	-3.05	-2.817	4.76	4.76	
-1.52		2.38	-1.40		
	-2.38		1.40		
	-1.24	-1.14	0.7	0.7	
0.62		0.35	-0.57		
	-0.35		0.57		
	-0.18	-0.16	0.28	0.28	
-0.09		0.14	-0.08		
	-0.14		0.08		
	-0.07	-0.07	0.04	0.04	
-0.035					
-31.065	14.6	-14.6	17.01	-17.01	0

BM@AB = 25.14 kN-m

BM@BC = 4.1 kN-m

BM @ CD = 19.57 kN-m





$\Sigma V = 0$, $V_A + V_B + V_C + V_D = 110\ kN$ —→(1)

$\Sigma M_B = 14.6\ kN\text{-}m$

$\Rightarrow V_A \times 5 - 40 \times 3 - 31.068 + 14.6 = 0$

$\boxed{V_A = 27.29\ kN}$

$\Sigma M_C = 17.01\ kN\text{-}m$

$\Rightarrow 27.29 \times 9 - 40 \times 7 + V_B \times 4 - 10 \times 4 \times 2 - 31.065 + 17.01 = 0$

$\boxed{V_B = +32.11\ kN}$

6.99
15.5 $\Rightarrow -V_D \times 4 + 30 \times 1.5 - 17.01 = 0$

$\boxed{V_D = 6.99\ kN}$

$\boxed{V_C = 48.61\ kN}$

SF @ A_R = 27.29 kN

SF @ 1_L = 27.29 kN

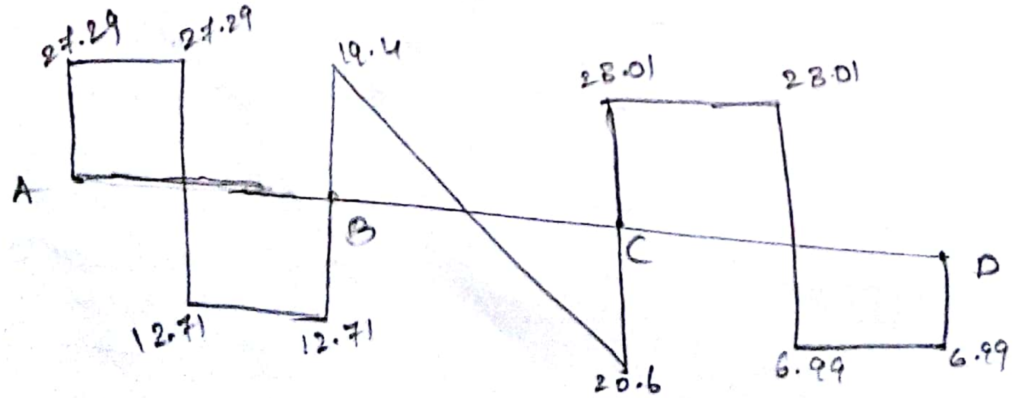
SF @ 1_R = 27.29 − 40 = −12.71 kN = SF @ B_L

SF @ B_R = −12.71 + 32.11 = 19.4 kN

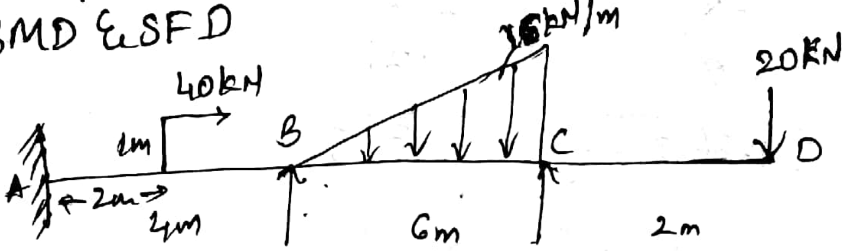
SF @ C_L = 19.4 − 10×4 = −20.6 kN

SF @ C_R = −20.6 + 48.61 = 28.01 kN = SF @ 2_L

SF @ 2_R = 28.01 − 30 = −6.99 kN = SF @ D_L

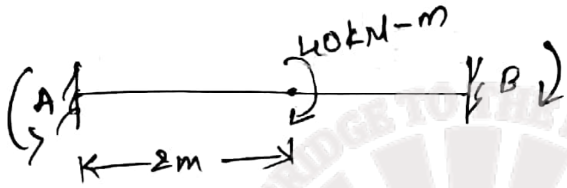


Analyse the continuous beam as shown in the figure by moment distribution method. Sketch the BMD & SFD



FEM's

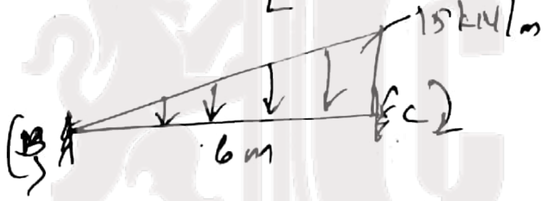
* AB



$$M_{FAB} = +\frac{Mb(2a-b)}{L^2} = \frac{+40\times 2(4-2)}{16} = +10\ kN\text{-}m$$

$$M_{FBA} = \frac{Ma(2b-a)}{L^2} = \frac{+40\times 2(4-2)}{16} = 10\ kN\text{-}m$$

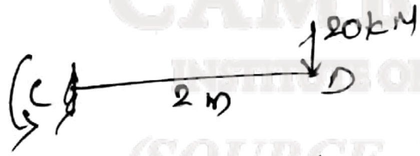
* BC



$$M_{FBC} = -\frac{wL^2}{30} = \frac{-15\times 6^2}{30} = -18\ kN\text{-}m$$

$$M_{FCB} = +\frac{wL^2}{20} = \frac{15\times 6^2}{20} = 27\ kN\text{-}m$$

* CD



$M_{CD} = -40\ kN$

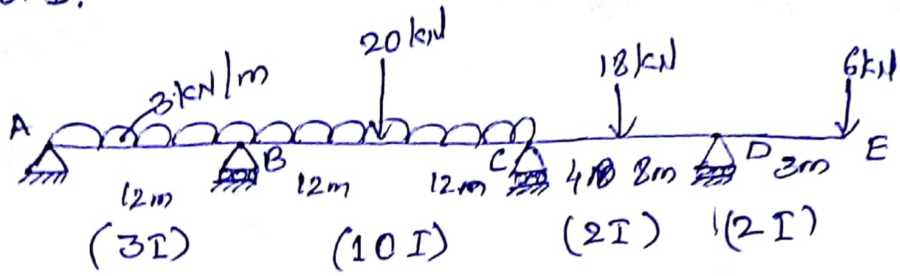
$M_{DC} = 0\ kN.$

DF's

Joint	member	stiffness	T.S.	DF
B	BA	$\frac{4EI}{L} = EI$	1.5EI	0.67
	BC	$\frac{3EI}{L} = 0.5EI$		0.33
C	CB	$\frac{3EI}{L} = 0.5EI$	0.5EI	1
	CD	0EI		

A	B		C		D
	0.67	0.33	1	0	
00	10	-18 6.5	27 13	-40	0
10 0.5	10 -1.05 1.05 1.005	-11.5 0.5	40		
10.5	+11	-11	40	-40	0

Analyse the beam as shown in the figure. also draw SFD & BMD.



$$M_{FAB} = \frac{-wL^2}{12} = \frac{-3\times12^2}{12} = -36\ kN\text{-}m$$

$$M_{FBA} = \frac{wL^2}{12} = \frac{3\times12^2}{12} = 36\ kN\text{-}m$$

$$M_{FBC} = \frac{-wL^2}{12} - \frac{PL}{8} = \frac{-3\times24^2}{12} - \frac{20\times24}{8} = -204\ kN\text{-}m$$

$$M_{FCB} = +\frac{wL^2}{12} + \frac{PL}{8} = +204\ kN\text{-}m$$

$$M_{FCD} = \frac{-Pb^2a}{L^2} = \frac{-18\times8^2\times4}{12^2} = -32\ kN\text{-}m$$

$$M_{FDC} = \frac{Pa^2b}{L^2} = \frac{18\times4^2\times8}{12^2} = 16\ kN\text{-}m$$

$$M_{FDE} = M_{DE} = 18\ kN\text{-}m.$$

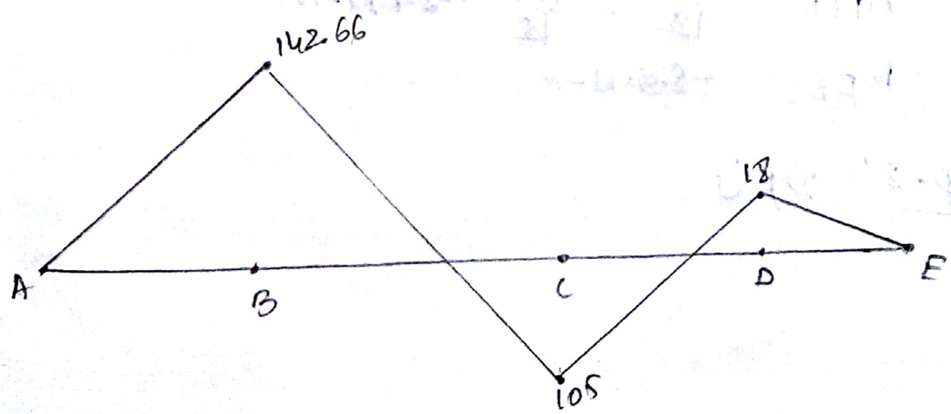
$$M_{ED} = 0\ kN\text{-}m$$

Step-2:-

~~joint~~	~~member~~	~~stiffness~~	~~T.S~~	~~DF~~
~~B~~	~~BA~~	~~$\frac{3EI}{L} = 0.25EI$~~	~~$0.375EI$~~	~~0.67~~
	~~BC~~	~~$\frac{3EI}{L} = 0.125EI$~~		~~0.33~~

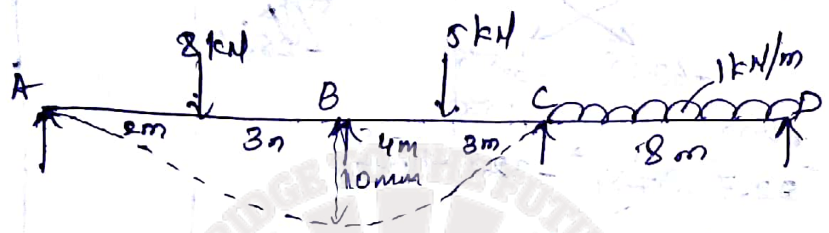
joint	member	stiffness	T.S	DF
A	AB	$\frac{3\times3EI}{L} = 0.75EI$	$0.75EI$	1
B	BA	$\frac{3\times3EI}{L} = 0.75EI$	$2EI$	0.375
	BC	$\frac{10\times3EI}{L} = 1.25EI$		0.625
C	CB	$\frac{10\times3EI}{L} = 1.25EI$	$1.75EI$	0.7
	CD	$\frac{2\times3EI}{L} = 0.5EI$		0.3
D	DE	$\frac{2\times3EI}{L} = 0.5EI$	$0.5EI$	1

	A	B		C		D		E
	1	0.375	0.625	0.7	0.3	1	0	
FEM	-36	36	-204	204	-32	16	-18	0
	36	18			1	2		
MF	0	54	-204	204	-31	18	-18	0
		-150		173				
		150		-173				
		56.25	93.75	-121.1	-51.9			
	~~28.125~~		-60.55	46.875		~~-28.95~~		
		60.55		-46.875				
		22.7	37.84	-32.8	-14.06			
			-16.4	18.92				
		+16.4		-18.42				
		6.15	10.25	-13.25	-5.67			
			-6.62	5.125				
		6.62		-5.125				
		2.48	4.14	-3.587	-1.537			
			-1.79	2.07				
		1.79		-2.07				
		0.67	1.12	-1.45	-0.14			
			-0.72	0.56				
		+0.72		-0.56				
		0.25	0.45	-0.35	-0.15			
			-0.18	0.22				
		0.18		-0.22				
		0.0675	0.1125	-0.154	-0.066			
			+0.077	0.056				
	0	142.56	-142.66	-105	105	18	-18	0



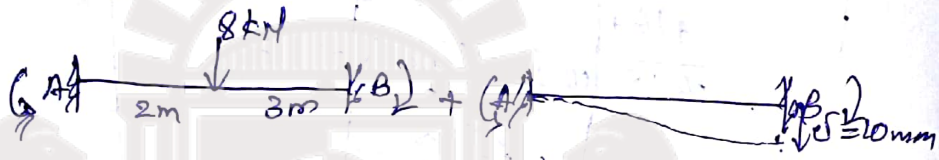
Problems on contineous beams with settlement.

1. A contineous beam ABCD 20m long is simply supported at its ends & is proped at the same level at B&C as shown in the figure. If support 'B' sinks by 10 mm, analyse the beam by moment distribution & sketch the BMD. Take $E = 2.1\times10^5\ N/mm^2$ & $I = 85\times10^6\ mm^4$.



Step-1:- FEM's

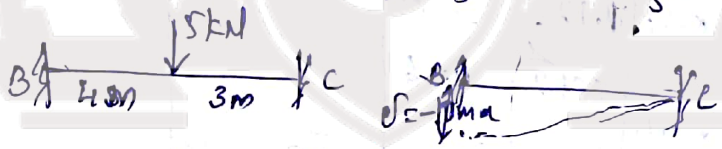
* AB



$$M_{FAB} = -\frac{Wb^2a}{L^2} - \frac{6EI\delta}{L^2} = -\frac{8\times3^2\times2}{5^2} - \frac{6\times1785\times10\times10^{-3}}{5^2} = -10.04\ kN\text{-}m$$

$$M_{FBA} = \frac{Wa^2b}{L^2} - \frac{6EI\delta}{L^2} = \frac{8\times2^2\times3}{5^2} - \frac{6\times1785\times10\times10^{-3}}{5^2} = -0.424\ kN\text{-}m$$

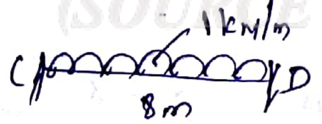
* BC



$$M_{FBC} = -\frac{5\times3^2\times4}{7^2} + \frac{6\times1785\times10\times10^{-3}}{7^2} = -1.487\ kN\text{-}m$$

$$M_{FCB} = \frac{5\times4^2\times3}{7^2} + \frac{6\times1785\times10\times10^{-3}}{7^2} = 7.084\ kN\text{-}m$$

* CD



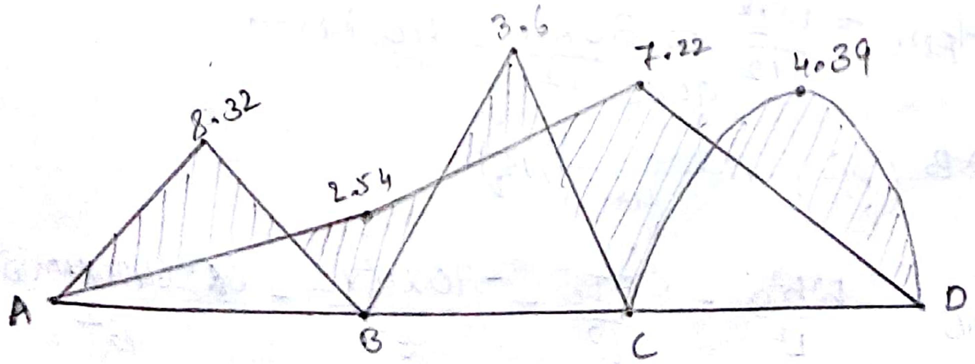
$$M_{FCD} = -\frac{WL^2}{12} = -\frac{1\times8^2}{12} = -5.33\ kN\text{-}m$$

$$M_{FDC} = +5.33\ kN\text{-}m$$

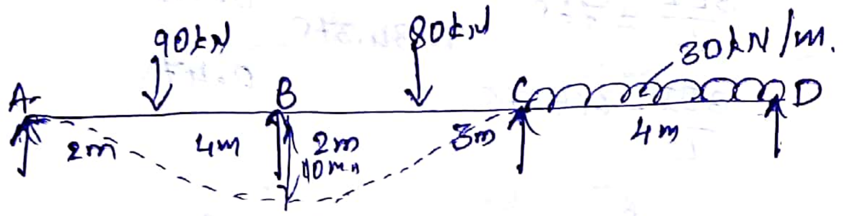
Step-2:- DF's

joint	member	stiffness	T.S	D.F
B	BA	$\frac{3EI}{L} = 1071$	1836	0.58
	BC	$\frac{3EI}{L} = 765$		0.42
C	CB	$\frac{3EI}{L} = 765$	1434.375	0.53
	CD	$\frac{3EI}{L} = 669.375$		0.47

A	B (BA)	B (BC)	C (CB)	C (CD)	D
1	0.58	0.42	0.53	0.47	1
-10.044	-0.44	-1.487	7.084	-5.33	5.33
10.044					-5.33
	5.022			-2.665	
0	4.582	-1.487	7.084	-7.995	0
	3.098		-0.911		
	-3.095		+0.911		
	-1.795	-1.29	0.483	0.428	
		0.2415	-0.65		
	-0.2415		0.65		
	-0.14	-0.101	0.344	0.305	
		0.172	-0.0505		
	-0.172		0.0505		
	-0.099	-0.072	0.0267	0.0237	
		0.0133	-0.036		
	-0.0133		0.036		
	-0.0077	-0.0056	0.019	0.017	
0	2.54	-2.5	7.2	-7.22	0



2. Draw the BMD for the beam loaded as shown in the figure when support B sinks by 10mm below the levels of A, C & D. Assume E = 200 GPa, $I = 132\times10^6\ mm^4$ for all the members. Use the MDM.

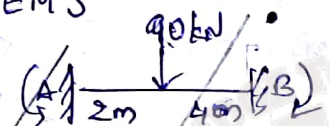


Step-1:- FEM's

$EI = 200\times10^9 \times 1.32\times10^6 \times 10^{-12}$

$\underline{EI = 26400\ kN\text{-}m^2}$

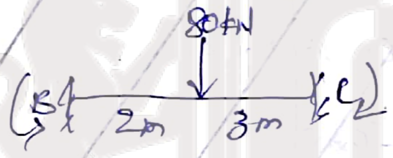
* AB



$$M_{FAB} = -\frac{Wb^2a}{L^2} = -\frac{90\times4^2\times2}{6^2} = -80\ kN\text{-}m$$

$$M_{FBA} = \frac{Wa^2b}{L^2} = \frac{90\times2^2\times4}{6^2} = 40\ kN\text{-}m$$

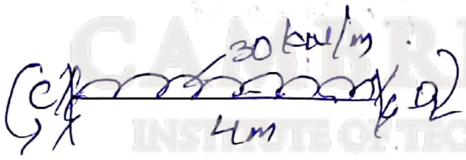
* BC



$$M_{FBC} = -\frac{Wb^2a}{L^2} = \frac{-80\times3^2\times2}{5^2} = -57.6\ kN\text{-}m$$

$$M_{FCB} = \frac{Wa^2b}{L^2} = \frac{80\times2^2\times3}{5^2} = +38.4\ kN\text{-}m$$

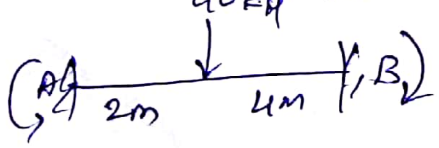
* CD



$$M_{FCD} = -\frac{WL^2}{12} = \frac{-30\times4^2}{12} = -40\ kN\text{-}m$$

$$M_{FDC} = \frac{WL^2}{12} = \frac{30\times4^2}{12} = 40\ kN\text{-}m$$

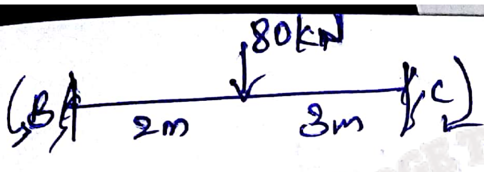
* AB



$$M_{FAB} = -\frac{Wb^2a}{L^2} - \frac{6EI\delta}{L^2} = \frac{-90\times4^2\times2}{6^2} - \frac{6\times26400\times10\times10^{-3}}{6^2} = -124\ kN\text{-}m$$

$$M_{FBA} = \frac{Wa^2b}{L^2} - \frac{6EI\delta}{L^2} = \frac{90\times2^2\times4}{6^2} - \frac{6\times26400\times10\times10^{-3}}{6^2} = -4\ kN\text{-}m$$

* BC



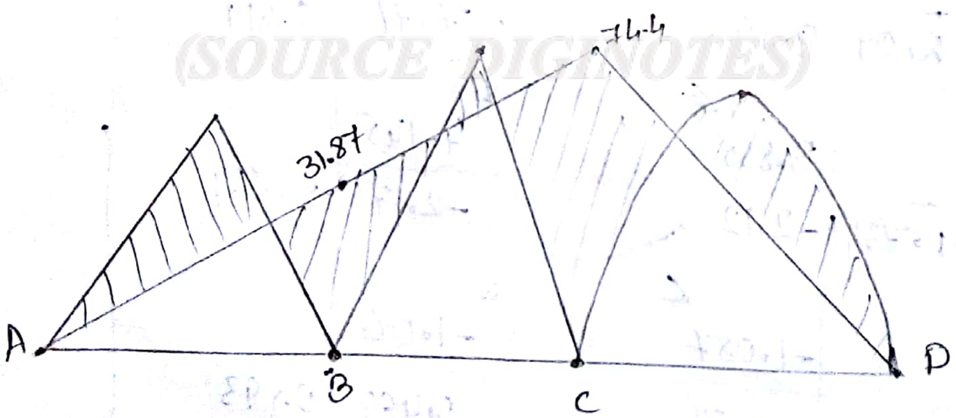
$$M_{FBC} = -\frac{Wb^2a}{L^2} + \frac{6EI\delta}{L^2} = \frac{-80\times3^2\times2}{5^2} + \frac{6\times26400\times10\times10^{-3}}{5^2} = 5.76 \text{ kN-m}$$

$$M_{FCB} = \frac{Wb^2a}{L^2} + \frac{6EI\delta}{L^2} = \frac{80\times2^2\times3}{5^2} + \frac{6\times26400\times10\times10^{-3}}{5^2} = 101.76 \text{ kN-m}$$

Step-2:- DF's

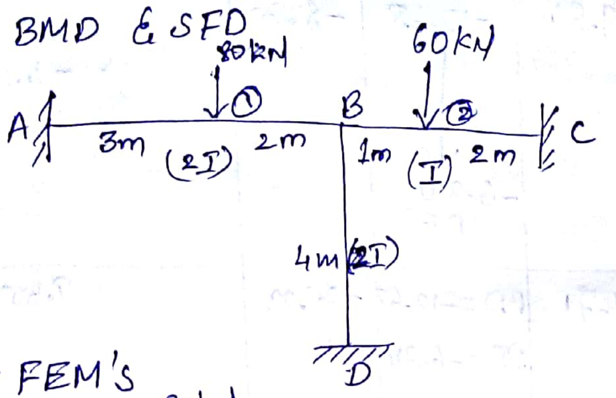
joint	member	stiffness	T.S	DF
B	BA	$\frac{3EI}{L} = 13200$	29040	0.45
	BC	$\frac{3EI}{L} = 15840$		0.55
C	CB	$\frac{3EI}{L} = 15840$	35640	0.44
	CD	$\frac{3EI}{L} = 19800$		0.56

1	0.45	0.55	0.44	0.56	1
-124	-4	8.76	101.76	-40	40
+124	62			-20	-40
0	58	5.76	101.76	-60	0
	63.76		41.76		
	-28.69	-35.07	-18.37	-23.38	
		-9.185	-17.535		
	4.133	5.05	7.71	9.82	
		+3.855	2.525		
	-1.732	-2.117	-1.111	-1.414	
		-0.555	-1.058		
	0.249	0.305	0.466	0.592	
		0.232	0.152		
	-0.104	-0.1276	-0.067	-0.085	
		-0.0335	-0.0638		
	0.015	0.018	0.028	0.035	
0	31.87	-31.86	74.43	-74.4	0



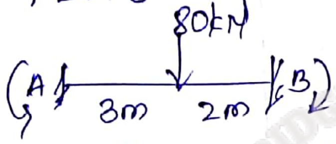
Problems on frames without sway :-

1. Analyse the structure loaded as shown in the figure Sketch BMD & SFD



Step-1 :- FEM's

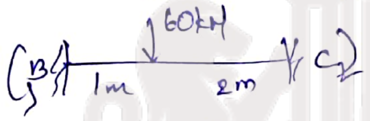
* AB



$$M_{FAB} = -\frac{Wb^2a}{L^2} = \frac{-80\times2^2\times3}{5^2} = -38.4 \text{ kN-m}$$

$$M_{FBA} = \frac{Wa^2b}{L^2} = \frac{80\times3^2\times2}{5^2} = 57.6 \text{ kN-m}$$

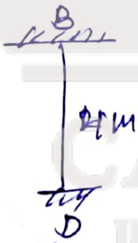
* BC



$$M_{FBC} = \frac{-Wb^2a}{L^2} = \frac{-60\times2^2\times1}{3^2} = -26.67 \text{ kN-m}$$

$$M_{FCB} = \frac{Wa^2b}{L^2} = \frac{60\times1^2\times2}{3^2} = 13.33 \text{ kN-m}$$

* BD



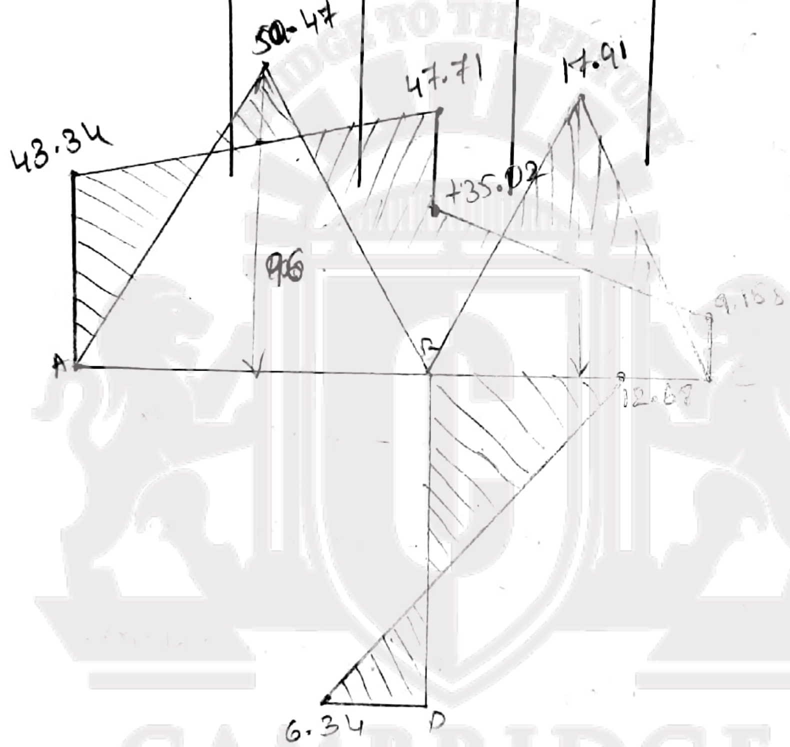
$$M_{FBD} = M_{FDB} = 0 \text{ kN-m.}$$

Step-2 :- DF's

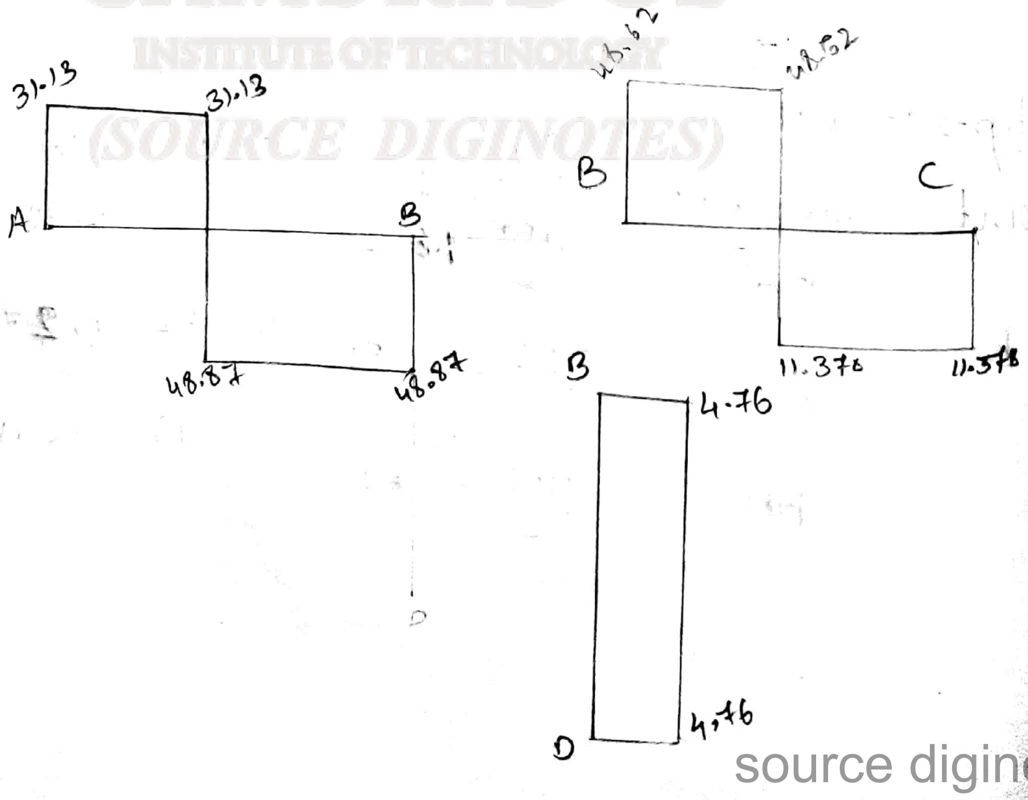
joint	member	stiffness	T.S	DF
	BA	$\frac{4EI}{L} = 1.6EI$ ($I=2I$)		0.32
B	BC	$\frac{4EI}{L} = 1.33EI$ ($I=I$)	4.93EI	0.27
	BD	$\frac{4EI}{L} = 2EI$ ($I=2I$)		0.41

A	BA	BD	BC	C
	0.82	0.41	0.27	
-38.4	87.6	0	-26.67	13.33
	-9.89	-12.68	-8.25	
-4.94		-6.34 (DB)		-4.17
-43.34	47.71	BD = -12.68 DB = -6.34	-35.02	9.158

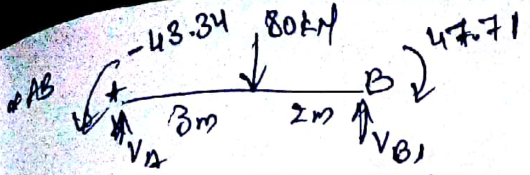
BMD



SFD



* AB



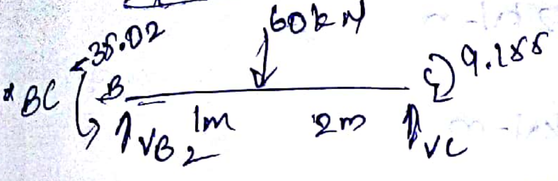
$\Sigma M_A = 0 \quad V_A + V_{B_1} = 80$

$-V_{B_1} \times 5 + 80 \times 3 - 43.34 + 47.71 = 0$

$\boxed{V_{B_1} = 48.87\,kN}$

$\boxed{V_A = 31.13\,kN}$

* BC



$V_{B_2} + V_C = 60$

$\Sigma M_B = 0$

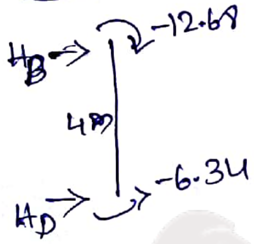
$-V_C \times 3 + 60 \times 1 - 35.02 + 9.155 = 0$

$\boxed{V_{B_2} = 48.62\,kN}$ $\quad \boxed{V_C = 11.378\,kN}$

$V_B = V_{B_1} + V_{B_2} = 48.87 + 48.62$

$\boxed{V_B = 97.5\,kN}$

* BD



$\Sigma M_B = 0, \quad H_B + H_D = 0$

$-H_D \times 4 - 12.68 - 6.36 = 0$

$\boxed{H_D = -4.76\,kN}$ $\quad \boxed{H_B = 4.76\,kN}$

SF@ A_R = 31.13 kN = SF@ 1_L ~~SF@ 1_L = 31~~

SF@ B_L = 31.13 − 80 = −48.87 kN = SF@ 1_R

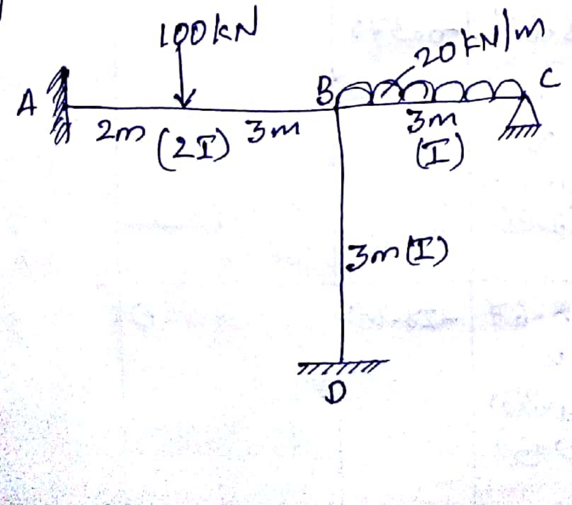
SF@ B_R = −11.378 + 60 = 48.62 kN = SF@ 2_L

SF@ C_L = −11.378 kN = SF@ 2_R

SF@ D_u = −4.76 kN

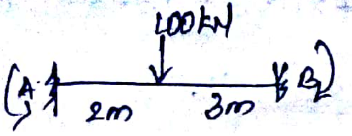
SF@ B_D = −4.76 kN

2. A contineous beam ABC is supported on an elastic column and is loaded as shown in figure treating joint B as rigid analyse the frame & sketch the BMD. Also sketch the deflected shape of the structure.



Step-1:- FEM's

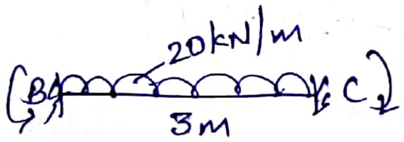
* AB



$$M_{FAB} = \frac{-Wb^2a}{L^2} = \frac{-100\times3^2\times2}{5^2} = -72\ kN\text{-}m$$

$$M_{FBA} = \frac{Wa^2b}{L^2} = \frac{100\times2^2\times3}{5^2} = 48\ kN\text{-}m$$

* BC



$$M_{FBC} = \frac{-Wl^2}{12} = \frac{-20\times3^2}{12} = -15\ kN\text{-}m$$

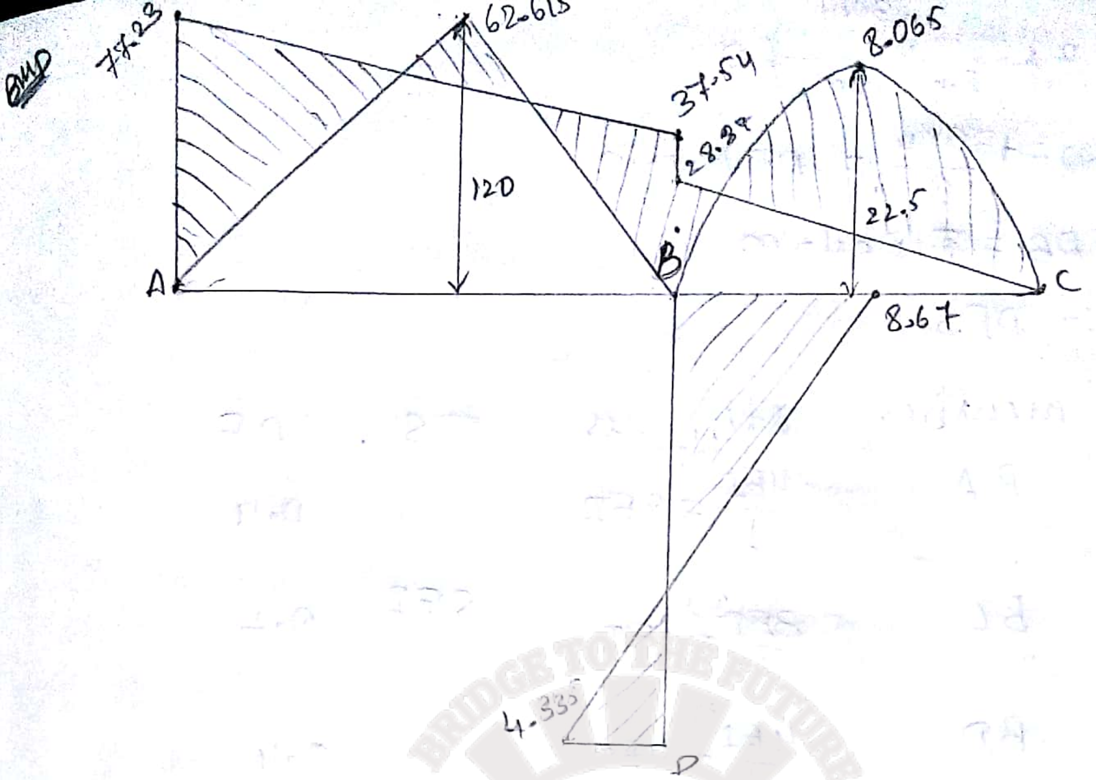
$$M_{FCB} = 15\ kN\text{-}m$$

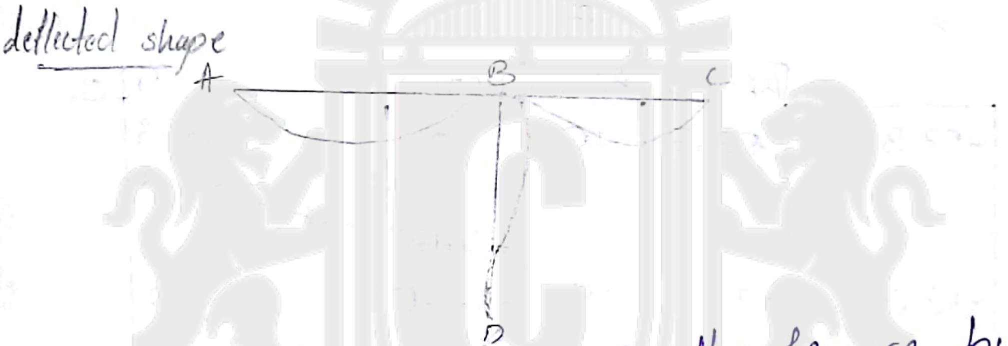
* $M_{FBD} = M_{FDB} = 0\ kN\text{-}m$

Step-2:- DF's

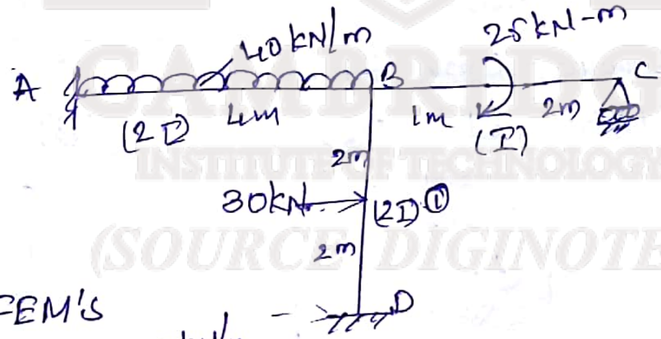
joint	member	stiffness	TS	DF
B	BA	$\frac{4EI}{L} = 1.6EI$		0.41
	BC	$\frac{3EI}{L} = 1EI$	3.93EI	0.25
	BD	$\frac{4EI}{L} = 1.33EI$		0.34

A	B: BA 0.41	B: BD 0.34	B: BC 0.25	C 1
-72	48	0	-15 -7.5	15 -15
-72 -5.237	48 -10.455	0 -8.67 -4.335 (DB)	-22.5 -6.375	0
-77.23	37.54	-8.67 (BD) -4.335 (DB)	-28.875	0



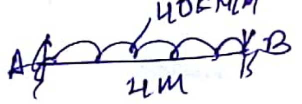


3. Analyse the frame shown in the figure by moment distribution method. Sketch the BMD & SFD



Step-1:- FEM's

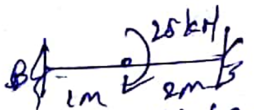
* AB



$$M_{FAB} = -\frac{wl^2}{12} = \frac{-40\times4^2}{12} = -53.3 \text{ kN-m}$$

$$M_{FBA} = +53.3 \text{ kN-m}$$

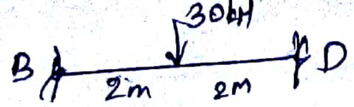
* BC



$$M_{FBC} = -\frac{Mb(2a-b)}{L^2} = \frac{-25\times2(2\times1-2)}{3^2} = 0 \text{ kN-m}$$

$$M_{FCB} = \frac{Ma(2b-a)}{L^2} = \frac{25\times1(2\times2-1)}{3^2} = 8.33 \text{ kN-m}$$

* BD



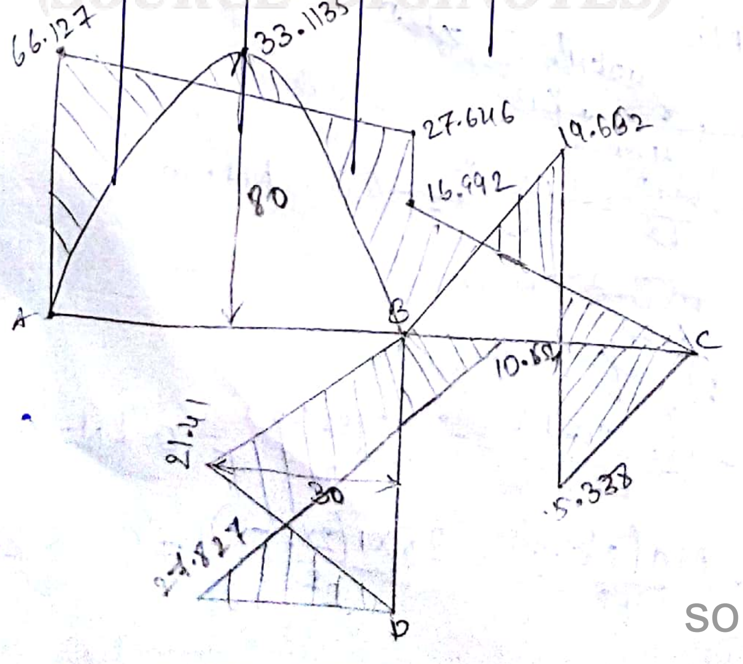
$M_{FBD} = +\frac{30 \times 4}{8} = +15\ kN\text{-}m$

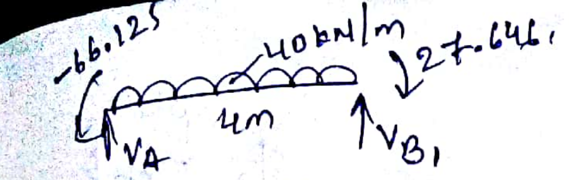
$M_{FDB} = -15\ kN\text{-}m.$

Step-2:- DF's

joint	member	stiffness	TS	DF
B	BA	$\frac{4E(2I)}{L} = 2EI$		0.4
	BC	$\frac{3E(I)}{L} = EI$	5EI	0.2
	BD	$\frac{4E(2I)}{L} = 2EI$		0.4

A	B: BA=0.4	BD=0.4	BC=0.2	C: 1
-53.3	53.3	15	0	8.33
				-8.33
			-4.165	
-53.3	53.3	15	-4.165	0
	-25.654	-25.654	-12.827	
-12.827		-12.827		
-66.127 (AB)	27.646 (BA)	-10.654 (BD)	-16.992 (BC)	0 (CB)
		-27.827 (DB)		

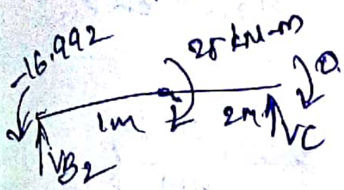




$V_A + V_{B1} = 40 \times 4 = 160\,kN$

$-V_{B1} \times 4 + 40 \times 4 \times 2 - 66.125 + 27.646 = 0$

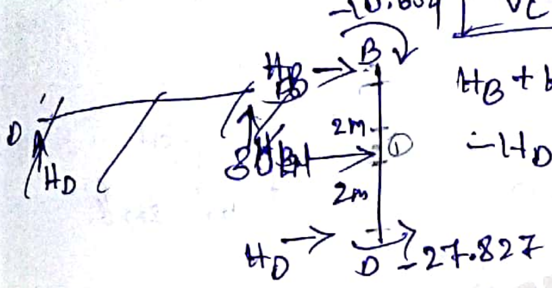
$\boxed{V_{B1} = 70.38\,kN}$ $\boxed{V_A = 89.62\,kN}$



$V_{B2} + V_C = 0$

$-V_C \times 3 + 25 - 16.992 = 0$

$\boxed{V_C = 2.669\,kN}$ $\boxed{V_{B2} = -2.669\,kN}$



$H_B + H_D = -30\,kN$

$-H_D \times 4 - 30 \times 2 - 10.684 - 27.827 = 0$

$\boxed{H_D = 24.6\,kN}$ $\boxed{H_B = 5.379\,kN}$

$\boxed{V_B = +67.7\,kN}$

BM @ $BC_R = -2.669 \times 2 = -5.338$ kN-m

BM @ $BC_L = 25 - 5.338 = 19.662$ kN-m

~~BM @ BD = +24.6×2 − 27.827~~

SF @ $A_R = 89.62$ kN

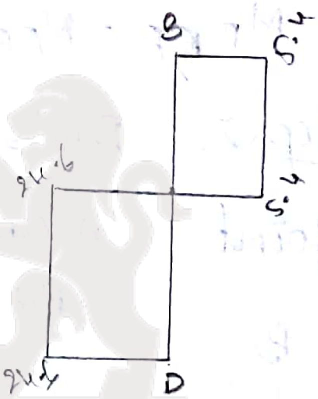
SF @ $B_L = 89.62 - 40 \times 4 = -70.38$ kN

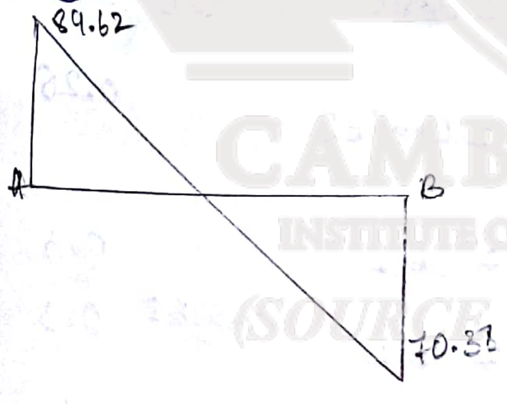
SF @ $C_L = -2.669$ kN

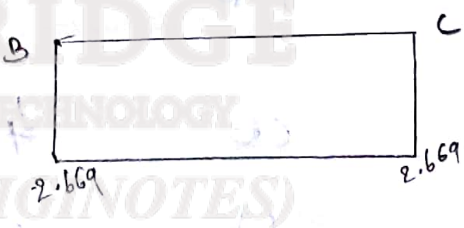
SF @ $B_R = -2.669$ kN

SF @ $D_U = -24.6$ kN $= 1_D$

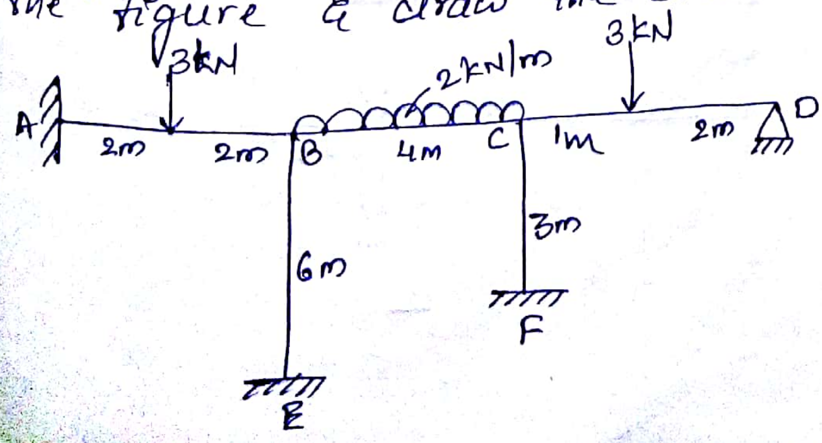
SF @ $1_U = B_D = +24.6 - 30 = -5.4$ kN





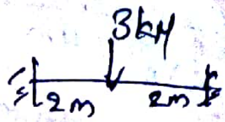


4. Analyse the structure loaded as shown in the figure & draw the BMD



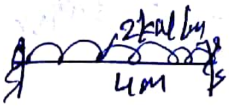
Step-1 :- FEM's

$$M_{FAB} = -\frac{3 \times 4}{8} = -1.5 \text{ kN-m}$$



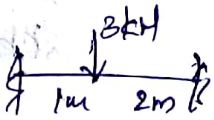
$$M_{FBA} = 1.5 \text{ kN-m}$$

$$M_{FBC} = -\frac{2 \times 4^2}{12} = -2.67 \text{ kN-m}$$



$$M_{FCB} = 2.67 \text{ kN-m}$$

$$M_{FCD} = -\frac{3 \times 1 \times 2^2}{3^2} = -1.33 \text{ kN-m}$$



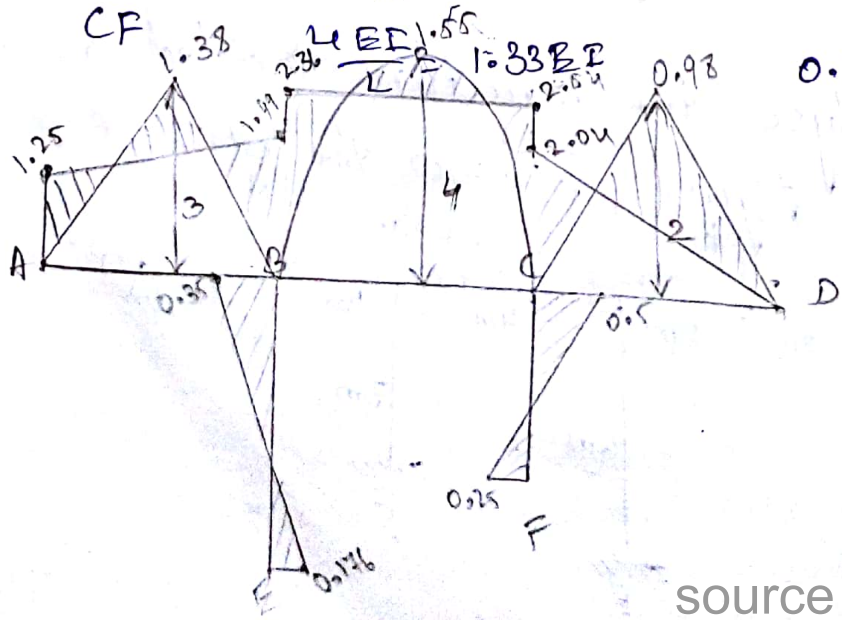
$$M_{FDC} = \frac{3 \times 2 \times 1^2}{3^2} = 0.67 \text{ kN-m}$$

$$M_{FBE} = M_{FEB} = 0$$

$$M_{FCF} = M_{FFC} = 0.$$

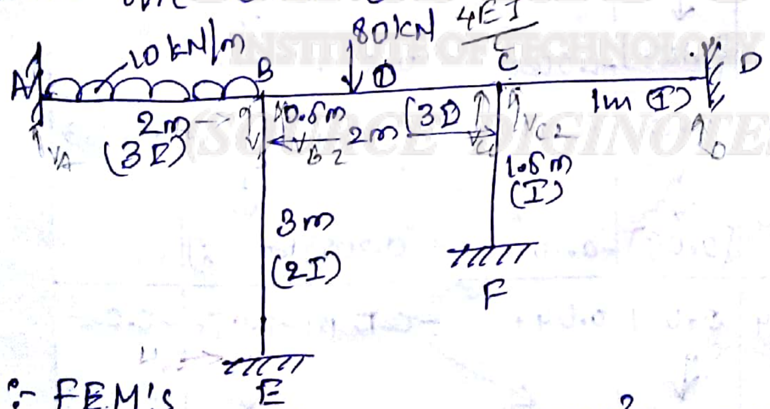
Step-2: DF's

joint	member	stiffness	TS	D.F
B	BA	$\frac{4EI}{L} = EI$		0.37
	BC	$\frac{4EI}{L} = EI$	2.67EI	0.37
	BE	$\frac{4EI}{L} = 0.67EI$		0.25
C	CB	$\frac{4EI}{L} = EI$		0.3
	CD	$\frac{3EI}{L} = EI$	3.33EI	0.3
	CF	$\frac{4EI}{L} = 1.33EI$		0.4



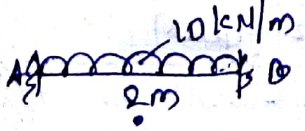
A	BA	BE	BC	CB	CF	CD	D
	0.37	0.26	0.37	0.3	0.4	0.3	1
-1.5	1.5	0	-2.67	2.67	0	~~-1.33~~	+0.67
						-0.335	~~+0.67~~
-1.5	1.5	0	-2.67	2.67	0	-1.665	0
	0.293	0.304	0.43	-0.3016	-0.402	-0.3015	
0.215		(0.152)	-0.151	0.215	(-0.201)		
	0.055	0.039	0.055	-0.064	-0.086	-0.064	
0.028		(0.02)	-0.032	0.028	(-0.043)		
	0.012	0.008	0.012	-0.008	-0.011	-0.008	
0.006		(0.004)	-0.004	0.006	(-0.0055)		
-1.25 (AB)	1.997 (BA)	0.351 (BE)	-2.38 (BC)	+2.54 (CB)	-0.5 (CF)	-2.04 (CD)	0 (DC)
		0.176 (EB)			-0.25 (FC)		

5. Analyse the continuous beam supported by columns & loaded as shown in the figure. Sketch the BMD & SFD



Step-1:- FEM's

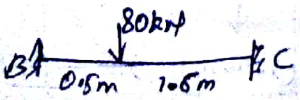
* AB



$M_{FAB} = \frac{-10 \times 2^2}{12} = -3.3$ kN-m

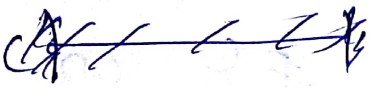
$M_{FBA} = 3.3$ kN-m

* BC



$M_{FBC} = \frac{-80(1.5^2) \times 0.5}{2^2} = -22.5$ kN-m

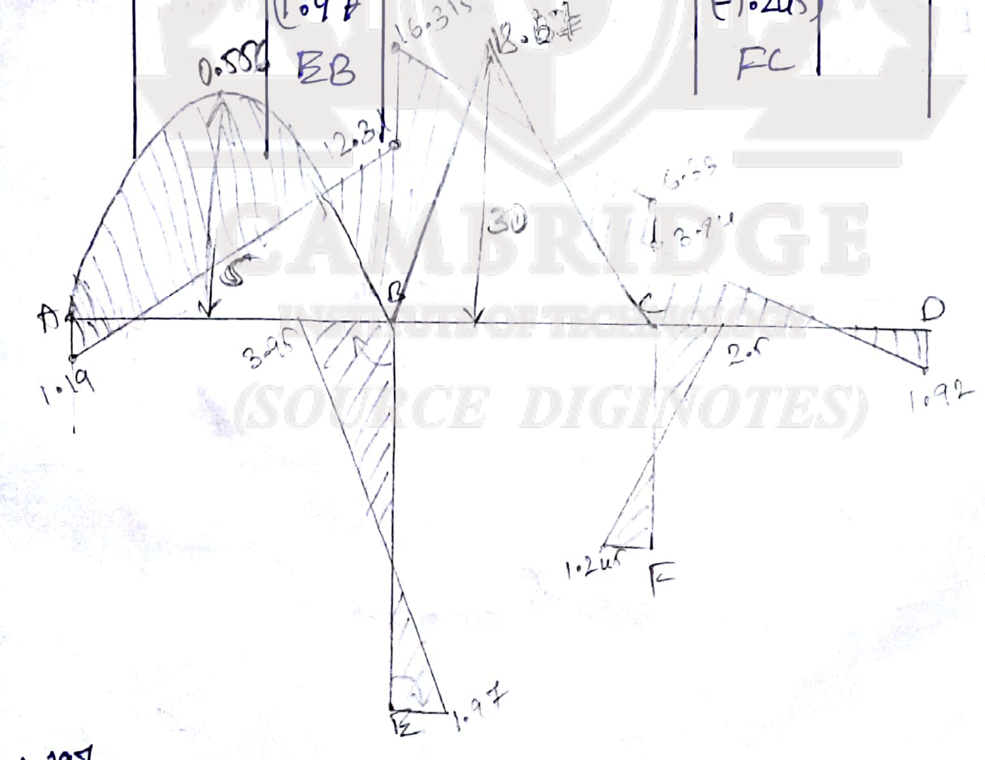
$M_{FCB} = \frac{80 \times 0.5^2 \times 1.5}{2^2} = 7.5$ kN-m

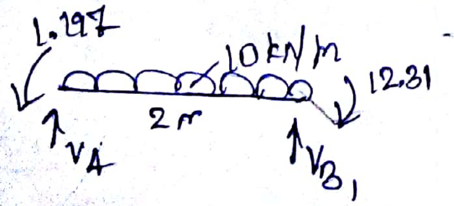
CD  $M_{FCD} = M_{FDC} = M_{FBE} = M_{FEB} = M_{FCF} = M_{FFC}$

Step-2:- DF's

joint	member	Stiffness	T.S	D.F
B	BA	(3I) $\frac{4EI}{L} = 6EI$		0.41
	BC	(3I) $\frac{4EI}{L} = 6EI$	14.67EI	0.41
	BE	(2I) $\frac{4EI}{L} = 2.67EI$		0.18
C	CB	(3I) $\frac{4EI}{L} = 6EI$		~~0.31~~ 0.47
	CD	(I) $\frac{4EI}{L} = 4EI$	12.67EI	~~0.26~~ 0.32
	CF	(I) $\frac{4EI}{L} = 2.67EI$		~~0.23~~ 0.21

A	B			C			D
	BA	BE	BC	CB	CF	CD	
	0.41	0.18	0.41	0.47	0.21	0.32	
-3.3	3.3	0	-22.5	7.5	0	0	0
	7.87	3.45	7.87	-3.52	-1.57	-2.4	
3.93		[1.72]	-1.76	3.93	[-0.78]		-1.2
	0.72	0.32	0.72	-1.86	-0.82	-1.26	
0.36		[0.16]	-0.93	0.36	[-0.41]		-0.63
	0.38	0.17	0.38	-0.17	-0.07	-0.12	
0.19		[0.085]	-0.085	0.19	[-0.035]		-0.06
	0.035	0.016	0.0035	-0.089	-0.04	-0.061	
0.00175		[0.0075]	-0.048	0.00175	[-0.02]		-0.031
1.197	12.31	3.965	-16.315	6.38	-2.5	-3.84	-1.92
AB	BA	BE	BC	CB	CF	CD	DC
		(1.97) EB			(-1.245) FC		



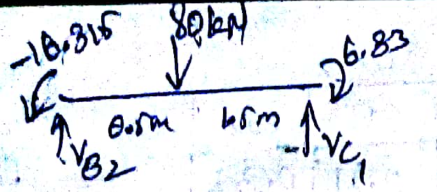


$V_A + V_{B1} = 20$ kN

$-V_{B1} \times 2 + 10 \times 2 \times 1 + 1.197 + 12.31 = 0$

$\boxed{V_{B1} = 16.75 \text{ kN}}$

$\boxed{V_A = +3.25 \text{ kN}}$



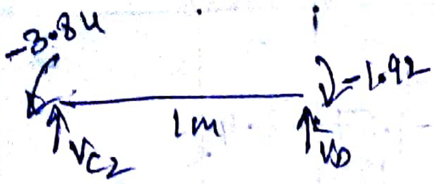
$V_{B_2}+V_{C_1}=80$

$-V_{C_1}\times 2+80\times 0.5-16.816+6.83=0$

$\boxed{V_{C_1}=15\text{ kN}}$

$\boxed{V_{B_2}=65\text{ kN}}$

$\boxed{V_B=81.75\text{ kN}}$



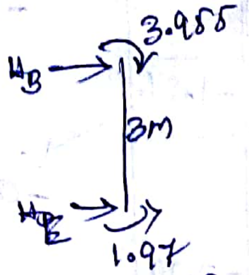
$V_{C_2}+V_D=0$

$-V_D\times 1-3.84-1.92=0$

$\boxed{V_D=-5.76\text{ kN}}$

$\boxed{V_{C_2}=5.76\text{ kN}}$

$\boxed{V_C=20.76\text{ kN}}$

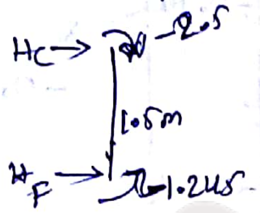


$H_B+H_E=0$

$-H_E\times 3+3.955+1.97=0$

$\boxed{H_E=1.975\text{ kN}}$

$\boxed{H_B=-1.975\text{ kN}}$



$H_C+H_F=0$

$-H_F\times 1.5-2.5-1.245=0$

$\boxed{H_F=2.5\text{ kN}}$

$\boxed{H_C=2.5\text{ kN}}$

SF@A_R = 3.25 kN

SF@B_L = 3.25 − 20 = −16.75 kN

SF@B_R = −16.76 + 81.75 = 65 kN = 1_L

SF@1_R = 65 − 80 = −15 kN = C_L

SF@C_R = −15 + 20.76 = 5.76 kN

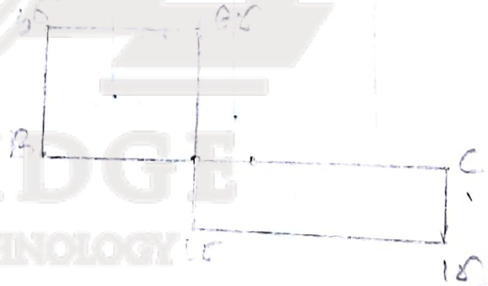
SF@D_L = 5.76 kN

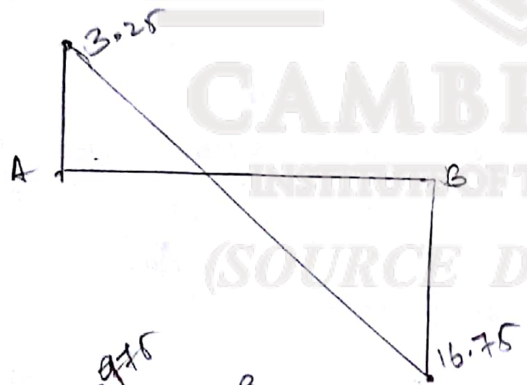
SF@E_U = −1.975 kN

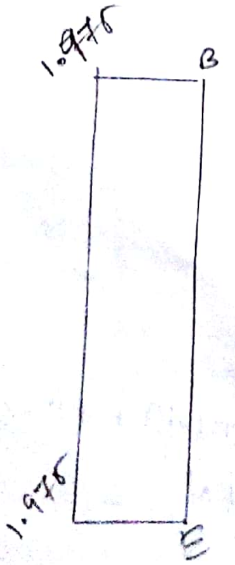
SF@B_D = −1.975 kN

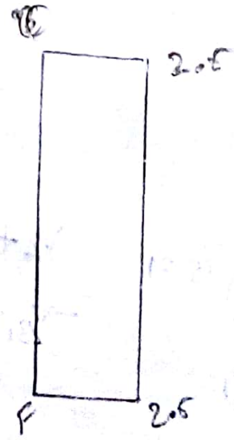
SF@F_U = 2.5 kN

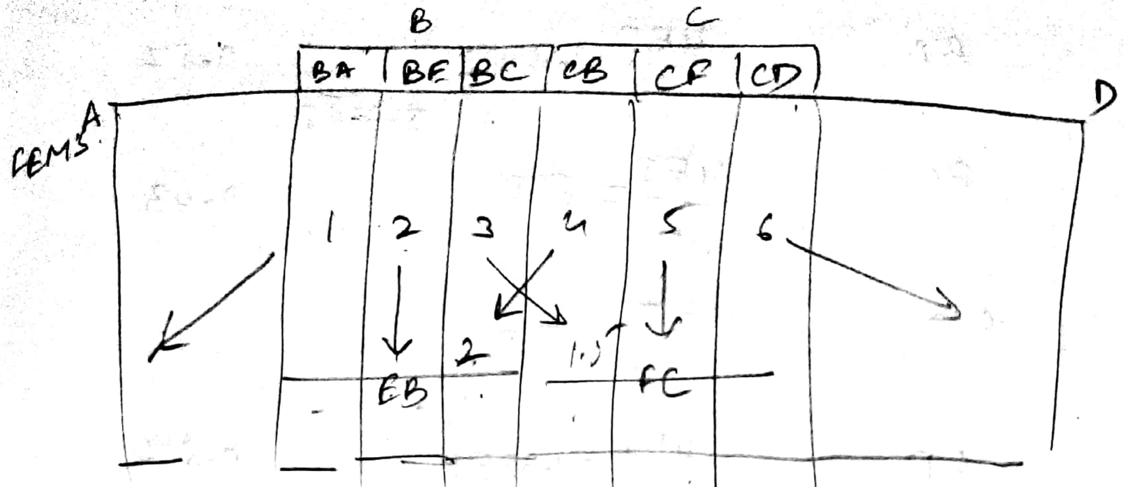
SF@C_D = 2.5 kN



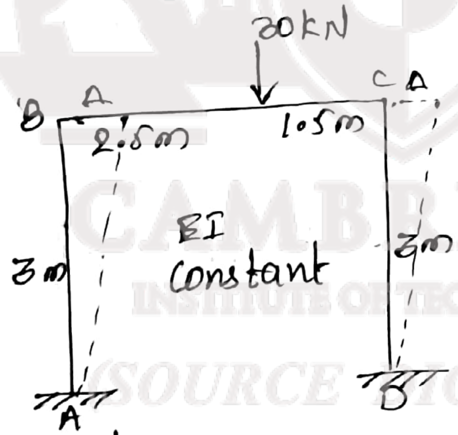








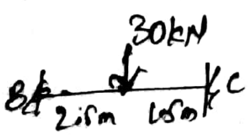
1. Analyse a portal frame as shown in the figure. Draw SFD & BMD.



Step-1 :- FEM's

* $M_{FAB} = M_{FBA} = 0$

* BC



$$M_{FBC} = -\frac{30 \times 1.5^2 \times 2.5}{4^2} = -10.55 \text{ kN-m}$$

$$M_{FCB} = +\frac{30 \times 2.5^2 \times 1.5}{4^2} = 17.57 \text{ kN-m}$$

* $M_{FCD} = M_{FDC} = 0.$

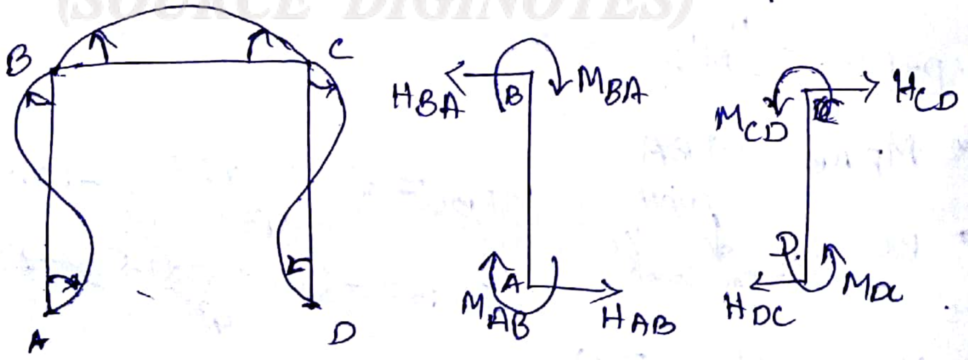
Step-2 :- DF's.

joint	member	stiffness	T.S	DF
B	BA	$\frac{4EI}{L} = 1.33EI$	2.33EI	0.57
	BC	$\frac{4EI}{L} = EI$		0.43
C	CB	$\frac{4EI}{L} = EI$		0.43
	CD	$\frac{4EI}{L} = 1.33EI$		0.57

Step-3 :- Non-sway moments.

A	B		C		D
	0.57	0.43	0.43	0.57	
0	0	-10.54	17.57	0	0
	6.01	4.53	-7.55	-10.01	
3.005		-3.775	2.265		-5.005
	2.15	1.62	-0.97	-1.29	
1.075		-0.485	0.81		-0.645
	0.276	0.21	-0.35	-0.46	
0.138		-0.175	0.105		-0.23
4.2	8.43	-8.6	11.88	-11.76	-5.88

Step-4:- Shear forces



* Span AB

$\Sigma H = 0, \; H_{BA} = H_{AB}$

$\Sigma M_B = 0, \Rightarrow (-H_{AB} \times 3) + M_{AB} + M_{BA} = 0$

$\boxed{H_{AB} = 4.28 kN}$ $\boxed{H_{BA} = 4.28 kN}$

• Span CD

$\Sigma H = 0$, $H_{CD} - H_{DC} = 0$.

$\Sigma M_C = 0$

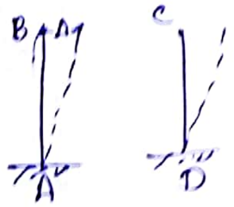
$\Rightarrow (H_{DC} \times 3) + M_{DC} + M_{CD} = 0$

$\boxed{H_{DC} = -5.88\ kN}$ $\boxed{H_{CD} = -5.88\ kN}$

difference in sway $= \underline{4.23 - 5.88}$

$\Rightarrow 1.65$

Step-5 :- joint ratios



$\dfrac{M_{AB}}{M_{DC}} = \dfrac{-6EI\Delta/L^2}{-6EI\Delta}$

$\dfrac{M_{AB}}{M_{DC}} = \dfrac{-1}{-1}$

$-1 : -1$

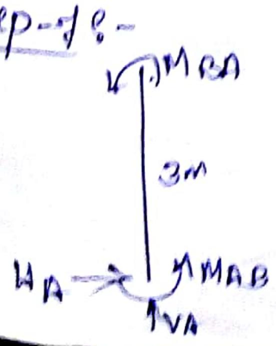
Consider sway moments $M_{AB} = M_{BA} = -100\ kN\text{-}m$

$M_{CD} = M_{DC} = -100\ kN\text{-}m$

Step-6 :- Sway moments

A	B		C		D
	0.57	0.43	0.43	0.57	
-100	-100	0	0	-100	-100
	57	43	43	57	
28.5		21.5	21.5		28.5
	-12.25	-9.24	-9.24	-12.25	
-6.13		-4.62	-4.62		-6.13
	2.63	1.98	1.98	2.63	
1.315		0.99	0.99		1.315
	-0.56	-0.42	-0.42	-0.56	
-0.28		-0.21	-0.21		-0.28
-76.59	-53.1	53.0	53	-53.1	-76.59

Step-7 :-



$\Sigma M_B = 0$

$(-H_A \times 3) - 76.5 - 53.06 = 0$

$\boxed{H_A = -43.2\ kN}$

$\therefore \boxed{H_D = 43.2\ kN}$ diff $= \dfrac{86.272}{100} = 0.863$

Step-8 :- Correction factor

$CF_{(k)} = \frac{\text{difference in SF due to non sway}}{\text{difference in 'H' reactions due to sway}}$

$= \frac{1.6}{0.863} = 1.85$

Step-9 :- Final moments.

F.M = Moment due to non-sway + k (sway moment)

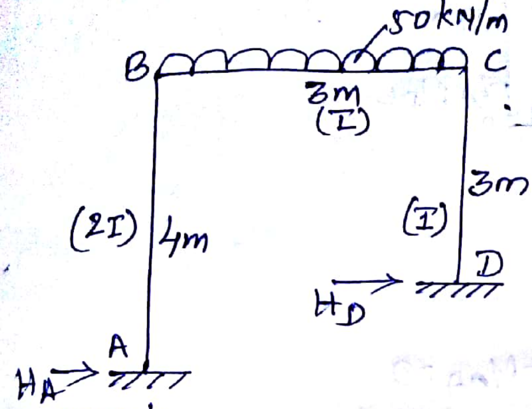
$M_{AB} = 4.23 + [(1.85) \times (-0.765)]$

$\boxed{M_{AB} = 2.81\ kN\text{-}m}$

$M_{BA} = 8.4 + [1.85 \times -0.53]$

$\boxed{M_{BA} = 7.42\ kN\text{-}m}$

2. Analyse the portal frame as shown in the figure & sketch the BMD & SFD.



Step-1:- FEM's

$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0$

$M_{FBC} = -\frac{wl^2}{12} = -37.5$ kN-m

$M_{FCB} = 37.5$ kN-m

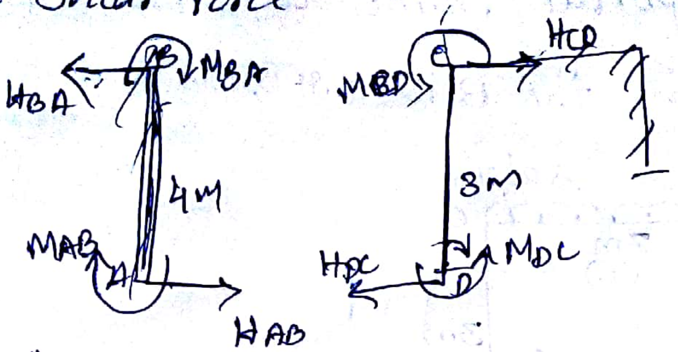
Step-2:- DF's

Joint	member	stiffness	T.S	DF
B	BA	$\frac{4E(2I)}{4} = 2EI$	3.33EI	0.6
	BC	$\frac{4EI}{3} = 1.33EI$		0.4
C	CB	$\frac{4EI}{3} = 1.33EI$	2.66EI	0.5
	CD	$\frac{4EI}{3} = 1.33EI$		0.5

Step-3:- Non sway moments

A	B		C		D
	0.6	0.4	0.5	0.5	
0	0	-37.5	37.5	0	0
	22.5	15	-18.75	-18.75	
11.25		-9.375	7.5		-9.375
	5.625	3.75	-3.75	-3.75	
2.81		-1.875	1.875		-1.875
	1.125	0.75	-0.94	-0.94	
0.56		-0.470	0.375		-0.47
	0.282	0.188	-0.188	-0.188	
0.141		-0.094	0.094		-0.094
14.76	29.5	-29.6	23.7	-23.63	-11.8

Step-4:- Shear force



* Span AB

$\Sigma H = 0$, $H_{AB} = H_{BA}$

$\Sigma M_B = 0$, $(-H_{AB} \times 4) + M_{AB} + M_{BA} = 0$

$$\boxed{H_{AB} = 11.07\ kN = H_{BA}}$$

* Span CD

$\Sigma H = 0$, $H_{DC} = H_{CD}$

$\Sigma M_C = 0$ $(H_{DC} \times 8) + M_{DC} + M_{CD} = 0$

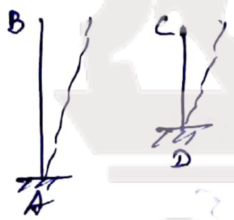
$$\boxed{H_{DC} = H_{CD} = 11.8\ kN}$$

difference in sway $= 11.8 - 11.07$

$= 0.73\ kN$

$\frac{a}{b} = \frac{c}{d}$ $\frac{a \times d}{c \times b}$

Step-5:- joint ratios



$$\frac{M_{AB}}{M_{DC}} = \frac{-6EI\Delta/L^2}{-6EI\Delta/L^2} = \frac{-6 \times E \times (2I)\Delta/4^2}{-6 \times E \times I \times \Delta/3^2}$$

$$= \frac{-0.75}{-0.67} = \frac{-9}{-8}$$

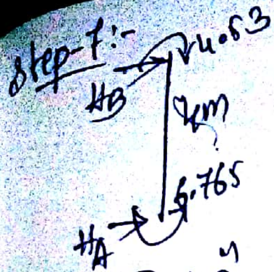
~~0.75~~ : $-9 : -8$

Consider fixed sway moments $M_{AB}'' = M_{BA}'' = 9\ kN\text{-}m$

Step-6:- Sway moments $M_{CD}'' = M_{DC}'' = 8\ kN\text{-}m$

A	B (0.6)	B (0.4)	C (0.5)	C (0.5)	D
9	9	0	0	8	8
	-5.4	-3.6	-4	-4	
-2.7		-2	-1.8		-2
	1.2	0.8	0.9	0.9	
0.6		0.45	0.4		0.45
	-0.27	-0.18	-0.2	-0.2	
-0.135		-0.1	-0.09		-0.1
6.765	4.53	-4.6	-4.8	4.7	6.35

Step-7:-

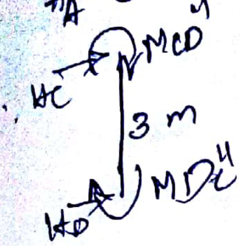


$\Sigma M_B = 0$

$-H_A \times 4 + 6.765 + 4.53 = 0$

$\boxed{H_A = 2.82\,kN}$ $\boxed{H_B = -2.82\,kN}$

$\boxed{\cancel{H_D = -2.82\,kN}}$



$\Sigma H = 0, H_C = -H_D$

$\Sigma M_C = 0$

$(-H_D \times 3) + 6.38 + 4.7 = 0$

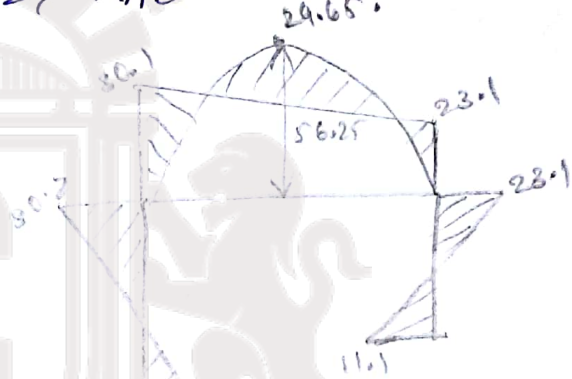
$\boxed{H_D = 3.69\,kN}$ $\boxed{H_C = -3.69\,kN}$

: difference = 6.525

Step-8:- Correction factor

$C.F = k = \frac{0.73}{6.525} = 0.1119$

Step-9:- final moments $\Rightarrow M_{AB} = M_{AB}' + k \times M_{AB}''$



$M_{AB} = 15.51\,kN\text{-}m$

$M_{BA} = 30.2\,kN\text{-}m$

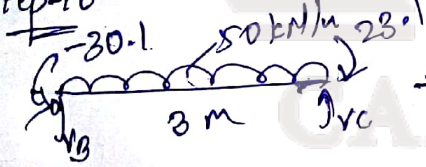
$M_{BC} = -30.1\,kN\text{-}m$

$M_{CB} = 23.10\,kN\text{-}m$

$M_{CD} = -23.1\,kN\text{-}m$

$M_{DC} = -11.1\,kN\text{-}m$

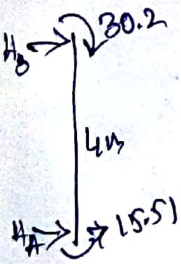
Step-10



$V_B + V_C = 180$

$-V_C \times 3 - 30.1 + 23.1 + 50 \times 3 \times 1.5 = 0$

$\boxed{V_C = 72.67\,kN}$ $\boxed{V_B = 77.33\,kN}$

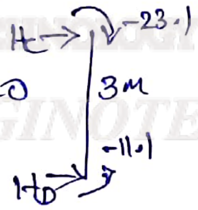


$\Rightarrow H_A + H_B = 0$

$-H_A \times 4 + 15.51 + 30.2 = 0$

$\boxed{H_A = 11.4\,kN}$

$\boxed{H_B = -11.4\,kN}$



$H_C + H_D = 0$

$-H_D \times 3 - 23.1 - 11.1 = 0$

$\boxed{H_D = -11.4\,kN}$

$\boxed{H_C = 11.4\,kN}$