MODULE: 2

LIMIT STATE ANALYSIS OF BEAMS

29/8/17

Singly reinforced beams:-

There are 3 types of beams

1. Singly reinforced beam.
2. Doubly reinforced beam
3. Flanged reinforced beam

① Singly reinforced beam:-

limit state of collapse - flexure

Step by step procedure:

* Assumptions - page no 69
* Partial safety factor.
* Ultimate load (or) factored load = 1.5 × working load

$W_u = 1.5 \times W$

Ultimate moment ← $M_u = 1.5 \times M$ → moment

Ultimate shear ← $V_u = 1.5 \times V$ → shear force

Limiting values (or) Balanced depth of neutral axis.

N/mm^2	x_{umax}/d
Fe_{415}	0.48
Fe_{500}	0.46

Types of section

1. Balanced section:- Moment of resistance $M_R = M_{u\,lim}$

$x_u = x_{umax}$

$$M_{u\,lim} = 0.36\,\frac{x_{u\,max}}{d}\left[1 - 0.42\,\frac{x_{u\,max}}{d}\right] bd^2 f_{ck}$$

2. Under reinforced section:-

$$\frac{x_u}{d} \quad M_u = 0.87\, f_y A_{st}\, d\left[1 - \frac{A_{st} f_y}{bd\, f_{ck}}\right]$$

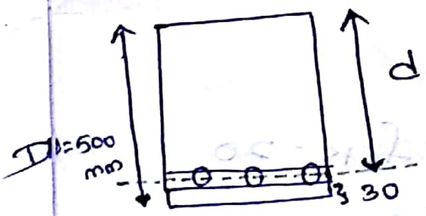
3. Over reinforced section. $x_u > x_{u\,max}$

$$M_{u\,lim} = 0.36\,\frac{x_{u\,max}}{d}\left[1 - 0.42\,\frac{x_{u\,max}}{d}\right] bd^2 f_{ck}.$$

Problems:

1. A rectangular RC beam of dimension 250 x 500mm is simply supported over a clear span of 5m. The beam consist of three number of 18mm dia was use M_{20} & Fe_{415}. Determine the moment of resistance.

Soln: clear cover for beam = 30mm [Assumed]



$$d = 500 - 30 - \frac{18}{2}$$

$$= 461\text{mm}$$

$$A_{st} = \frac{\pi d^2}{4} \times 3$$

$$= 763.40\ \text{mm}^2$$

$f_{ck} = 20\,N/mm^2$ $\quad f_y = 415\,N/mm^2$

Step ② Depth of neutral axis:

$$x_u = \frac{0.87\, f_y\, A_{st}}{0.362\ b\ f_{ck}}$$

$$= \frac{0.87 \times 415 \times 763.40}{0.362 \times 250 \times 20} = 153.128$$

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$$Fe_{415} = \frac{x_{umax}}{d} = \quad x_{umax} > x_u \qquad M = \frac{M_u}{1.5}$$

$$\boxed{M = 7.39 \times 10^7 \text{ N-mm}}$$

$$x_{umax} = 0.48 \times 4\cancel{6}1$$

$$= \cancel{8664}\ 221.28$$

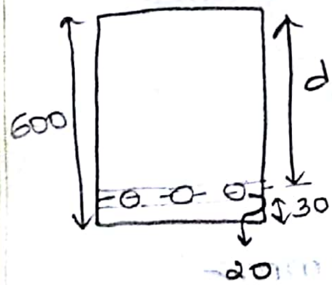
$$M_u = 0.87 f_y A_{st} d \left[1 - \frac{A_{st} f_y}{b d f_{ck}}\right]$$

$$= 0.87 \times 415 \times 763.4 \times 461 \left[1 - \frac{763.4 \times 415}{250 \times 500 \times 20}\right]$$

$$M_u = 110.96 \times 10^6 \text{ N-mm}$$

② A rectangular RC beam of dimension 350 × 600 is simply supported over a clear span of 8m. The beam consists of 3 NO of 20mm dia tension bars used M_{20} & Fe_{415}. Determine the moment of resistance & UDL live load that the beam can carry.

Soln: Clear cover assume as 30mm



$$d = 600 - 30 - \frac{20}{2}$$

$$\boxed{d = 560\text{mm}}$$

$f_{ck} = 20$

$f_y = 415$

$$A_{st} = \frac{\pi d^2}{4} \times 3 = \frac{3.14(\cancel{2}20)^2}{4} \times 3 = 942.\cancel{47}\ \text{mm}^2$$

depth of neutral axis

$$x_u = \frac{0.87 f_y A_{st}}{0.36\, b\, f_{ck}}$$

$$= \frac{0.87 \times 415 \times 942.\cancel{47}}{0.36 \times \cancel{2}350 \times 20} = \boxed{134.96 = x_u} \text{ mm}$$

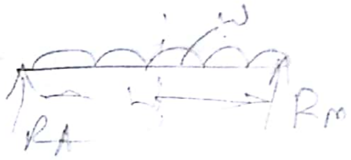
$$x_u max = 0.48 \times 560$$
$$= 268.8 \text{ mm}$$

$$M_U = 0.87 f_y A_{st} d \left[1 - \frac{A_{st} f_y}{b d f_{ck}}\right]$$

$$= 0.87 \times 415 \times 942 \times 560 \left[1 - \frac{942 \times 415}{350 \times 600 \times 20}\right]$$

$$\boxed{M_U = 17.27 \times 10^7 \text{ N-mm}} =$$

$$M = \frac{M_U}{1.5} = \underline{11.51 \times 10^7 \text{ N-mm}}$$



$R_A + R_B = wL$

$\Sigma M_A = 0$

$-R_B \times L + wL \times \frac{L}{2}$

$R_B = \frac{wL}{2}$

$R_A = \frac{wL}{2}$

$BM = R_A \times \frac{L}{2} - w\frac{L}{2} \times \frac{L}{2 \cdot 2}$

$BM = \frac{wL^2}{4} - \frac{wL^2}{8}$

$BM = \frac{wL^2}{8}$

$350 \times 600 = 0.35 \times 0.6$

$w_g = D L = \text{Area} \times \text{Density of concrete}$

$= 0.21 \text{ m}^2 \times 25 \frac{kN}{m^3}$

$$w_g = \boxed{D.L = 5.25 \text{ kN/m} = w_g}$$

$$M = \frac{w l^2}{8}$$

$$116.1 \times 10^6 = \frac{w_q l^2}{8} + \frac{w_g l^2}{8}$$

$$115.1 \times 10^6 = \frac{l^2}{8}[w_q + w_g]$$

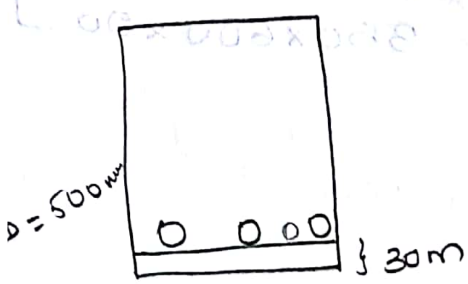
$$115.1 \times 10^6 = 8[w_q + 5.25]$$

$$\boxed{w_q = 9.13 \text{ kN/m}}$$

③ A Cantilever RC beam of size 250 × 500 mm consists of 4 bars of 22mm dia in tension zone the clear span of beam is 6m. Determine the UDL L.L M_{25} & Fe_{415}

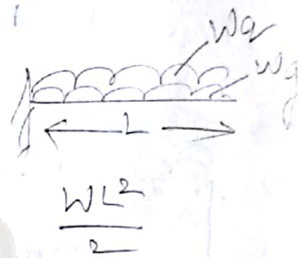
$$d = 500 - 30 - \frac{22}{2}$$

$$d = 459 mm$$



$$A_{st} = \frac{\pi d^2}{4} \times \cancel{3} = \frac{\pi}{4} \times (22)^2 \times \cancel{4}$$

$$A_{st} = 1520.5 \ mm^2$$



$$x_u = \frac{0.87 f_y A_{st}}{0.3642 f_{ck} \times b}$$

$$= \frac{0.87 \times 415 \times 1520.5}{0.3642 \times 25 \times 250}$$

$$x_u = 241.17$$

$$x_{umax} = fe415 \times d$$

$$= 0.48 \times 459$$

$$x_{umax} = 220.32$$

$$x_u > x_{umax}$$

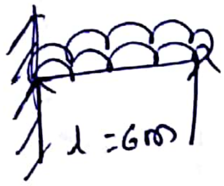
$$M_{u\,limit} = 0.36 \frac{x_{umax}}{d} \left[1 - 0.42 \frac{x_{umax}}{d}\right] bd^2 f_{ck}$$

$$= 0.36 \times \frac{220.32}{459} \left[1 - 0.42 \times \frac{220.32}{459}\right] 250 \times (459)^2 \times 25$$

$M_{u\,lim} = 181.66$ KN-m

$$M = \frac{M_{u\,lim}}{1.5} =$$

$$= 121.1 \text{ KN-m}$$



$DL = A \times$ Density of concrete

$$= 0.125 \times 25$$

$$DL = 3.125$$

$$M = \frac{w_q l^2}{2} + \frac{w_g l^2}{2}$$

$$121.1 = \frac{L^2}{2}[w_q + w_g]$$

$$121.1 = \frac{6^2}{2}[w_q + 3.125]$$

$$\boxed{w_q = 3.60 \text{ N-mm}}$$

$$M = \frac{w_2 l^2}{8} + w_q \frac{l}{3}$$

(4) A rectangular beam of size 250x500mm overall consists of 4 bars of 22mm dia. In tension zone the beam is simply supported with 6m clear span. Determine moment of resistance & a point load at 1/3 span.

$$d = 500 - 30 - \frac{22}{2}$$

$$A_{st} = \frac{\pi}{4} \times 4 \times (22)^2$$

$$\boxed{d = 459 \text{ mm}}$$

$$A_{st} = 1520.53 \text{ mm}^2$$

$$x_u = \frac{0.87 \times 415 \times 1520.53}{0.36 \times 20 \times 250}$$

$x_u = 304.99$ mm

$x_{u\,max} = 0.48 \times 459$
$= 220.32$ mm

$x_u > x_{u\,max} \rightarrow$ Over reinforcement section

$$M_{u\,lim} = 0.36 \frac{x_{u\,max}}{d}\left[1 - 0.42 \frac{x_{u\,max}}{d}\right] b d^2 f_{ck}$$

$$= 0.36 \times \frac{220.32}{459}\left[1 - 0.42 \times \frac{220.32}{459}\right] 250 \times (459)^2 \times 20$$

$\boxed{M_{u\,lim} = 145.33 \text{ N-mm}}$

$\boxed{M = 96.887 \text{ kN-m}}$

NOTE:-
only for cantilever beam
Tension will be at top
& compression in bottom
In cantilever bars will
be up

$D_L = 0.125 \times 25$
$= 3.125$ kN/m

$$M = W_a \times L/3 + \frac{W_d l^2}{8}$$

$$96.887 = W_a \times 2 + \frac{3.125 \times 6 \times 6}{8}$$

$W_a = 41.41$ N-mm

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(5) A rectangular cantilever beam of sides 250mm × 550mm overall consists of 3 bars of 25mm dia in tension zone. Use M_{25} & Fe_{415} scale. Determine moment of resistance & point load & free End. The span of the beam is 3m.

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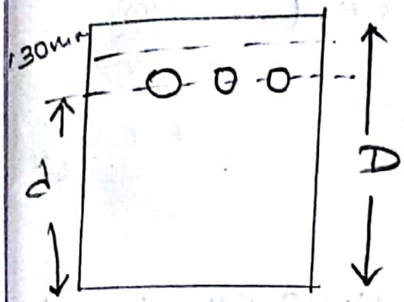


$f_{ck} = 25 N/mm^2$

$f_y = 415 N/mm^2$

$A_{st} = \frac{3}{4} \times \pi (25)^2 = 1472.62 \ mm^2$

$d = 550 - 30 - \frac{25}{2}$

$d = 507.5mm$

$x_u = \frac{0.87 f_y A_{st}}{0.3642 f_{ck} \times b}$

$= \frac{0.87 \times 415 \times 1472.62}{0.3642 \times 25 \times 250}$

$\boxed{x_u = 236.30 \ mm}$

$x_{umax} = fe_{415} \times d$

$= 0.48 \times 507.5$

$\boxed{x_{umax} = 243.6mm}$

$x_{u_{max}} > x_u$

$x_u \ lim = 0.36 \frac{x_{umax}}{d} \left[1 - 0.42 \frac{x_{umax}}{d}\right]$

$= 0.36 \times \frac{243.6}{507.5} \left[1 - 0.42 \frac{243.6}{507.5}\right]$

$x_u \ lim = 0.1379$

$M_u \ lim = 0.36 \frac{x_{umax}}{d} \left[1 - 0.42 \frac{x_{umax}}{d}\right] bd^2 f_{ck}$

$= 0.36 \frac{243.6}{507.5} \left[1 - 0.42 \frac{243.6}{507.5}\right] 250 \times (507.5)^2 \times 25$

$M_u \ lim =$

$x_u < x_{umax} \longrightarrow$ under reinforced

$$M_u = 0.87 f_y A_{st} d \left[1 - \frac{A_{st} f_y}{bd f_{ck}}\right]$$

$$M_u = 0.87 \times 415 \times 1472.62 \times 507.5 \left[1 - \frac{1472.62 \times 415}{250 \times 507.5 \times 25}\right]$$

$M_u = \underline{217.8 \times 10^6}$ $M_d = \frac{M_u}{1.5} = \underline{145.2 \times 10^6 \text{ N-mm}^2}$

~~D_L = A × f_c conc~~ :

$$M = \frac{w_g l^2}{2} + w_q l$$

$w_g = 0.1375 \times 25$

$\boxed{w_g = 3.43 \text{ kN/m}}$

$$145.23 = \frac{3.43 \times 3^2}{2} + w_q \times 3$$

$\boxed{w_q = 43.26 \text{kN/m}}$

Type - 2

1. A rectangular simply supported beam of span 5m & size 250 x 400mm consists of 3 bars of 20mm dia use M_{20} & Fe_{415} with moderate exposure.
[Page 47]

Determine (i) Tensile force in the steel (T)
(ii) Compression force in the concrete (c)
(iii) lever arc.
(iv) moment of resistance

step ① $f_{ck} = 20 \text{ N/mm}^2$

$f_y = 415 \text{ N/mm}^2$

$$A_{st} = \frac{3}{4} \times \pi \times (20)^2 = 942.47 \text{ mm}^2$$

$$d = 400 - 30 - \frac{20}{2} = 360 \text{mm}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times 942.47}{0.36 \times 20 \times 250}$$

$\boxed{x_u = 189.04 \text{ mm}}$

$x_{u\,max} = fe_{415} \times d$

$= 0.48 \times 360$

$\boxed{x_{u\,max} = 172.8\ mm.}$

$T = f_{st} \cdot A_{st}$

$f_{st} = 0.87 f_y$

$= 0.87 \times 415$

$\boxed{f_{st} = 361.05}$

$T = 361.05 \times 942.47$

$\boxed{T = 340.27 \times 10^3\ N}$

$C = 0.364\ x_u\, b\, f_{ck}$ *for calculating C take x_u max value.

$= 0.36 \times 172.8 \times 250 \times 20$

$C = 311.04 \times 10^3\ N$ (or) 311.04 kN.

z = $x_u \propto \frac{1}{\text{eccentricity}}$ only for this condition

lever arc $z = (d - 0.432 \times x_{u\,max})$

$\boxed{z = 285.35}$

→ Take x_u $x_u < x_{u\,max}$

→ Take this $x_{u\,max} < x_u$

$M_{u\,lim} = 0.36 \times \frac{x_{u\,max}}{d}\left[1 - 0.42\frac{x_{u\,max}}{d}\right] b d^2 f_{ck}$

$= 0.36 \times \frac{172.8}{360}\left[1 - 0.42 \times \frac{172.8}{360}\right] 250 \times (360)^2 \times 20$

$\boxed{M_{u\,lim} = 89.4 \times 10^6\ N\text{-}mm} = 89.4\ kN\text{-}m$

$\boxed{M = \frac{M_u}{1.5} = 59.6\ kN\text{-}m}$

$T = 0.87 \times f_y \times A_{st}$

$\boxed{T = 340.278\ kN}$

$c = 0.36 \times f_{ck} \times b \times x_{u\,max}$

$\boxed{C = 311.09\ kN}$

$z = (d - 0.42\ x_{u\,max}) = 287.424$

2. A rectangular cantilever beam of span 6m & dimension 300x500 Consists of 3 bars of 18mm dia Use M_{25} & Fe_{415} for severe Exposure. Determine

i) Tensile force in steel

ii) compression force in Concrete

iii) Lever arm.

iv) Moment of resistance.

$f_{ck} = 25$

$f_y = 415$

$A_{st} = \frac{3}{4} \times \pi (18)^2 = 763.40$ N-m

$d = 500 - \cancel{45} - \frac{18}{2}$

$d = 446$ mm

$x_u = \frac{0.87 f_y \; A_{st}}{0.36 \; f_{ck} \; b}$

$= \frac{0.87 \times 415 \times 763.4}{0.36 \times 25 \times 300}$

$\boxed{x_u = 102.08}$ mm

$x_{umax} = fe_{415} \times d$

$= 0.48 \times 461$

$\boxed{x_{umax} = 221.28 mm}$

$x_{umax} > x_u$

$T = f_{st} \cdot A_{st}$

$T = 0.87 f_y . A_{st}$

$T = 275.62 \times 10^3$

compression loses $C = 0.364\, x_u\, b\, f_{ck}$

$$\boxed{C = 278.67 \times 10^3\ N}$$

(iii) Lever ram

severe Exposure

$$z = [d - 0.432 \times x_{u\,max}]$$

$$= (446 - 0.432 \times 102.08)$$

$$\boxed{z = 401.9\ N}$$

iv) moment of resistance

$$x_u < x_{u\,max}$$

$$M_u = 0.87\, f_y\, A_{st}\, d \left[1 - \frac{A_{st} f_y}{b d f_{ck}}\right]$$

$$= 0.87 \times 415 \times 763.4 \times 446 \left[1 - \frac{763.4 \times 415}{300 \times 446 \times 25}\right]$$

$$\boxed{M_u = 111.28 \times 10^6\ N-mm^2}$$

$$M = \frac{M_u}{1.5} = 741.9 \times 10^5\ N-mm^2 = \underline{74.19\ kN-m}$$

$$T = 0.87 \times f_y \times A_{st}$$

$$= 0.87 \times 415 \times 763.4$$

$$\boxed{T = 275.6\ kN}$$

$$z = d - 0.42 x_u$$

$$= 446 - (0.42 \times 102.08)$$

$$\boxed{z = 403.12\ mm}$$

$$C = 0.36 \times f_{ck} \times x_u \times b$$

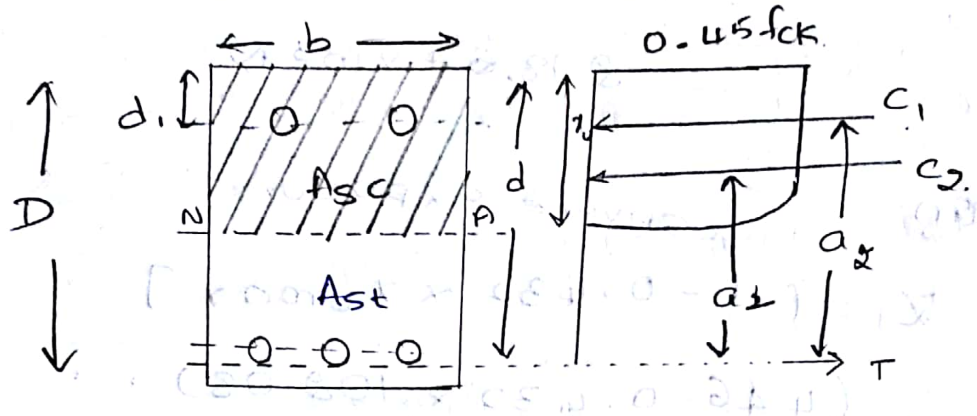
$$= 0.36 \times 25 \times 102.08 \times 300$$

$$\boxed{C = 276.6\ kN}$$

The reason for designing doubly reinforced beam.

1. When fixed depth is given [Architech decide depth of beam is restricted].
2. To reduce vertical deflection
3. To increase Tortional resistance

Problems on doubly reinforced beam - flexure.



A_{st} = Area of tension steel

A_{sc} = Area of compression steel

f_{sc} = stress in compression steel.

d' = Effective cover to compression steel.

C_1 & C_2 = compression in concrete & steel.

$C_1 = 0.36\ f_{ck}\, b\, x_u$

$C_2 = (f_{sc} - 0.45 f_{ck}) A_{sc}$

$T = 0.87 f_y . A_{st}$

a_1 & a_2 = Lever arm.

$a_1 = (d - 0.42 \times x_u)$

$a_2 = (d - d')$

$C = T$

$C_1 + C_2 = T$

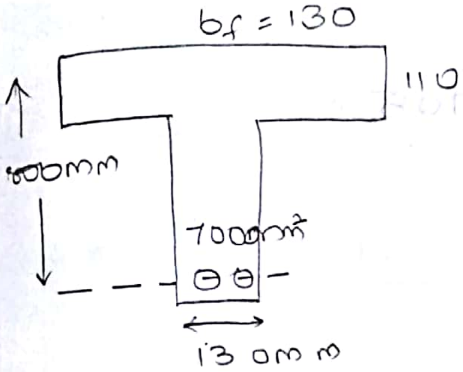
$(0.36 f_{ck}\, b\, x_u) + (f_{sc} - 0.46 f_{ck}) A_{sc} = 0.87 f_y$

Equation Both compression & tension ↑ Eq^n is got.

$M_R = (C_1 a_1 + C_2 a_2)$

↳ moment of resistance

Determine the live load applied at the distance of $1/5$th from the starting point of the span. If breadth of the flange is 2000 mm and depth of the flange is 110 mm, Effective depth is 800mm.
Actual Area of steel = 7000mm². , Breadth of the rib = 130mm
M_{20}, Fe_{500} Span 7 8m



Step ①:- Assume that neutral axis lies within the flange.

(1) $x_u < d_f$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} \cdot b_f}$$

$$x_u = \frac{0.87 \times 500 \times 7000}{0.36 \times 20 \times 2000}$$

$$\boxed{x_u = 211.45\,mm}$$

∴ Hence the assumption is wrong.

Step ②:- $x_u < d_w$

$$C_1 + C_2 = T$$

$$(0.36 f_{ck} x_u b_w) + (0.45 f_{ck} y_f)(b_f - b_w) = 0.87 f_y A_{st}$$

$$y_f = 0.15 x_u + 0.65 D_f$$

$$0.36 f_{ck} x_u b_w + [0.45 \times f_{ck} \times \underbrace{0.15 x_u + 0.65 D_f}](b_f - b_w) = 0.87 f_y A_{st}$$

$$0.36 f_{ck} x_u b_w = 0.87 f_y A_{st} \cancel{-0.65 D_f (b_f - b_w)} - 0.45 f_{ck} \times 0.15 x_u + 0.65 D_f (b_f - b_w)$$

$$x_u = \frac{0.87 f_y A_{st} - 0.45 f_{ck} \times 0.15 x_u + 0.65 D_f (b_f - b_w)}{0.36 f_{ck} b_w}$$

$$x_u = \frac{(0.87\times 500\times 7000) - [(0.45\times 20\times 0.15\times x_u) + (0.65\times 110(2000-130)}{0.36\times 20\times 130}$$

$$x_u = \frac{(3045000) - [1.35x_u + 133705]}{0.36\times 20\times 130}$$

$$x_u = \frac{3045000 - 1.35x_u - 133705}{936}$$

$$x_u = \frac{2911295 - 1.35x_u}{936}$$

$$936x_u + 1.35x_u = 2911295$$

$$\boxed{x_u = 532.15mm}$$

$$x_{umax} = 0.46\times 800 = 368mm$$

$$x_u > x_{umax} \quad [\text{over Reinforced}]$$

$$Mu = 0.36\left[\frac{x_{umax}}{d}\right]\left[1-0.42\frac{x_{umax}}{d}\right]bd^2f_{ck}$$

$$= 0.36\times 0.46[1-(0.42\times 0.46)]130\times 800^2\times 20$$

$$\boxed{Mu = 222.32 KN-m}$$

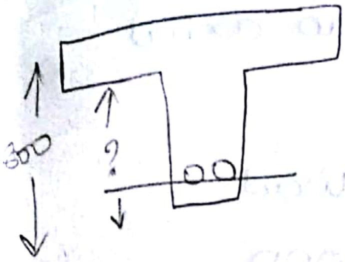
Load calculation

DL = Area of beam x density of concrete

$$A_{st} = \frac{\pi d^2}{4}\times 2$$

$$7000 = \frac{2\times\pi d^2}{4}$$

$$\boxed{d = 66.75mm}$$



$800 - 110 + 33 + 30$

$\boxed{D_w = 753mm}$

$$M_u = \frac{w_q l^2}{8} + w_q \frac{l}{5}$$

~~Dl = (0.13 × 0.753) + 2.~~

$$M_u = \frac{w_q l^2}{8} + \frac{w_q l}{5}$$

$w_q = [2 \times 0.11) + 0.13 \times 0.753] \times 25$

$w_q = 7.94$ kN/m

$$222.32 = \frac{7.94 \times 8^2}{8} + \frac{w_q \times 8}{5}$$

$$222.32 = 63.52 + w_q\, 1.6$$

$\boxed{w_q = 99.25kN}$

Check for deflection

$$P_f = \frac{100\, A_{st}}{bd} = \frac{100 \times 7000}{800 \times 130} = 6.73 \quad x\text{-axis}$$

$$f_{sc} = \frac{0.58\, f_y\, A_{st}}{A_{ct}\, A_{st}}$$

$$A_{st} = \frac{\pi d^2}{4} \times 2$$

$$= \frac{0.58 \times 500 \times \cancel{6842}.388}{7000}$$

$$= \frac{\pi (66)^2}{4} \times 2$$

$\boxed{A_{st} = 6842.388\, mm^2}$

$\boxed{f_{sc} = 283.47}$

$F_1 = 0.7$

$F_2 =$

(1) Design a square column to carry an axial load of 1000 kN use M_{25} & Fe_{415}

Factored load / ultimate load = 1.5 × load

= 1.5 × 1000

P_u = 1500 kN

If the column dimensions is not given then assume % of steel b/w 0.8 to 4%

Assume 1% of steel = $\frac{1}{100} \times b \times d$

$= \frac{1}{100} \times b \times b$

$= 0.01 b^2$

$$P_u = 0.4 f_{ck} A_{sc} + 0.65 f_y A_{sc}$$

$$1500 \times 10^3 = 0.4 \times 25 \times (b^2 - 0.01 b^2) + 0.67 \times 415 \times 0.01 b^2$$

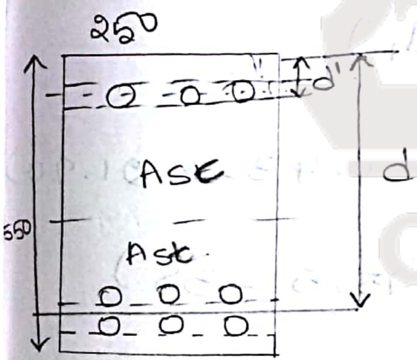
$M_R = (0.36 f_{ck} b x_u \times d - 0.42 \times x_u) +$

$(f_{sc} - 0.45 f_{ck}) A_{sc} \times (d - d_1) \times \cancel{D}$

M_R = Under reinforced section's moment of resistance

Note:- For balanced & over reinforced section replace x_u by x_{umax}.

Determine the moment of resistance of simply supported rectangular beam of dimension 250×550 mm which consists of 3 bars of 16mm dia in compression zone and 6 bars of 12mm dia arranged in two layers in the tension zone use M20 & Fe415 also determine the UDL live load.



$d' = 30 + \frac{16}{2} = 38 \text{mm}$

$d = 550 - 30 - 12 - \frac{25}{2}$

$d = 495.5 \text{mm}$

$A_{st} = \cancel{\frac{3}{4} \times \pi (16)^2} = \frac{6}{4} \times \pi (12)^2$

$A_{st} = 678.58$

$A_{sc} = \frac{3}{4} \times \pi (16)^2 = 603.18$

Formula for strain

$$f_{sc} = 0.0035 \frac{(x_{umax} - d')}{x_{umax}}$$

$x_{umax} = 0.48 \times d$

$= 0.48 \times 495.5$ $\qquad x_u < x_{umax}$

$= 237.84$

$$\text{Strain} = \frac{0.0035\,(x_{umax} - d')}{x_{umax}}$$

$= 0.0029$

$0.87 f_y A_{st} = (0.36 f_{ck}\, b\, x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc}$

$\cancel{0.87\times}\ 415 \times 678.58 = (0.36 \times 20 \times 250 \times x_u) + (350 - 0.45 \times 20)\, 603.18$

$\boxed{x_u = 21.90}$

$M_R = (0.36 f_{ck}\, b\, x_u \times (d - 0.432 \times x_u)) + (f_{sc} - 0.45 f_{ck}) A_{sc} \times (d - d_1)$

$= (0.36 \times 20 \times 250 \times 21.9) \times 495.5 - 0.432 \times 21.90)$

$+ (350 - 0.45 \times 20)\, 603.18 \times (495.5 - 38)$

$= 19532610 - 9.4608 + 2483243.65$

$M_R = 113.26 \times 10^6$ $\quad = \quad M = \dfrac{113.26 \times 10^6}{\underset{\text{FOS}}{1.5}} = 75.5$ kN-m

$$M = \frac{w_g l^2}{8} + \frac{w_q l^2}{8}$$

g = DL = A of Beam $\times$ (Density) Strength of Concrete

$= (0.25 \times 0.55) \times 25$

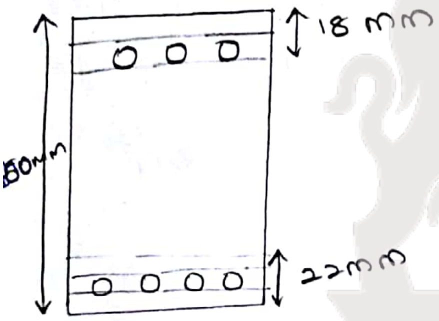
DL $= 3.43$ kN/m

$$M_R = \frac{w_q l^2}{8} + \frac{w_q l^2}{8}$$

$$75.5 = \frac{w_q (6)^2}{8} + \frac{3.43 (6)^2}{8}$$

$$\boxed{w_q = 13.34 \text{ kN/m}}$$

② Determine the M_R for rectangular simply supported beam 300×600mm consists of 3 bars of 18mm in compression & 4 bars of 22mm in Tension of span 8m. M_{20} & Fe_{415}, calculate the point load at the distance $l/3$.



$$d' = 30 + \frac{18}{2} = 39 \text{ mm}$$

$$d = 600 - 30 - \frac{22}{2}$$

$$\boxed{d = 559 \text{ mm}}$$

$$A_{sc} = \frac{\pi}{4} d^2 \times 3 = 763.40$$

$$A_{st} = \frac{\pi}{4} d^2 \times 4 = 1520.5$$

$$f_{sc} = 0.0035 \frac{(x_{umax} - d')}{x_{umax}}$$

$$x_{umax} = 0.48 \times d$$

$$= 0.48 \times 559$$

$$x_{umax} = 268.32$$

$$\text{Strain} = 0.0035 \left[\frac{x_{umax} - d'}{x_{umax}}\right] = 0.00296$$

$$0.87 f_y A_{st} = (0.36 f_{ck}\, b\, x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc}$$

$$0.87 \times 415 \times 763.4 = (0.36 \times 20 \times 300 \times x_u) + (350 - 0.45 \times 20)\, 1520.5$$

$$\boxed{x_u = 133.77} \qquad x_u < x_{umax}$$

$$M_R = (0.36 \times f_{ck} \times b \times x_u) \times (d - 0.432 \times x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} \times (d - d')$$

$$= (0.36 \times 20 \times 300 \times 133.77) \times (\cancel{535.5}\ 559 - 0.432 \times 133.77) + (350 - 0.45 \times 20)\, \cancel{1520.5}\ 763.40 \times (\cancel{535.5}\ 559 - 39)$$

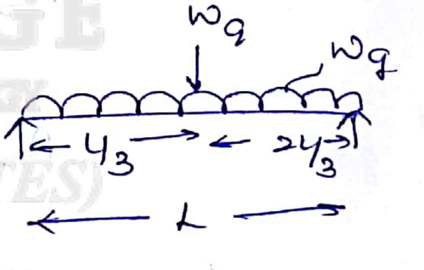
$$M_R = 280.526 \text{ kN-m}$$

$$M = \frac{M_R}{1.5} = 187.017 \text{ kN-m}$$

Dead load $w_g = 0.3 \times 0.6 \times 25$

$$w_g = 4.5 \text{ kN/m}$$

$$M = \frac{w_g l^2}{8} + w_q \times \frac{L}{3}$$



$$187.017 = \frac{4.5 \times 8^2}{8} + w_q \times \frac{8}{3}$$

$$\boxed{w_q = 56.6 \text{ kN/m}}$$

3. Rectangular cantilever RC beam of size 250 × 500 m as 3 No of 20mm bars in bottom and 4 No of 25mm in top. span is 3m use M_{20} & Fe_{415}. Determine the UDL live load entire the span.

This fig is only for Cantilever beam

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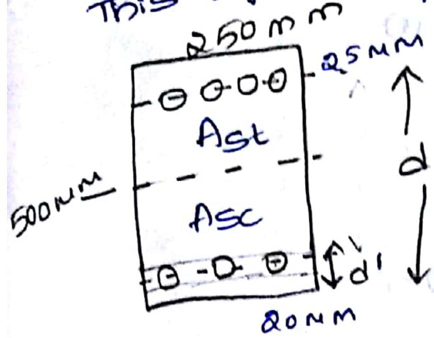


$d' = 40mm = 30 + \frac{20}{2}$

$d = 457.5mm = 500 - 30 - \frac{25}{2} =$

$A_{st} = \frac{\pi}{4} \times d^2 \times 4 = 1963.49 m^2$

$A_{sc} = \frac{\pi}{4} \times d^2 \times 3 = 942.47 m^2$

$$\text{Strain} = 0.0035\left[\frac{x_{u max} - d'}{x_{u max}}\right]$$

$x_{u max} = 0.48 \times d$

$= 0.48 \times 457.5$

$\boxed{x_{u max} = 219.6}$

$$\text{Strain} = 0.0035\left[\frac{219.6 - 40}{219.6}\right]$$

$\text{strain} = 0.00286$

$f_{ck} = 20$

$f_{sc} = 350 mpa.$ $f_y = 415 = 0.48$

$0.87 f_y A_{st} = (0.36 f_{ck} b x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc}$

$0.87 \times 415 \times 1963.49 = (0.36 \times 20 \times 250 \times x_u) +$

$(350 - 0.45 \times 20) \times 942.47$

$\boxed{x_u = 215.29}$

$x_{u max} > x_u$

$M_R = (0.36 \times f_{ck} \times b \times x_u) \times (d - 0.432 \times x_u) +$

$(f_{sc} - 0.45 f_{ck}) A_{sc} \times (d - d')$

$= (0.36 \times 20 \times 250 \times 215.29) \times (457.5 - 0.432 \times 215.29) +$

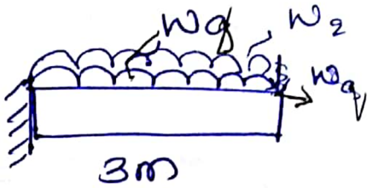
$(350 - 0.45 \times 20) 942.47 \times (457.5 - 40)$

M_u (or) $M_R = 275.42$ kN-m

$$M = \frac{M_u}{1.5} = 183.61 \text{ kN-m}$$

Dead load $w_g = 0.25 \times 0.5 \times 25$

$= 3.125$ kN

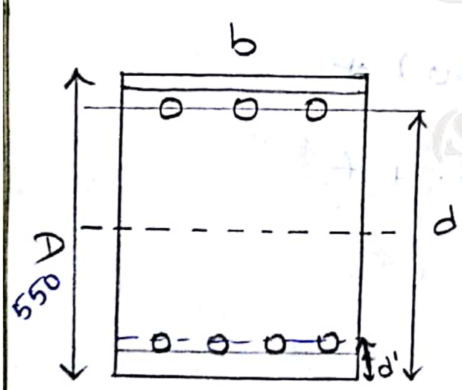


$$M = \frac{w_g l^2}{2} + \frac{w_q l^2}{2}$$

$$183.61 = \frac{3.125 \times 3^2}{2} + \frac{w_q (3)^2}{2}$$

$$\boxed{w_q = 37.67 \text{ kN/m}}$$

4. A rectangular cantilever beam of dimension 300 × 550 mm has 3 No of 22 mm dia bars and in tension and 4 No of 25mm dia bars in compression. Use M_{25} & Fe_{500} for the span of 4.5m. Determine the live load varying from 0 at one end to maximum at the other end.



$$d' = 30 + \frac{25}{2} = 42.5 \text{ m}$$

$$d = 550 - 30 - \frac{22}{2} = 509 \text{ m}$$

$$A_{st} = \frac{\pi}{4} \times d^2 \times 3 = 1140.39 \text{ mm}^2$$

$(22)^2$

$$A_{sc} = \frac{\pi}{4} \times d^2 \times 4 = 1963.49 \text{ mm}^2$$

$$\text{Strain} = 0.0035 \left[\frac{x_{u\,max} - d'}{x_{u\,max}}\right]$$

Strain = 0.0028

$x_{u\,max} = 0.46 \times d$

$x_{u\,max} = 234.14$ $\qquad f_{ck} = 25$

$\boxed{\cancel{x_{u\,max} = 0.0028}}$ $\qquad f_y = 500$

$f_{sc} = 420$ mpa

$0.87 f_y A_{st} = (0.36 f_{ck}\, b\, x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc}$

$0.87 \times 500 \times 1140.39 = (0.36 \times 25 \times 300 \times x_u) +$

$(4\cancel{25} - 0.45 \times 25)\ 1963.49$

$\boxed{x_u = -109.88\ mm}$ → Value is -ve bcoz compression zone has more steel bar

Depth of the beam should be Increased.

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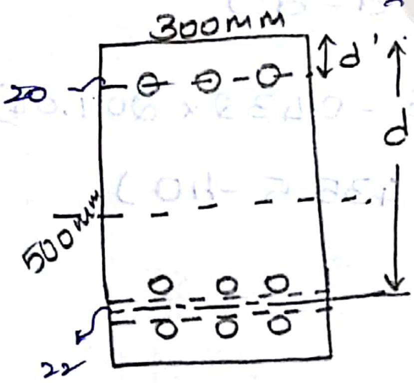
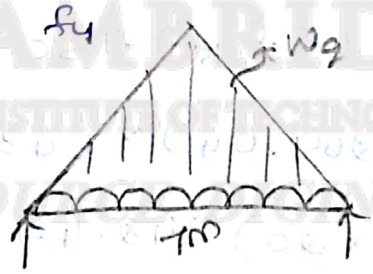
5. A rectangular simply supported beam of dimension 300×500mm & span length 7m has 3 bars of 20 mm dia in top & 6 bars of 22mm dia arranged in two layers in bottom. Determine the live load which is zero at end span & maximum at mid Point. Use M_{20} (f_{ck}) & Fe_{415} (f_y)





$d' = 30 + \frac{20}{2} = 40m$

$d = 500 - 30 - \frac{22}{\cancel{2}} - \frac{25}{2}$

$d = 435.5\ m$

$A_{st} = \frac{\pi}{4}(d^2) \times 6 = \frac{\pi}{4} \times (22)^2 \times 6 = 2280.79\ m^2$

$A_{sc} = \frac{\pi}{4}(d^2) \times 3 = \frac{\pi}{4} \times (20)^2 \times 3 = 942.47\ m^2$

$x_{umax} = 0.48 \times 435.5$

$= 209.04$

$\text{Strain} = 0.0035 \left[\frac{x_{umax} - d'}{x_{umax}}\right]$

$= 0.0035 \left[\frac{209.04 - 40}{209.04}\right]$

$\text{Strain} = 0.0028$

$f_{sc} = 350\,\text{mpa}$

$0.87 f_y A_{st} = (0.36 f_{ck}\ b\ x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc}$

$0.87 \times 415 \times 2280.79 = (0.36 \times 20 \times 300 \times x_u) +$

$(350 - 0.45 \times 20)\, 942.47$

$\boxed{x_u = 232.4\,\text{mm}}$

$x_u > x_{umax}$

$M_R = (0.36 \times f_{ck} \times b \times x_{umax}) \times (d - 0.432 \times x_{umax}) +$

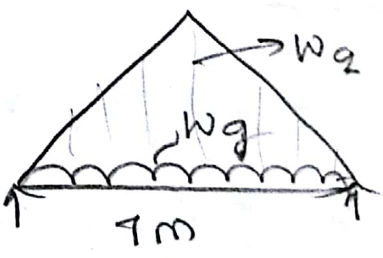
$(f_{sc} - 0.45 f_{ck}) A_{sc} \times (d - d')$

$= 0.36 \times 20 \times 300 \times 209.04) \times (435.5 - 0.432 \times 209.04)$

$+ (350 - 0.45 \times 20)\, 942.47 \times (435.5 - 40)$

$M_R = 282.97$

$M = \frac{M_R}{1.5} = 188.64$



DL $w_q = 0.3 \times 0.5 \times 25$

$w_q = 3.75\,\text{KN/m}$

$$M = \frac{w_q l^2}{8} + \frac{5 w_q l^2}{96}$$

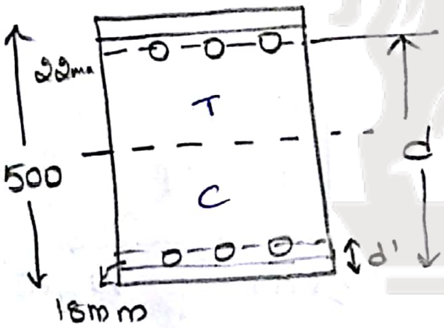
$$188.64 = \frac{3.75 \times (7)^2}{8} + \frac{5 w_q (7)^2}{96}$$

$$165.67 = w_q (2.55)$$

$$\boxed{w_q = 64.91 \text{ KN/m}}$$

(6) A rectangular cantilever beam of dimension 250 x 500 of span 4m, consist of 3 number of 22mm dia bars in tension & 3 no's of 18mm dia bar in compression. Determine live load which is increasing linearly from 0 at one End to max at ~~fixed~~ other End

M_{20} & Fe_{415}
f_{ck} f_y



$$d' = 30 + \frac{18}{2} = 39 \text{ m}$$

$$d = 500 - 30 - \frac{22}{2} = 459 \text{ m}$$

$$A_{st} = \frac{\pi}{4} \times (22)^2 \times 3 = 1140.39 \text{ m}^2$$

$$A_{sc} = \frac{\pi}{4} \times (18)^2 \times 3 = 763.40 \text{ m}^2$$

$$x_{u\,max} = 0.48 \times 459$$
$$= 220.32$$

$$\text{Strain} = 0.0035 \left[\frac{x_{u\,max} - d'}{x_{u\,max}}\right]$$

$$= 0.0035 \left[\frac{220.32 - 39}{220.32}\right]$$

$$\text{Strain} = 0.0028$$

$$f_{sc} = 350 \text{ mpa.}$$

$$0.87 f_y A_{st} = (0.36 f_{ck}\, b\, x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc}$$

$$0.87 \times 415 \times 1140.39 = (0.36 \times 20 \times 250 \times x_u) + (350 - 0.45 \times 20)\, 763.4$$

$$\boxed{x_u = 84.121}$$

$$x_{u\,max} > x_u$$

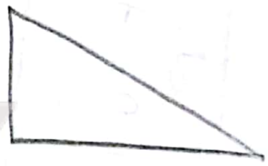
$$M_R = (0.36 \times f_{ck} \times b \times x_{u\,\cancel{max}}) \times (d - 0.432 \times x_{u\,\cancel{max}}) + (f_{sc} - 0.45 f_{ck}) A_{sc} \times (d - d')$$

$$= (0.36 \times 20 \times 250 \times 84.121) \times (459 - 0.432 \times 84.12) + (350 - 0.45 \times 20)\, 763.4 \times (459 - 39)$$

$$\boxed{M_R = 173.33}$$

$$M = \frac{M_R}{1.5} = 115.55$$

$$w_g = 0.25 \times 0.5 \times \frac{25}{\cancel{2}}$$



$$w_g = 3.125$$

$$M = \frac{w_g l^2}{8} + \frac{w_q l^2}{6}$$

$$115.55 = \frac{3.125(4)^2}{8} + \frac{w_q (4)^2}{6}$$

$$w_q = 40.98 \text{ kN/m}$$

<u>Problems on shear</u> 9/9/17

1. An RCC beam of dimension 250 x 450mm tbis reinforced with 3 NO of 20mm dia bars on tension side with effective cover of 50mm. If the shear reinforcement of 2 legged 8mm stirrups at spacing of 160mm centre to centre is provided at a section. Determine the ultimate strength of the section. use M_{20} & $Fe_{415} \rightarrow f_y$

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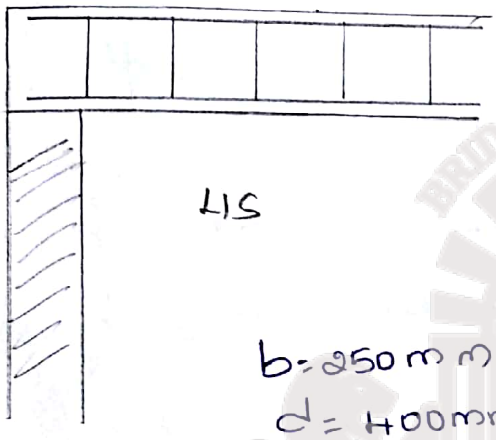
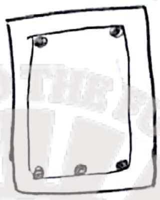


L/S



C/S

τ_c = shear value

$b = 250mm$ $D = 450mm$

$d = 400mm$

Mode

$A_{st} = 3 \times \pi \times \frac{20^2}{4} = 942.47 mm^2$

$\frac{100 A_{st}}{bd} = \underline{\underline{0.94}}$

Interpolation

$0.75 \rightarrow 0.56$

$0.94 \rightarrow x$

$1.0 \rightarrow 0.62$

$\frac{0.56 \times 0.62}{0.56 \times x} = \frac{0.75 \times 1}{0.75 \times 0.94}$

$\boxed{x = 0.58}$

$\tau_c = 0.61 N/mm^2 \rightarrow$ shear value

Shear strength in concrete $V_{uc} = \tau_c . b . d$

$V_{uc} = 61000 N$

$\boxed{V_{uc} = 61 kN}$

Shear strength in Entire steel $V_{us} = \frac{0.87 f_y A_{sv} . d}{S_v}$

$$V_{us} = \frac{0.87 \times f_y \cdot A_{sv} \cdot d}{S_v}$$

$$A_{sv} = \frac{2\pi d^2}{4} = 100.48\, m^2 \quad (8)^2$$

S_v = spacing of stirrups from centre to centre = 160 mm

$$V_{us} = \frac{0.87 \times 415 \times 100.48 \times 400}{160}$$

$$\boxed{V_{us} = 90.69\, kN}$$

$$V_u = V_{uc} + V_{us}$$

$$= 61 + 90.69$$

$$\boxed{V_u = 151.69\, kN}$$

$\tau_{max} = 2.8\, MPa$ $\quad V_{umax} = \tau_{max} \cdot b \cdot d$

$$= 2.8 \times 250 \times 400$$

$$\boxed{V_{umax} = 280\, kN}$$

$$V_{umax} > V_u$$

Hence the beam is safe.

Note :- If $V_u > V_{umax}$ Increase the depth of a beam.

In the above problem if one of the tensile bar is bent up by 45° Then what is the design Strength of the section in the shear.

Given $\alpha = 45°$

$$A_{sb} = \frac{\pi \times 8^2}{4} = 50.24\, m^2$$

If main bar is 20 then $A_{sb} = \frac{\pi \times 20^2}{4}$

$= 314.15 \, m^2$

strength of bent up bars $V_{ub} = 0.87 \times f_y \times A_{sb} \sin\alpha$

$= 0.87 \times 415 \times 314.15 \times \sin(45)$

$V_{ub} = \underline{80.2 \text{ kN}}$

$V_{us} = \frac{90.71}{2}$ $V_{ub} = 80.02$

$\boxed{V_{us} = 45.35 \text{ kN}}$

If V_{ub} is half of V_{us} then the beam is safe

$V_{uc} = 61 \text{ kN}$

$V_u = 90.71 + 80.2 + 61$

$V_{u\,max} = 231.91$

Just for reference no need to write

$V_u = 90.71 + 45.35 + 61$

$\boxed{V_u = 197.06}$

(2) RC beam of dimension 250 × 600 mm is reinforced with 2 legged 10mm Inclined stirups with $\alpha = 60°$ at 250mm centre to centre. Longitudinal steel consist of 4 bars of 20mm with cover of 40mm. Use M_{25} & Fe_{415} & determine the strength of section for shear

Effective cover $= 40 + \frac{d}{2} = 40 + \frac{d}{2}$

$d = 600 - 40 - \frac{20}{2} = 550$

$$A_{st} = \frac{\pi}{4} \times (20)^2 \times 4 = 1256.6\,mm^2$$

$$\frac{100 A_{st}}{bd} = \frac{100 \times 1256 \times}{250 \times (550)^2} = 0.91$$

$\tau_c =$

0.75	0.57
0.9	x
1.00	0.64

$$\frac{0.57 \times 0.64}{0.57 \times x} = \frac{0.75 \times 1}{0.75 \times 0.9}$$

$$x = 0.57 \simeq 0.6\,N/mm^2$$

$$V_{uc} = \tau_c \times b \times d$$

$$= 0.61 \times 250 \times 550$$

$$\boxed{V_{uc} = 83.87\,kN}$$

$$A_{sv} = \frac{2 \times \pi \times 10^2}{4}$$

$$= 157.07\,mm^2$$

$$V_{us} = \frac{0.87 \times f_y \cdot A_{sv}\, d\,(\sin\alpha + \cos\alpha)}{S_v}$$

$$= \frac{0.87 \times 415 \times 157.07 \times 550\,(\sin 60 + \cos 60)}{250}$$

$$\boxed{V_{us} = 170.42\,kN}$$

available in code book

$$V_{umax} = \tau_{max} \cdot b \cdot d$$

$$= 3.1 \times 250 \times 550$$

$$\boxed{V_{umax} = 426.25\,kN}$$

$$V_u = V_{uc} + V_{us}$$

$$\boxed{V_u = 254.2\,kN}$$