

Note:-

[30/10/19]

- ① While finding out the rotation factor (or) the distribution factor consider finding the relative stiffness  
i.e. Relative stiffness of a member with far

$$\text{End fixed} = \frac{I}{L}$$

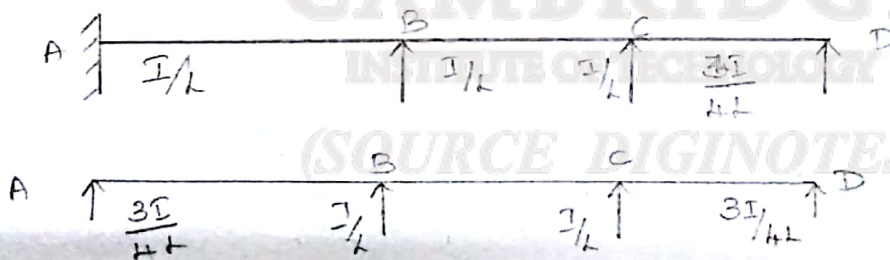
Relative stiffness for a member with far

$$\text{End hinged} = \frac{3I}{4L}$$

Joint	Member	Relative stiffness	Total stiffness	DF	RF
-------	--------	--------------------	-----------------	----	----

- ② For Beams having Internal supports to be simply supported (or) hinged (or) roller

Eg:- In the member ABCD as shown below support 'B' and 'c' [Internal] are simply supported but the relative stiffness of the members BC & CB should be considered as the relative stiffness for a member for far end fixed.  $\left[\frac{I}{L}\right]$



- ③ If the moment of Inertia of the member is  $2I, 3I$ , any constant  $\times I$  has to be considered as the total moment of Inertia of the member ( $I$ )

### Important Notes

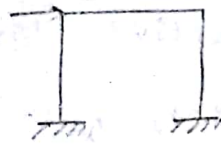
Kani's method is also called as

Rotation Contribution method

Source: diginotes.in

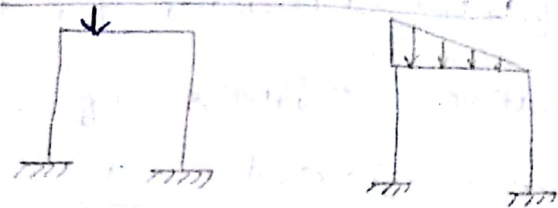
# Kani's method for portal frames with sway.

Sway  
Case (i) Horizontal load



Case (ii)

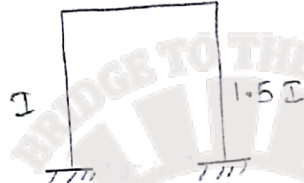
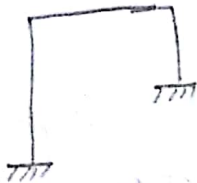
Unsymmetrical loading



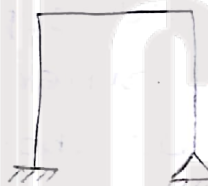
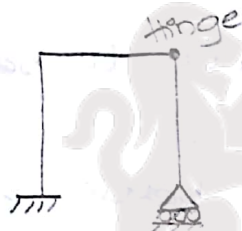
Case (iii)

Unsymmetrical supports

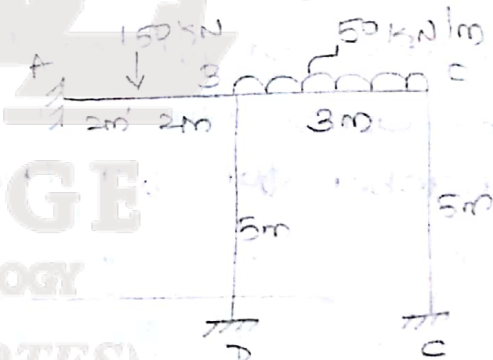
(a) unsymmetrical columns



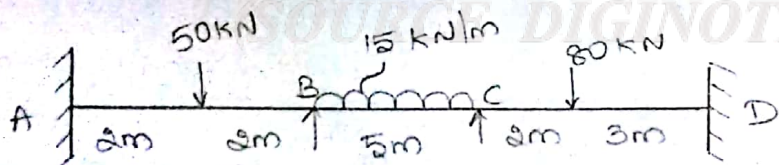
Case (iv) Hinged support / Roller.



Non Sway



①



Step ①: FEM's

$$M_{FAB} = -\frac{WL}{8} = -25 \text{ kN-m}$$

$$M_{FBA} = 25 \text{ kN-m}$$

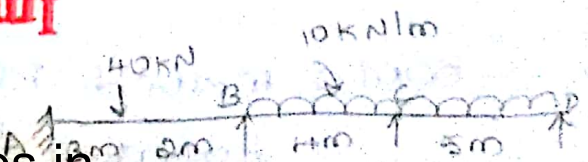
$$M_{FBC} = -\frac{WL^2}{12} = -31.25 \text{ kN-m}$$

$$M_{FCB} = 31.25 \text{ kN-m}$$

$$M_{PCD} = -57.6 \text{ kN-m}$$

$$M_{FDC} = 57.6 \text{ kN-m}$$

**Important Notes**



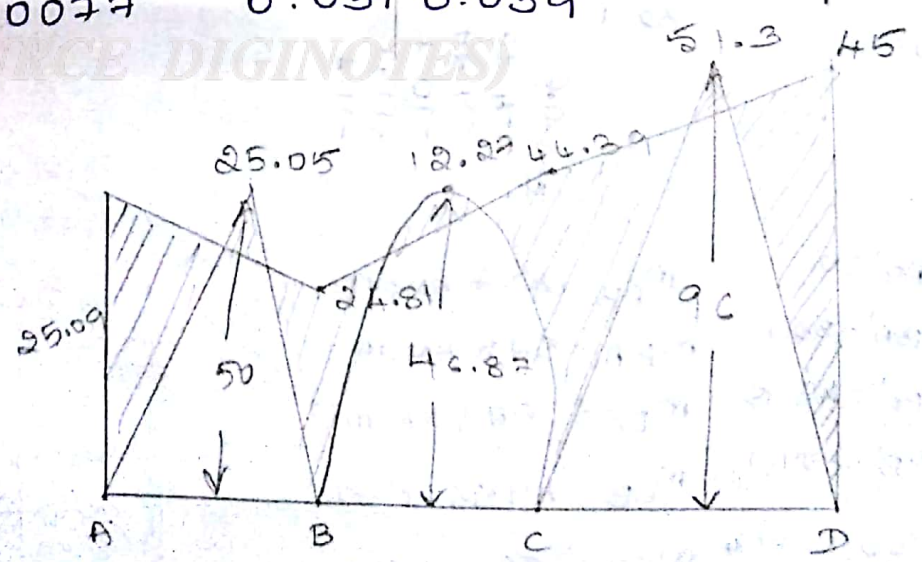


Step 2:-

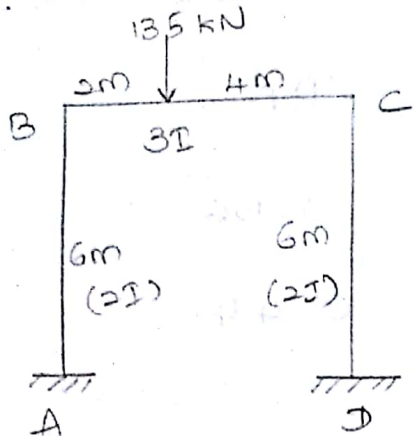
Joint	Member	Relative Stiffness	Total Stiffness	DF
B	BA	$K = \frac{I}{L} = 0.25I$	0.45I	0.56
	BC	$K = 0.2I$		0.44
C	CB	$K = \frac{I}{L} = 0.2I$	0.4I	0.5
	CD	$K = 0.2I$		0.5

	B		C		
	0.56	0.44	0.5	0.5	
A	-25	25	-31.25	31.25	-57.6
	3.5	2.75	13.17	13.17	6.58
	1.75	6.58	1.375		
	-3.68	-2.89	-0.68	-0.68	
	-1.84	-0.34	-1.44		-0.34
	0.19	0.14	0.72	0.72	0.36
	0.095	0.36	0.07		
	-0.20	-0.158	-0.035	-0.035	
	-0.1	-0.0175	-0.079		-0.0175
	0.0098	0.0077	0.039	0.039	

$M_{AB} = -25.095 \text{ KN-m}$   
 $M_{BA} = 24.81 \text{ KN-m}$   
 $M_{BC} = -24.81 \text{ KN-m}$   
 $M_{CB} = 44.39 \text{ KN-m}$   
 $M_{CD} = -44.39 \text{ KN-m}$   
 $M_{DC} = 45 \text{ KN-m}$



2



# FEM'S

$$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0$$

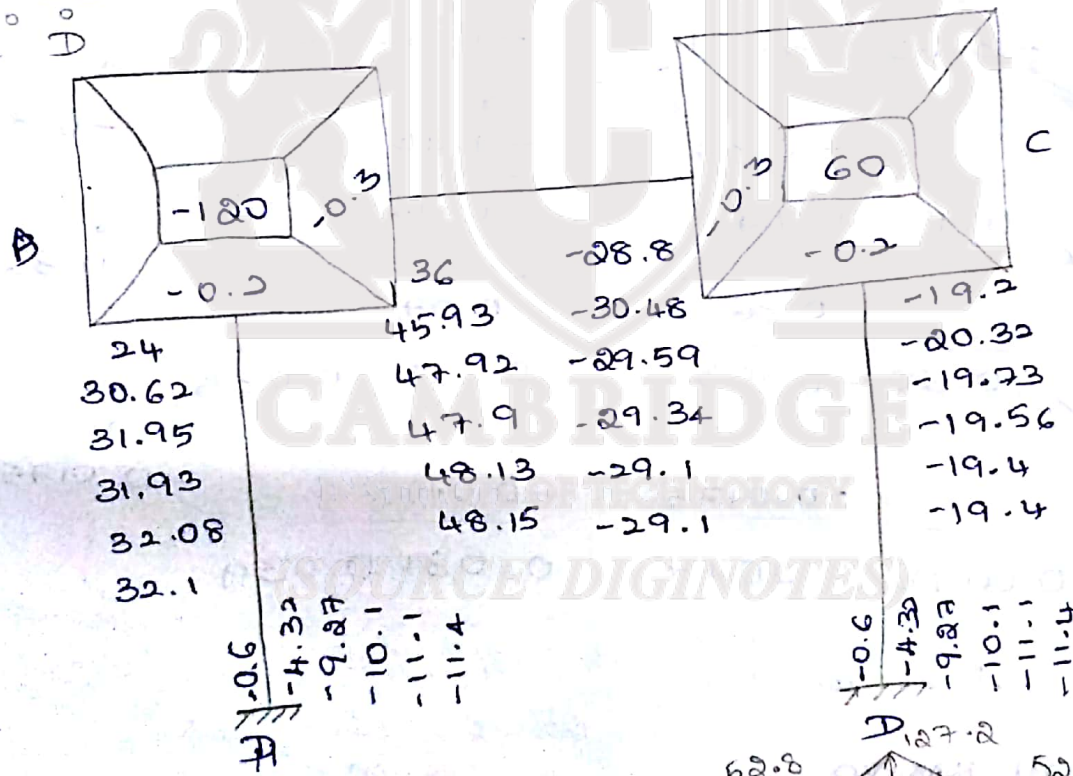
$$M_{FBC} = -120 \text{ kN-m}$$

$$M_{FCB} = 60 \text{ kN-m}$$

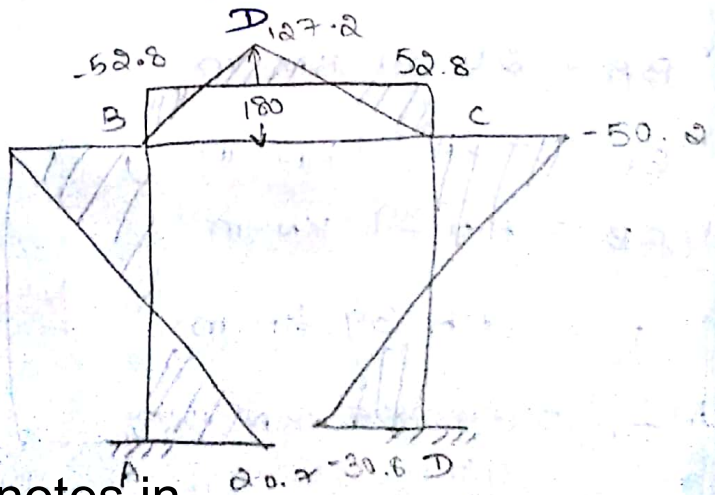
(only for col)

Joint	member	Relative stiffness	Total stiffness	DF	RF	$\Delta F$
B	BA	$I/L = 0.33 I_0$	$0.83 I_0$	0.4	-0.2	-0.6
	BC	$I/L = 0.5 I_0$		0.6	-0.3	
C	CB	$I/L = 0.5 I_0$	$0.83 I_0$	0.6	-0.3	
	CD	$I/L = 0.33 I_0$		0.4	-0.2	-0.6

0.00600  
A B C D



$$\begin{aligned} M_{AB}' &= 0 & M_{AB} &= 20.7 \text{ kN-m} \\ M_{BA}' &= 32.1 & M_{BA} &= 52.8 \text{ kN-m} \\ M_{BC}' &= 48.15 & M_{BC} &= -52.8 \text{ kN-m} \\ M_{CB}' &= -29.1 & M_{CB} &= 49.95 \text{ kN-m} \\ M_{CD}' &= -19.4 & M_{CD} &= -50.2 \text{ kN-m} \\ M_{DC}' &= 0 & M_{DC} &= -30.8 \text{ kN-m} \end{aligned}$$





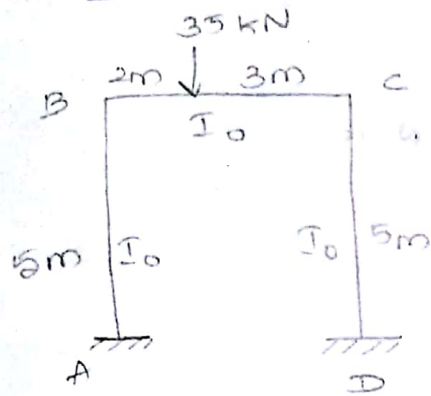
# Frames with sway

## FEM's

$$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0$$

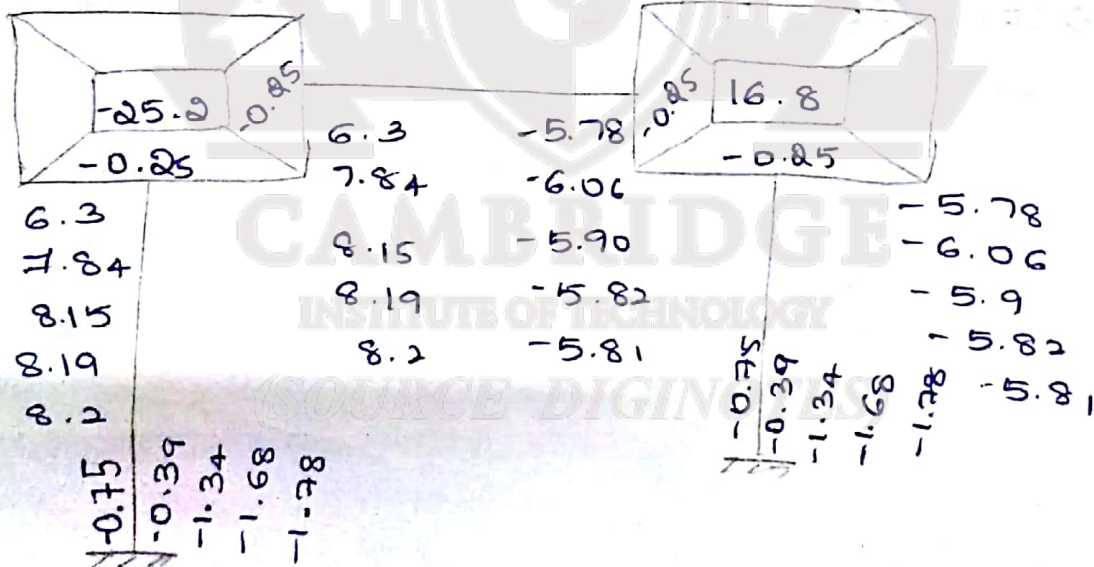
$$M_{FBC} = -\frac{wb^2a}{L^2} = -25.2 \text{ kN-m}$$

$$M_{FCB} = 16.8 \text{ kN-m}$$



Joint	member	Relative stiffness	Total stiffness	DF	RF	AF
B	BA	$I/L = 0.2 I_0$	$0.4 I_0$	0.5	-0.25	-0.75
	BC	$I/L = 0.2 I_0$		0.5	-0.25	
C	CD	$I/L = 0.2 I_0$	$0.4 I_0$	0.5	-0.25	-0.75
	CB	$I/L = 0.2 I_0$		0.5	-0.25	

Resultant Restraint moments



$$M'_{AB} = 0$$

$$M'_{BA} = 8.2 \text{ kN-m}$$

$$M'_{BC} = 8.2 \text{ kN-m}$$

$$M'_{CB} = -5.81 \text{ kN-m}$$

$$M'_{CD} = -5.81 \text{ kN-m}$$

$$M'_{DC} = 0$$

$$M''_{AB} = M''_{BA} = -1.78 \text{ kN-m}$$

$$M''_{CD} = M''_{DC} = -1.78 \text{ kN-m}$$

135 kN

Final moments:-

$$M_{AB} = M_{FAB} + 2m'_{AB} + m'_{BA} + m''_{AB}$$

$$= -0.75 + 2(0) + 8.2 + (-1.78) = 6.42 \text{ kN-m}$$

$$M_{BA} = M_{FBA} + 2m'_{BA} + m'_{AB} + m''_{BA}$$

$$= 2(8.2) + (-1.78) = 14.62 \text{ kN-m}$$

$$M_{BC} = M_{FBC} + 2m'_{BC} + m'_{CB}$$

$$= -25.2 + 2(8.2) + (-5.81) = -14.61 \text{ kN-m}$$

$$M_{CB} = M_{FCB} + 2m'_{CB} + m'_{BC} + 0$$

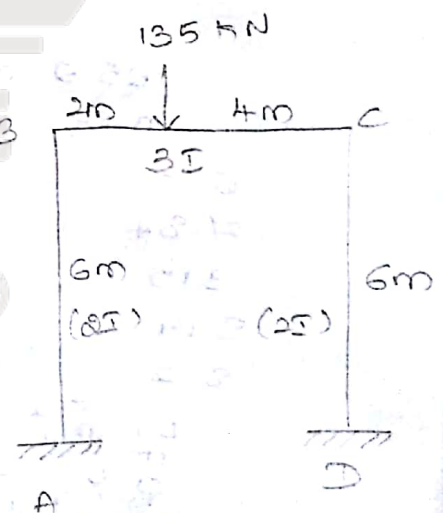
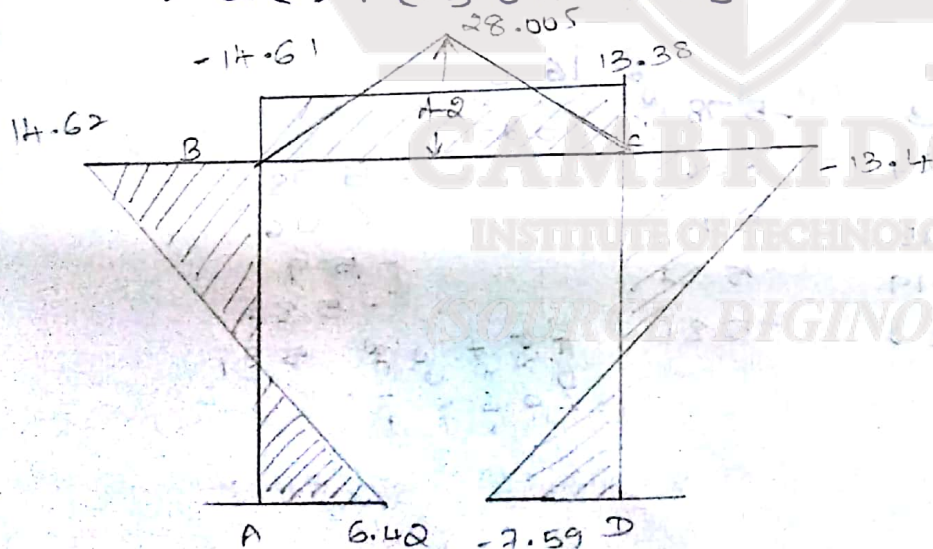
$$= 16.8 + 2(-5.81) + 8.2 = 13.38 \text{ kN-m}$$

$$M_{CD} = M_{FCD} + 2m'_{CD} + m'_{DC} + m''_{CD}$$

$$= 2(-5.81) + (-1.78) = -13.4 \text{ kN-m}$$

$$M_{DC} = M_{FDC} + 2m'_{DC} + m'_{CD} + m''_{DC}$$

$$= 2(0) + (-5.81) - 1.78 = -7.59 \text{ kN-m}$$



3/11/17

Case ② Unequal Column Height.

① Draw the BMD & sketch the deflected shape of the frame as shown in the figure.

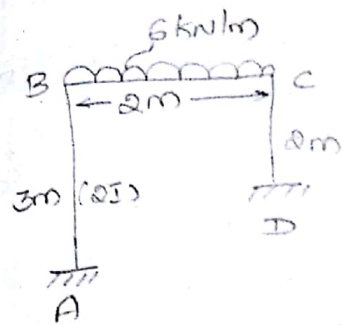


Step ①: FEM's

$$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0$$

$$M_{FBC} = -2 \text{ kN-m}$$

$$M_{FCB} = 2 \text{ kN-m}$$



Joint	member	Relative stiffness	Total stiffness	DF	RF	Displacement factor $\Delta F$
B	BA	$K = I/L$ $K = 0.67I$	$1.167I$	0.57	-0.285	
	BC	$K = I/L$ $K = 0.56I$		0.43	-0.215	
C	CB	$K = I/L$ $K = 0.56I$	$I$	0.5	-0.25	
	CD	$K = 0.5I$		0.5	-0.25	

Step ②: Displacement factor

$$D_{BA} = \frac{-1.5 C_{BA} K_{BA}}{C_{BA}^2 K_{BA} + C_{CD}^2 K_{CD}}$$

$$D_{CD} = \frac{-1.5 C_{CD} K_{CD}}{C_{BA}^2 K_{BA} + C_{CD}^2 K_{CD}}$$

$h_A = 3\text{m} \rightarrow$  consider higher value only

$h_A = 2\text{m}$

$$C_{BA} = \frac{h_A}{h_{BA}}$$

$$C_{CD} = \frac{h_A}{h_{CD}}$$

$$C_{BA} = \frac{3}{3} = 1$$

$$C_{CD} = \frac{3}{2} = 1.5$$

~~$$C_{CD} = \frac{h_A}{h_{CD}} = \frac{2}{2} = 1$$

$$C_{BA} = \frac{h_A}{h_{BA}} = \frac{2}{3}$$~~

$$D_{BA} = \frac{-1.5 \times 1 \times 0.67I}{0.67I + 1.5^2 \times 0.5I}$$

$$D_{CD} = \frac{-1.5 \times 1.5 \times 0.5I}{0.67I + 1.5^2 \times 0.5I}$$

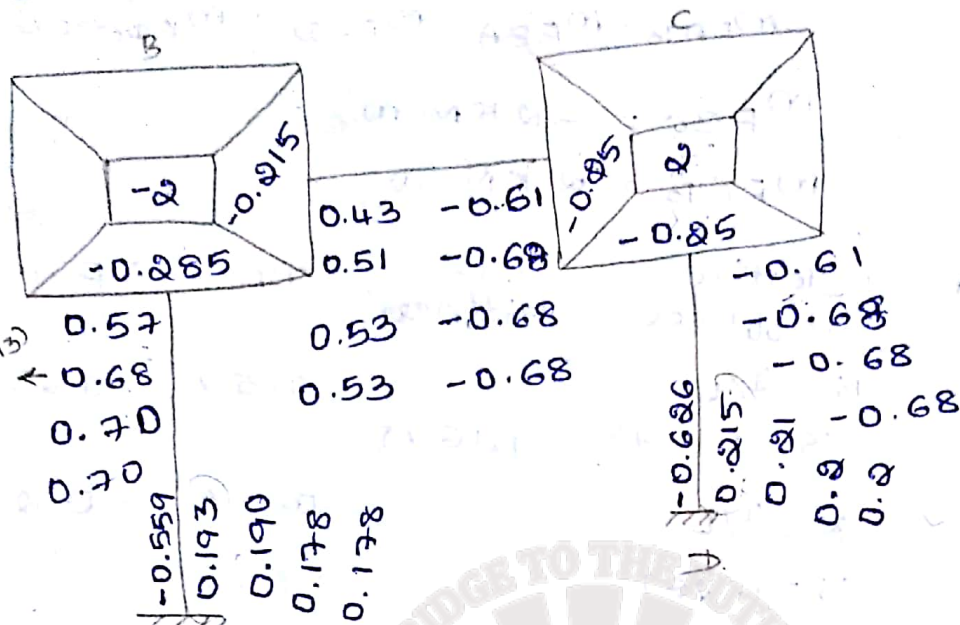
$$= \frac{-1.005I}{1.795I}$$

$$= \frac{-1.125I}{1.795I}$$

$$D_{BA} = -0.559$$

$$D_{CD} = -0.626$$

# Step 4: Resultant Restraint moments



$M''_{BA}$   
(or)

$$M''_{AB} = -0.559 \times [C_{BA} (M'_{AB} + M'_{BA}) + C_{CD} (M'_{CD} + M'_{DC})]$$

$$= -0.559 [1 (0.57) + 1.5 (-0.61)] = 0.193$$

$$M''_{CD} \text{ (or) } M''_{DC} = -0.626 \times [C_{BA} (M'_{AB} + M'_{BA}) + C_{CD} (M'_{CD} + M'_{DC})]$$

$$= -0.626 [1 (0.57) + 1.5 \times (-0.61)] = 0.215$$

$$M_{AB} = M_{FAB} + 2M'_{AB} + M'_{BA} + M''_{AB}$$

$$M_{AB} = 0 + 2(0) + 0.7 + 0.193 = 0.893 \text{ kN-m}$$

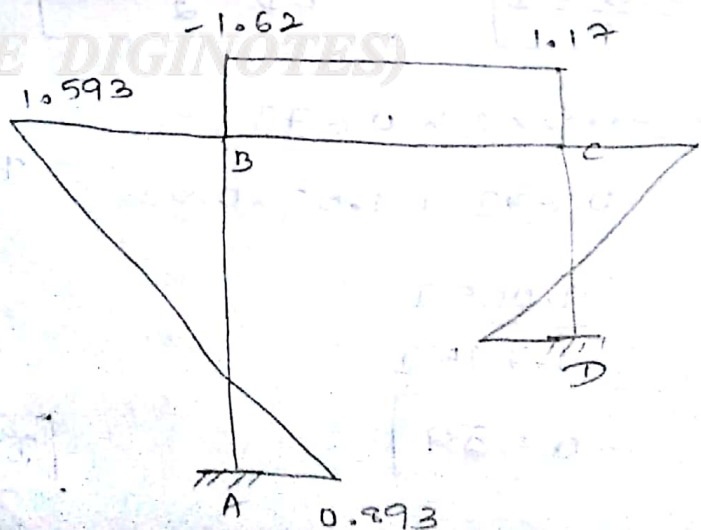
$$M_{BA} = 1.593 \text{ kN-m}$$

$$M_{BC} = -1.62 \text{ kN-m}$$

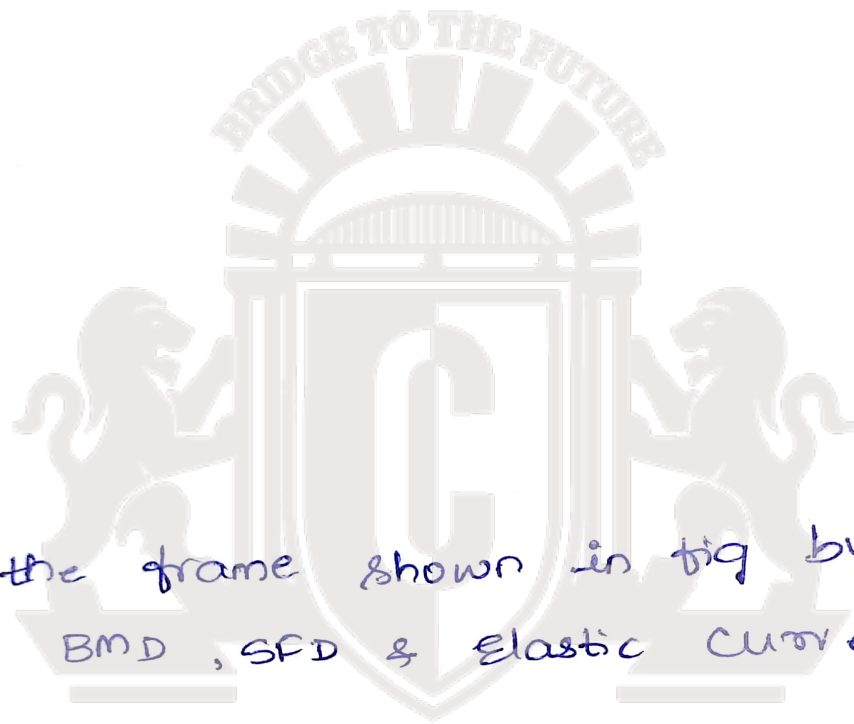
$$M_{CB} = 1.17 \text{ kN-m}$$

$$M_{CD} = -1.17 \text{ kN-m}$$

$$M_{DC} = -0.49 \text{ kN-m}$$







Type -

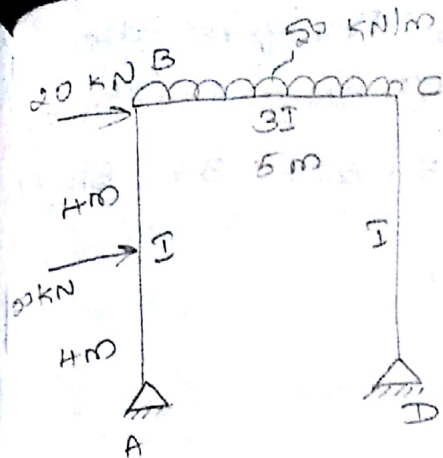
① Analyse the frame shown in fig by Kani's method.

Draw the BMD, SFD & Elastic Curve.

**CAMBRIDGE**

INSTITUTE OF TECHNOLOGY

(SOURCE DIGINOTES)



### Fixed End moments

$$M_{FAB} = -50 \text{ kN-m} \quad M_{FI}$$

$$M_{FBA} = 50 \text{ kN-m}$$

$$M_{FBC} = -104.16 \text{ kN-m}$$

$$M_{FCB} = 104.16 \text{ kN-m}$$

$$M_{FCD} = M_{FDC} = 0$$

Joint	member	Relative stiffness	Total stiffness	DF	RF
B	BA	$k = 3I/4L$	0.69	0.13	-0.065
	BC	$k = I/L$		0.87	-0.435
C	CB	$k = 0.6$	0.69	0.87	-0.435
	CD	$k = 0.895$		0.13	-0.065
A	AB	$k = 0.09I$	0.09I	1	-0.5
D	DC	$k = 0.09I$	0.09I	1	-0.5

$$C_{CD} = C_{BA} = 1$$

### Step ③ Displacement factor

$$D_{BA} = \frac{-1.5 C_{BA} \cdot k_{BA}}{C_{BA}^2 \cdot k_{BA} + C_{CD}^2 \cdot k_{CD}}$$

$$D_{CD} = \frac{-1.5 \times C_{CD} \times k_{CD}}{C_{BA}^2 \cdot k_{BA} + C_{CD}^2 \cdot k_{CD}}$$

$$C_{BA} = \frac{h_{A1}}{h_{BA}} = \frac{8}{8} = 1 = C_{CD}$$

$$D_{BA} = -0.75$$

$$D_{CD} = -0.75$$

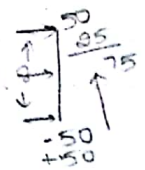
### Step ④: storey moment ( $M_{st}$ )

Top shear  $P_h$  is calculated as follows.

The concentrated load at the centre 50 kN is distributed as 25 kN at the top & 25 kN at the bottom. Also at the top there is horizontal load of 20 kN. The support A is hinged & hence the moment should be zero. To make the moment zero at source.diginotes.in the moment is released



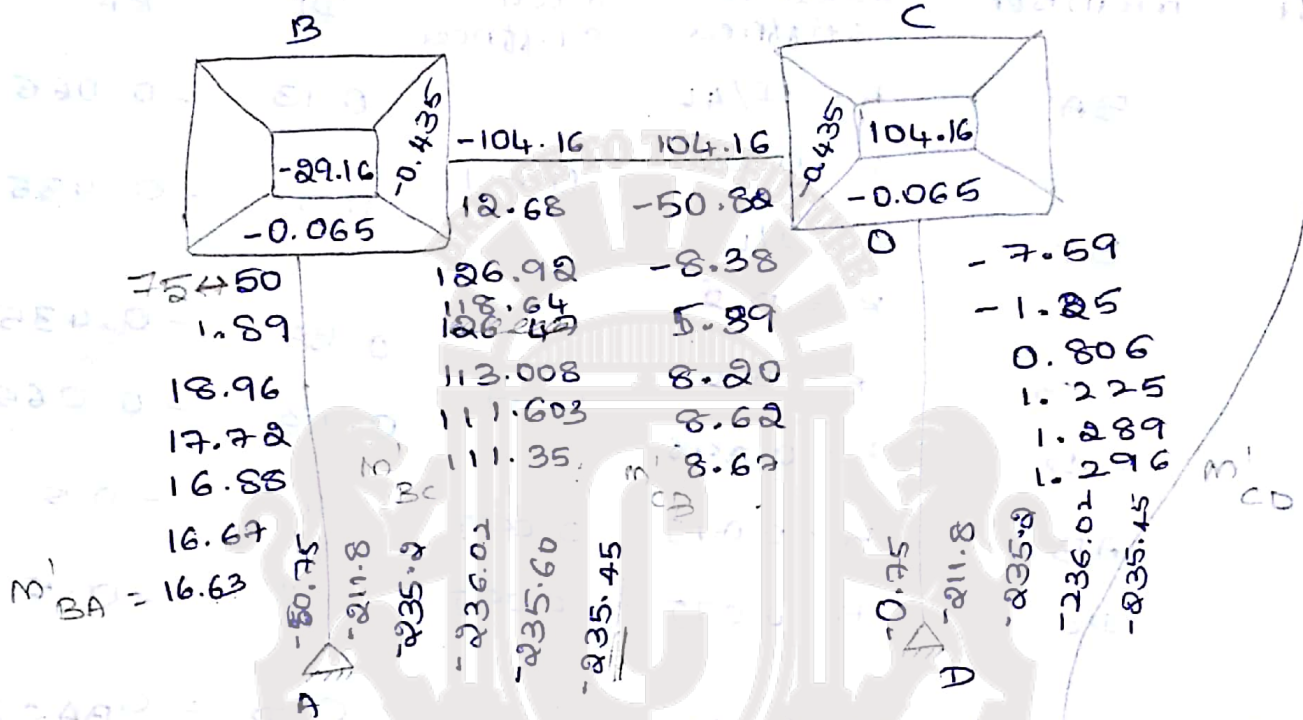
& half of the moment is carried over to B. Therefore the shear at the top due to the end moment. 75 kN-m is  $(75 \div 8)$  Hence the total shear force will be  $P = 25 + 80 + 9.37 = 54.37 \text{ kN}$



$$M_{gr} = \frac{Ph_{gr}}{3} = \frac{54.375 \times 8}{3} = 145 \text{ kN-m}$$
 (AB, BC, CD)

$$m''_{AB} = 2 \times \Delta_{BA} \left[ M_{gr} + \frac{1}{1.5} (M_{AB} + M_{BA}) \right]$$

### Step 5: Resultant Restraint Moments



$$m''_{AB} = 2 \Delta_{AB} \left[ \frac{C_{BA}}{1.5} (m'_{AB} + m'_{BA}) + \frac{C_{CD}}{1.5} (m'_{CD} + m'_{DC}) + M_{gr} \right]$$

$$= 2 (-0.75) \left[ 1 (145) + \frac{1}{1.5} (10.89) + \frac{1}{1.5} (-7.59) \right]$$

$$m''_{AB} = -211.8 \text{ kN-m}$$

$$M_{AB} = M_{FAB} + 2m'_{AB} + m'_{BA} + m''_{AB} = 0$$

$$M_{AB} = -50 + 2(0) + 16.63 + (-235.45) = -268.82$$

$$M_{BA} = -152.19$$

$$M_{BC} = -108.24$$

$$M_{CB} = -2.6$$

$$M_{CD} = -232.85$$