MODULE - 4: [13/11/17]

Flexibility Matrix method

Flexibility is the deformation at a point due to a unit load at the same (or) a different point.

WKT $\delta = \frac{PL}{AE}$ for an axial member

$$[a][F] = (\Delta - \Delta_L)$$

where $[a]$: Flexibility matrix of size $(n \times n)$

$[F]$: Redundant force vector of size $(n \times 1)$

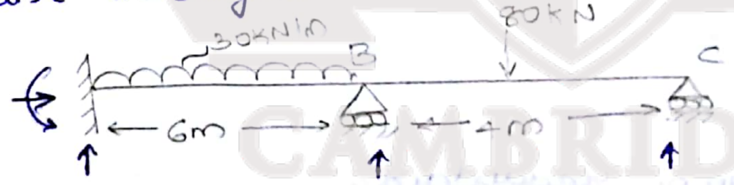
Δ = Final displacement vector of size $(n \times 1)$

where $\Delta_i = \Delta_{iL} + \delta_{i1} \cdot F_{01} + \delta_{i2} F_2 + \delta_{i3} F_3 + \ldots\ldots \delta_{in} F_n$

(OR) $\Delta_i = \Delta_{iL} + a_{i1} F_1 + a_{i2} F_2 + a_{i3} F_3 + \ldots\ldots a_{in} F_n$

Δ_L = Displacement vector of size $(n \times 1)$ due to loads.

① Analyze the continuous Beam as shown in the fig by flexibility matrix method. Flexural rigidity is constant throughout.



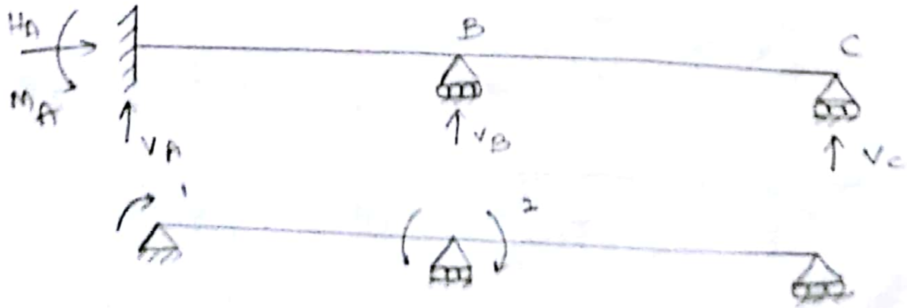
Step ①:- Degree of static Indeterminacy

No of unknown reactions = 5

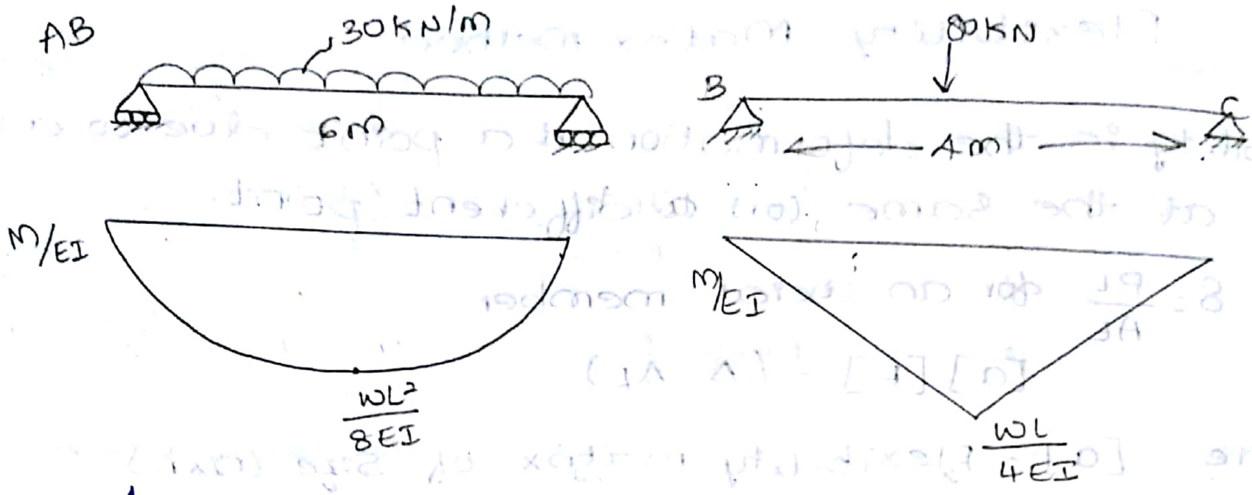
Equation of Equilibrium = 3

Degree of static Indeterminacy = 5 - 3 = 2.

Step ②:- Identify Reduntant forces.



Step ③:- Deflection due to given loading



$$\Delta_{1L} = \text{Area b/w } A \,\&\, B = \frac{2}{3} \times 6 \times \frac{wL^2}{8EI} \times \frac{1}{2} = \frac{270}{EI}$$

$$\Delta_{2L} = \text{Area b/w } B \,\&\, C = \left[\frac{1}{2} \times 4 \times \frac{80}{EI} \times \frac{1}{2}\right] + \left[\frac{270}{EI}\right] = \frac{350}{EI}$$

Step ④: Displacement [Final]

$$\Delta_1 = \Delta_{1L} + a_{11} F_1 + a_{12} F_2 = 0$$ — It is considered as zero becoz no settlement takes place

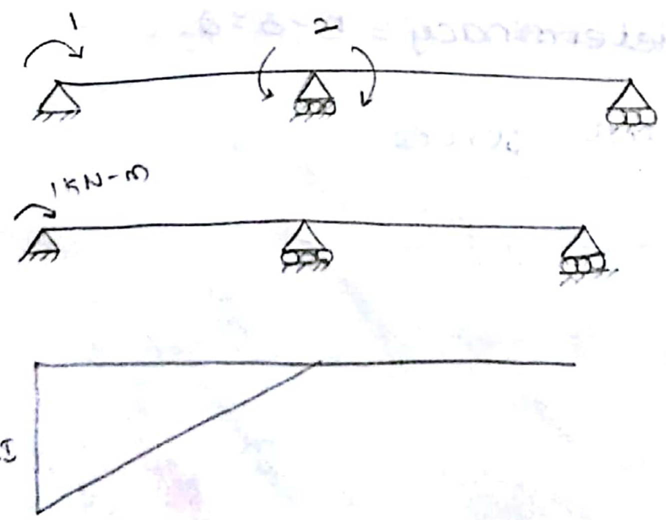
$$\Delta_2 = \Delta_{2L} + a_{21} F_1 + a_{22} F_2 = 0$$

Step ⑤ Flexibility matrix

$$[a] = \begin{array}{c} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \\ \begin{matrix} ① & ② \end{matrix} \end{array}$$

Size of matrix $\longrightarrow$ No of Redundants.

Step ⑤a Unit moment at ①



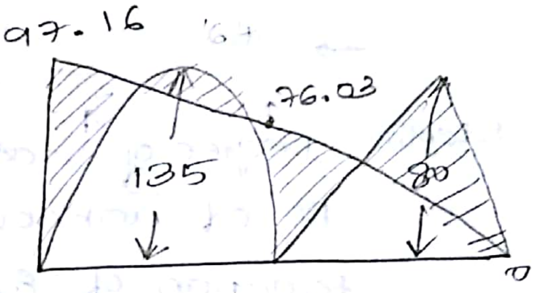
a_{11} = Reaction at A about B

$$= \left[\frac{1}{2}\times 6\times\frac{1}{EI}\right]\frac{2}{3} = \frac{2}{EI}$$

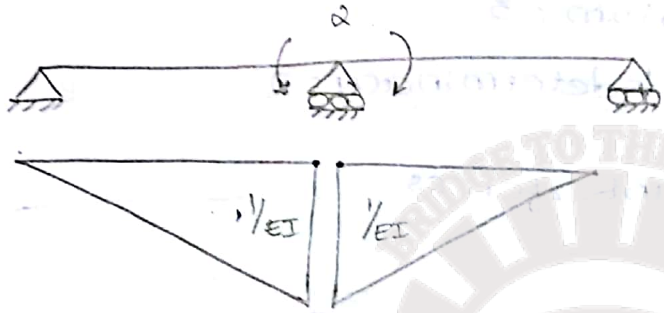
a_{21} = Reaction at B about A

$$= \left[\frac{1}{2}\times 6\times\frac{1}{EI}\right]\frac{1}{3} = \frac{1}{EI}$$

Step 7 BMD



step 5b Unit moment at ②



$$a_{12} = \left(\frac{1}{2}\times 6\times\frac{1}{EI}\right)\times\left(\frac{1}{3}\right) = \frac{1}{EI}$$

$$a_{22} = \left[\frac{1}{2}\times 6\times\frac{1}{EI}\times\frac{2}{3}\right]+\left[\frac{1}{2}\times 4\times\frac{1}{EI}\right]\frac{2}{3} = \frac{10}{3EI} = 3.33EI$$

$$[a] = \frac{1}{EI}\begin{bmatrix} 2 & 1 \\ 1 & 3.33 \end{bmatrix}$$

$$[a] = [a]^T$$

Step 6: To find unknown Redundants

$$[a][F] = [\Delta - \Delta_L]$$

$$\frac{1}{EI}\begin{bmatrix} 2 & 1 \\ 1 & 3.33 \end{bmatrix}\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{Bmatrix} 0-\Delta_{1L} \\ 0-\Delta_{2L} \end{Bmatrix}$$

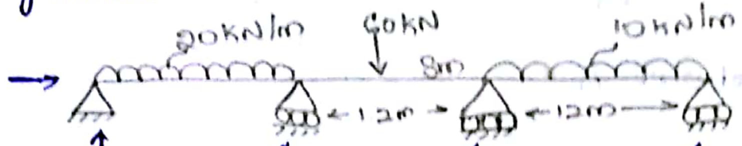
$$\frac{1}{EI}\begin{bmatrix} 2 & 1 \\ 1 & 3.33 \end{bmatrix}\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} -270/EI \\ -350/EI \end{bmatrix}$$

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \left[\frac{1}{EI}\begin{pmatrix} 2 & 1 \\ 1 & 3.33 \end{pmatrix}\right]^{-1}\begin{bmatrix} -270/EI \\ -350/EI \end{bmatrix}$$

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} 0.58/EI & -0.176/EI \\ -0.176/EI & 0.353/EI \end{bmatrix}\begin{bmatrix} -270/EI \\ -350/EI \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{156.76}{EI^2} + \frac{61.6}{EI^2} \\ \frac{47.52}{EI^2} - \frac{123.55}{EI^2} \end{bmatrix} = \begin{bmatrix} -97.16 \\ -76.03 \end{bmatrix}$$

② Analyse the continous beam as shown in the fig. Use the flexibility matrix method. Take EI, constant throughout.



Step ①:- Degree of Static Indeterminacy

No of unknown reactions = 5

Equation of Equilibrium = 3

Degree of static Indeterminacy = 2

Step ②:- Identify Redundant forces.

Step ③:- Deflection due to given

Δ_{1L} = SF at A and Area b/w A & B & C.

$$= \left(\frac{2}{3} \times 12 \times \frac{\overset{(20)(12)^2}{wL^2}}{8EI} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{12+8}{3} \times \frac{60 \times 4 \times 8}{12EI}\right)$$

$$\text{Ans} = \frac{1973.33}{EI}$$

$$\Delta_{2L} = \left(\frac{1}{2} \times \frac{12+4}{3} \times \frac{60 \times 4 \times 8}{12EI}\right) + \left(\frac{2}{3} \times 12 \times \frac{10 \times 12^2}{8EI} \times \frac{1}{2}\right)$$

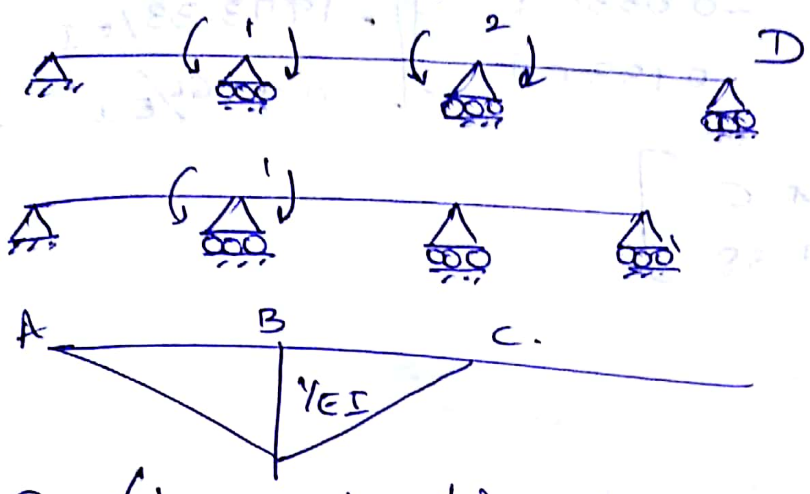
$$\Delta_{2L} = \frac{1146.66}{EI}$$

④ Final displacement

$\Delta_1 = 0$

$\Delta_2 = 0$

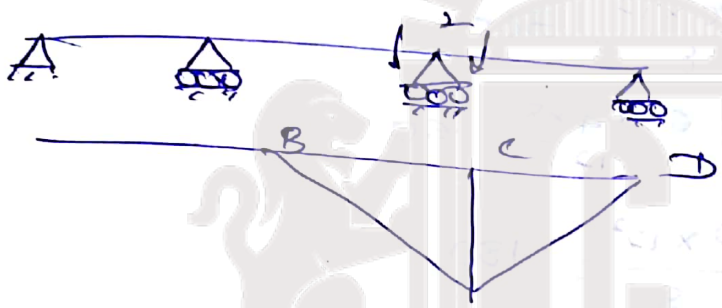
(2) at unit moment (1)



$$a_{11} = \left(\frac{1}{2} \times 12 \times \frac{1}{EI} \times \frac{2}{3}\right) + \cancel{\left(\frac{1}{2} \times 12 \times \frac{1}{EI} \times \frac{2}{3}\right)} = 8/EI$$

$$a_{22} = \left(\frac{1}{2} \times 12 \times \frac{1}{EI} \times \frac{2}{3}\right) + \cancel{\left(\frac{1}{2} \times 12 \times \frac{1}{EI} \times \frac{2}{3}\right)} = 8/EI$$

unit moment (2)



$$a_{12} = \left(\frac{1}{2} \times 12 \times \frac{1}{EI}\right)\frac{1}{3} = 2/EI$$

$$a_{22} = \left(\frac{1}{2} \times 12 \times \frac{1}{EI} \times \frac{2}{3}\right) + \left(\frac{1}{2} \times 12 \times \frac{1}{EI} \times \frac{2}{3}\right) = 8/EI$$

$$[a] = \begin{bmatrix} 8/EI & 2/EI \\ 2/EI & 8/EI \end{bmatrix}$$

unknown redundant

$$[a][F] = [\Delta - \Delta_L]$$

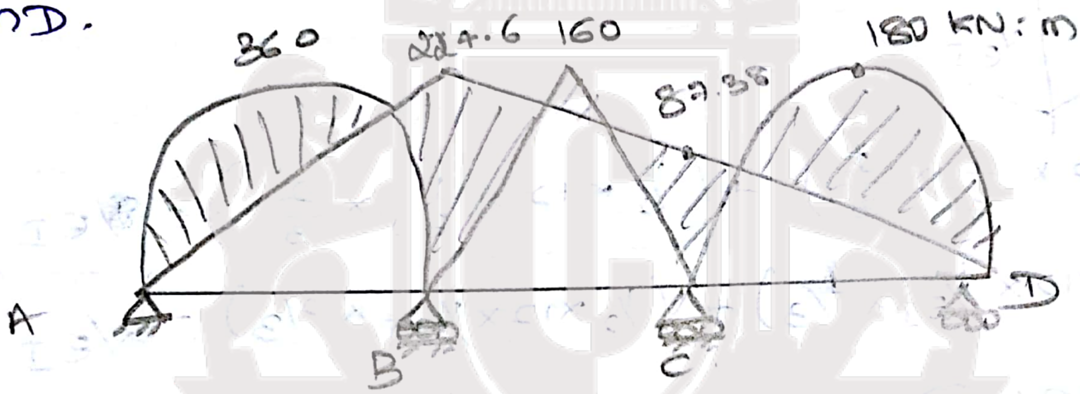
$$\begin{bmatrix} 8/EI & 2/EI \\ 2/EI & 8/EI \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} -\frac{1973.33}{EI} \\ -\frac{1146.66}{EI} \end{bmatrix}$$

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \frac{\begin{bmatrix} 8/EI & -2/EI \\ -2/EI & 8/EI \end{bmatrix}}{64/EI^2 - 4/EI^2} \begin{bmatrix} -1973.33/EI \\ -1146.66/EI \end{bmatrix}$$

$$= \begin{bmatrix} 0.133EI & -0.033EI \\ -0.033EI & 0.133EI \end{bmatrix} \begin{bmatrix} -1973.33/EI \\ -\cancel{cc}4.66/EI \end{bmatrix}$$

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} -224.6 \\ -87.38 \end{bmatrix}$$

7 BMD.



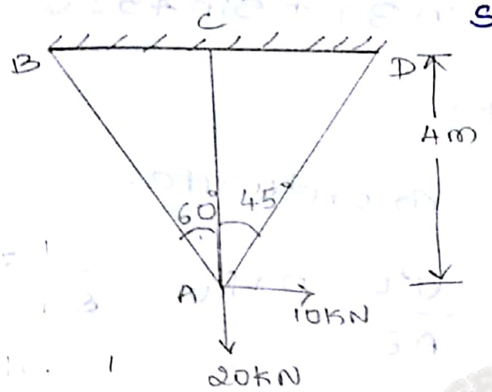
$$\text{Span } AB = \frac{wL^2}{8} = \frac{20\times 12^2}{8} = 360$$

$$BC = \frac{wab}{L} = \frac{60\times 4\times 8}{12} = 160$$

$$CD = \frac{wL^2}{8} = \frac{10\times 12^2}{8} = 180$$

Analysis of pin jointed frames.

1) Analyse the truss shown in the fig by flexibility matrix method. Choosing the force member AD as the redundant. Assume EA is constant for all members.



Step ①:- Find degree of static Indeterminacy Equations of Equilibrium

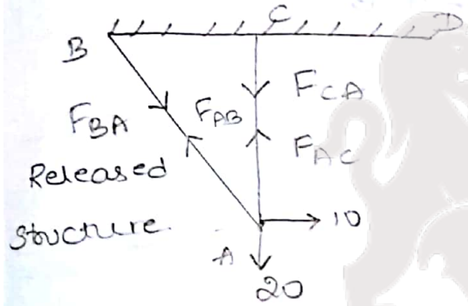
DSI = 3-3 = 0

Internal static Determinacy

m = 3 $m-2j+r$
j = 1
r = 3   Internal = 2
External = 0

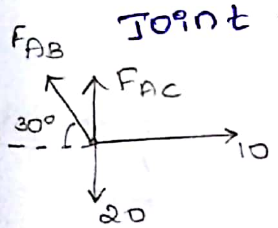
Step ②:- Identify Redundant force/member.

AD - Redundant --- makes it statically Indeterminate



Released structure.

Step ③: To find member forces.



Joint A,

$\Sigma H = 0$

$10 - F_{AB}\cos 30 = 0$

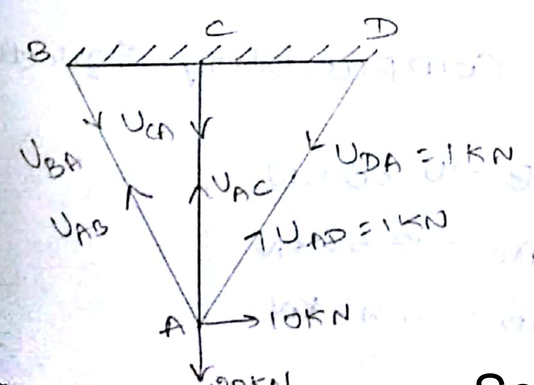
$\boxed{F_{AB} = 11.54\ kN}$

$\Sigma V = 0$

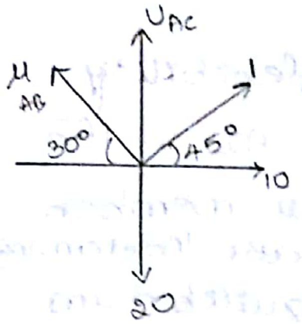
$F_{AC} - 20 + F_{AB}\sin 30 = 0$

$\boxed{F_{AC} = 14.23\ kN}$

Step ④: Apply unit load & find forces in members.



Joint A



$\Sigma H = 0.$

$10 + \cos 45° - \mu_{AB} \cos 30 = 0$

$\mu_{AB} = \cancel{12.36\,kN}$ $\boxed{U_{AB} = 0.816\ kN}$

$U_{AC} - 20 + U_{AB} \sin 30 + \sin 45 = 0$

$\boxed{U_{AC} = -1.115\,kN}$

⑤ Flexibility & displacement co-ordinate.

Member	P(F) forces	u	L	$\frac{PuL}{AE}$	$\frac{u^2L}{AE}$	P+Ru
AB	11.55	0.816	8	$\frac{75.39}{AE}$	$\frac{5.32}{AE}$	10.82
AC	14.85	-1.115	4	$\frac{-63.55}{AE}$	$\frac{4.97}{AE}$	13.52
AD		1	5.65	~~5.65/AE~~	5.65	-0.73
				Σ = 11.84	Σ = 15.94	

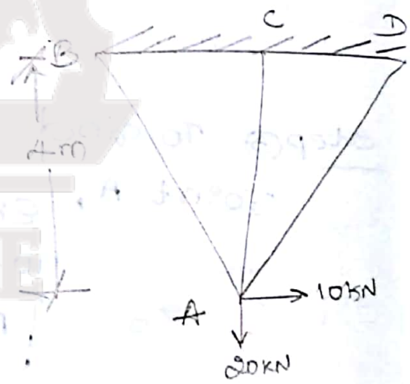
$\cos 60 = \frac{AC}{AB} = \frac{4}{AB}$

$AB = \frac{4}{\cos 60}$

$AB = 8$

$\cos 45 = \frac{AC}{AD} = \frac{4}{AD}$

$AD = 5.65$



δL_1 (Disp co-ordinate) $\delta L_1 = \frac{\Sigma PuL}{AE} = \frac{11.75}{AE}$

f_{11} (flexibility co-efficient) $= \frac{\Sigma u^2 L}{AE} = \frac{15.97}{AE}$

Step ⑥: Solve for unknowns using Compatibility Equation

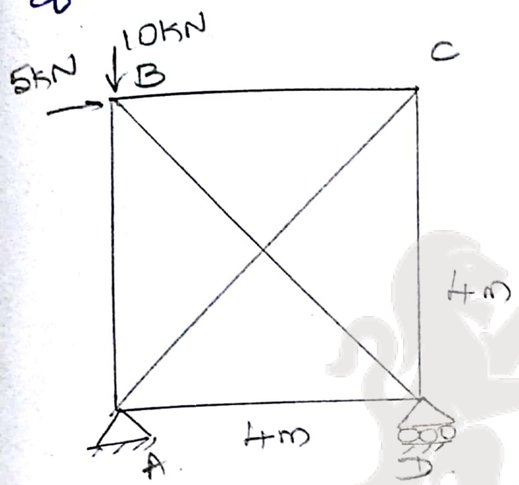
$\delta L_1 + f_{11} R = 0$

$\frac{11.75}{AE} + \frac{15.97}{AE} \cdot R = 0$

$\boxed{R = -0.735}$

$F_{AB} = 10.80\,kN$
$F_{AC} = 13.52\,kN$
$F_{AD} = -0.73\,kN$

① Analyse the pin jointed plane frame as shown in the fig by flexibility matrix method & tabulate the member forces. Assume L/AE of each member = 0.025 mm/KN



Step ①:

No of members = 6

No of joints = 4 External 3-3=0

$m - 2j + r = 1$

Considering AC as a reductant

② Identify Reduntant member